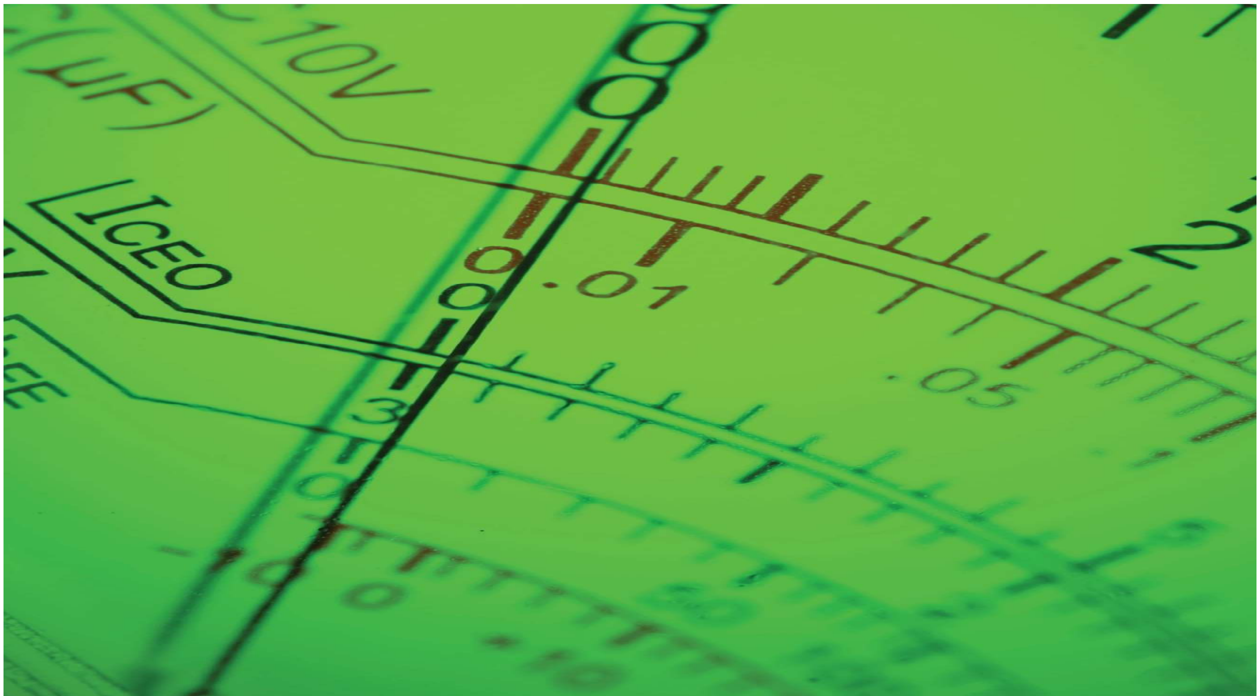


Chapter P1

Practical skills at AS Level

LEARNING INTENTIONS

In this chapter you will learn how to:

- recognise random, systematic and zero errors
- calculate uncertainties in measurements made with a range of instruments
- distinguish between precision and accuracy
- estimate absolute uncertainties and combine uncertainties when quantities are added, subtracted, multiplied and divided
- set up apparatus, follow instructions and make a variety of measurements
- present data in an adequate table, produce best fit straight-line graphs and obtain the intercept and gradient
- use readings to draw conclusions from an experiment and to test a relationship
- identify limitations in an experiment and identify the main sources of uncertainty
- suggest changes to an experiment to improve accuracy and extend an investigation.

BEFORE YOU START

- What are physical properties of materials?
- What quantities do all these instruments measure: protractor, 30 cm ruler, metre rule, micrometer screw gauge, calipers, newton-meter, balance, measuring cylinder, thermometer, stopwatch, ammeter and voltmeter?
- Can you suggest, for each instrument in the list, what is its range and its smallest scale division, and suggest a simple experimental problem in using it?

P1.1 Practical work in physics

Throughout your A Level physics course, you will develop your skills in practical work, and they will be assessed at both AS & A Level. This chapter outlines the skills you will develop in the first year of the course; it includes some activities to test your understanding as you go along.

The sciences differ from most other subjects in that they involve not only theory but also practical work. The very essence of science is that theory can be tested by practical experiment. So, the ability to carry out practical exercises in a logical and scientific manner is essential.

P1.2 Using apparatus and following instructions

You need to familiarise yourself with the use of simple measuring instruments such as metre rules, balances, protractors, stopwatches, ammeters and voltmeters, and even more complicated ones such as a micrometer screw gauge and calipers.

When using measuring instruments like these you need to ensure that you are fully aware of what each division on a scale represents. If you look at Figure P1.1 you will see that on the first ruler each division is 1 mm, and on the second each division is 2 mm.

If you use instruments incorrectly, you may introduce errors into your readings. For example, when taking a reading your line of sight should always be perpendicular to the scale that you are using. Otherwise, you will introduce a parallax error; this is shown in Figure P1.2. Looking from point A the length of the rod appears to be 21 mm, from point C it appears to be 25 mm and from point B, the correct position, the length is 23 mm.

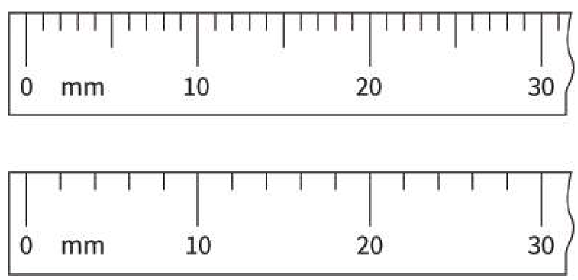


Figure P1.1: When reading from a scale, make sure that you know what each division on the scale represents.

A rule, for example, a metre rule, or a ruler, for example, an ordinary school ruler of length 30 cm, are simple measuring instruments with a smallest division of 1 mm. Other instruments have a greater precision because their smallest scale division is less than 1 mm. Here, we will look at two of them.

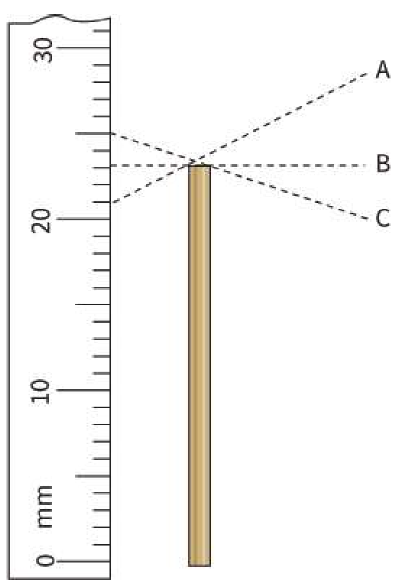


Figure P1.2: Parallax error.

Calipers

Calipers are designed to grip an object with two jaws and, in the example shown in Figure P1.3, to measure the diameter of the object. They can also be used to measure the internal diameter of a tube, for example, if the two prongs are placed inside the tube and the moving part of the calipers is adjusted until the prongs just grip the inside of the tube.

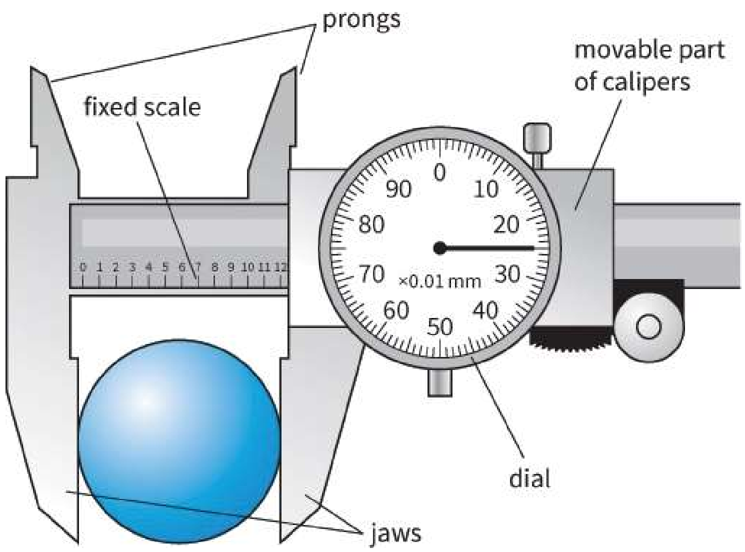


Figure P1.3: Using dial calipers.

The calipers shown in Figure P1.3 are dial calipers, although other versions such as vernier calipers are still sometimes used. As the sliding scale moves along, one rotation of the dial moves the jaws 1 mm further apart. Since the dial shown has 100 divisions, each of these divisions is $\frac{1}{100} = 0.01\ \mathrm{mm}$. The object shown has a diameter of 12 mm on the fixed scale and 25 divisions or 0.25 mm on the dial, so the diameter of the object is 12.25 mm.

Micrometer screw gauge

A micrometer screw gauge, or more simply a micrometer, is shown in Figure P1.4. This also has two scales. The main scale is on the shaft and the fractional scale is on the rotating barrel. One rotation of the barrel moves the end of the barrel 0.50 mm along the shaft. The barrel has 50 divisions so each division represents $\frac{0.50}{50} = 0.01\ \mathrm{mm}$.

