Chapter 1

Velocity and acceleration

- Work with scalar and vector quantities for distance and speed.
- Use equations of constant acceleration.
- Sketch and read displacement–time graphs and velocity–time graphs.
- Solve problems with multiple stages of motion.

1.1 Displacement and velocity

WORKED EXAMPLE 1.1

A plane flies from Warsaw to Athens, a distance of 1600 km, at an average speed of 640 km h^{-1}.

How long does the flight take?

Answer

$\text{speed} = \frac{\text{distance}}{\text{time}}$	State the equation to use.
$\text{so time} = \frac{\text{distance}}{\text{speed}}$	Rearrange the equation to make time the subject.
$= \frac{1600}{640}$ $= 2.5$	Use consistent units, substitute values into the equation.
Flight takes 2 hours 30 minutes.	Convert the decimal answer into hours and minutes.

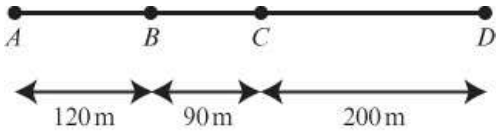
EXERCISE 1A

1 How long will an athlete take to run 1500 metres at 7.5 m s^{-1}?

2 A train maintains a constant velocity of 60 m s^{-1} due south for 20 minutes. What is its displacement in that time? Give the distance in kilometres.

3 Some Antarctic explorers walking towards the South Pole expect to average 1.8 kilometres per hour. What is their expected displacement in a day in which they walk for 14 hours?

Questions 4 and 5 refer to the four points, A, B, C and D, which lie in a straight line with distances between them shown in the diagram. The displacement is measured from left to right.



4 Find:

a **i** the displacement from D to A

ii the displacement from D to B

b **i** the distance from D to B

ii the distance from C to A

c **i** the total displacement when a particle travels from B to C and then to A

ii the total displacement when a particle travels from C to D and then to A.

TIP

Remember displacement is a vector quantity, and distance is a scalar quantity.

PS **5** **a** **i** A particle travels from A to C in 23 seconds and then from C to B in 18 seconds. Find its average speed and average velocity.

ii A particle travels from B to D in 38 seconds and then from D to A in 43 seconds.

Find its average speed and average velocity.

b i A particle travels from B to D in 16 seconds and then back to B in 22 seconds. Find its average speed and average velocity.

ii A particle travels from A to C in 26 seconds and then back to A in 18 seconds. Find its average speed and average velocity.

TIP

Remember speed is a scalar quantity, and velocity is a vector quantity.

6 Here is an extract from the diary of Samuel Pepys for 4 June 1666, written in London.

'We find the Duke at St James's, whither he is lately gone to lodge. So walking through the Parke we saw hundreds of people listening to hear the guns.'

These guns were at the battle of the English fleet against the Dutch off the Kent coast, a distance of between 110 and 120 km away. The speed of sound in air is 344 m s^{-1}. How long did it take the sound of the gunfire to reach London?

7 Light travels at a speed of 3.00×10^8 m s^{-1}. Light from the star Sirius takes 8.65 years to reach the Earth. What is the distance of Sirius from the Earth in kilometres?

TIP

Consider how many seconds there are in 8.65 years.

1.2 Acceleration

WORKED EXAMPLE 1.2

A skateboarder travels down a hill in a straight line with constant acceleration. She starts with speed 1.5 m s^{-1} and finishes with speed 9.5 m s^{-1}. The length of the hill is 22 m.

a Find the time taken.

b Find the acceleration of the skateboarder.

Answer

a $s = 22$, $u = 1.5$, $v = 9.5$	Begin by listing the information given.
$s = \frac{1}{2}(u+v)t$ so $22 = \frac{1}{2}(1.5+9.5)t$	State the equation to be used and substitute in the known values.
$t = 4$	Rearrange the equation to find the time.
Time taken = 4 seconds	Include the units in the final answer.
b $\alpha = \frac{v-u}{t}$	Choose the equation to be used.
$= \frac{9.5-1.5}{4}$ $= 2$	Substitute in the known values.
Acceleration = 2 m s^{-2}	Include the units in the final answer.

EXERCISE 1B

1 Write the following quantities in the specified units, giving your answers to 3 significant figures.

a **i** 3.6 km h^{-1} in m s^{-1}

ii 62 km h^{-1} in m s^{-1}

b **i** 5.2 m s^{-1} in km h^{-1}

ii 0.26 m s^{-1} in km h^{-1}

c **i** 120 km h^{-2} in m s^{-2}

ii 450 km h^{-2} in m s^{-2}

d **i** 0.82 m s^{-2} in km h^{-2}

ii 2.7 m s^{-2} in km h^{-2}

TIP

Use velocities not speeds.

