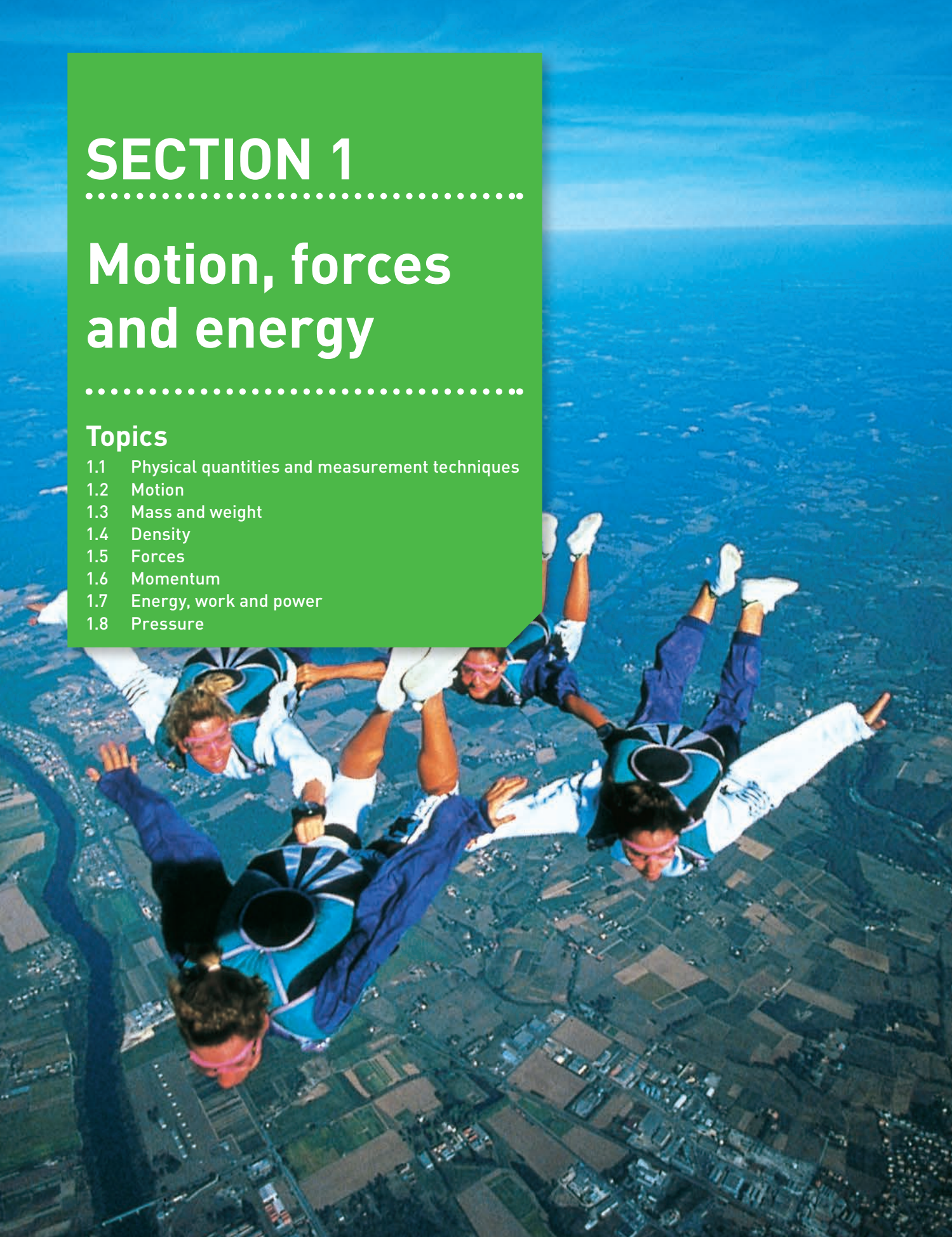


SECTION 1

Motion, forces and energy

Topics

- 1.1 Physical quantities and measurement techniques
- 1.2 Motion
- 1.3 Mass and weight
- 1.4 Density
- 1.5 Forces
- 1.6 Momentum
- 1.7 Energy, work and power
- 1.8 Pressure



1.1

Physical quantities and measurement techniques

FOCUS POINTS

- ★ Describe how to measure length, volume and time intervals using simple devices.
- ★ Know how to determine the average value for a small distance and a short time interval.
- ★ Understand the difference between scalar and vector quantities, and give examples of each.
- ★ Calculate or determine graphically the resultant of two perpendicular vectors.

This topic introduces the concept of describing space and time in terms of numbers together with some of the basic units used in physics. You will learn how to use simple devices to measure or calculate the quantities of length, area and volume. Accurate measurements of time will be needed frequently in the practical work in later topics and you will discover how to choose the appropriate clock or timer for the measurement of a time interval. Any single measurement will not be entirely accurate and will have an error associated with it. Taking the average of several measurements, or measuring multiples, reduces the size of the error.

Many physical quantities, such as force and velocity, have both magnitude and direction; they are termed vectors. When combining two vectors to find their resultant, as well as their size, you need to take into account any difference in their directions.

Units and basic quantities

Before a measurement can be made, a standard or *unit* must be chosen. The size of the quantity to be measured is then found with an instrument having a scale marked in the unit.

Three basic quantities we measure in physics are **length, mass and time**. Units for other quantities are based on them. The SI (Système International d'Unités) system is a set of metric units now used

in many countries. It is a decimal system in which units are divided or multiplied by 10 to give smaller or larger units.

Measuring instruments on the flight deck of a passenger jet provide the crew with information about the performance of the aircraft (see Figure 1.1.1).



▲ Figure 1.1.1 Aircraft flight deck

Powers of ten

This is a useful way of writing numbers, especially if they are large or small. The example below shows how it works.

$$4000 = 4 \times 10 \times 10 \times 10 = 4 \times 10^3$$

$$400 = 4 \times 10 \times 10 = 4 \times 10^2$$

$$40 = 4 \times 10 = 4 \times 10^1$$

$$4 = 4 \times 1 = 4 \times 10^0$$

$$0.4 = 4/10 = 4/10^1 = 4 \times 10^{-1}$$

$$0.04 = 4/100 = 4/10^2 = 4 \times 10^{-2}$$

$$0.004 = 4/1000 = 4/10^3 = 4 \times 10^{-3}$$

The small figures 1, 2, 3, etc. are called **powers of ten**. The power shows how many times the number has to be multiplied by 10 if the power is greater than 0 or divided by 10 if the power is less than 0. Note that 1 is written as 10^0 .

This way of writing numbers is called **standard form** or **standard notation**.

The number in front of the power of ten could be a decimal. For example, 45 500 in standard form is 4.55×10^4 .

Length

The unit of length is the **metre** (m) and is the distance travelled by light in a vacuum during a specific time interval. At one time it was the distance between two marks on a certain metal bar. Submultiples are:

$$1 \text{ decimetre (dm)} = 10^{-1} \text{ m}$$

$$1 \text{ centimetre (cm)} = 10^{-2} \text{ m}$$

$$1 \text{ millimetre (mm)} = 10^{-3} \text{ m}$$

$$1 \text{ micrometre } (\mu\text{m}) = 10^{-6} \text{ m}$$

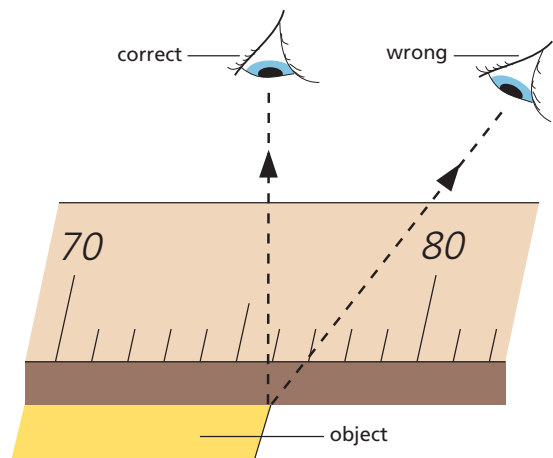
$$1 \text{ nanometre (nm)} = 10^{-9} \text{ m}$$

Multiples for large distances are

$$1 \text{ kilometre (km)} = 10^3 \text{ m } \left(\frac{5}{8} \text{ mile approx.}\right)$$

$$1 \text{ gigametre (Gm)} = 10^9 \text{ m} = 1 \text{ billion metres}$$

Many length measurements are made with rulers; the correct way to read one is shown in Figure 1.1.2. The reading is 76 mm or 7.6 cm. Your eye must be directly over the mark on the scale or the thickness of the ruler causes a **parallax error**.



▲ **Figure 1.1.2** The correct way to measure with a ruler

To obtain an average value for a small distance, multiples can be measured. For example, in ripple tank experiments (Topic 3.1), measure the distance occupied by five waves then divide by 5 to obtain the average wavelength.

Significant figures

Scientists try to make sure a measurement is **accurate** (close to the true value). However, the apparatus and the experimental procedure may have sources of error. The number of digits, called **significant figures**, given for a measurement indicates how accurate we think it is. You should not give more digits in a calculated answer than are justified by the apparatus and how it was used.

For example, a value of 4.5 for a measurement has two significant figures; 0.0385 has three significant figures, 3 being the most significant and 5 the least, i.e. it is the one we are least sure about since it might be 4 or it might be 6. Perhaps it had to be estimated by the experimenter because the reading was between two marks on a scale.

When doing a calculation your answer should have the same number of significant figures as the measurements used in the calculation. For example, if your calculator gave an answer of 3.4185062, this would be written as 3.4 if the measurements had two significant figures. It would be written as 3.42 for three significant figures. Note that in deciding the least significant figure, you look at the following digit. If it is less than 5, you round down (so 3.41 becomes 3.4), but if it is 5 or above you round up (so 3.418 becomes 3.42).

If a number is expressed in standard notation, the number of significant figures is the number of digits before the power of ten. For example, 2.73×10^3 has three significant figures.