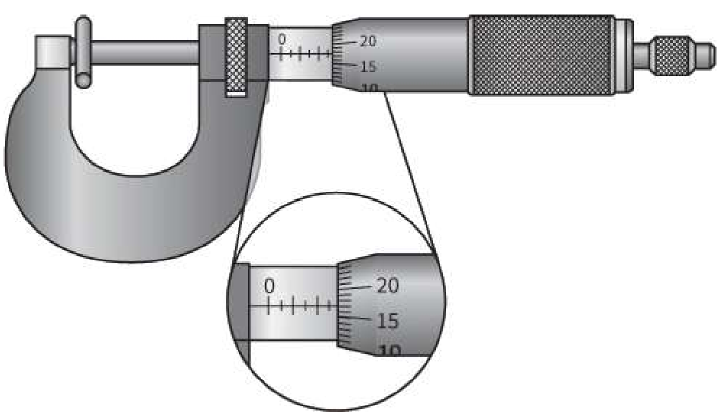


Figure P1.4: Using a micrometer screw gauge.

To use the micrometer, turn the barrel until the jaws just tighten on the object. Some micrometers have a ratchet or slip mechanism to prevent the user from tightening too hard and damaging the micrometer or object. Read the main scale to the nearest 0.5 mm, then read the number of divisions on the sleeve, which will be in 0.01 mm, and finally add the two readings. You should realise that the smallest division on the micrometer is 0.01 mm.

Before you start to use a micrometer or dial calipers, it is usual to check if there is a zero error. This is done by bringing the jaws together without any object between them. Obviously, the reading should be zero, but if the instrument is worn or has been used badly the reading may not be zero. When you have taken this zero error reading, it should be added to or subtracted from every other reading that you take with the instrument. If the jaws do not quite close to the zero mark, there is a positive zero error, and this zero error reading should be subtracted. The zero error is an example of a systematic error, which is dealt with later in this chapter.

It is also important that you become familiar with setting up apparatus. When instructions are given, the only way to become confident is through practice. You may face a variety of tasks, from setting up a pendulum system to measuring the angle at which a tilted bottle falls.

You should also learn to set up simple circuits from circuit diagrams. The most common error in building circuits comes where components need to be connected in parallel. A good piece of advice here is to build the main circuit first, and then add the components that need to be connected in parallel.

P1.3 Gathering evidence

When gathering evidence, you should take into account the range of results that you are going to obtain. If you are investigating the extension of a spring with load, for loads of between 0 N and 20 N, you should take a fair spread of readings throughout that range. For instance, six readings between 12 N and 20 N would not be sensible because you are not investigating what happens with smaller loads. Equally, taking three readings below 5 N and three more between 15 N and 20 N does not test what happens with intermediate loads.

A sensible set of readings might be at 0 N, 4 N, 8 N, 12 N, 16 N and 20 N. This covers the whole range in equal steps.

Question

1 You are investigating how the current through a resistor depends on its resistance when connected in a circuit. You are given resistors of the following values:

50Ω, 100Ω, 150Ω, 200Ω, 250Ω, 300Ω, 350Ω, 400Ω, 450Ω, 500Ω

You are asked to take measurements with just six of these resistors. Which six resistors would you choose? Explain your choice.

P1.4 Precision, accuracy, errors and uncertainties

Whenever you make a measurement, you are trying to find the true value of a quantity. This is the value you would find if your measurement was perfect. However, no measurement can ever be perfect; there will always be some **uncertainty**. Your equipment may be imperfect or your technique may be capable of improvement. So, whenever you carry out practical work, you should think about two things:

- how the equipment or your technique could be improved to give better results, with less uncertainty
- how to present the uncertainty in your findings.

As you will see later in this chapter, both of these need to be reflected in the way you present your findings.

We will first consider the **precision** of a measurement. The level of precision is high if you make several measurements of a quantity and they are all very similar. A precise measurement, when repeated, will be the same, or nearly so. However, if your measurements are spread widely around the average, they are less precise. This can arise because of practical difficulties in making the measurements.

Precision is reflected in how the results are recorded. If a distance is quoted as '15 m' then it implies that it was only measured to the nearest metre, whereas if it is quoted as '15.0 m' then it suggests that it was measured to the nearest 0.1 m.

Take care not to confuse precision with **accuracy**. A measurement is described as 'accurate' if the value obtained is close to the true value. Even if a measurement is precise, and always produces the same result, it may not be accurate because every reading may have the same error. For example, you can make very precise measurements of the diameter of a wire using a micrometer screw gauge to the nearest 0.01 mm, but every reading may be inaccurate if the gauge has a zero error.

Figure P1.5 shows two attempts at making holes in the centre of a target. Imagine that the positions of the holes represent readings, with the true value at the centre. On the left, the readings are close together so we can say that they are precise. However, they are not accurate as the average is far from the centre. In the second, the measurement can be said to be accurate as the average position of the holes is close to the centre, but the readings are not precise as the holes are spread out.

Whenever you make a measurement, you should be aware of the uncertainty in the measurement. It will often, but not always, be determined by the smallest division on the measuring instrument. On a metre rule, which is graduated in millimetres, we should be able to read to the nearest half millimetre, but beware! If we are measuring the length of a rod there are two readings to be taken, one at each end of the rod. Each of these readings has an uncertainty of 0.5 mm, giving a total uncertainty of 1 mm.

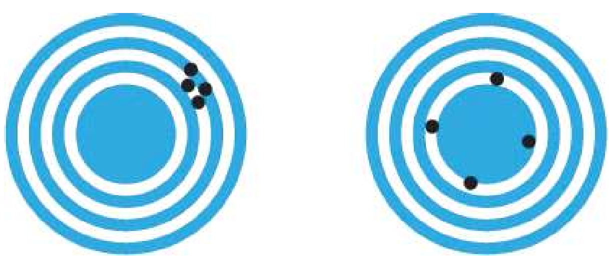


Figure P1.5: The left-hand diagram represents readings that are precise but not accurate; the right-hand diagram represents readings that are accurate but without precision.

The uncertainty will depend not only on the precision of the calibrations on the instrument you are using, but also on your ability to observe and on **errors** introduced by less than perfect equipment or poor technique in taking the observations. Here are some examples of where uncertainties might arise:

Systematic error – A spring on a force meter might, over time, become weaker so that the force meter reads consistently high. Similarly, the magnet in an ammeter might, over the years, become weaker and the needle may not move quite as far round the scale as might be expected. Parallax errors, described earlier, may be another example of a systematic error if one always looks from the same angle, and not directly from above, when taking a measurement. In principle, systematic errors can be corrected for by recalibrating the instrument or by correcting the technique being used.

Zero error – The zero on a ruler might not be at the very beginning of the ruler. This will introduce a fixed error into any reading unless it is allowed for. This is a type of systematic error.

Random errors – When a judgement has to be made by the observer, a measurement will sometimes be above and sometimes below the true value. Random errors can be reduced by making multiple measurements and averaging the results.

Good equipment and good technique will reduce the uncertainties introduced, but difficulties and judgements in making observations will limit the precision of your measurements. Here are two examples of how difficulties in observation will determine the uncertainty in your measurement.

Example 1: Using a stopwatch

Tambo has a digital stopwatch that measures to the nearest one-hundredth of a second. He is timing his sister Nana in a 100 metre race (Figure P1.6). He shows her the stopwatch, which reads 11.87 s. She records in her notebook the time 11.9 s. She explains to Tambo that he cannot possibly measure to the nearest one-hundredth of a second as he has to judge both when the starting pistol was fired and the exact moment at which she crossed the finishing line. To do this to any closer than the nearest one-tenth of a second is impossible. In addition, sometimes he will press the button too early and sometimes too late.

Figure P1.6: Uncertainty in timing using a stopwatch.

Example 2: Measuring displacement of a pendulum

Fatima is asked to measure the maximum displacement of a pendulum bob as it oscillates, as shown in Figure P1.7. She uses a ruler calibrated in millimetres. She argues that she can measure the displacement to the nearest millimetre. Joanne, however, correctly argues that she can only measure it to the nearest two millimetres, as not only is there the uncertainty at either end (0.5 mm) but she also has to judge precisely the point at which the bob is at its greatest displacement, which adds an extra millimetre to the uncertainty.

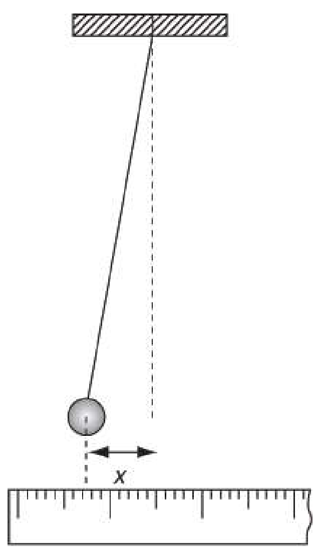


Figure P1.7: Displacement of a pendulum bob.

Questions

2 Look at Figure P1.5. Draw similar diagrams to represent:

- **a** a target where the holes are both precise and accurate
- **b** a target where the holes are neither precise nor accurate.