2 A police car accelerates from 15 m s^{-1} to 35 m s^{-1} in 5 seconds. The acceleration is constant. Illustrate this with a velocity–time graph. Use the equation $v = u + at$ to calculate the acceleration. Find also the distance travelled by the car in that time.

3 A marathon competitor running at 5 m s^{-1} puts on a sprint when she is 100 metres from the finish, and covers this distance in 16 seconds. Assuming that her acceleration is constant, use the equation $s = \frac{1}{2}(u+v)t$ to find how fast she is running as she crosses the finishing line.

4 Starting from rest, an aircraft accelerates to its take-off speed of 60 m s^{-1} in a distance of 900 metres. Assuming constant acceleration, find how long the take-off run lasts. Hence calculate the acceleration.

TIP

'Rest' means not moving, so the velocity is zero.

5 A train is travelling at 80 m s^{-1} when the driver applies the brakes, producing a deceleration of 2 m s^{-2} for 30 seconds. How fast is the train then travelling, and how far does it travel while the brakes are on?

PS **6** A balloon at a height of 300 m is descending at 10 m s^{-1} and decelerating at a rate of 0.4 m s^{-2}. How long will it take for the balloon to stop descending, and what will its height be then?

7 A train goes into a tunnel at 20 m s^{-1} and emerges from it at 55 m s^{-1}. The tunnel is 1500 m long. Assuming constant acceleration, find how long the train is in the tunnel for, and the acceleration of the train.

PS **8** A cyclist riding at 5 m s^{-1} starts to accelerate, and 200 metres later she is riding at 7 m s^{-1}. Find her acceleration, assumed constant.

1.3 Equations of constant acceleration

WORKED EXAMPLE 1.3

A train is travelling at 55 m s^{-1}. The driver needs to reduce the speed to 35 m s^{-1} to pass through a junction. The deceleration must not exceed 0.6 m s^{-2}. How far ahead of the junction should the driver begin to slow down the train?

Answer

Using the maximum deceleration:	
$u = 55$, $v = 35$, $a = -0.6$	Begin by listing the information given. As we have deceleration, the acceleration is a negative value.
$v^2 = u^2 + 2as$ so $1225 = 3025 - 1.2s$	State the equation to be used and substitute in the known values.
$s = 1500$	Rearrange to find the distance.
The driver should start to slow down at least 1500 m ahead of the junction.	Include the units in the final answer and clarify the answer in the contextof the question.

EXERCISE 1C

P **1** Use the formulae $v = u + at$ and $s = \frac{1}{2}(u + v)t$ to prove that $s = ut + \frac{1}{2}at^2$.

TIP

Decide which variable to eliminate.

P **2** **a** Use the formulae $s = ut + \frac{1}{2}at^2$ and $v = u + at$ to derive the formula $s = vt + \frac{1}{2}at^2$.

b A particle moves with constant acceleration 3.1 m s^{-2}. It travels 300 m in the first 8 seconds. Find its speed at the end of the 8 seconds.

P **3** Use the formulae $v = u + at$ and $s = ut + \frac{1}{2}at^2$ to derive the formula $v^2 = u^2 + 2as$.

4 An ocean liner leaves the harbour entrance travelling at 3 m s^{-1}, and accelerates at 0.04 m s^{-2} until it reaches its cruising speed of 15 m s^{-1}.

a How far does it travel in accelerating to its cruising speed?

b How long does it take to travel 2 km from the harbour entrance?

5 A boy kicks a football up a slope with a speed of 6 m s^{-1}. The ball decelerates at 0.3 m s^{-2}. How far up the slope does it roll?

6 A cyclist comes to the top of a hill 165 metres long travelling at 5 m s^{-1}, and free-wheels down it with an acceleration of 0.8 m s^{-2}. Write expressions for his speed and the distance he has travelled after t seconds. Hence find how long he takes to reach the bottom of the hill, and how fast he is then travelling.

PS **7** A particle reduces its speed from 20 m s^{-1} to 8.2 m s^{-1} while travelling 100 m. Assuming it continues to move with the same constant acceleration, how long will it take to travel another 20 m?

PS **8** A particle moves with constant deceleration of 3.6 m s^{-2}. It travels 350 m while its speed halves. Find the time it takes to do this.