Test yourself

- How many millimetres are there in these measurements?
 - 1 cm
 - 4 cm
 - 0.5 cm
 - 6.7 cm
 - 1 m
- What are these lengths in metres?
 - 300 cm
 - 550 cm
 - 870 cm
 - 43 cm
 - 100 mm
- Write the following as powers of ten with one figure before the decimal point.
100 000 3500 428 000 000 504 270 56
 - Write out the following in full.
 10^3 2×10^6 6.92×10^4 1.34×10^2 10^9
- Write these fractions as powers of ten.
 $1/1000$ $7/100\,000$ $1/10\,000\,000$ $3/60\,000$
 - Express the following decimals as powers of ten with one figure before the decimal point.
0.5 0.084 0.000 36 0.001 04

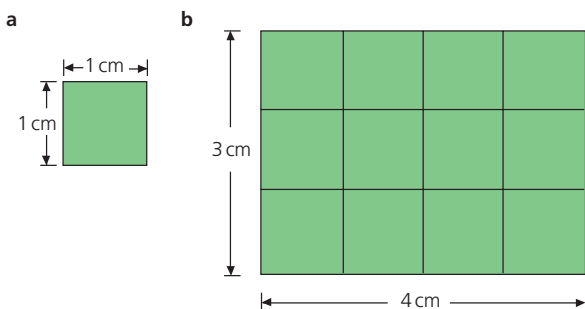
Area

The **area of the square** in Figure 1.1.3a with sides 1 cm long is 1 square centimetre (1 cm^2). In Figure 1.1.3b the rectangle measures 4 cm by 3 cm and has an area of $4 \times 3 = 12\text{ cm}^2$ since it has the same area as twelve squares each of area 1 cm^2 . The area of a square or rectangle is given by

$$\text{area} = \text{length} \times \text{breadth}$$

The SI unit of area is the square metre (m^2) which is the area of a square with sides 1 m long. Note that

$$1\text{ cm}^2 = \frac{1}{100}\text{ m} \times \frac{1}{100}\text{ m} = \frac{1}{10000}\text{ m}^2 = 10^{-4}\text{ m}^2$$



▲ Figure 1.1.3

Sometimes we need to know the area of a triangle. It is given by

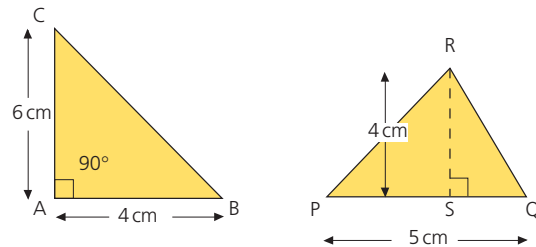
$$\text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

The **area of a circle** of radius r is πr^2 where $\pi = 22/7$ or 3.14; its circumference is $2\pi r$.

? Worked example

Calculate the area of the triangles shown in Figure 1.1.4.

- $\text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$
 so area of triangle ABC = $\frac{1}{2} \times \text{AB} \times \text{AC}$
 $= \frac{1}{2} \times 4\text{ cm} \times 6\text{ cm} = 12\text{ cm}^2$
- $\text{area of triangle PQR} = \frac{1}{2} \times \text{PQ} \times \text{SR}$
 $= \frac{1}{2} \times 5\text{ cm} \times 4\text{ cm} = 10\text{ cm}^2$



▲ Figure 1.1.4

Now put this into practice

- Calculate the area of a triangle whose base is 8 cm and height is 12 cm.
- Calculate the circumference of a circle of radius 6 cm.

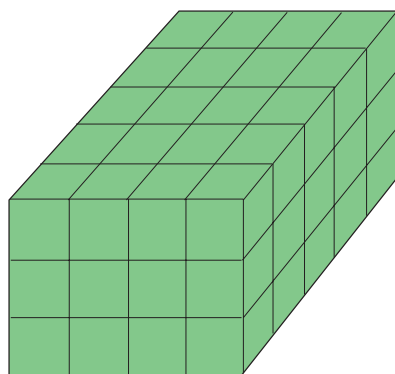
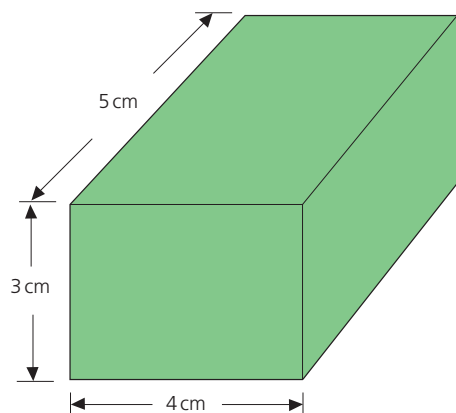
Volume

Volume is the amount of space occupied. The unit of volume is the **cubic metre** (m^3) but as this is rather large, for most purposes the **cubic centimetre** (cm^3) is used. The volume of a cube with 1 cm edges is 1 cm^3 . Note that

$$\begin{aligned} 1\text{ cm}^3 &= \frac{1}{100}\text{ m} \times \frac{1}{100}\text{ m} \times \frac{1}{100}\text{ m} \\ &= \frac{1}{1000000}\text{ m}^3 = 10^{-6}\text{ m}^3 \end{aligned}$$

For a regularly shaped object such as a rectangular block, Figure 1.1.5 shows that

$$\text{volume} = \text{length} \times \text{breadth} \times \text{height}$$



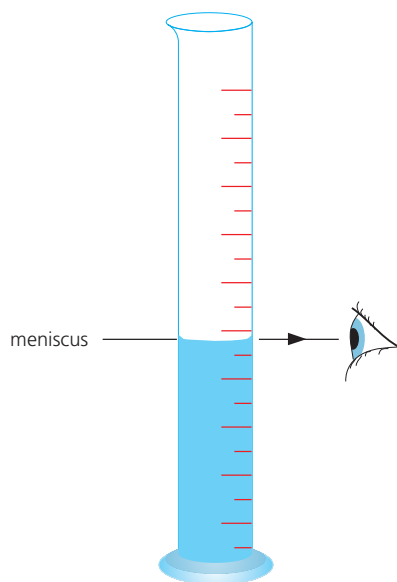
$3 \times 4 \times 5$ cubes

▲ Figure 1.1.5

The **volume of a cylinder** of radius r and height h is $\pi r^2 h$.

The volume of a liquid may be obtained by pouring it into a measuring cylinder (Figure 1.1.6). When making a reading the cylinder must be upright and, to avoid parallax error, your eye must be level with the bottom of the curved liquid surface, i.e. the **meniscus**. The meniscus formed by mercury is curved oppositely to that of other liquids and the top is read.

Measuring cylinders are often marked in millilitres (ml) where $1 \text{ ml} = 1 \text{ cm}^3$; note that $1000 \text{ cm}^3 = 1 \text{ dm}^3 (= 1 \text{ litre})$.



▲ Figure 1.1.6 A measuring cylinder

? Worked example

- a Calculate the volume of a block of wood which is 40 cm long, 12 cm wide and 5 cm high in cubic metres.

$$\begin{aligned} \text{volume } V &= \text{length} \times \text{breadth} \times \text{height} \\ &= 40 \text{ cm} \times 12 \text{ cm} \times 5 \text{ cm} \\ &= 2400 \text{ cm}^3 \\ &= 2400 \times 10^{-6} \text{ m}^3 \\ &= 2.4 \times 10^{-3} \text{ m}^3 \end{aligned}$$

- b Calculate the volume of a cylinder of radius 10 mm and height 5.0 cm in cubic metres.

$$\begin{aligned} \text{volume of cylinder } V &= \pi r^2 h \\ r &= 10 \text{ mm} = 1.0 \text{ cm} \text{ and } h = 5.0 \text{ cm} \\ \text{so } V &= \pi r^2 h \\ &= \pi \times (1.0 \text{ cm})^2 \times 5.0 \text{ cm} \\ &= 16 \text{ cm}^3 = 16 \times 10^{-6} \text{ m}^3 = 1.6 \times 10^{-5} \text{ m}^3 \end{aligned}$$

Now put this into practice

- 1 Calculate the volume of a rectangular box which is 30 cm long, 25 cm wide and 15 cm high in cubic metres.
- 2 Calculate the volume of a cylinder of radius 50 mm and height 25 cm in cubic metres.

Time

The unit of time is the **second** (s), which used to be based on the length of a day, this being the time for the Earth to revolve once on its axis. However, days are not all of exactly the same duration and the second is now defined as the time interval for a certain number of **energy** changes to occur in the caesium atom.

Time-measuring devices rely on some kind of constantly repeating oscillation. In traditional clocks and watches a small wheel (the balance wheel) oscillates to and fro; in digital clocks and watches the oscillations are produced by a tiny quartz crystal. A swinging pendulum controls a pendulum clock.

To measure an interval of time in an experiment, first choose a timer that is precise enough for the task. For short times, your own **reaction time** will affect the measurements. A stopwatch that records times with a precision of 1 s or 0.1 s is suitable for finding the period in seconds of a pendulum (see Figure 1.1.7 opposite), but to measure the speed of sound (Topic 3.4), a clock that can time in milliseconds is needed. To measure very short time intervals, a digital clock that can be triggered to start and stop by an electronic signal from a

microphone, photogate or mechanical switch is useful. Tickertape timers or dataloggers are often used to record short time intervals in motion experiments. Accuracy can be improved by measuring longer time intervals. Several oscillations (rather than just one) are timed to find the period of a pendulum; the average value for the period is found by dividing the time by the number of oscillations. Ten ticks, rather than single ticks, are used in tickertape timers.

Test yourself

- The pages of a book are numbered 1 to 200 and each leaf is 0.10 mm thick. If each cover is 0.20 mm thick, what is the thickness of the book?
- How many significant figures are there in a length measurement of
 - 2.5 cm
 - 5.32 cm
 - 7.180 cm
 - 0.042 cm?
- A rectangular block measures 4.1 cm by 2.8 cm by 2.1 cm. Calculate its volume giving your answer to an appropriate number of significant figures.
- What type of timer would you use to measure the period of a simple pendulum? How many oscillations would you time?



Practical work

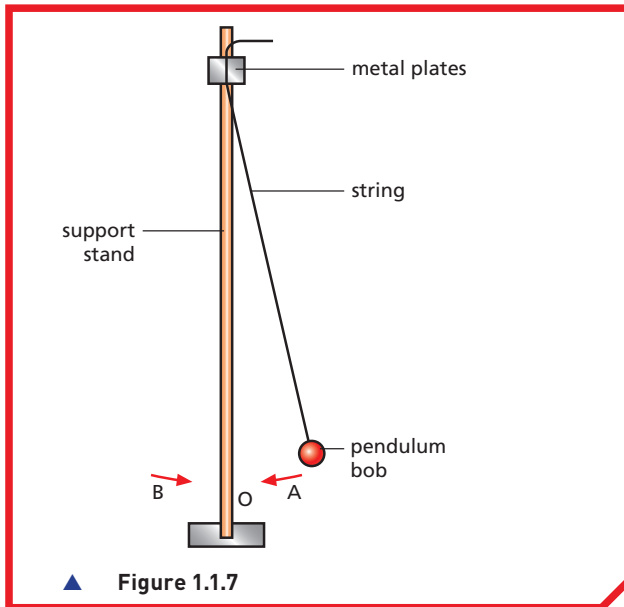
Period of a simple pendulum

In this investigation you have to make time measurements using a stopwatch or clock. A motion sensor connected to a datalogger and computer could be used instead of a stopwatch for these investigations.

Attach a small metal ball (called a bob) to a piece of string, and suspend it as shown in Figure 1.1.7 opposite. Pull the bob a small distance to one side, and then release it so that it oscillates to and fro through a small angle.

Find the time for the bob to make several complete oscillations; one oscillation is from A to O to B to O to A (Figure 1.1.7). Repeat the timing a few times for the same number of oscillations and work out the average.

- The time for one oscillation is the **period** T . Determine the period of your pendulum.
- The **frequency** f of the oscillations is the number of complete oscillations per second and equals $1/T$. Calculate a value for f for your pendulum.
- Comment on how the amplitude of the oscillations changes with time.
- Plan an investigation into the effect on T of (i) a longer string and (ii) a larger bob.
- What procedure would you use to determine the period of a simple pendulum?
- In Figure 1.1.7 if the bob is first released at B, give the sequence of letters which corresponds to one complete oscillation.
- Explain where you would take measurements from to determine the length of the pendulum shown in Figure 1.1.7.