3 The position of the holes in Figure P1.5 represents attempts at measuring the position of the centre of the circle. Which one shows more random error and which shows more systematic error?

P1.5 Finding the value of an uncertainty

We have used the terms uncertainty and error; they are not quite the same thing. In general, an ‘error’ is just a problem that causes the reading to be different from the true value (although a zero error can have an actual value). The uncertainty, however, is an actual range of values around a measurement, within which you expect the true value to lie. The uncertainty is an actual number with a unit.

For example, if you happen to know that the true value of a length is 21.0 cm and an ‘error’ or problem causes the actual reading to be 21.5 cm, then, since the true value is 0.5 cm away from the measurement, the uncertainty is ±0.5 cm.

But how do you estimate the uncertainty in your reading without knowing the true value? Obviously, if a reading is 21.5 cm and you know the true value is 21.0 cm, then the uncertainty in the reading is 0.5 cm. However, you may still have to estimate the uncertainty in your reading without knowing the true value. So how is this done?

KEY IDEA

You can find the uncertainty from whichever is the largest out of:

- the smallest division on the instrument used, or
- half the range of a number of readings of the measurement.

First, it should be understood that the uncertainty is only an estimate of the difference between the actual reading and the true value. We should not feel too worried if the difference between a single measurement and the true value is as much as twice the uncertainty. Because it is an estimate, the uncertainty is likely to be given to only one significant figure. For example, we write the uncertainty as 0.5 cm and not 0.50 cm.

The uncertainty can be estimated in two ways.

Using the division on the scale – Look at the smallest division on the scale used for the reading. You then have to decide whether you can read the scale to better than this smallest division. For example, what is the uncertainty in the level of point B in Figure P1.2? The smallest division on the scale is 1 mm but is it possible to measure to better than 1 mm? This will depend on the instrument being used and whether the scale itself is accurate. In Figure P1.2, the width of the line itself is quite small but there may be some parallax error that would lead you to think that 0.5 mm or 1 mm is a reasonable uncertainty. In general, the position of a mark on a ruler can generally be measured to an uncertainty of ±0.5 mm. In Figure P1.8, the smallest division on the scale is 20 g. Can you read more accurately than this? In this case, it is doubtful that every marking on the scale is accurate and so 20 g would be reasonable as the uncertainty.

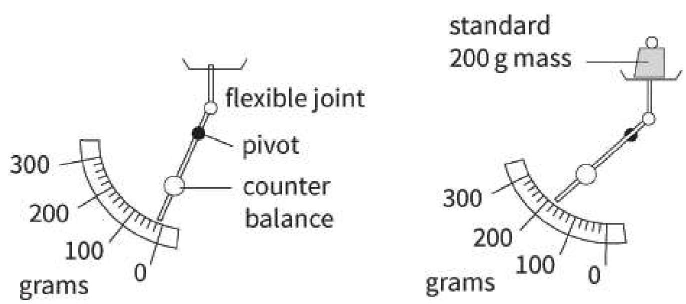


Figure P1.8: The scales on a lever-arm balance.

You need to think carefully about the smallest division you can read on any scale. As another example, look at a protractor. The smallest division is probably 1° but it is unlikely you can use a protractor to measure an angle to better than ±0.5° with your eye.

Repeating the readings – Repeat the reading several times. The uncertainty can then be taken as half of the range of the values obtained; in other words, the smallest reading is subtracted from the largest and the result is halved. This method deals with random errors made in the readings but does not account for systematic errors. This method should always be tried, wherever possible, because it may reveal random errors and gives an easy way to estimate the uncertainty. However, if the repeated readings are all the same, do not think that the uncertainty is zero. The uncertainty can never be less than the value you obtained by looking at the smallest scale division.

Which method should you actually use to estimate the uncertainty? If possible, readings should be repeated and the second method used. But if all the readings are the same, you have to try both methods!

The uncertainty in using a stopwatch is something of a special case as you may not be able to repeat the measurement. Usually, the smallest division on a stopwatch is 0.01 s, so can you measure a time interval with this uncertainty? You may know that your own reaction time is larger than this and is likely to be at least 0.1 s. The stopwatch is recording the time when you press the switch but this is not pressed at exactly the correct moment. If you do not repeat the reading then the uncertainty is likely to be at least 0.1 s, as shown in Figure P1.7. If several people take the reading at the same time, you are likely to see that 0.01 s is far too small to be the uncertainty.

Even using a digital meter is not without difficulties. For example, if a digital ammeter reads 0.35 A, then, without any more information, the uncertainty is ±0.01 A, the smallest digit on the meter. But if you look at the handbook for the ammeter, you may well find that the uncertainty is ±0.02 or 0.03 A (although you cannot be expected to know this).

WORKED EXAMPLE

1 A length is measured five times with a ruler whose smallest division is 0.1 cm and the readings obtained, in cm, are: 22.9, 22.7, 22.9, 23.0, 23.1. What is the reading obtained and the uncertainty?

Step 1 Find the average by adding the values and dividing by the number of values:

$$\frac{22.9+22.7+22.9+23.0+23.1}{5} = 22.92 \text{ cm}$$

This is written to four significant figures. At this stage, you are not sure how many figures to write in the answer.

Step 2 The maximum value is 23.1 and the minimum value is 22.7. Use these values to find half the range.

$$\text{half the range} = \frac{23.1-22.7}{2} = 0.2 \text{ cm}$$

Step 3 Check that the uncertainty calculated in Step 2 is larger than the smallest division you can read on the scale.

Step 4 Write down the average value, the uncertainty to a reasonable number of significant figures and the unit. Obviously, the last digit in 22.92 is meaningless as it is much smaller than the uncertainty; it should not be written down.

The final value is (22.9 ± 0.2) cm.

You do not usually write down the final value of the answer to a greater number of decimal places than the uncertainty. Uncertainties are usually quoted to one or perhaps two significant figures.

Questions

4 Figure P1.8 shows a lever-arm balance, initially with no mass in the pan and then with a standard 200 g mass in the pan.

Explain what types of error might arise in using this equipment.

5 Estimate the uncertainty when a student measures the length of a room using a steel tape measure calibrated in millimetres.

6 Estimate the uncertainty when a girl measures the temperature of a bath of water using the thermometer in Figure P1.9.

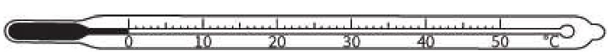


Figure P1.9: For Question 6.

7 A student is asked to measure the wavelength of waves on a ripple tank using a metre rule that is graduated in millimetres. Estimate the uncertainty in his measurement.

8 Estimate the uncertainty when a student attempts to measure the time for a single swing of a pendulum.

9 What is the average value and uncertainty in the following sets of readings? All are quoted to be consistent with the smallest scale division used.

a 20.6, 20.8

b 20, 30, 36

c 0.6, 1.0, 0.8, 1.2

d 20.5, 20.5.

P1.6 Percentage uncertainty

The uncertainties we have found so far are sometimes called absolute uncertainties, but percentage uncertainties are also very useful.

The percentage uncertainty expresses the absolute uncertainty as a fraction of the measured value and is found by dividing the uncertainty by the measured value and multiplying by 100%.

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

For example, suppose a student times a single swing of a pendulum. The measured time is 1.4 s and the estimated uncertainty is 0.2 s. Then we have:

$$\begin{aligned}\text{percentage uncertainty} &= \frac{\text{uncertainty}}{\text{measured value}} \times 100\% \\ &= \frac{0.2}{1.4} \times 100\% \\ &= 14\%\end{aligned}$$

This gives a percentage uncertainty of 14%. We can show our measurement in two ways:

- with absolute uncertainty: time for a single swing = 1.4 s ± 0.2 s
- with percentage uncertainty: time for a single swing = 1.4 s ± 14%

(Note that the absolute uncertainty has a unit whereas the percentage uncertainty is a fraction, shown with a % sign.)

A percentage uncertainty of 14% is very high. This could be reduced by measuring the time for 20 swings. In doing so, the absolute uncertainty remains 0.2 s (it is the uncertainty in starting and stopping the stopwatch that is the important thing here, not the accuracy of the stopwatch itself), but the total time recorded might now be 28.4 s.

$$\begin{aligned}\text{percentage uncertainty} &= \frac{0.2}{28.4} \times 100\% \\ &= 0.7\%\end{aligned}$$

So measuring 20 oscillations rather than just one reduces the percentage uncertainty to less than 1%. The time for one swing is now calculated by dividing the total time by 20, giving 1.42 s. Note that, with a smaller uncertainty, we can give the result to two decimal places. The percentage uncertainty remains at 0.7%:

time for a single swing = 1.42 s ± 0.7%

Questions

10 The depth of water in a bottle is measured as 24.3 cm, with an uncertainty of 0.2 cm. (This could be written as (24.3 ± 0.2) cm.) Calculate the percentage uncertainty in this measurement.

11 The angular amplitude of a pendulum is measured as (35 ± 2)°.

a Calculate the percentage uncertainty in the measurement of this angle.

b The protractor used in this measurement was calibrated in degrees. Suggest why the user only feels confident to give the reading to within 2°.