9 A car reduces its speed from 18 m s^{-1} to 9 m s^{-1} while travelling 200 m. Assuming the car continues to move with the same uniform acceleration, how much further will it travel before it stops?

10 **a** A particle moves in a straight line with constant acceleration $a = -3.4$ m s^{-2}. At $t = 0$ its velocity is $u = 6$ m s^{-1}. Find its maximum displacement from the starting point.

b Explain why this is not the maximum distance from the starting point.

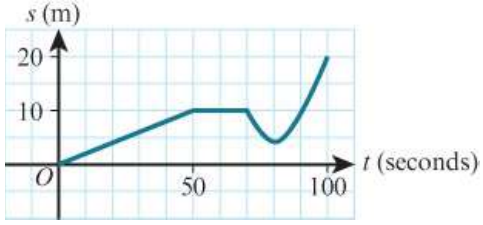
11 A car travelling at 10 m s^{-1} is 25 metres from a pedestrian crossing when the traffic light changes from green to amber. The light remains at amber for 2 seconds before it changes to red. The driver has two choices: to accelerate so as to reach the crossing before the light changes to red, or to try to stop at the light. What is the least acceleration which would be necessary in the first case, and the least deceleration which would be necessary in the second?

12 A cheetah is pursuing an impala. The impala is running in a straight line at a constant speed of 16 m s^{-1}. The cheetah is 10 m behind the impala, running at 20 m s^{-1} but tiring, so that it is decelerating at 1 m s^{-2}. Find an expression for the gap between the cheetah and the impala t seconds later. Will the impala get away?

1.4 Displacement–time graphs and multi-stage problems

WORKED EXAMPLE 1.4

The diagram shows the displacement–time graph for a cyclist moving in a straight line.



a Find the velocity of the cyclist over the first 50 seconds.

b Estimate the times when the velocity of the cyclist is 0 m s^{-1}.

c Find the greatest (positive) velocity over the 100 seconds.

Answer

a $\frac{10}{50} = 0.2 \text{ms}^{-1}$

The gradient of a displacement-time graph is equal to the velocity.

b t = 50 to 70 and at approximately 82 seconds

As the gradient represents the velocity, we look at when the gradient is zero.

c Velocity = gradient of graph

Greatest (positive) velocity is t = 90 to 100

This is where the graph is steepest.

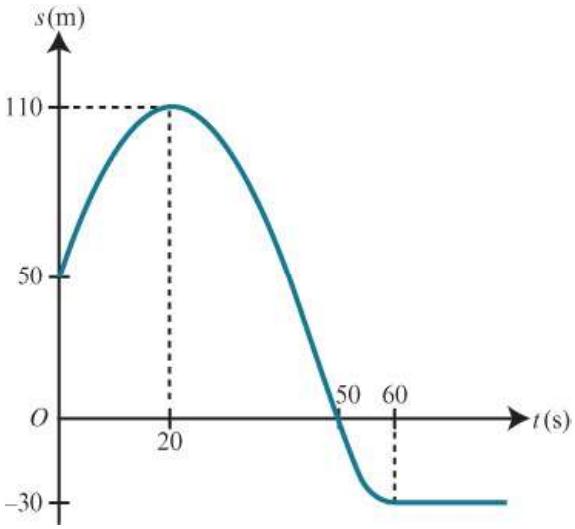
In this time s increases from 10 to 20

Velocity $= \frac{20-10}{100-90} = 1 \text{ms}^{-1}$

Gradient = increase in height/change in horizontal distance.

EXERCISE 1D

1 A particle moves in a straight line. Its displacement from point P is shown on the displacement–time graph.



a How far from P does the particle start?

b In the first 20 seconds, is the particle moving towards P or away from it?

c What happens when t = 20 seconds?

d What happens after 60 seconds?

e At what time does the particle pass P?

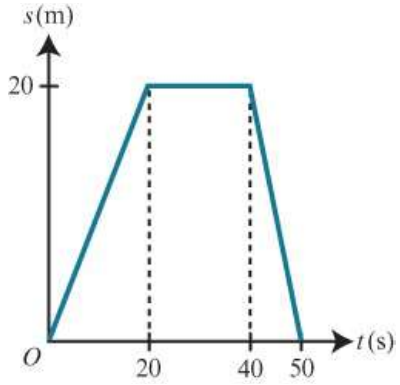
f Is the particle's speed increasing or decreasing during the first 20 seconds?

g Is the particle's speed increasing or decreasing between 50 and 60 seconds? What about its velocity?

h Find the total distance travelled by the particle in the first 60 seconds.