Systematic errors

Figure 1.1.8 shows a part of a ruler used to measure the height of a point P above the bench. The ruler chosen has a space before the zero of the scale. This is shown as the length x . The height of the point P is given by the scale reading added to the value of x . The equation for the height is

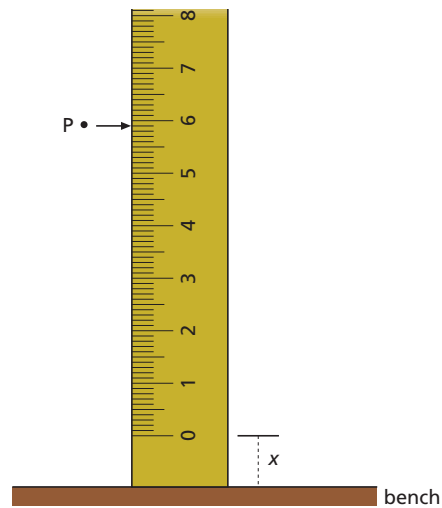
$$\text{height} = \text{scale reading} + x$$

$$\text{height} = 5.9 + x$$

By itself the scale reading is not equal to the height. It is too small by the value of x .

This type of zero error is known as a **systematic error**. The error is introduced by the system. A half-metre ruler has the zero at the *end of the ruler* and so can be used without introducing a systematic error.

When using a ruler to determine a height, the ruler must be held so that it is vertical. If the ruler is at an angle to the vertical, a systematic error is introduced.

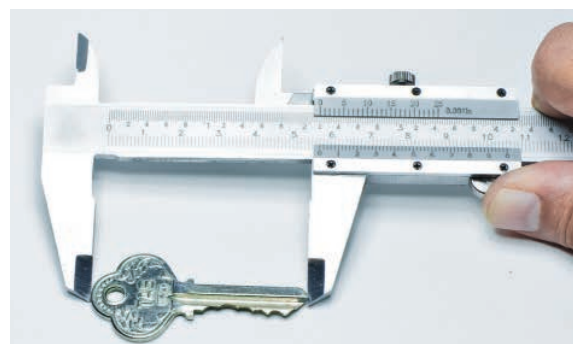


Vernier scales and micrometers

Lengths can be measured with a ruler to a precision of about 0.5 mm. Some investigations may need a more precise measurement of length, which can be achieved by using vernier calipers (Figure 1.1.9) or a micrometer screw gauge.

➔ Going further

Vernier scale



1.1 PHYSICAL QUANTITIES AND MEASUREMENT TECHNIQUES

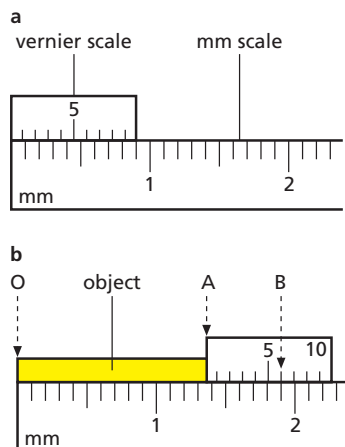
The calipers shown in Figure 1.1.9 use a vernier scale. The simplest type enables a length to be measured to 0.01 cm. It is a small sliding scale which is 9 mm long but divided into ten equal divisions (Figure 1.1.10a) so

$$\begin{aligned} 1 \text{ vernier division} &= \frac{9}{10} \text{ mm} \\ &= 0.9 \text{ mm} \\ &= 0.09 \text{ cm} \end{aligned}$$

One end of the length to be measured is made to coincide with the zero of the millimetre scale and the other end with the zero of the vernier scale. The length of the object in Figure 1.1.10b is between 1.3 cm and 1.4 cm. The reading to the second place of decimals is obtained by finding the vernier mark which is exactly opposite (or nearest to) a mark on the millimetre scale. In this case it is the 6th mark and the length is 1.36 cm, since

$$\begin{aligned} OA &= OB - AB \\ OA &= (1.90 \text{ cm}) - (6 \text{ vernier divisions}) \\ &= 1.90 \text{ cm} - 6(0.09) \text{ cm} \\ &= (1.90 - 0.54) \text{ cm} = 1.36 \text{ cm} \end{aligned}$$

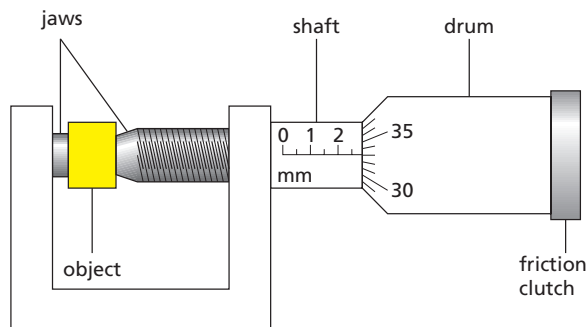
Vernier scales are also used on barometers, travelling microscopes and spectrometers.



▲ Figure 1.1.10 Vernier scale

Micrometer screw gauge

This measures very small objects to 0.001 cm. One revolution of the drum opens the flat, parallel jaws by one division on the scale on the shaft of the gauge; this is usually mm, i.e. 0.05 cm. If the drum has a scale of 50 divisions round it, then rotation of the drum by one division opens the jaws by $0.05/50 = 0.001$ cm (Figure 1.1.11). A friction clutch ensures that the jaws exert the same force when the object is gripped.



▲ Figure 1.1.11 Micrometer screw gauge

The object shown in Figure 1.1.11 has a length of

$$\begin{aligned} &2.5 \text{ mm on the shaft scale} + 33 \text{ divisions on the drum scale} \\ &= 0.25 \text{ cm} + 33(0.001) \text{ cm} \\ &= 0.283 \text{ cm} \end{aligned}$$

Before making a measurement, check to ensure that the reading is zero when the jaws are closed. Otherwise the zero error must be allowed for when the reading is taken.

Scalars and vectors

Length and time can be described by a single number specifying size, but many physical quantities also have a direction.

A **scalar** quantity has magnitude (size) only. Time is a scalar and is completely described when its value is known. Other examples of scalars are distance, speed, time, mass, energy and **temperature**.

A **vector** quantity is one such as force which is described completely only if both its size (magnitude) and direction are stated. It is not enough to say, for example, a force of 10 N, but rather a force of 10 N acting vertically downwards. Gravitational field strength and electric field strength are vectors, as are displacement (distance in a stated direction), weight, velocity, acceleration and momentum.

A vector can be represented by a straight line whose length represents the magnitude of the quantity and whose direction gives its line of action. An arrow on the line shows which way along the line it acts.

Scalars are added by ordinary arithmetic; vectors are added by taking account of their directions as well as their magnitudes. In the case of two vectors F_X and F_Y acting at right angles to each other at

at a point, the magnitude of the resultant F , and the angle θ between F_x and F can be calculated from the following equations:

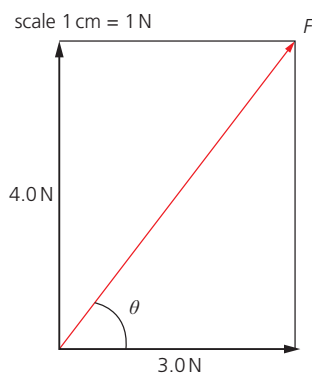
$$F = \sqrt{F_x^2 + F_y^2}, \quad \tan \theta = \frac{F_y}{F_x}$$

The resultant of two vectors acting at right angles to each other can also be obtained graphically.

? Worked example

Calculate the resultant of two forces of 3.0 N and 4.0 N acting at right angles to each other.

Let $F_x = 3.0$ N and $F_y = 4.0$ N as shown in Figure 1.1.12.



▲ **Figure 1.1.12** Addition of two **perpendicular** vectors

Then $F = \sqrt{F_x^2 + F_y^2} = \sqrt{3.0^2 + 4.0^2} = \sqrt{9 + 16} = \sqrt{25} = 5.0$ N

and $\tan \theta = \frac{F_y}{F_x} = \frac{4.0}{3.0} = 1.3$

so, using \tan^{-1} , $\theta = 53^\circ$.

The resultant is a force of 5.0 N acting at 53° to the force of 3.0 N.

Graphical method

The values for F and θ can be found graphically by drawing the vectors to scale on a piece of graph paper as shown in Figure 1.1.12.

First choose a scale to represent the size of the vectors (1 cm could be used to represent 1.0 N).

Draw the vectors at right angles to each other. Complete the rectangle as shown in Figure 1.1.12 and draw the diagonal from the origin as shown. The diagonal then represents the resultant force, F . Measure the length of F with a ruler and use the scale you have chosen to determine its size. Measure the angle θ , the direction of the resultant, with a protractor.

Check that the values for F and θ you obtain are the same as those found using the algebraic method.

Now put this into practice

- Calculate the following square roots:
 - $\sqrt{6^2 + 8^2}$
 - $\sqrt{5^2 + 7^2}$
 - $\sqrt{2^2 + 9^2}$
- Calculate
 - $\tan 30^\circ$
 - $\tan 45^\circ$
 - $\tan 60^\circ$.
- Calculate the resultant of two forces of 5.0 N and 7.0 N which are at right angles to each other.
- A girl walks 600 m north and then 800 m east. What is the displacement from her starting point?

Revision checklist

After studying Topic 1.1 you should know and understand the following:

- ✓ how to make measurements of length and time intervals, minimise the associated errors and use multiple measurements to obtain average values
- ✓ how to measure length with appropriate precision
- ✓ the difference between scalars and vectors and recall examples of each.

After studying Topic 1.1 you should be able to:

- ✓ write a number in powers of ten (standard notation) and recall the meaning of standard prefixes
- ✓ measure and calculate lengths, areas and volumes of regular objects and give a result with the correct units and an appropriate number of significant figures
- ✓ read the vernier scale on a micrometer
- ✓ determine by calculation or graphically the resultant of two vectors at right angles.

Exam-style questions

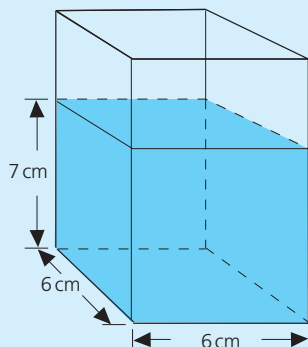
- 1 A chocolate bar measures 10 cm long by 2 cm wide and is 2 cm thick.
- a** Calculate the volume of one bar. [3]
- b** How many bars each 2 cm long, 2 cm wide and 2 cm thick have the same total volume? [3]
- c** A pendulum makes 10 complete oscillations in 8 seconds. Calculate the time period of the pendulum. [2]

[Total: 8]

- 2 **a** A pile of 60 sheets of paper is 6 mm high. Calculate the average thickness of a sheet of the paper. [2]
- b** Calculate how many blocks of ice cream each 10 cm long, 10 cm wide and 4 cm thick can be stored in the compartment of a freezer measuring 40 cm deep, 40 cm wide and 20 cm high. [5]

[Total: 7]

- 3 A Perspex container has a 6 cm square base and contains water to a height of 7 cm (Figure 1.1.13).
- a** Calculate the volume of the water. [3]
- b** A stone is lowered into the water so as to be completely covered and the water rises to a height of 9 cm. Calculate the volume of the stone. [4]



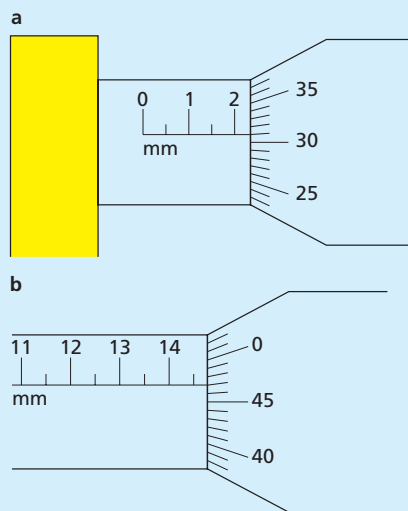
▲ Figure 1.1.13

[Total: 7]

- 4 **a** State the standard units of length and time. [2]
- b** A measurement is stated as 0.0125 mm. State the number of significant figures. [1]
- c** Give expressions for
- i** the area of a circle [1]
 - ii** the circumference of a circle [1]
 - iii** the volume of a cylinder. [2]

[Total: 7]

- 5 What are the readings on the micrometer screw gauges in Figures 1.1.14a and 1.1.14b?