12 A student measures the potential difference across a battery as 12.4 V and states that his measurement has a percentage uncertainty of 2%. Calculate the absolute uncertainty in his measurement.

P1.7 Recording results

It is important that you develop the skill of recording results in a clear and concise manner.

Generally, numerical results will be recorded in a table. The table should be neatly drawn using a ruler and each heading in the table should include both the quantity being measured and the unit it is measured in.

KEY IDEA

Each column of a table must be labelled with a quantity / unit, and, if a reading be given to the precision of the instrument, usually to the same number of decimal places. Calculated quantities may have one more significant figure than the readings used.

Table P1.1 shows how a table may be laid out. The measured quantities are the length of the wire and the current though it; both have their units included. Similarly, the calculated quantity, $\frac{1}{\text{current}}$, is included and this too has a unit, A^{-1}.

When recording your results, you need to think once more about the precision to which the quantities are measured. In the example in Table P1.1, the length of the wire might be measured to the nearest millimetre and the current might be measured to the nearest milliampere.

Note how '.0' is included in the second result for the length of the wire, to show that the measurement is to the nearest millimetre, not the nearest centimetre. Similarly the zero after the 0.35 shows that it is measured to the nearest milliampere or $\frac{1}{1000}$ of an ampere.

The third column is calculated and should show the same number of significant figures, or one more than the quantity (or quantities) it is calculated from. In this example, the current is measured to three significant figures so the inverse of the current is calculated to three significant figures.

Length of wire / cm	Current / A	$\frac{1}{\text{current}} / \text{A}^{-1}$
10.3	0.682	1.47
19.0	0.350	2.86

Table P1.1: A typical results table.

Question

13 A ball is allowed to roll down a ramp from different starting points. Figure P1.10 shows the apparatus used. The ramp is placed at a fixed height above the floor. You are asked to measure the vertical height *h* of the starting point above the bottom of the ramp and the horizontal distance *d* the ball travels after it leaves the ramp.

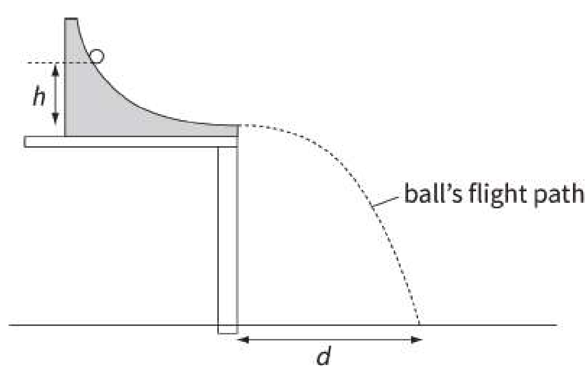


Figure P1.10: For Question 13.

You are also asked to find the square of the horizontal distance the ball travels after it leaves the ramp.

Table P1.2 shows the raw results for the experiment. Copy and complete the table.

h / cm	*d* / cm	d^2 /

1.0	18.0	
2.5	28.4	
4.0	35.8	
5.5	41.6	
7.0	47.3	
9.0	53.6	

Table P1.2: For Question 13.

P1.8 Analysing results

When you have obtained your results, the next thing to do is to analyse them. Very often this will be done by plotting a graph.

You may be asked to plot a graph in a particular way, however, the general rule is that the variable you control or alter (the **independent variable**) is plotted on the *x*-axis and the variable that changes as a result (the **dependent variable**) is plotted on the *y*-axis.

In the example in Table P1.1, the length of the wire would be plotted on the *x*-axis and the current (or $\frac{1}{\text{current}}$) would be plotted on the *y*-axis.

You should label your axes with both the quantities you are using and their units. You should then choose your scales to use as much of the graph paper as possible. However, you also need to keep the scales simple. Never choose scales that are multiples of 3, 7, 11 or 13. Try and stick to scales that are simple multiples of 1, 2 or 5.

Plot your points carefully using small crosses; dots tend to disappear into the page and larger dots become blobs, the centre of which is difficult to ascertain.

Many, but not all, graphs you meet will be straight lines. The points may not all lie exactly on the straight line and it is your job to choose the **best fit line**. Choosing this line is a skill that you will develop through the experience of doing practical work.

Generally, there should be equal points either side of the line (but not three on one side at one end and three on the other at the other end). Sometimes, all the points, bar one, lie on the line. The point not on the line is often referred to as an anomalous point, and it should be checked, if possible. If it still appears to be off the line it might be best to ignore it and use the remaining points to give the best line. It is best to mark it clearly as ‘anomalous’.

In Figure P1.11, the line chosen on the first graph is too shallow. By swinging it round so that it is steeper, it goes closer to more points and they are more evenly distributed above and below the line.

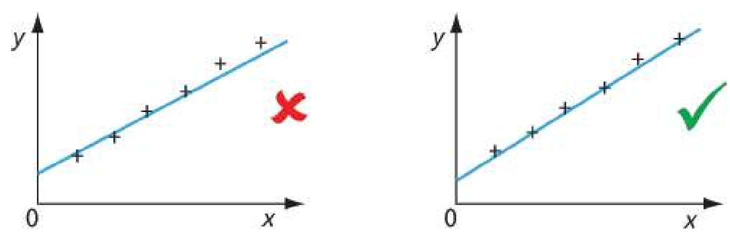


Figure P1.11

Deductions from graphs

There are two major points of information that can be obtained from straight-line graphs: the gradient and the intercept with the *y*-axis. When measuring the gradient, a triangle should drawn, as in Figure P1.12, using at least half of the line that has been drawn.

$$\text{gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

In the mathematical equation $y = mx + c$, m is equal to the gradient of the graph and c is the intercept with the *y*-axis. If c is equal to zero, the graph passes through the origin, the equation becomes $y = mx$ and we can say that y is proportional to x.

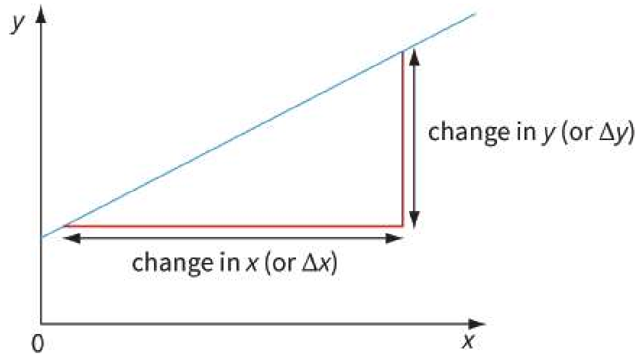


Figure P1.12

Question

14 **a** Use your results from Question 13 to plot a graph of the square of the horizontal distance d^2 (on the y-axis) against the height h (on the x-axis). Draw the best fit line.

b Determine the gradient of the line on your graph and the intercept with the y-axis. Remember, both the gradient and the intercept have units; these should be included in your answer.

Curves and tangents

You also need to develop the skill of drawing smooth curves through a set of points, and drawing tangents to those points. When drawing curves, you need to draw a single smooth curve, without any jerks or feathering. As with a straight line, not every point will lie precisely on the curve, and there should be a balance of points on either side.

In the first graph of Figure P1.13, the student has joined each of the points using a series of straight lines. This should never be done. The second graph is much better, although there is some feathering at the left-hand side, as two lines can be seen. The third graph shows a well-drawn curve.

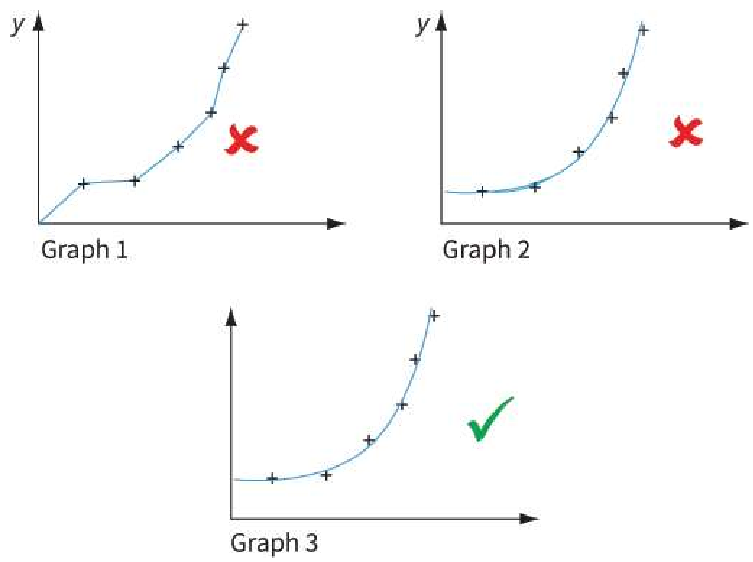


Figure P1.13: For Question 14.

P1.9 Testing a relationship

The readings from an experiment are often used to test a relationship between two quantities, typically whether two quantities are proportional or inversely proportional.

You should know that if two quantities y and x are directly proportional:

- the formula that relates them is $y = kx$, where k is a constant
- if a graph is plotted of y against x then the graph is a straight line through the origin and the gradient is the value of k.

If the two quantities are inversely proportional then $y = \frac{k}{x}$ and a graph of y against $\frac{1}{x}$ gives a straight line through the origin.