PS **2** For each displacement–time graph, draw the corresponding straight line velocity–time graph:

a



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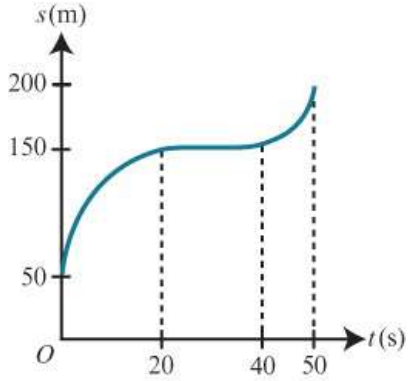

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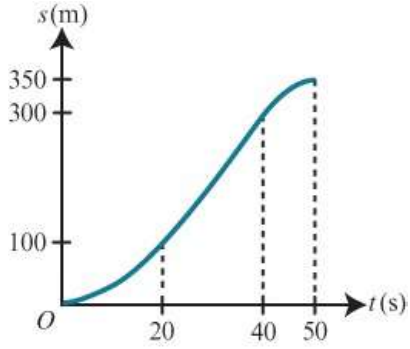

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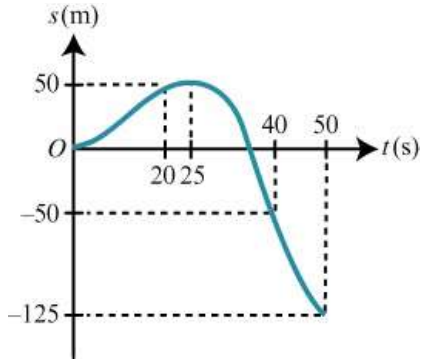
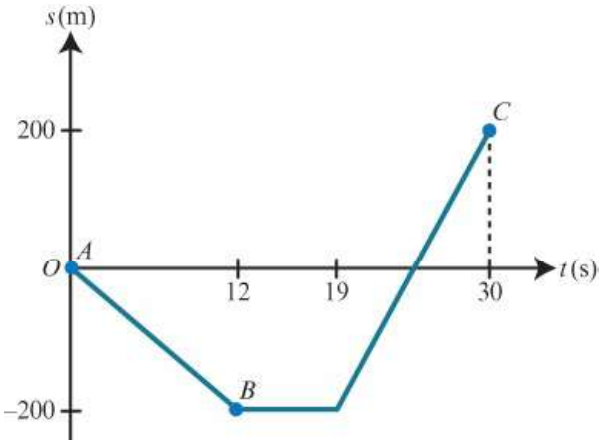


3 This displacement–time graph represents the motion of a particle moving in a straight line. The particle passes point A when $t = 0$.



The particle is at point B when $t = 12$ and at point C when $t = 30$.

a Describe what happens between $t = 12$ and $t = 19$.

b Write down the displacement of C from A. Hence find the average velocity of the particle during the 30 seconds.

c Find the average speed of the particle during the 30 seconds.

PS **4** A cyclist is free-wheeling down a long straight hill. The times between passing successive kilometre posts are 100 seconds and 80 seconds. Assuming his acceleration is constant, find this acceleration.

TIP

Each individual km can be considered, or the combined motion for 2 km can be considered.

5 A train is slowing down with constant deceleration. It passes a signal at A, and after successive intervals of 40 seconds it passes points B and C, where $AB = 1800$ m and $BC = 1400$ m.

a How fast is the train moving when it passes A?

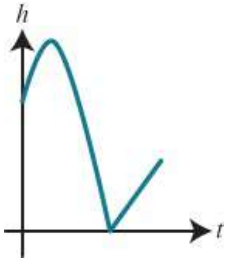
b How far from A does it come to a stop?

PS **6** A particle is moving along a straight line with constant acceleration. In an interval of T seconds it moves D metres; in the next interval of $3T$ seconds it moves $9D$ metres.

How far does it move in a further interval of T seconds?

7 A ball is thrown vertically upwards, travels up and then down; when the ball hits the ground it bounces up again.

The following graph shows the height of the ball above the ground against time.



Sketch a graph to show the distance that the ball has travelled against time.

8 Amir and Sofia start side by side on the starting line of a 100 m track. Amir runs the 100 metres at a constant speed of v m s^{-1}. Sofia starts from rest 1 second after Amir and accelerates at a constant 0.5 m s^{-2}.

a When $v = 4$, how far has Amir run when Sofia overtakes him?

b What happens when $v = 5$?