▲ Figure 1.1.14

[Total: 4]

- 6 **a** Select which of the following quantities is a vector:
- | | |
|----------------------|----------------|
| A length | C force |
| B temperature | D time. |
- [1]

- b** Two forces of 5 N and 12 N act at right angles to each other. Using a piece of graph paper determine the magnitude and direction of the resultant force graphically. State the scale you use to represent each vector. You will need a protractor to measure the angle the resultant makes with the 5 N force. [7]

[Total: 8]

1.2

Motion

FOCUS POINTS

- ★ Define speed and velocity and use the appropriate equations to calculate these and average speed.
- ★ Draw, plot and interpret distance–time or speed–time graphs for objects at different speeds and use the graphs to calculate speed or distance travelled.
- ★ Define acceleration and use the shape of a speed–time graph to determine constant or changing acceleration and calculate the acceleration from the gradient of the graph.
- ★ Describe examples of uniform acceleration and non-uniform acceleration.
- ★ Know the approximate value of the acceleration of freefall, g , for an object close to the Earth’s surface.
- ★ Describe the motion of objects falling with and without air/liquid resistance.

The concepts of speed and acceleration are encountered every day, whether it be television monitoring of the speed of a cricket or tennis ball as it soars towards the opposition or the acceleration achieved by an athlete or racing car. In this topic you will learn how to define speed in terms of distance and time. Graphs of distance against time will enable you to calculate speed and determine how it changes with time; graphs of speed against time allow acceleration to be studied. Acceleration is also experienced by falling objects as a result of gravitational attraction. All objects near the Earth’s surface experience the force of gravity, which produces a constant acceleration directed towards the centre of the Earth.

Speed

The **speed** of a body is the distance that it has travelled in unit time. When the distance travelled s is over a short time period t , the speed v is given by

$$v = \frac{s}{t}$$

or

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Key definition

Speed distance travelled per unit time

If a car travels 300 km in five hours, its **average speed** is $300 \text{ km}/5 \text{ h} = 60 \text{ km/h}$. The speedometer would certainly not read 60 km/h for the whole journey and might vary considerably from this value. That is why we state the average speed. If a car could travel at a constant speed of 60 km/h for 5 hours, the distance covered would still be 300 km. It is *always* true that

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

To find the actual speed at any instant we would need to know the distance moved in a very short interval of time. This can be done by multiframe photography. In Figure 1.2.1 the golfer is photographed while a flashing lamp illuminates him 100 times a second. The speed of the club-head as it hits the ball is about 200 km/h.



▲ **Figure 1.2.1** Multiframe photograph of a golf swing

Velocity

As we saw in Topic 1.1, distance moved in a stated direction is called the **displacement**. It is a *vector*, unlike distance which is a *scalar*. Similarly, direction can be important when we talk about speed. If two trains travel due north at 20 m/s, they have the same speed of 20 m/s and the same **velocity** of 20 m/s *due north*. If one travels north and the other south, their speeds are the same but not their velocities since their directions of motion are different.

Key definition

Velocity change in displacement per unit time

The velocity of a body is **uniform** or constant if it moves with a steady speed in a straight line. It is not uniform if it moves in a curved path. Why?

The units of speed and velocity are the same, km/h, m/s.

$$60 \text{ km/h} = \frac{60\,000 \text{ m}}{3600 \text{ s}} = 17 \text{ m/s}$$

Speed is a *scalar* quantity and velocity a *vector* quantity. Speed is the distance travelled in unit time and velocity is defined as change in displacement per unit time.

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

Acceleration

When the velocity of an object changes, we say the object *accelerates*. If a car starts from rest and moving due north has velocity 2 m/s after 1 second, its velocity has increased by 2 m/s in 1 s and its acceleration is 2 m/s per second due north. We write this as 2 m/s².

Acceleration is defined as the change in velocity in unit time, or

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta v}{\Delta t}$$

Key definition

Acceleration change in velocity per unit time

For a steady increase of velocity from 20 m/s to 50 m/s in 5 s

$$\text{acceleration} = \frac{(50 - 20) \text{ m/s}}{5 \text{ s}} = 6 \text{ m/s}^2$$

Acceleration is also a vector and both its magnitude and direction should be stated. However, at present we will consider only motion in a straight line and so the magnitude of the velocity will equal the speed, and the magnitude of the acceleration will equal the change of speed in unit time.

The speeds of a car accelerating on a straight road are shown below.

Time/s	0	1	2	3	4	5	6
Speed/m/s	0	5	10	15	20	25	30

The speed increases by 5 m/s every second and the acceleration of 5 m/s² is constant.

An acceleration is positive if the velocity increases, and negative if it decreases. A negative acceleration is also called a **deceleration** or **retardation**.

Test yourself

- What is the average speed of
 - a car that travels 400 m in 20 s
 - an athlete who runs 1500 m in 4 minutes?
- A train increases its speed steadily from 10 m/s to 20 m/s in 1 minute.
 - What is its average speed during this time, in m/s?
 - How far does it travel while increasing its speed?
- A motorcyclist starts from rest and reaches a speed of 6 m/s after travelling with constant acceleration for 3 s. What is his acceleration?
 - The motorcyclist then decelerates at a constant rate for 2 s. What is his acceleration?
- An aircraft travelling at 600 km/h accelerates steadily at 10 km/h per second. Taking the speed of sound as 1100 km/h at the aircraft's altitude, how long will it take to reach the 'sound barrier'?

Speed-time graphs

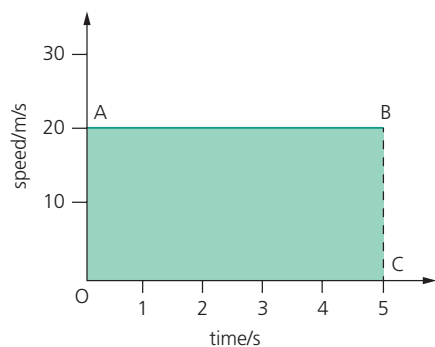
If the speed of an object is plotted against the time, the graph obtained is a **speed-time graph**. It provides a way of solving motion problems.

In Figure 1.2.2, AB is the speed-time graph for an object moving with a **constant speed** of 20 m/s.

Values for the speed of the object at 1 s intervals can be read from the graph and are given in Table 1.2.1. The data shows that the speed is constant over the 5 s time interval.

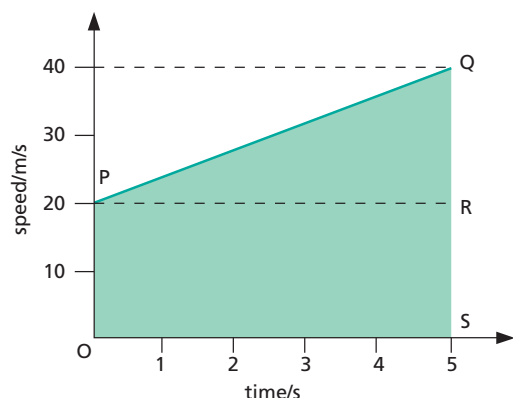
▼ Table 1.2.1

Speed/m/s	20	20	20	20	20	20
Time/s	0	1	2	3	4	5



▲ Figure 1.2.2 Constant speed

The linear shape (PQ) of the speed-time graph shown in Figure 1.2.3a means that the gradient, and hence the acceleration of the body, are constant over the time period OS.



▲ Figure 1.2.3a Constant acceleration

Values for the speed of the object at 1 s intervals can be read from the graph and are given in Table

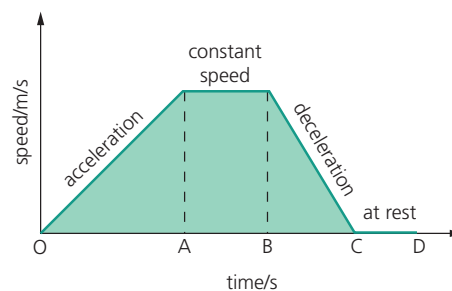
1.2.2. The data shows that the speed increases by the same amount (4 m/s) every second.

▼ Table 1.2.2

Speed/m/s	20	24	28	32	36	40
Time/s	0	1	2	3	4	5

You can use the data to plot the speed-time graph. Join up the data points on the graph paper with the best straight line to give the line PQ shown in Figure 1.2.3a. (Details for how to plot a graph are given on pp. 299–300 in the *Mathematics for physics* section.)

Figure 1.2.3b shows the shape of a speed-time graph for an object accelerating from rest over time interval OA, travelling at a constant speed over time interval AB and then decelerating (when the speed is decreasing) over the time interval BC. The steeper gradient in time interval BC than in time interval OA shows that the deceleration is greater than the acceleration. The object remains at rest over the time interval CD when its speed and acceleration are zero.



▲ Figure 1.2.3b Acceleration, constant speed and deceleration

Figure 1.2.3c overleaf shows a speed-time graph for a changing acceleration. The curved shape OX means that the gradient of the graph, and hence the acceleration of the object, change over time period OY – the acceleration is changing.

Values for the speed of the object at 1 s intervals are given in Table 1.2.3. The data shows that the speed is increasing over time interval OY, but by a smaller amount each second so the acceleration is decreasing. This is shown on the graph by the gradient of the curve between 0 and X decreasing continuously.

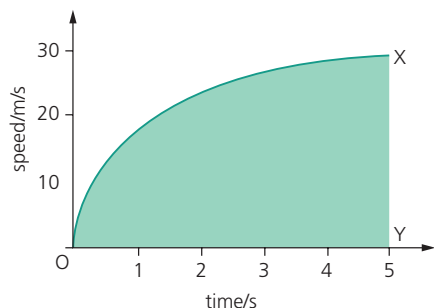
▼ Table 1.2.3

Speed/m/s	0	17.5	23.0	26.0	28.5	30.0
Time/s	0	1	2	3	4	5

1.2 MOTION

You can use the data to plot the speed–time graph. Join up the data points on the graph paper with a smooth curve as shown in Figure 1.2.3c.

Note that an object *at rest* will have zero speed and zero acceleration; its speed–time graph is a straight line along the horizontal axis.