These statements can be used as a basis for a test. If a graph of y against x is a straight line through the origin, then y and x are directly proportional. If you know the values of y and x for two points, you can then calculate two values of k with the formula $y = \frac{k}{x}$ and see whether these two values of k are actually the same. But what if the points are not exactly on a straight line or the two values of k are not exactly the same – is the relationship actually false or is it just that errors caused large uncertainties in the readings?

Later in this chapter, we will look at how to combine the uncertainties in the values for y and x to find an uncertainty for k. However, you can use a simple check to see whether the difference in the two values of k may be due to the uncertainties in the readings. For example, if you found that the two values of k differ by 2% but the uncertainties in the readings of y and x are 5%, then you cannot say that the relationship is proved false. Indeed, you are able to say that the readings are consistent with the relationship.

You should first write down a criterion for checking whether the values of k are the same. This criterion is just a simple rule you can invent for yourself and use to compare the two values of k with the uncertainties in the readings. If the criterion is obeyed you can then write down that the readings are consistent with the relationship.

KEY IDEA

Write down a criterion.

Calculate the percentage difference between two values of the constant.

Compare the percentage difference with the percentage uncertainty in one of the variables.

Write a conclusion as to whether the criterion is obeyed or not.

Criterion 1

A simple approach is to assume that the percentage uncertainty in the value of k is about equal to the percentage uncertainty in either x or y; choose the larger percentage uncertainty of x or y.

You first look at the percentage uncertainty in both x and y and decide which is bigger. Let us assume that the larger percentage uncertainty is in x. Your stated criterion is then that 'if the difference in the percentage uncertainty in the two values of k is less than the percentage uncertainty in x, then the readings are consistent with the relationship'.

If the percentage difference in k values is less than the percentage uncertainty in x (or y), the readings are consistent with the relationship.

KEY IDEA

If the percentage difference in k values is less than the percentage uncertainty in x (or y), the readings are consistent with the relationship.

Criterion 2

Another criterion is to state that the k values should be the same within 10% or 20%, depending on the experiment and the uncertainty that you think sensible. It is helpful if the figure of 10% or 20% is related to some uncertainty in the actual experiment.

Whatever criterion you use, it should be stated clearly and a clear conclusion given. The procedure to check whether two values of k are reasonably constant is as follows:

- Calculate two values of the constant k. The number of significant figures chosen when writing down these values should be equal to the least number of significant figures in the data used. If you are asked to justify the number of significant figures you give for your value of k, state the number of significant figures that x and y were measured to and that you will choose the smallest. Do not quote your values of k to one significant figure to make them look equal when x and y were measured to two significant figures.

- Calculate the percentage difference in the two calculated values of k. It is worthwhile using one more significant figure in each actual value of k than is completely justified in this calculation.
- Compare the percentage difference in the two values of k with your clearly stated criterion. You could compare your percentage difference in k values with the larger of the percentage differences in x and y.

WORKED EXAMPLES

1 A student investigates the depth D of a crater made when ball-bearings of different diameters d are dropped into sand. He drops two ball bearings from the same height and measures the depth of the craters using a 30 cm ruler. The results are shown in Table P1.3.

Diameter of ball bearing d / mm	Depth of the crater D / mm	D/d
5.42 ± 0.01	36 ± 2	6.64
3.39 ± 0.01	21 ± 2	6.19

Table P1.3: For Worked example 2.

It is suggested that the depth D of the crater is directly proportional to the diameter d of the ball-bearing, that is:

$D = kd$ or $\frac{D}{d} = k$

Do the readings support this hypothesis?

Step 1 Calculate the values of $k = \frac{D}{d}$. These values are shown in the third column in Table P1.3, although they should only be given to two significant figures as values of D are given to two significant figures and values of d to three significant figures. The more precise values for k are to be used in the next step.

Step 2 Calculate the percentage difference in the k values. The percentage difference is: $\frac{0.45}{6.19} \times 100\% = 7.2\%$

So the k values differ by 7% of the smaller value.

Step 3 State a criterion and check it.

'My criterion is that, if the hypothesis is true, then the percentage difference in the k values will be less than the percentage uncertainty in D. I chose D as it obviously has the higher percentage uncertainty.'

The uncertainty in the smaller measurement of D can be calculated as:

uncertainty in $D = \frac{2}{21} \times 100\% = 9.5\%$

The percentage difference in the k values is less than the uncertainty in the experimental results; therefore, the experiment is consistent with the hypothesis.

Of course, we cannot say for sure that the hypothesis is correct. To do that, we would need to greatly reduce the percentage uncertainties.

3 A student obtains data shown in Table P1.4.

x / cm	d / cm
2.0	3.0
3.5	8.0

Table P1.4: For Worked example 3.

The first reading of x was found to have an uncertainty of ±0.1. Do the results show that d is proportional to x?

Step 1 Calculate the ratio of $\frac{d}{x}$ in both cases:

$$\left(\frac{d}{x}\right)_1 = 1.50 \quad \left(\frac{d}{x}\right)_2 = 2.29$$

Step 2 Calculate how close to each other the two ratios are: 2.29 − 1.50 = 0.79

So the two values of $\left(\frac{d}{x}\right)$ are $\frac{0.79}{1.5} = 53\%$ different.

Step 3 Compare the values and write a conclusion.

The uncertainty in the first value of x is 5% and, since the percentage difference between the ratios of 53% is much greater, the evidence does not support the suggested relationship.

Questions

15 A student obtains the following data for two variables T and m (Table P1.5).

T / s	m / kg
4.6	0.90
6.3	1.20

Table P1.5: Data for Question 15.

The first value of T has an uncertainty of ±0.2 s. Do the results show that T is proportional to m?

16 A student obtains the following values of two variables r and t (Table P1.6).

r / cm	t / s
6.2	4.6
12.0	6.0

Table P1.6: Data for Question 16.

The first value of r has an uncertainty of ±0.2 cm, which is much greater than the percentage uncertainty in t. Do the results show that t^2 is proportional to r?

P1.10 Combining uncertainties

When quantities are combined, for example, multiplied or divided, what is the uncertainty in the final result?

KEY IDEA

If quantities are added or subtracted, add absolute uncertainties.

If quantities are multiplied or divided, add percentage uncertainties.

Suppose that quantity $A = 1.0 \pm 0.1$ and that $B = 2.0 \pm 0.2$, so that the value of $A + B$ is 3.0. The maximum likely value of $A + B$, taking into account the uncertainties, is 3.3 and the minimum likely value is 2.7. You can see that the combined uncertainty is ±0.3, so $A + B = 3.0 \pm 0.3$. Similarly, $B - A = 1.0 \pm 0.3$.

When quantities are added or subtracted, their absolute uncertainties are added. A simple example is measuring the length of a stick using a millimetre scale. There is likely to be an uncertainty of 0.5 mm at both ends, giving a total uncertainty of 1.0 mm.

When quantities are multiplied or divided, combining uncertainties is a little more complex. To find the combined uncertainty in this case, we add the percentage uncertainties of the two quantities to find the total percentage uncertainty.

Remember, you always add uncertainties; never subtract.

Where quantities are:

- added or subtracted, then add **absolute** uncertainties
- multiplied or divided, then add **percentage** or fractional uncertainties.

WORKED EXAMPLES

3 The potential difference across a resistor is measured as (6.0 ± 0.2) V, while the current is measured as (2.4 ± 0.1) A.

Calculate the resistance of the resistor and the absolute uncertainty in its measurement.

Step 1 Find the percentage uncertainty in each of the quantities:

$$\begin{aligned}\text{percentage uncertainty in p.d.} &= \frac{0.2}{6.0} \times 100\% \\ &= 3.3\% \\ \text{percentage uncertainty in current} &= \frac{0.1}{2.4} \times 100\% \\ &= 4.2\%\end{aligned}$$

Step 2 Add the percentage uncertainties. Sum of uncertainties:

(3.3 + 4.2)% = 7.5%

Step 3 Calculate the resistance value and find the absolute uncertainty:

$$\begin{aligned}R &= \frac{V}{I} \\ &= \frac{6.0}{2.4} \\ &= 2.5\Omega\end{aligned}$$

7.5% of 2.5 = 0.1875 ≈ 0.2 Ω

The resistance of the resistor is 2.5 ± 0.2 Ω.

When you calculate the uncertainty in the square of a quantity, since this is an example of multiplication, you should double the percentage uncertainty. For example, if $A = (2.0 \pm 0.2)$ cm, then A has a percentage uncertainty of 10% so $A^2 = 4.0$ cm^2 ± 20%; or giving the absolute uncertainty, $A^2 = (4.0 \pm 0.8)$ cm^2.

Questions

17 You measure the following quantities:

$A = (1.0 \pm 0.4)$ m

$B = (2.0 \pm 0.2)$ m

$C = (2.0 \pm 0.5)$ m s^{-1}

$D = (0.20 \pm 0.01)$ s

Calculate the result and its uncertainty for each of the following expressions. You may express your uncertainty either as an absolute value or as a percentage.

a $A + B$

b $B - A$

c $C \times D$

d $\frac{B}{D}$

e A^2

f $2 \times A$

g the square root of ($A \times B$). (Recall that the square root of x can be written as $x^{\frac{1}{2}}$.)