9 A particle starts from rest and moves with constant acceleration.

a Sketch the displacement–time graph.

In the first 4 seconds the particle moves 16 metres.

b Find how far the particle travels in the next 4 seconds.

PS **10** A motorbike and a car are waiting side by side at traffic lights. When the lights turn to green, the motorbike accelerates at 2.5 m s^{-2} up to a top speed of 20 m s^{-1}, and the car accelerates at 1.5 m s^{-2} up to a top speed of 30 m s^{-1}. Both then continue to move at constant speed.

a Using the same axes, sketch the displacement–time graphs.

b After what time will the motorbike and the car again be side by side?

c What is the greatest distance that the motorbike is in front of the car?

M **11** The displacement, in metres, of a particle is plotted against time, in seconds. The resulting displacement–time graph is modelled as a quadratic equation that passes through (0, 20), (10, 5) and (12, 20).

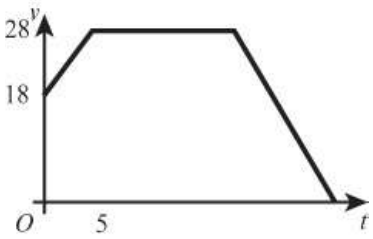
At some time the displacement is x metres and 4 seconds later the displacement is again x metres. Find the value of x.

1.5 Velocity–time graphs and multi-stage problems

WORKED EXAMPLE 1.5

A train moves along a straight track. The train passes a signal at time $t = 0$ seconds and is moving with a velocity of 18 m s^{-1}. The train accelerates at 2 m s^{-2} for 5 seconds, travels at a constant velocity for some time, and then decelerates at 1.4 m s^{-2} until it stops at a station. The signal is 1000 m from the station. Find the time at which the train reaches the station.

Answer

When $t = 0$, $v = 18$	State the initial values.
When $t = 5$, $v = 18 + 2 \times 5 = 28$	Use $v = u + at$ to find the velocity after 5 seconds.
The velocity–time graph looks like this: 	Although we don't have all the informationfor a velocity–time graph, drawing a sketchof what we do know may help to answer the question.
While accelerating, the train travels 0.5(18 + 28) × 5 =115 metres	Use $s = \frac{1}{2}(u+v)t$ to find the distance travelled during the first section of the motion.
The deceleration phase takes $\frac{0-28}{-1.4} = 20$ seconds	Use $\frac{v-u}{\alpha} = t$ to find the time taken to deceleration to rest.
The distance travelled while decelerating is0.5(28 + 0) × 20 = 280 metres	Use $s = \frac{1}{2}(u+v)t$ to find the distance travelled during the final section of the motion.
So the distance travelled at 28 m s^{-1} is1000 −115 − 280 = 605 metres	Use the total distance to find the distance travelled during the constant speed section of motion.
This takes 605 ÷ 28 = 21.6 seconds	Use time $= \frac{\text{distance}}{\text{speed}}$.
5 + 21.6 + 20 = 46.6 The train reaches the station at time t = 46.6 seconds	Finally add the time for each of the threesections.

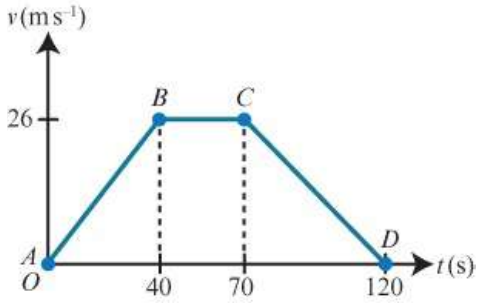
EXERCISE 1E

1 For each velocity–time graph, find:

a the acceleration from A to B and from C to D

b the total distance travelled.

i



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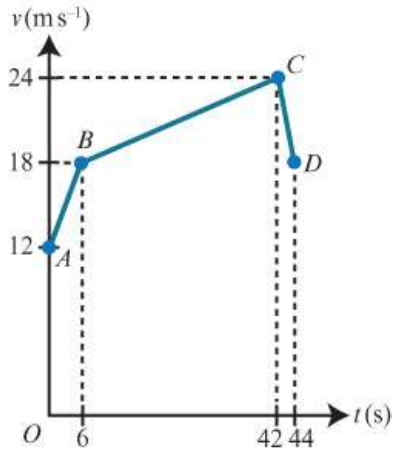