▲ **Figure 1.2.3c** Changing acceleration

The acceleration of free fall (see p. 18) is constant; acceleration against a changing resistive force (such as air resistance) is changing (non-uniform).

Using the gradient of a speed–time graph to calculate acceleration

The gradient of a speed–time graph represents the acceleration of the object.

In Figure 1.2.2, the gradient of AB is zero, as is the acceleration. In Figure 1.2.3a, the gradient of PQ is $QR/PR = 20/5 = 4$: the acceleration is constant at 4 m/s^2 . In Figure 1.2.3c, when the gradient along OX changes, so does the acceleration.

An object is accelerating if the speed increases with time and decelerating if the speed decreases with time, as shown in Figure 1.2.3b. In Figure 1.2.3c the speed is increasing with time and the acceleration of the object is decreasing.

Distance–time graphs

An object travelling with constant speed covers equal distances in equal times. Its **distance–time graph** is a straight line, like OL in Figure 1.2.4a for a constant speed of 10 m/s . The gradient of the graph is $LM/OM = 40 \text{ m}/4 \text{ s} = 10 \text{ m/s}$, which is the value of the speed. The following statement is true in general:

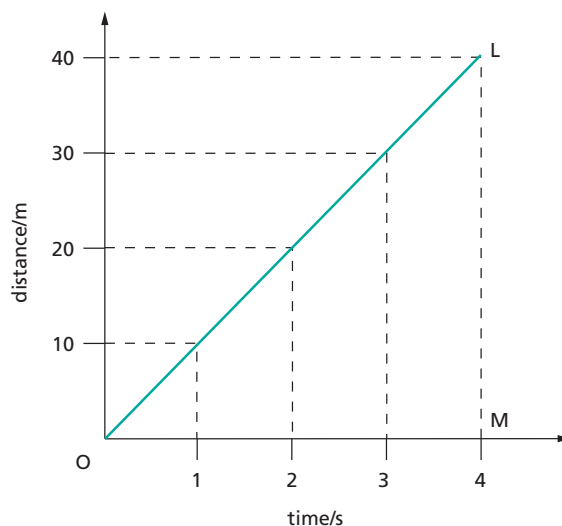
The gradient of a distance–time graph represents the speed of the object.

Values for the distance moved by the object recorded at 1 s intervals are given in Table 1.2.4. The data shows it moves 10 m in every second so the speed of the object is constant at 10 m/s .

▼ **Table 1.2.4**

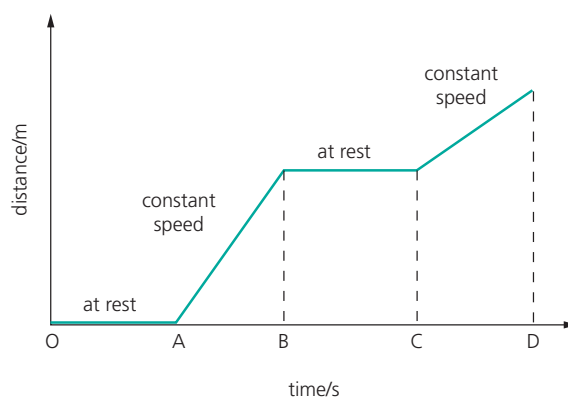
Distance/m	10	20	30	40
Time/s	1	2	3	4

You can use the data to plot the distance–time graph shown in Figure 1.2.4a.



▲ **Figure 1.2.4a** Constant speed

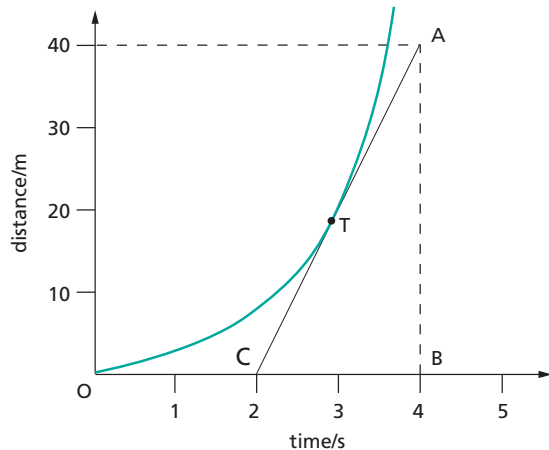
Figure 1.2.4b shows the shape of a distance–time graph for an object that is at rest over time interval OA and then moves at a constant speed in time interval AB. It then stops moving and is at rest over time interval BC before moving at a constant speed in time interval CD.



▲ **Figure 1.2.4b** Constant speed

The speed of the object is higher when the gradient of the graph is steeper. The object is travelling faster in time interval AB than it is in time interval CD; it is at rest in time intervals OA and BC when the distance does not change.

When the speed of the object is changing, the gradient of the distance–time graph varies, as in Figure 1.2.5.



▲ Figure 1.2.5 Non-constant speed

Speed at any point equals the gradient of the **tangent**. For example, the gradient of the tangent at T is $AB/BC = 40\text{ m}/2\text{ s} = 20\text{ m/s}$. The speed at the instant corresponding to T is therefore 20 m/s.

Area under a speed–time graph

The area under a speed–time graph measures the distance travelled.

In Figure 1.2.2, AB is the speed–time graph for an object moving with a constant speed of 20 m/s. Since distance = average speed \times time, after 5 s it will have moved $20\text{ m/s} \times 5\text{ s} = 100\text{ m}$. This is the shaded area under the graph, i.e. rectangle OABC.

In Figure 1.2.3a, PQ is the speed–time graph for an object moving with *constant acceleration*. At the start of the timing the speed is 20 m/s but it increases steadily to 40 m/s after 5 s. If the distance

covered equals the area under PQ, i.e. the shaded area OPQS, then

$$\begin{aligned} \text{distance} &= \text{area of rectangle OPRS} + \text{area of triangle PQR} \\ &= OP \times OS + \frac{1}{2} \times PR \times QR \\ &\quad (\text{area of a triangle} = \frac{1}{2} \text{ base} \times \text{height}) \\ &= 20\text{ m/s} \times 5\text{ s} + \frac{1}{2} \times 5\text{ s} \times 20\text{ m/s} \\ &= 100\text{ m} + 50\text{ m} = 150\text{ m} \end{aligned}$$

Note that when calculating the area from the graph, the unit of time must be the same on both axes.

The rule for finding distances travelled is true even if the acceleration is not constant. In Figure 1.2.3c, the distance travelled equals the shaded area OXY.

Test yourself

- 5 The speeds of a bus travelling on a straight road are given below at successive intervals of 1 second.

Time/s	0	1	2	3	4
Speed/m/s	0	4	8	12	16

- Sketch a speed–time graph using the values.
 - Choose two of the following terms which describe the acceleration of the bus: constant changing positive negative
 - Calculate the acceleration of the bus.
 - Calculate the area under your graph.
 - How far does the bus travel in 4 s?
- 6 The distance of a walker from the start of her walk is given below at successive intervals of 1 second.
- Sketch a distance–time graph of the following values.

Time/s	0	1	2	3	4	5	6
Distance/m	0	3	6	9	12	15	18

- How would you describe the speed at which she walks?
constant changing increasing
accelerating
 - Calculate her average speed.
- 7 Use Figure 1.2.3c to determine the acceleration at a time of 1 s.

Equations for constant acceleration

Problems involving bodies moving with constant acceleration in a straight line can often be solved quickly using some *equations of motion*.

First equation

If an object is moving with constant acceleration a in a straight line and its speed increases from u to v in time t , then

$$a = \frac{\text{change in speed}}{\text{time taken}} = \frac{v - u}{t}$$

$$\therefore at = v - u$$

or

$$v = u + at \quad (1)$$

Note that the initial speed u and the final speed v refer to the start and the finish of the *timing* and do not necessarily mean the start and finish of the motion.

Second equation

The speed of an object moving with constant acceleration in a straight line increases steadily. Its average speed therefore equals half the sum of its initial and final speeds, that is,

$$\text{average speed} = \frac{u + v}{2}$$

If s is the distance moved in time t , then since average speed = total distance/total time = s/t ,

$$\frac{s}{t} = \frac{u + v}{2}$$

or

$$s = \frac{(u + v)}{2}t \quad (2)$$

? Worked example

A sprint cyclist starts from rest and accelerates at 1 m/s^2 for 20 seconds. Find her final speed and the distance she travelled.

$$\text{Since } u = 0 \quad a = 1 \text{ m/s}^2 \quad t = 20 \text{ s}$$

Using $v = u + at$, we have her maximum speed

$$v = 0 + 1 \text{ m/s}^2 \times 20 \text{ s} = 20 \text{ m/s}$$

and distance travelled

$$\begin{aligned} s &= \frac{(u + v)}{2}t \\ &= \frac{(0 + 20) \text{ m/s} \times 20 \text{ s}}{2} = \frac{400}{2} = 200 \text{ m} \end{aligned}$$

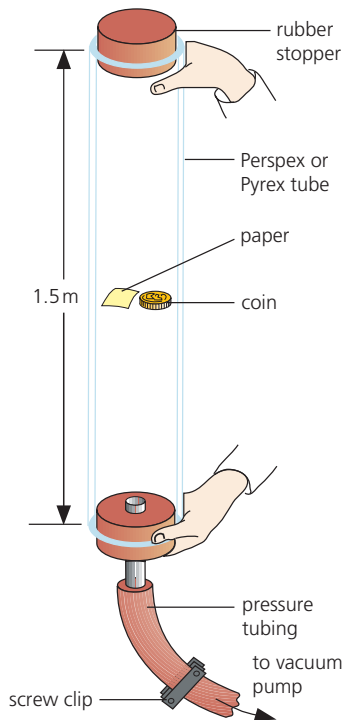
Now put this into practice

- 1 An athlete accelerates from rest at a constant rate of 0.8 m/s^2 for 4 s. Calculate the final speed of the athlete.
- 2 A cyclist increases his speed from 10 m/s to 20 m/s in 5 s. Calculate his average speed over this time interval.
- 3 Calculate the distance moved by a car accelerating from rest at a constant rate of 2 m/s^2 for 5 s.