18 A rifle bullet is photographed in flight using two flashes of light separated by a time interval of (1.00 ± 0.02) ms. The first image of the bullet on the photograph appears to be at a position of (22.5 ± 0.5) cm on a scale underneath the flight path. The position of the second image is (37.5 ± 0.7) cm on the same scale. Find the speed of the bullet and its absolute uncertainty.

P1.11 Identifying limitations in procedures and suggesting improvements

No experiment is perfect and the ability to see weaknesses in the experimental setup and the techniques used is an important skill. You should also take the opportunity to think of ways to improve the experimental technique, thereby reducing the overall percentage uncertainty.

In this topic, we will look at five experiments and discuss **problems** that might arise and the **improvements** that might be made to overcome them. It will help if you try out some of the experiments yourself so that you get a feel for the methods described. The table for each experiment is a summary of ideas that you might use in your answer.

Experiment 1: Ball-bearings and craters

In Worked example 2, the student dropped a ball-bearing of diameter *d* into sand and measured the depth *D* of the crater produced. He dropped two ball-bearings of different diameters from the same height and measured the depth of the crater using a 30 cm ruler. Table P1.7 suggests some of the problems with the simple method used, together with some improvements.

Suggestion	Problem	Improvement
1	'Two results are not enough to draw a valid conclusion.'	'Take more results and plot a graph of *D* against *d*.'
2	'The ruler is too wide to measure the depth of the crater.'	'Use a knitting needle and mark the sand level on the needle and then measure with a ruler.'
3	'There may be a parallax error when measuring the top level of the crater.'	'Keep the eye parallel to the horizontal level of the sand, or use a stiff card.'
4	'It is difficult to release the ball-bearing without giving it a sideways velocity, leading to a distorted crater.'	'Use an electromagnet to release the ball.'
5	'The crater lip is of varying height.'	'Always measure to the highest point.'

Table P1.7: Suggestions for improving Experiment 1.

It is worth making some points regarding these suggestions.

1 This is a simple idea, but it is important to explain how the extra results are to be used. In this case, a graph is suggested – alternatively the ratio $\frac{D}{d}$ could be calculated for each set of readings.

2 The problem is clearly explained. It is not enough to just say that the depth is difficult to measure.

3 It is not enough to just say 'parallax errors'. We need to be specific as to where they might occur. Likewise, make sure you make it clear where you look from when you suggest a cure.

4 There is no evidence that this will affect the crater depth, but it is a point worthy of consideration.

5 An interesting point: does the crater depth include the lip or is it just to the horizontal sand surface? Consistency in measurement is what is needed here.

Experiment 2: Timing with a stopwatch

Many years ago, Galileo suggested that heavy and light objects take the same time to fall to the ground from the same height, as illustrated in Figure P1.14. Imagine that you want to test this hypothesis.

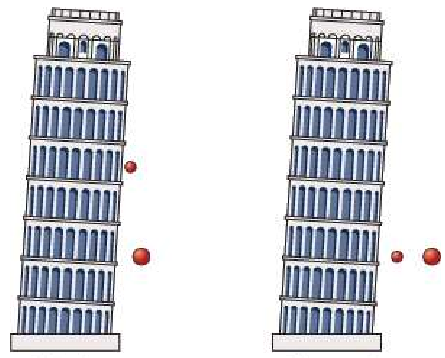
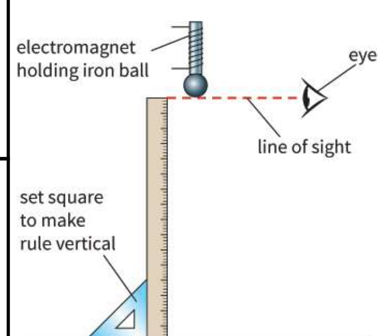


Figure P1.14: It was believed that Galileo dropped two different masses from the top of the Leaning Tower of Pisa to prove his idea. But people now think it probably didn't happen. He just did a 'thought experiment'.

This is an experiment you can do yourself with two objects and a stopwatch, or even a digital wrist watch or a cell phone with a timing app. Drop two different objects, for example two stones, and measure the time they take to fall the same distance to the ground.

Of course, the times you obtain are likely to be different. Does this prove Galileo wrong? You can test the relationship and establish whether your readings are consistent with his hypothesis. However, if you improve the experiment and reduce the uncertainties, the conclusion will be much more useful.

When you consider improving an experiment, first consider any practical difficulties and possible sources of inaccuracy. Write them down in detail. Do not just write, for example, 'reaction time' or 'parallax error'. It is always a good idea to start with the idea that more readings need to be taken, possibly over a greater range (for example, in this case, if the masses of the stones were almost equal). Table P1.8 gives other possibilities.

Problem	**Improvement**
'Taking readings for just two masses was not enough.'	'I should use a great range of different masses and plot a graph of the average time to fall to the ground against the mass of the object.'
'It was difficult to start the stopwatch at the same instant that I dropped the stone and to stop it exactly as it hit the ground. I may have been late because of my reaction time.'	'Film the fall of each stone with a video camera which has a timer in the background. When the video is played back, frame by frame, I will see the time when the ball hits the ground on the timer. '(Alternatively, you can use light gates connected to a timer to measure the time electronically. You should draw a diagram, explaining that the timer starts when the first light gate is broken and stops when the second is broken.)
'My hand was not steady and so I may not have dropped the stones from exactly the same height each time.'	'Use iron objects which hang from an electromagnet. When the current in the electromagnet is switched off, the object falls.' (A diagram would help – see Figure P.15.)  **Figure P1.15:** Using an electromagnet to release iron objects. The line of sight is clearly shown.
'The heavier stone was larger in size and it was important that the bottom of each stone started at the same height. There may have been parallax error.'	'Clamp a metre rule vertically and start the bottom of each stone at exactly the top of the ruler each time. To avoid parallax error, I will make sure my line of sight is horizontal, at right angles to the rule.' (A diagram will show this clearly – see Figure P1.15.)
'The times that I measured were very short – not much greater than	'Increase the distance of fall so that the times are larger. This will make the uncertainty in each time measurement smaller in

my reaction time – so reaction time had a great effect.’	proportion to the time being measured.’

Table P1.8: Suggestions for improving Experiment 2.

Question

19 Use a stopwatch and a metre rule to measure the average speed as an object falls from a table to the ground. What are the difficulties and how might they be reduced? Some of the suggestions will be the same as those in Experiment 2, but you should also consider difficulties in measuring the distance to the ground and how they can be avoided. Remember, rules have battered ends and the ends may not be at 0 and 100 cm.

Experiment 3: Timing oscillations

In physics, the study of oscillations is of great importance. Indeed, the observation of a pendulum led Galileo to study time intervals and allowed pendulum clocks to be developed.

One skill you will need to develop is finding the time for an oscillation. Figure P1.16 shows a simple pendulum and one complete oscillation. The pendulum is just a small weight, the bob, which hangs on a string.

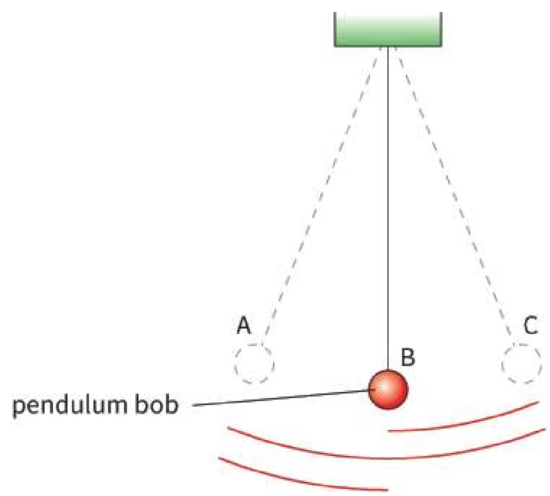


Figure P1.16: One complete oscillation is either from A to C and then back to A, or from B to C then back to B, then to A and back to B, as shown.

Figure P1.16 shows that one complete oscillation can be measured in two ways. Which way is better? In fact, the second way is better. This is because it is difficult to judge exactly when the pendulum bob is at the end of its swing. It is easier to start timing when the bob is moving quickly past a point; this happens in the middle of the swing. To time from the middle of the swing, you should use a **fiducial** mark. This can be a line on the bench underneath the bob at the centre of the swing, or it can be another object in the laboratory that appears to be in line with the bob when it hangs stationary, as seen from where you are standing. As long as you do not move your position, every time the bob passes this point it passes the centre.

Another way to reduce the uncertainty in the time for one oscillation is to time more than one swing, as explained in the topic on percentage uncertainty.

A simple practical task is to test the hypothesis that the time for one oscillation T is related to the length l of a simple pendulum by the formula $T^2 = kl$, where k is a constant.

What difficulties would you face and what are possible improvements? Table P1.9 gives some possibilities.