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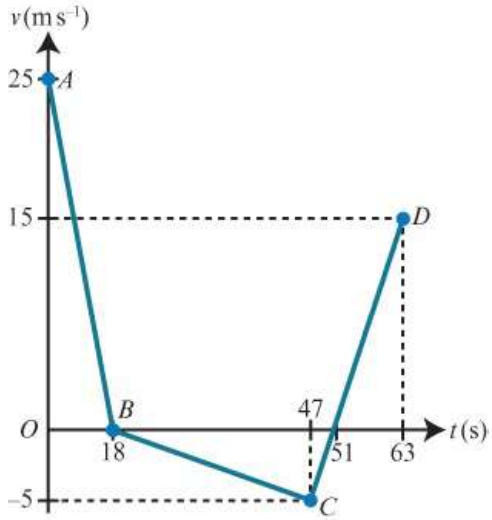


2 For each of these descriptions of motion, draw the velocity–time graph and find the total distance travelled.

a A particle accelerates uniformly from 20 m s^{-1} to 32 m s^{-1} in 15 seconds, then moves with constant speed for 25 seconds and finally decelerates uniformly and comes to rest in another 10 seconds.

b An object starts from rest and accelerates at 2.5 m s^{-2} for 12 seconds. It then moves with a constant velocity for 8 seconds and finally decelerates at 6 m s^{-2} until it comes to rest.

c A particle accelerates uniformly from 11 m s^{-1} to 26 m s^{-1} with acceleration 0.4 m s^{-2}. It then decelerates at 2 m s^{-2} until it comes to rest.

3 A train travelling at 20 m s^{-1} starts to accelerate with constant acceleration. It covers the next kilometre in 25 seconds. Use the equation $s = ut + \frac{1}{2}at^2$ to calculate the acceleration. Find also how fast the train is moving at the end of this time. Illustrate the motion of the train with a velocity–time graph. How long does the train take to cover the first half kilometre?

4 A long-jumper takes a run of 30 metres to accelerate to a speed of 10 m s^{-1} from a standing start. Find the time he takes to reach this speed, and hence calculate his

acceleration. Illustrate his run-up with a velocity–time graph.

PS **5** A particle moves in a straight line, starting from rest at point P. It accelerates for 5 seconds, until it reaches a speed of 16 m s^{-1}. It maintains this speed for T seconds and then decelerates at 2 m s^{-2} until it comes to rest at point Q.

a Sketch the velocity–time graph to represent the motion of the particle.

b Given the average speed of the particle on the journey from P to Q is 12 m s^{-1}, find the value of T.

6 A cyclist travels from A to B, a distance of 240 metres. He passes A at 12 m s^{-1}, maintains this speed for as long as he can, and then brakes so that he comes to a stop at B. If the maximum deceleration he can achieve when braking is 3 m s^{-2}, what is the least time in which he can get from A to B?

PS **7** Two villages are 900 metres apart. A car leaves the first village travelling at 15 m s^{-1} and accelerates at $\frac{1}{2}$ m s^{-2} for 30 seconds.

How fast is it then travelling, and what distance has it covered in this time?

The driver now sees the next village ahead, and decelerates so as to enter it at 15 m s^{-1}. What constant deceleration is needed to achieve this? How much time does the driver save by accelerating and decelerating, rather than covering the whole distance at 15 m s^{-1}?

M **8** A cyclist starts at the bottom of a hill moving at a speed of 13.5 m s^{-1}. She moves with a constant deceleration of 0.9 m s^{-2}, reaching the top of the hill 9.2 seconds later. She then accelerates down the hill at 1.6 m s^{-2} for 86 m. Find the speed of the cyclist when she reaches the bottom of the hill.

9 A car comes to a stop from a speed of 30 m s^{-1} in a distance of 804 m. The driver brakes so as to produce a deceleration of $\frac{1}{2}$ m s^{-2} to begin with, and then brakes harder to produce a deceleration of $\frac{3}{2}$ m s^{-2}. Find the speed of the car at the instant when the deceleration is increased, and the total time the car takes to stop.

10 A ball is dropped from a height of 2.6 m above the surface of a water well and falls freely under gravity. After it enters the water, the ball's acceleration decreases to 1.2 m s^{-2}. It reaches the bottom of the well 0.9 seconds after reaching the surface of the water. Assuming the acceleration through the water is constant over a short period of time, find the depth of the water in the well.

TIP

Use a value of 10 m s^{-2} for the acceleration whilst the ball is above the surface of the water.