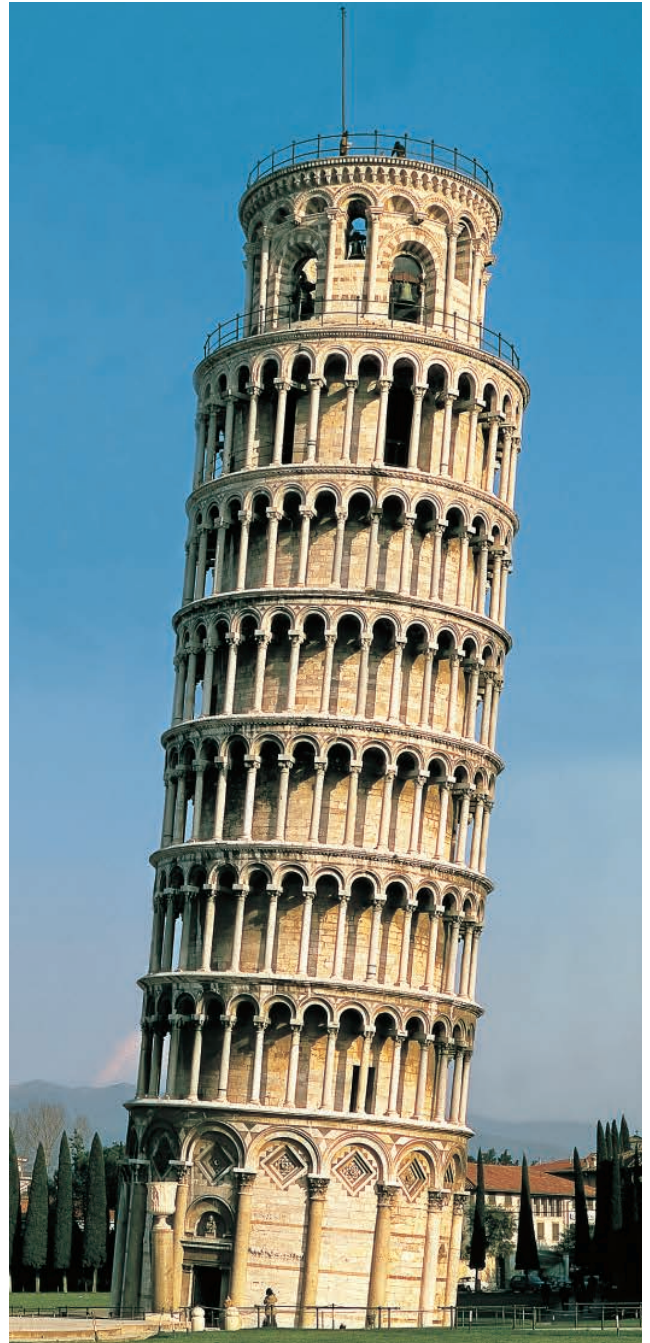
Falling bodies

In air, a coin falls faster than a small piece of paper. In a vacuum they fall at the same rate, as may be shown with the apparatus of Figure 1.2.6. The difference in air is due to **air resistance** having a greater effect on light bodies than on heavy bodies. The air resistance to a light body is large when compared with the body's weight. With a dense piece of metal, the resistance is negligible at low speeds.

There is a story, untrue we now think, that in the sixteenth century the Italian scientist Galileo Galilei dropped a small iron ball and a large cannonball ten times heavier from the top of the Leaning Tower of Pisa (Figure 1.2.7). And we are told that, to the surprise of onlookers who expected the cannonball to arrive first, they reached the ground almost simultaneously.



▲ **Figure 1.2.6** A coin and a piece of paper fall at the same rate in a vacuum.



▲ **Figure 1.2.7** The Leaning Tower of Pisa, where Galileo is said to have experimented with falling objects



Practical work

Motion of a falling object

Safety

- Place something soft on the floor to absorb the impact of the masses.
- Take care to keep feet well away from the falling masses.

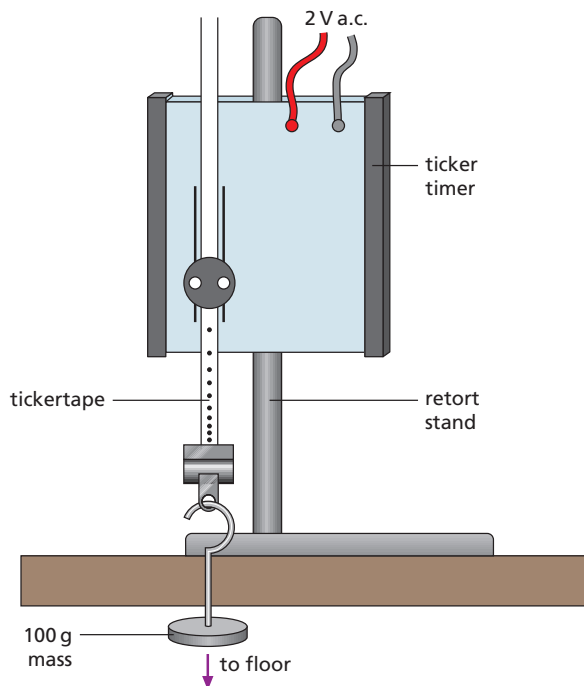
Arrange your experimental apparatus as shown in Figure 1.2.8 and investigate the motion of a 100 g mass falling from a height of about 2 m.

A tickertape timer has a marker that vibrates at 50 times a second and makes dots at $1/50$ s intervals on a paper tape being pulled through it. Ignore the start of the tape where the dots are too close.

Repeat the experiment with a 200 g mass and compare your results with those for the 100 g mass.

- 1 The spacing between the dots on the tickertape increases as the mass falls. What does this tell you about the speed of the falling mass?
- 2 The tape has 34 dots on it by the time the mass falls through 2 m. Estimate how long it has taken the mass to fall through 2 m.

- 3 Why would a stopwatch not be chosen to measure the time of fall in this experiment?
- 4 How would you expect the times taken for the 100 g and 200 g masses to reach the ground to differ?



▲ Figure 1.2.8

Acceleration of free fall

All bodies falling freely under the force of gravity do so with **uniform acceleration** if air resistance is negligible (i.e. the 'steps' on the tape chart from the practical work should all be equally spaced).

This acceleration, called the **acceleration of free fall**, is denoted by the italic letter g . Its value varies slightly over the Earth but is constant in each place; on average it is about 9.8 m/s^2 , or near enough 10 m/s^2 . The velocity of a free-falling body therefore increases by about 10 m/s every second. A ball shot straight upwards with a velocity of 30 m/s decelerates by about 10 m/s every second and reaches its highest point after 3 s.

Key definition

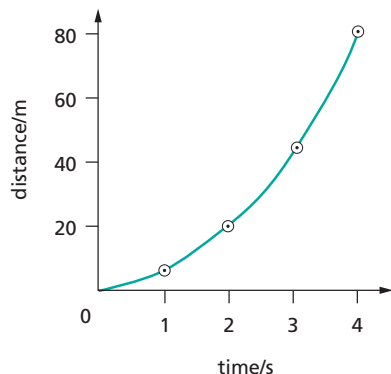
Acceleration of free fall g for an object near to the surface of the Earth, this is approximately constant and is approximately 9.8 m/s^2

In calculations using the equations of motion, g replaces a . It is given a positive sign for falling bodies (i.e. $a = g = +9.8 \text{ m/s}^2$) and a negative sign for rising bodies since they are decelerating (i.e. $a = -g = -9.8 \text{ m/s}^2$).

Distance–time graph for a falling object

For an object falling freely from rest (without air resistance), there will be constant acceleration g .

A graph of distance s against time t is shown in Figure 1.2.9. The gradually increasing slope indicates the speed of the object increases steadily.



▲ **Figure 1.2.9** A graph of distance against time for a body falling freely from rest

Test yourself

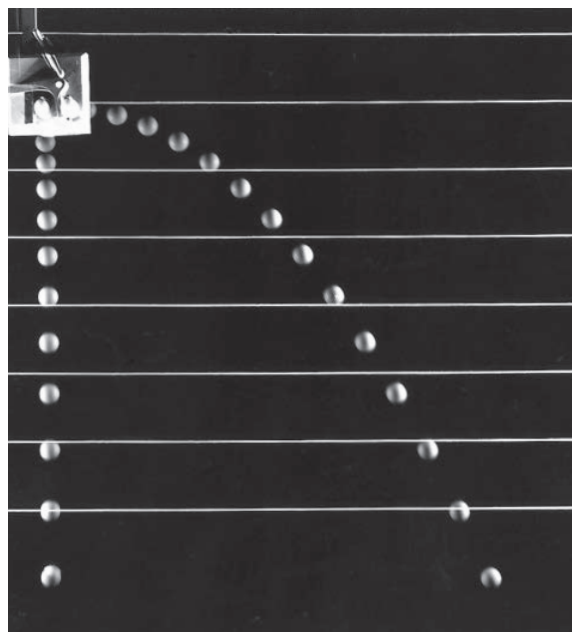
- 8 An object falls from a hovering helicopter and hits the ground at a speed of 30 m/s . How long does it take the object to reach the ground and how far does it fall? Sketch a speed–time graph for the object (ignore air resistance).
- 9 A stone falls from rest from the top of a high tower. Ignore air resistance and take $g = 9.8\text{ m/s}^2$. Calculate
 - a the speed of the stone after 2 seconds
 - b how far the stone has fallen after 2 seconds.
- 10 At a certain instant a ball has a horizontal velocity of 12 m/s and a vertical velocity of 5 m/s . Calculate the resultant velocity of the ball at that instant.

Going further

Projectiles

The photograph in Figure 1.2.10 was taken while a lamp emitted regular flashes of light. One ball was *dropped from rest* and the other, a projectile, was *thrown sideways* at the same time. Their vertical accelerations (due to gravity) are equal, showing that a projectile falls like a body which is dropped from rest. Its horizontal velocity does not affect its vertical motion.

The horizontal and vertical motions of a body are independent and can be treated separately.



▲ **Figure 1.2.10** Comparing free fall and projectile motion using multiflash photography

Revision checklist

After studying Topic 1.2 you should know and understand the following:

- ✓ that a negative acceleration is a deceleration.

After studying Topic 1.2 you should be able to:

- ✓ define speed and velocity, and calculate average speed from total distance travelled/total time taken; sketch, plot, interpret and use speed–time and distance–time graphs to solve problems
- ✓ define and calculate acceleration and use the fact that deceleration is a negative acceleration in calculations

- ✓ give examples of uniform acceleration and non-uniform acceleration
- ✓ state that the acceleration of free fall, g , for an object near to the Earth is constant and use the given value of 9.8 m/s^2
- ✓ describe the motion of objects falling freely, without air resistance.

Exam-style questions

- 1 The speeds of a car travelling on a straight road are given below at successive intervals of 1 second.

Time/s	0	1	2	3	4
Speed/m/s	0	2	4	6	8

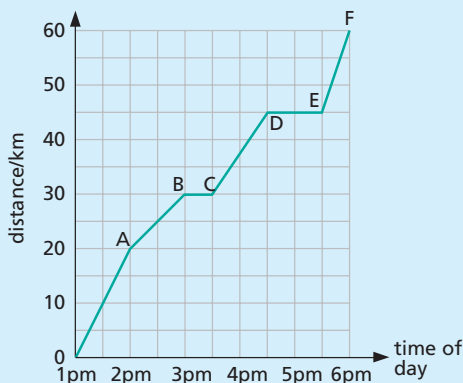
Calculate

- a the average speed of the car in m/s [2]
 - b the distance the car travels in 4 s [3]
 - c the constant acceleration of the car. [2]
- [Total: 7]

- 2 If a train travelling at 10 m/s starts to accelerate at 1 m/s^2 for 15 s on a straight track, calculate its final speed in m/s.
- [Total: 4]

- 3 The distance–time graph for a girl on a cycle ride is shown in Figure 1.2.11.