Problem	Improvement
‘Taking readings for just two lengths was not enough.’	‘Use more than two lengths and plot a graph of the average time squared against the length of the string.’
‘It was difficult to judge the end of the swing.’	‘Use a fiducial mark at the centre of the oscillation as the position to start and stop the stopwatch.’ ‘Use an electronic timer placed at the centre of the oscillation to

	measure the time.' 'Make a video of the oscillation with a timer in the background and play it back frame by frame.'
'The oscillations died away too quickly.'	'Use a heavier mass which swings longer.'
'The times were too small to measure accurately, as my reaction time was a significant fraction of the total time.'	'Use longer strings.' 'Time 20 rather than 10 oscillations.'
'It was difficult to measure the length to the centre of gravity of the weight accurately.'	'Use a longer string so any errors are less important.' 'Measure the length to the top of the weight and use a micrometer to measure the diameter of the bob and add on half the diameter to the length of the string.'

Table P1.9: Suggestions for improving Experiment 3.

Question

20 Hang a mass from a spring or from a rubber band. Use a stopwatch to time the mass as it oscillates up and down. Measure the time for just one oscillation, the time for 10 oscillations and the time for 20 oscillations. Repeat each reading several times. Use your readings to find the time for one complete oscillation and the uncertainty in each time. Draw up a table to show the problems of such measurements and how to reduce them.

Experiment 4: Using force meters

You need to be able to read instruments, estimating the uncertainty, looking for sources of error and trying to improve their use. One such instrument is a force meter or newton-meter, shown in Figure P1.17.

In this experiment, the block is pulled using the force meter to find the force F needed to make a block just start to move. An extra mass is added on top of the block to see whether the relationship $F = km$ is obeyed, where m is the total mass of the block and k is a constant.

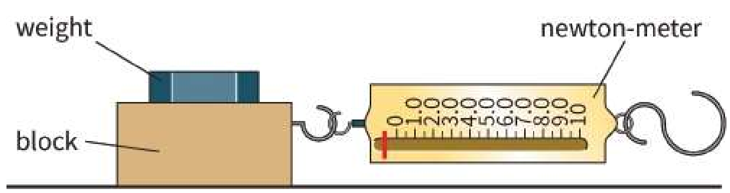


Figure P1.17: A newton-meter, just before it pulls a block along the bench. Look closely at Figure P1.17. When reading the meter, the uncertainty is the smallest scale division on the meter, unless one can reasonably read between the markings. This is difficult and so an uncertainty of 0.5 N, the smallest scale division, is reasonable.

Another problem in using the meter is that it reads less than zero before it is pulled. It needs a small force to bring the meter to zero. This is a zero error and all the actual readings will be too large by the same amount. This is probably because the meter was adjusted to read zero when hanging vertically and it is now being used horizontally.

Fortunately, the meter can be adjusted to read zero before starting to pull.

Table P1.10 describes the problems that may be encountered with this experiment, together with suggested improvements.

Problem	**Improvement**
'Taking readings for just two masses was not enough.'	'Use more than two masses and plot a graph of the force against the mass.'
'It was difficult to zero the newton-meter used horizontally.'	'Use a force sensor and computer.' 'Use a pulley and string to connect a tray to the block. Then tip sand onto a tray until the block starts to move. The weight of the sand and

	tray is then the force.'
'The reading of *F* was very low on the scale and gave a large percentage uncertainty.'	'Use heavier masses on top of the block.'
'The block starts to move suddenly and it is difficult to take the reading as this happens.'	'Video the experiment and play back frame by frame to see the largest force.' 'Use a force sensor and computer.'
'Different parts of the board are rougher than others.'	'Mark round the block with a pencil at the start and put it back in the same place each time.'

Table P1.10: Suggestions for improving Experiment 4.

Question

21 If you grip the bulb of a thermometer gently in your fingers, the reading rises to a new value. The reading will be different depending on whether you cover the bulb entirely or only partially with your fingers.

A laboratory thermometer can be used to measure the increase in temperature.

a Suggest a value for the uncertainty in such a reading. (You may need to look at some different thermometers.)

b Describe how you would test whether the temperature rise is proportional to the area of the bulb covered by your fingers You can take the surface area of the bulb to be 1 cm^2 and when you cover half of the bulb the area covered is 0.5 cm^2. The exact value of the surface area is not important; just the ratio is important.

c Suggest difficulties with this experiment, and how it might be improved. One problem with a thermometer is that it takes time for the reading to rise. What can you do about this?

Experiment 5: Electrical measurements

Electrical experiments have their own problems. Figure P1.18 shows an apparatus used to test the hypothesis that the resistance R of a wire is related to its length l by the formula $R = kl$, where k is a constant. The current is kept constant and the voltmeter reading is taken at two different values of l, for l = 0.30 m and 0.50 m.

What problems are likely to arise when using this apparatus? Table P1.11 identifies some possible problems with this experiment, and some suggestions for improvement.

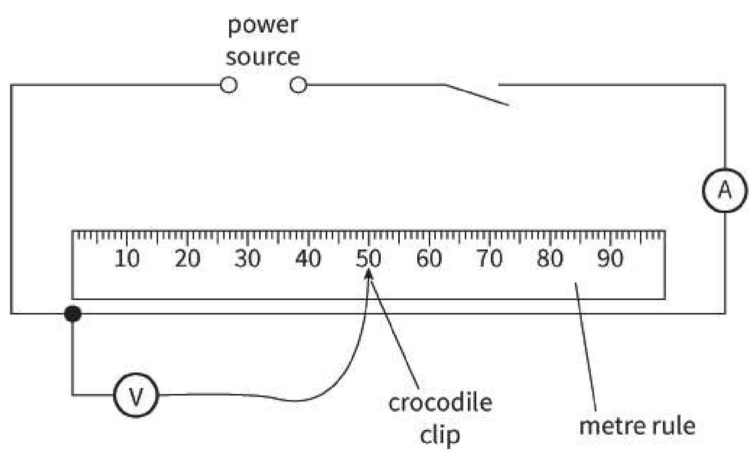


Figure P1.18: Apparatus used to check the hypothesis $R = kl$.

REFLECTION

Without looking at your textbook, produce a list of the problems and improvements that can be encountered in mechanics experiments, light experiments and electrical experiments.

Check your list against someone else's list.

Problem	Improvement
'Taking readings for just two lengths was not enough.'	'Use more than two lengths and plot a graph of the voltmeter reading against the length.' 'Calculate more than just two values of k.'
'Difficult to measure the length of the wire as the clips have width and I don't know where inside they grip the wire.'	'Use narrower clips.' 'Solder the contacts onto the wire.'
'The scale is not sensitive enough and can only measure to 0.05 V.'	'Use a voltmeter that reads to 0.01 V.' 'Use a digital voltmeter.'
'The values of voltage are small, particularly at 0.30 m.'	'Use a larger current so that when l = 0.50 m the voltmeter reading is at the top of the scale.'
'The voltmeter reading fluctuates because of contact resistance.'	'Clean the wires with wire wool first.'
'Other factors may have changed the resistance; for example, the temperature may have increased because of the current.'	'Wait a long time until the wire has reached a constant temperature.' 'Use smaller currents, but with a more sensitive voltmeter.'

Table P1.11: Suggestions for improving Experiment 5.

SUMMARY

A **precise** reading is one in which there is very little spread about the mean value.
The **uncertainty** in a reading is an estimate of the difference between the reading and true value of the quantity being measured.
A **systematic error** cause readings to differ from the true value by a consistent amount each time the reading is made.
Random errors cause readings to vary around the mean value in an unpredictable way from one reading to another.
A **zero error** is caused when an instrument gives a non-zero reading when the true value of the quantity is zero.
Find the uncertainty from the largest of the smallest division on the instrument used or half the range of a number of readings of the same measurement.
Each column of a table must be labelled with a quantity / unit, and, if a reading be given to the precision of the instrument, usually to the same number of decimal places. Calculated quantities may have one more significant figure than the readings used.
The **independent** variable is the one that the experimenter alters or selects.
The **dependent** variable is the quantity that changes as a result of the independent variable being altered by the experimenter.
$\text{gradient} = \frac{\text{change in } y}{\text{change in } x}$ Use a large triangle to show the values used in calculating the gradient.
In testing a relationship, write down a criterion. Calculate the percentage difference between two values of the constant. Compare the percentage difference with the percentage uncertainty in one of the variables and write a conclusion as to whether the criterion is obeyed or not
If quantities are added or subtracted, then add absolute uncertainties. If quantities are multiplied or divided, add percentage uncertainties.
A **problem** is a difficulty you experience during the experiment.
An **improvement** is a suggestion that will reduce the problem. You should have experience of a range of these problems and improvements. For more details, consult the Practical Workbook.

EXAM-STYLE QUESTIONS

1 Quantity P has a fractional uncertainty ***p***. Quantity Q has a fractional uncertainty ***q***.

What is the fractional uncertainty in $\frac{P^2}{Q^3}$? **[1]**

A $p - q$

B $p + q$

C $2p - 3q$

D $2p + 3q$

2 The p.d. V across a wire of length l is given by the formula $V = \frac{4I\rho l}{d^2}$ where d is the diameter of the wire, ρ is the resistivity and there is a current I in the wire.