M **11** A roller-skater increases speed from 4 m s^{-1} to 10 m s^{-1} in 10 seconds at a constant rate.

a What is her average velocity over this period?

b For what proportion of the time is she moving at less than her average velocity?

c For what proportion of the distance is she moving at less than her average velocity?

12 A car starts from rest at time $t = 0$. It accelerates uniformly until its speed reaches V m s^{-1}. It travels at constant speed for 12 seconds and then decelerates uniformly, coming to rest when $t = 26$. The total distance travelled by the car is 840 m. Find the value of V.

1.6 Graphs with discontinuities

WORKED EXAMPLE 1.6

In a game of miniature golf, a player hits the ball from the tee so that it rolls along the ground with an initial speed of 10 m s^{-1}. The ball is subject to a deceleration of constant magnitude D m s^{-2} throughout its motion.

The ball hits a small wall 5 metres from the tee and returns past the tee. It hits another wall 1 metre beyond the tee and returns to the tee where it stops, having been rolling for T seconds. The ball loses 50% of its speed each time it hits a wall.

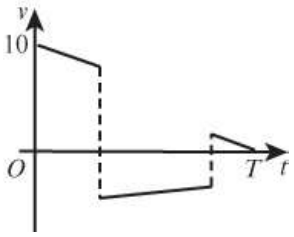
a Sketch the velocity–time graph for the motion.

b Find the value of D.

c Find the value of T.

Answer

a Taking the direction from the tee to the first wall as the positive direction:



Each time the ball hits a wall the velocity changes sign.

b From the tee to the first wall:

$s = 5$, $u = 10$, $a = -D$

Begin by listing the information given,noting the acceleration is negative asit is a deceleration.

Using $v^2 = u^2 + 2as$:

Choose the most suitable equation.

$v^2 = 100 - 10D$

Substitute in the known values.

Ball hits the wall with velocity $\sqrt{100 - 10D}$ and leaves with velocity $0.5\sqrt{100 - 10D}$.

The ball leaves the wall with half the previous velocity and the velocitychanges sign.

From the first wall to the second wall:

$s = -6$, $u = -0.5\sqrt{100 - 10D}$, $a = D$

List the new information known.

Using $v^2 = u^2 + 2as$:

Choose the most suitable equation.

$v^2 = 0.25(100 - 10D) - 12D = 25 - 14.5D$

Substitute in the known values andsimplify the answer.

Ball hits the second wall with velocity $-\sqrt{25 - 14.5D}$ and leaves with velocity $-0.5\sqrt{25 - 14.5D}$.

Again the ball leaves the wall with halfthe previous velocity and the velocitychanges sign.

From the second wall to the tee:

$s = 1$, $u = -0.5\sqrt{25 - 14.5D}$, $v = 0$, $a = -D$

List the new information known.

Using $v^2 = u^2 + 2as$:	Choose the most suitable equation.
$0 = 0.25(25 - 14.5D) - 2D$	Substitute in the known values.
$D = \frac{10}{9} = 1.11$	Solve to find D.

c From the tee to the first wall:

$s = 5$, $u = 10$, $v = 9.43$, $a = -1.11$	List the new information known.
Using $v = u + at$:	Choose the most suitable equation.
$9.43 = 10 - 1.11t$ so $t = 0.514$	Substitute in the known values and findthe time.

From the first wall to the second wall:

$s = -6$, $u = -4.72$, $v = -2.98$, $a = 1.11$	List the new information known.
Using $v = u + at$:	Choose the most suitable equation.
$-2.98 = -4.72 + 1.11t$ so $t = 1.559$	Substitute in the known values and findthe time.

From the second wall to the tee:

$s = 1$, $u = 1.49$, $v = 0$, $a = -1.11$	List the new information known.
Using $v = u + at$:	Choose the most suitable equation.
$0 = 1.49 - 1.11t$ so $t = 1.342$	Substitute in the known values and findthe time.
T = 0.518 +1.559 +1.342 = 3.42 seconds	Add together the times for each of thethree sections of motion.

EXERCISE 1F

M **1** An ice hockey puck slides across the ice with a constant speed of 15 m s^{-1}. The puck travels 10 metres, bounces off the board at the edge of the ice, losing 20% of its speed, and returns with constant velocity to the point where it was hit.

How long does the player who hit the puck have to get out of the way, to avoid being hit by the puck?

PS **2** A car is travelling at 20 m s^{-1} when the driver brakes heavily. The car is instantly subject to a constant deceleration that lasts for 1.5 seconds and slows the speed of the car to 5 m s^{-1}. The driver then eases off the brakes so the deceleration changes to a different constant value that brings the car to rest after travelling a further 50 metres.