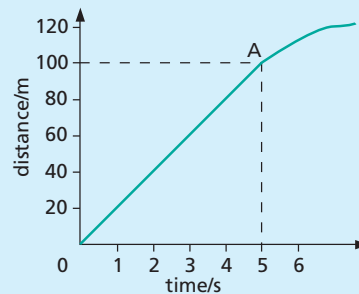
- a Calculate
 - i how far the girl travelled [1]
 - ii how long the ride took [1]
 - iii the girl’s average speed in km/h [1]
 - iv the number of stops the girl made [1]
 - v the total time the girl stopped [1]
 - vi the average speed of the girl *excluding* stops. [2]
- b Explain how you can tell from the shape of the graph when the girl travelled fastest. Over which stage did this happen? [2]



▲ Figure 1.2.11

[Total: 9]

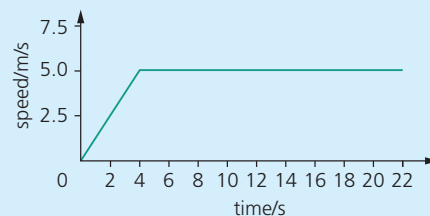
- 4 The graph in Figure 1.2.12 represents the distance travelled by a car plotted against time.
- a State how far the car has travelled at the end of 5 seconds. [1]
 - b Calculate the speed of the car during the first 5 seconds. [1]
 - c State what has happened to the car after A. [2]
 - d Sketch a graph showing the speed of the car plotted against time during the first 5 seconds. [3]



▲ Figure 1.2.12

[Total: 7]

- 5 Figure 1.2.13 shows an incomplete speed–time graph for a boy running a distance of 100 m.
- a Calculate his acceleration during the first 4 seconds. [2]
 - b Calculate how far the boy travels during
 - i the first 4 seconds [2]
 - ii the next 9 seconds? [2]
 - c Copy and complete the graph, showing clearly at what time he has covered the distance of 100 m. Assume his speed remains constant at the value shown by the horizontal portion of the graph. [4]



▲ Figure 1.2.13

[Total: 10]

6 The approximate speed–time graph for a car on a 5-hour journey is shown in Figure 1.2.14. (There is a very quick driver change midway to prevent driving fatigue!)

a State in which of the regions OA, AB, BC, CD, DE the car is

i accelerating

ii decelerating

iii travelling with constant speed.

[3]

b Calculate the value of the acceleration, deceleration or constant speed in each region.

[3]

c Calculate the distance travelled over each region.

[3]

d Calculate the total distance travelled.

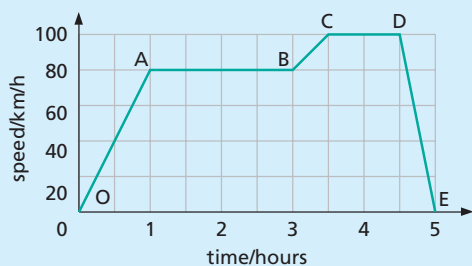
[1]

e Calculate the average speed for the whole journey.

[1]

f State what times the car is at rest.

[1]



▲ Figure 1.2.14

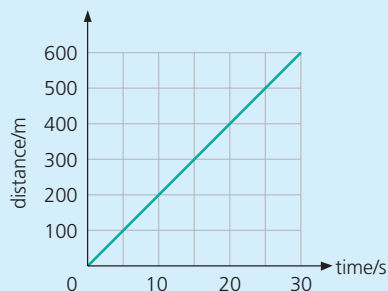
[Total: 12]

7 The distance–time graph for a motorcyclist riding off from rest is shown in Figure 1.2.15.

a Describe the motion. [2]

b Calculate how far the motorbike moves in 30 seconds. [1]

c Calculate the speed. [2]



▲ Figure 1.2.15

[Total: 5]

8 A ball is dropped from rest from the top of a high building. Ignore air resistance and take $g = 9.8 \text{ m/s}^2$.

a Calculate the speed of the ball after

i 1 s [2]

ii 3 s. [2]

b Calculate how far it has fallen after

i 1 s [2]

ii 3 s. [2]

[Total: 8]

1.3

Mass and weight

FOCUS POINTS

- ★ Define mass and weight and know that weights (and therefore masses) may be compared using a balance or force meter.
- ★ State that the mass of an object resists change from its state of rest or motion (inertia).
- ★ Define gravitational field strength and know that this is equivalent to the acceleration of free fall.
- ★ Understand that weight is the effect of a gravitational field on mass.
- ★ Describe, and use the concept of, weight as the effect of a gravitational field on a mass.

Images of astronauts walking on the surface of the Moon show them walking with bouncing steps. The force of gravity is less on the Moon than it is on the Earth and this accounts for their different movements. In the previous topics you have used measurements of space and time, and the rates of change that define speed and acceleration. You will now learn about a further fundamental property, the mass of an object. Mass measures the quantity of matter in a body. In the presence of gravity, mass acquires weight in proportion to its mass and the strength of the gravitational force. Although the mass of an object on the Moon is the same as it is on the Earth, its weight is less on the Moon because the force of gravity there is less.

Mass

The **mass** of an object is the measure of the amount of matter in it. It can be stated that mass is a measure of the quantity of matter in an object at rest relative to an observer.

The standard unit of mass is the **kilogram** (kg) and until 2019 was the mass of a piece of platinum-iridium alloy at the Office of Weights and Measures in Paris. It is now based on a fundamental physical constant which can be measured with great precision. The gram (g) is one-thousandth of a kilogram.

$$1 \text{ g} = \frac{1}{1000} \text{ kg} = 10^{-3} \text{ kg} = 0.001 \text{ kg}$$

The term **weight** is often used when mass is really meant. In science the two ideas are distinct and have different units. The confusion is not helped by the fact that mass is found on a **balance** by a process we unfortunately call 'weighing'!

Key definitions

Mass a measure of the quantity of matter in an object at rest relative to an observer

Weight a gravitational force on an object that has mass

There are several kinds of balance used to measure mass. In the beam balance (equal-arm balance) the unknown mass in one pan is balanced against known masses in the other pan. In the lever balance a system of levers acts against the mass when it is placed in the pan. A direct reading is obtained from the position on a scale of a pointer joined to the lever system. An electronic balance is shown in Figure 1.3.1.



▲ **Figure 1.3.1** An electronic or digital top-pan balance

Mass resists a change in its motion; if an object is at rest, it remains at rest unless it is acted on by a force. This property of mass is called **inertia** and we will discuss it further in Topic 1.5.

Key definition

Inertia the mass of an object resists change from its state of rest or motion

Weight

We all constantly experience the force of *gravity*, in other words, the pull of the Earth. It causes an unsupported body to fall from rest to the ground. Weight is a gravitational force on an object that has mass.

For an object above or on the Earth's surface, the nearer it is to the centre of the Earth, the more the Earth attracts it. Since the Earth is not a perfect sphere but is flatter at the poles, the weight of a body varies over the Earth's surface. It is greater at the poles than at the equator.

Gravity is a force that can act through space. In other words, there does not need to be contact between the Earth and the object on which it acts as there does when we push or pull something. Other action-at-a-distance forces which, like gravity, decrease with distance are

- (i) **magnetic forces** between magnets and
- (ii) **electric forces** between electric charges.

When a mass experiences a gravitational force we say it is in a **gravitational field**. Weight is the result of a gravitational field acting on a mass: weight is a vector quantity and is measured in newtons (N).

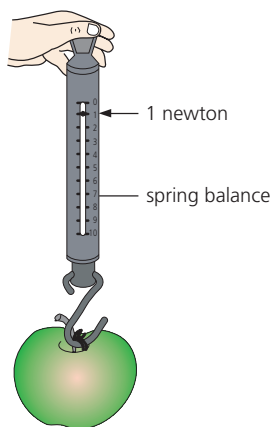
Key definition

Gravitational field a region in which a mass experiences a force due to gravitational attraction

The newton

The unit of force is the **newton**. It will be defined later (Topic 1.5); the definition is based on the change of speed a force can produce in a body. Weight is a force and therefore should be measured in newtons.

The weight of an object can be measured by hanging it on a spring balance (force meter) marked in newtons (Figure 1.3.2) and letting the pull of gravity stretch the spring in the balance. The greater the pull, the more the spring stretches.



▲ **Figure 1.3.2** The weight of an average-sized apple is about 1 newton.

On most of the Earth's surface:

The weight of an object of mass 1 kg is 9.8 N.

Often this is taken as 10 N. A mass of 2 kg has a weight of 20 N, and so on. The mass of an object is the same wherever it is and, unlike weight, does not depend on the presence of the Earth.

Test yourself

- 1 An object of mass 1 kg has weight of 10 N at a certain place. What is the weight of
 - a 100 g
 - b 5 kg
 - c 50 g?
- 2 The force of gravity on the Moon is said to be one-sixth of that on the Earth. What would a mass of 12 kg weigh
 - a on the Earth
 - b on the Moon?

Weight and gravity

The weight W of an object is the force of gravity acting on it which gives it an acceleration g when it is falling freely near the Earth's surface. If the object has mass m , then W can be calculated from $F = ma$ (Newton's second law, see p. 36). We put $F = W$ and $a = g$ to give

$$W = mg$$

Taking $g = 9.8 \text{ m/s}^2$ and $m = 1 \text{ kg}$, this gives $W = 9.8 \text{ N}$, that is an object of mass 1 kg has weight 9.8 N, or near enough 10 N. Similarly, an object of mass 2 kg has weight of about 20 N, and so on.

Gravitational field

The force of gravity acts through space and can cause an object, not in contact with the Earth, to fall to the ground. We try to explain its existence by saying that the Earth is surrounded by a gravitational field which exerts a force on any object in the field. Later, magnetic and electric fields will be considered.

The **gravitational field strength** is defined as the force acting per unit mass. Gravitational field strength is a vector; it has both magnitude and direction. Its direction is the same as the gravitational force.

Rearranging the equation $W = mg$ gives

$$g = \frac{W}{m}$$

or

$$\text{gravitational field strength} = \frac{\text{weight}}{\text{mass}}$$

Key definition

Gravitational field strength force per unit mass

Measurement shows that on the Earth's surface a mass of 1 kg experiences a force of 9.8 N, i.e. its weight is 9.8 N. The strength of the Earth's field is therefore 9.8 N/kg (near enough 10 N/kg). It is denoted by g , the letter also used to denote the acceleration of free fall. Hence

$$g = 9.8 \text{ N/kg} = 9.8 \text{ m/s}^2$$

We now have two ways of regarding g . When considering objects *falling freely*, we can think of it as an acceleration of 9.8 m/s^2 . When an object of known mass is *at rest* and we wish to know the force of gravity (in N) acting on it, we think of g as the Earth's gravitational field strength of 9.8 N/kg . The gravitational field strength is equivalent to the acceleration of free fall.

The weight of an object is directly **proportional** to its mass, which explains why g is the same for all objects. The greater the mass of an object, the greater is the force of gravity on it but it does not accelerate faster when falling because of its greater inertia (i.e. its greater resistance to acceleration).

While the mass of an object is always the same, its weight varies depending on the value of g . On the Moon the acceleration of free fall is only about 1.6 m/s^2 , and so a mass of 1 kg has a weight of just 1.6 N there.

Test yourself

- 3 An astronaut has a mass of 80 kg.
- Calculate the weight of the astronaut on the Moon where the gravitational field strength is 1.6 N/kg .
 - On the journey back to Earth, the astronaut reaches a point X where the gravitational field strengths due to the Earth and the Moon are equal in magnitude but opposite in direction. State
 - the resultant value of the gravitational field strength at X
 - the weight of the astronaut at X.