Which quantity provides the largest contribution to the percentage uncertainty in ***V***? **[1]**

	Quantity	Value of quantity	Absolute uncertainty
A	l / cm	250	±10
B	d/mm	1.4	±0.1
C	ρ/Ω m	1.5×10^{-8}	$\pm 0.2 \times 10^{-8}$
D	I / A	2.0	±0.2

Table P1.12

3 What is the uncertainty in the following sets of readings? All of them are written down to the smallest division on the instrument used in their measurement.

a 24.6, 24.9, 30.2, 23.6 cm **[1]**

b 2.66, 2.73, 3.02 s **[1]**

c 24.0, 24.0, 24.0 g **[1]**

[Total: 3]

4 Electrical experiments usually involve the reading of meters such as the voltmeters shown.

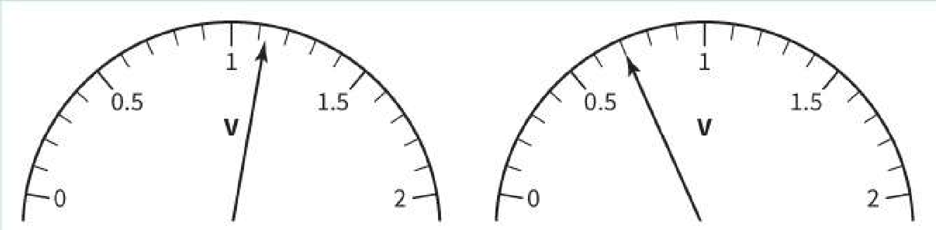


Figure P1.19

a What is the reading shown by each voltmeter, and the uncertainty in each reading? **[2]**

b The voltmeters show the readings obtained when they were connected across two wires that were identical apart from their different lengths. The current in each wire was 0.500 A and the length l of the wire was 30.0 cm in the right diagram and 50.0 cm in the left diagram.

Use the scale readings to test the hypothesis that the resistance R of the wire is proportional to length l. Consider the effect of the uncertainties on your conclusion. **[4]**

[Total: 6]

5 This apparatus can be used to test the hypothesis that T, the time taken for a ball to roll down a plane from rest, is related to the distance *s* by the formula $T^2 = ks$, where *k* is a constant.

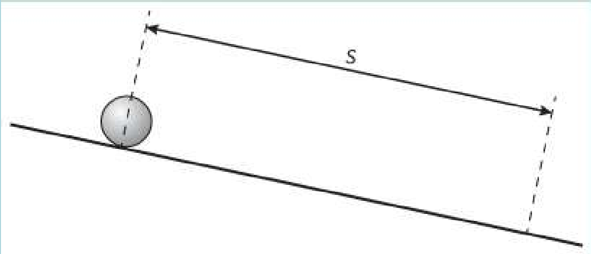


Figure P1.20

The ball is timed using a stopwatch over two different values of *s*.

Suggest problems with the experiment and how they might be overcome. You should consider problems in measuring the distance as well as the time. Also note what happens to the ball; it may not roll in the way that you expect. **[8]**

Questions 6–8 are designed to illustrate some aspects of practical questions. They are not formal practical questions as, ideally, you should perform the experiment yourself and take some readings. This helps you to see the problems.

6 An experiment explores the relationship between the period of a vibrating spring and the mass *m* in a pan holder. The student is instructed to set up the apparatus as shown here, with a mass of 200 g in the pan.

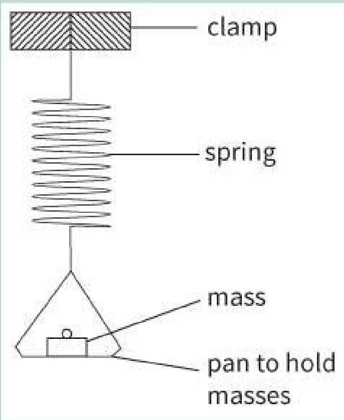


Figure P1.21

The student is then told to move the pan downwards by approximately 1 cm and to release it so that it vibrates in a vertical direction.

The student is asked to record the time taken for 20 oscillations of the spring, and then to repeat the procedure, using masses between 20 g and 200 g until She has six sets of readings. Columns are provided in the table for $\sqrt{m}$ and *T*, the period of the pendulum.

This table shows the readings taken by a student with the different masses.

Mass / g	**Time for 20 oscillations / s**	$\sqrt{m}$	***T***
20	12.2		
50	15.0		
100	18.7		
150	21.8		
200	24.5		
190	24.0		

Table P1.13

a Copy the table and include values for $\sqrt{m}$ and T. **[2]**

b Plot a graph of T on the y-axis against $\sqrt{m}$ on the x-axis. Draw the straight line of best fit. **[4]**

c Determine the gradient and y-intercept of this line. **[2]**

d The quantities T and m are related by the equation:

$$T = C + k\sqrt{m}$$

where C and k are constants.

Find the values of the two constants C and k. **Give** appropriate units. **[2]**

[Total: 10]

7 A student releases a toy car to roll down a ramp, as shown.

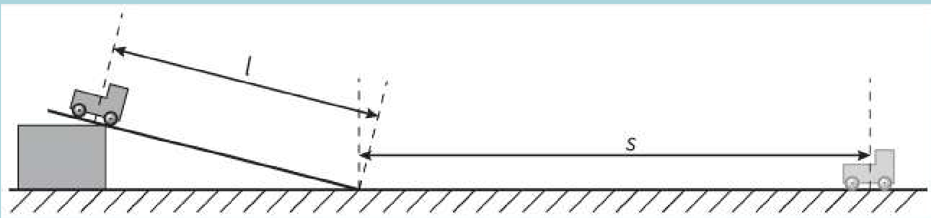


Figure P1.22

The student measures the distance l from the middle of the car as it is released to the bottom of the ramp and the distance s travelled along the straight section before the car stops. He also measures the time t taken to travel the distance s. He then repeats the experiment using a different value of l.

The student obtained readings with l = 40 and 60 cm, taking each reading for s and t twice. The readings were:

l = 40.0 cm: values for s were 124 and 130 cm; values for t were 4.6 and 4.8 s

l = 60.0 cm: values for s were 186 and 194 cm; values for t were 4.9 and 5.2 s.

a For the smaller value of l, obtain a value for:

i the average value of s **[1]**

ii the absolute and percentage uncertainty in the value of s **[2]**

iii the average value of t **[1]**

iv the absolute and percentage uncertainty in the value of t. **[2]**

b **i** For both values of l, calculate the average speed v of the car along the straight section of track using the relationship $v = \frac{s}{t}$. **[1]**

ii **Justify** the number of significant figures that you have given for your values of v. **[1]**

c **i** It is suggested that s is proportional to l. Explain whether the readings support this relationship. **[2]**

ii (HARDER) It is suggested that v^2 is proportional to l. Explain whether the readings support this relationship. **[2]**

d Describe **four** sources of uncertainty or limitations of the procedure for this experiment. **[4]**

e Describe **four** improvements that could be made to this experiment. You may suggest the use of other apparatus or different procedures. **[4]**

[Total: 20]

8 This apparatus shows a resistor in some water.

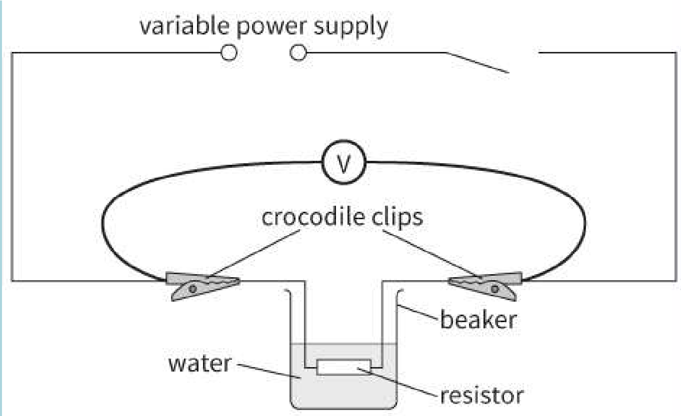


Figure P1.23

A student measures the rise in temperature θ of the water in 100 s using two different values of voltage.

The student wrote:

'When the voltage was set at 6.0 V, the rise in temperature of the water in 100 s was 14.5 °C. The voltmeter reading decreased by about 0.2 V during the experiment, and so the final voltmeter reading was 5.8 V.

'The reading fluctuated from time to time by about 0.2 V. The smallest scale division on the thermometer was 1 °C, but I could read it to 0.5 °C. I did not have time to repeat the reading.

'When the voltage was set at 12.0 V, the rise in temperature in 100 s was 51.0 °C and the voltage was almost the same at the end, but fluctuated by about 0.2 V.'

a Estimate the percentage uncertainty in the measurement of the first voltage. **[1]**

b It is suggested that θ is related to V according to the formula $\theta = kV^2$, where k is a constant.

i Calculate **two** values for k. Include the units in your answer. **[2]**

ii Justify the number of significant figures you have given for your value of k. **[1]**

iii Explain whether the results support the suggested relationship. **[1]**

c Describe **four** sources of uncertainty or limitations of the procedure for this experiment. **[4]**

d Describe **four** improvements that could be made to this experiment. You may suggest the use of other apparatus or different procedures. **[4]**

[Total: 13]