Find:

a the total time taken

b the total distance travelled from when the brakes are applied to when the car stops.

PS **3** A ball is dropped from a window that is 10 m above the ground. The initial speed of the ball is 0 m s^{-1} and it accelerates at 9.8 m s^{-2} while it is falling. The ball bounces and loses 10% of its speed in the bounce.

a How high does the ball rise after the first bounce?

b How long does it take until the ball hits the ground for the second time?

A person spots the bouncing ball and catches it as it approaches the ground for the first time that is more than 10 seconds from when the ball first bounced.

c How many more bounces does the ball make after the first bounce before it is caught?

END-OF-CHAPTER REVIEW EXERCISE 1

TIP

Graphs and equations of motion may both be helpful.

1 A train leaves a station, starting from rest, with a constant acceleration of a m s^{-2}. It reaches a signal 100 seconds later at a speed of 40 m s^{-1}. Find:

a the value of a

b the distance between the station and the signal.

M **2** A woman skis down a slope with constant acceleration. She starts from rest and is travelling at 25 m s^{-1} when she reaches the bottom of the slope. The slope is 125 m long. Find:

a her acceleration down the slope

b the time taken to reach the bottom of the slope.

PS **3** A particle moves along a straight line ABC with constant acceleration. AB = 50 cm and BC = 150 cm. After passing through A, the particle travels for 2 seconds before passing through B, and for a further 3 seconds before passing through C. Find the acceleration of the particle and the speed with which it reaches C.

TIP

Remember to use SI units throughout.

4 A car is travelling at V m s^{-1} along a straight road and passes point A at t = 0. When the car is at point A the driver sees a pedestrian crossing the road at a point B ahead and decelerates at 1 m s^{-2} for 6 seconds. The car then travels at a constant speed and reaches B after a further 6 seconds. The distance AB is at 180 m.

a Sketch a velocity–time graph for the car's journey.

b Determine the value of V.

5 A car is travelling along a road. It passes point A at a constant speed of V m s^{-1}, and drives for T seconds at this speed. It then accelerates at a constant rate for 6 seconds until it reaches a speed of $2V$ m s^{-1}. Maintaining this speed, it arrives at point B after a further $2T$ seconds. The total distance travelled between A and B was 528 m and the average speed was 20 m s^{-1}. Find V and T.

6 A goods train travels along a straight track between two signals. The train has speed 3 m s^{-1} as it passes through the first signal. The train moves with constant acceleration 0.5 m s^{-2} for the first 30 s, then with constant speed for 180 s, and finally with constant deceleration 0.12 m s^{-2} to come to rest at the second signal.

a Find the total time taken for the train to travel between the two signals.

b Calculate the distance between the two signals.

c Find, to the nearest whole number, the percentage of the distance between the two signals that the train travels at more than half the maximum speed.

M **7** Two runners, Ayesha and Fatima, are completing a long-distance race. They are both running at 5 m s^{-1}, with Ayesha 10 m behind Fatima. When Fatima is 50 m from the finish line, Ayesha accelerates but Fatima doesn't. What is the least acceleration Ayesha must produce to overtake Fatima?

If instead Fatima accelerates at 0.1 m s^{-2} up to the finish line, what is the least acceleration Ayesha must produce?

M **8** A woman stands on the bank of a frozen lake with a dog by her side. She slides a bone across the ice at a speed of 3 m s^{-1} and at the same instant the dog begins to chase the bone. The bone slows down with deceleration 0.4 m s^{-2}, and the dog chases it with

acceleration 0.6 m s^{-2}. How far out from the bank does the dog catch up with the bone?

9 A man is running for a bus at 3 m s^{-1}. When he is 100 m from the bus stop, the bus passes him going at 8 m s^{-1}. If the deceleration of the bus is constant, at what constant rate should the man accelerate so as to arrive at the bus stop at the same instant as the bus?

10 The distance from A to B is 10 000 m. A car starts from rest at A and accelerates uniformly until it has travelled 2000 m. The car travels at a constant speed V m s^{-1} for 300 s and then decelerates uniformly for T s to come to rest at B.

The average speed of the car between A and B is 20 m s^{-1}.

a Find the value of V.

b Find the value of T.

PS **11** If a ball is placed on a straight sloping track and then released from rest, the distances that it moves in successive equal intervals of time are found to be in the ratio 1 : 3 : 5 : 7 : .... Show that this is consistent with the theory that the ball rolls down the track with constant acceleration.