Revision checklist

After studying Topic 1.3 you should know and understand the following:

- ✓ what is meant by the mass of a body
- ✓ that mass is a property that resists a change in motion
- ✓ the difference between mass and weight and that weights (and masses) may be compared using a balance.

After studying Topic 1.3 you should be able to:

- ✓ state the units of mass and weight and recall how to measure mass or weight
- ✓ recall and use the equation

$$\text{gravitational field strength} = \frac{\text{weight}}{\text{mass}} \text{ or } g = \frac{W}{m}$$
- ✓ state that gravitational field strength, g , is equivalent to the acceleration of free fall
- ✓ describe and use the concept of a gravitational field and its effect on a mass.

Exam-style questions

- 1 a i Explain what is meant by the mass of an object.
- ii Explain what is meant by the weight of an object.
- iii Describe how weights may be compared. [4]
- b State which of the following definitions for weight W is correct.
- A $W = g/\text{mass}$
- B $W = \text{mass}/g$
- C $W = \text{mass} \times g$
- D $W = \text{force} \times g$ [1]
- c Which of the following properties is the same for an object on the Earth and on the Moon?
- A weight
- B mass
- C acceleration of free fall
- D gravitational field strength [1]
- d State the SI units of
- i weight
- ii acceleration of free fall
- iii gravitational field strength. [3]
- [Total: 9]
- 2 a Define gravitational field strength. [2]
- b On the Earth the acceleration of free fall is 9.8 m/s^2 . On Mars the acceleration of free fall is about 3.7 m/s^2 .
The weight of the Mars Rover Opportunity on the Earth was 1850 N.
- i Calculate the mass of the Rover. [2]
- ii Calculate the weight of the Rover on Mars. [2]
- [Total: 6]
- 3 a Explain what is meant by a gravitational field. [2]
- b State the effect of a gravitational field on a mass. [1]
- c Define gravitational field strength. [2]
- d The gravitational field strength on Venus is 8.8 N/kg . The mass of a rock is 200 kg. Calculate the weight of the rock on Venus. [2]
- [Total: 7]

1.4

Density

FOCUS POINTS

★ Define density and calculate the density of a liquid and both regular-and irregular-shaped solid objects.

A pebble thrown into a pond will sink to the bottom of the pond, but a wooden object will float. Objects of the same shape and size but made from different materials have different masses. In this topic you will see how you can quantify such differences with the idea of density. Density specifies the amount of mass in a unit volume. To measure the density of a material you will need to know both its mass and its volume. The mass can be found using an electronic balance and the volume by measurement.

In everyday language, lead is said to be heavier than wood. By this it is meant that a certain volume of lead is heavier than the same volume of wood. In science such comparisons are made by using the term **density**. This is the *mass per unit volume* of a substance and is calculated from

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

For a mass m of volume V , the density $\rho = m/V$.

Key definition

Density mass per unit volume

The density of lead is 11 grams per cubic **centimetre** (11 g/cm^3) and this means that a piece of lead of volume 1 cm^3 has mass 11 g. A volume of 5 cm^3 of lead would have mass 55 g. If the density of a substance is known, the mass of *any* volume of it can be calculated. This enables engineers to work out the weight of a structure if they know from the plans the volumes of the materials to be used and their densities. Strong enough foundations can then be made.

The SI unit of density is the kilogram per cubic metre. To convert a density from g/cm^3 , normally the most suitable unit for the size of sample we use, to kg/m^3 , we multiply by 10^3 . For example, the density of water is 1.0 g/cm^3 or $1.0 \times 10^3 \text{ kg/m}^3$.

The approximate densities of some common substances are given in Table 1.4.1.

▼ **Table 1.4.1** Densities of some common substances

Solids	Density/g/cm ³	Liquids	Density/g/cm ³
aluminium	2.7	paraffin	0.80
copper	8.9	petrol	0.80
iron	7.9	pure water	1.0
gold	19.3	mercury	13.6
glass	2.5	Gases	Density/kg/m ³
wood (teak)	0.80	air	1.3
ice	0.92	hydrogen	0.09
polythene	0.90	carbon dioxide	2.0

Calculations

Using the symbols ρ (rho) for density, m for mass and V for volume, the expression for density is

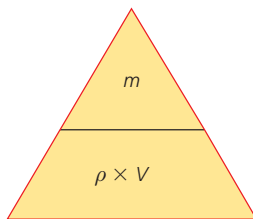
$$\rho = \frac{m}{V}$$

Rearranging the expression gives

$$m = V \times \rho \text{ and } V = \frac{m}{\rho}$$

These are useful if ρ is known and m or V have to be calculated. If you do not see how they are obtained refer to the *Mathematics for physics* section on p. 297.

The triangle in Figure 1.4.1 is an aid to remembering them. If you cover the quantity you want to know with a finger, such as m , it equals what you can still see, i.e. $\rho \times V$. To find V , cover V and you get $V = m/\rho$.



▲ Figure 1.4.1

? Worked example

Taking the density of copper as 9 g/cm^3 , find **a** the mass of 5 cm^3 and **b** the volume of 63 g .

a $\rho = 9 \text{ g/cm}^3$, $V = 5 \text{ cm}^3$ and m is to be found.

$$m = V \times \rho = 5 \text{ cm}^3 \times 9 \text{ g/cm}^3 = 45 \text{ g}$$

b $\rho = 9 \text{ g/cm}^3$, $m = 63 \text{ g}$ and V is to be found.

$$\therefore V = \frac{m}{\rho} = \frac{63 \text{ g}}{9 \text{ g/cm}^3} = 7 \text{ cm}^3$$

Now put this into practice

- 1 A sheet of aluminium has a mass of 200 g and a volume of 73 cm^3 . Calculate the density of aluminium.
- 2 Taking the density of lead as 11 g/cm^3 , calculate
 - a** the mass of 4 cm^3
 - b** the volume of 55 g .

Simple density measurements

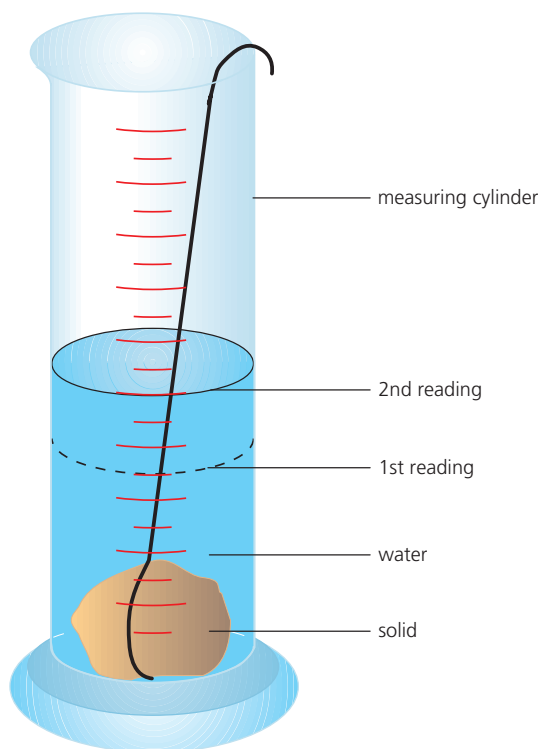
If the mass m and volume V of a substance are known, its density can be found from $\rho = m/V$.

Regularly shaped solid

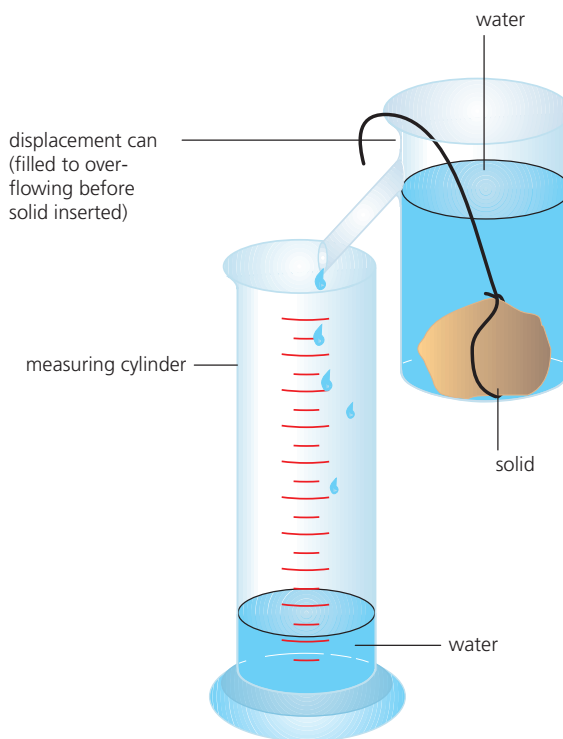
The mass is found on a balance and the volume by measuring its dimensions with a ruler.

Irregularly shaped solid: volume by displacement

Use one of these methods to find the volume of a pebble or glass stopper, for example. The mass of the solid is found on a balance. Its volume is measured by one of the displacement methods shown in Figure 1.4.2. In Figure 1.4.2a the volume is the difference between the first and second readings. In Figure 1.4.2b it is the volume of water collected in the measuring cylinder.



▲ Figure 1.4.2a Measuring the volume of an irregular solid: method 1



▲ Figure 1.4.2b Measuring the volume of an irregular solid: method 2

1.4 DENSITY

Liquid

The mass of an empty beaker is found on a balance. A known volume of the liquid is transferred from a burette or a measuring cylinder into the beaker. The mass of the beaker plus liquid is found and the mass of liquid is obtained by subtraction.

Air

Using a balance, the mass of a 500 cm³ round-bottomed flask full of air is found and again after removing the air with a vacuum pump; the difference gives the mass of air in the flask. The volume of air is found by filling the flask with water and pouring it into a measuring cylinder.



Going further

Floating and sinking

An object sinks in a liquid of lower density than its own; otherwise it floats, partly or wholly submerged. For example, a piece of glass of density 2.5 g/cm³ sinks in water (density 1.0 g/cm³) but floats in mercury (density 13.6 g/cm³).

An iron nail sinks in water but an iron ship floats because its average density is less than that of water, due to the low-density air enclosed in the hull.

A liquid of low density will float on a liquid of higher density if the two liquids do not mix.

Test yourself

- Calculate the density of a substance of
 - mass 100 g and volume 10 cm³
 - volume 3 m³ and mass 9 kg.
 - The density of gold is 19 g/cm³. Calculate the volume of
 - 38 g
 - 95 g of gold.
- A rectangular steel bar is 4 cm long, 3 cm wide and 1 cm thick. When weighed it is found to have a mass of 96 g. Calculate its density in
 - g/cm³
 - kg/m³.
- The water in a measuring cylinder is at the 50 cm³ level. A pebble is dropped into the water and the water level rises to 60 cm³. The pebble is completely covered by water. Calculate
 - the volume of the pebble
 - the density of the pebble, if it weighs 60 g.

Revision checklist

After studying Topic 1.4 you should know and understand the following:

- ✓ how density is defined and how to perform calculations using $\rho = m/V$.

After studying Topic 1.4 you should be able to:

- ✓ describe methods to measure the density of a liquid and a regularly shaped solid
- ✓ describe the method of displacement to measure the density of an irregularly shaped solid.

Exam-style questions

- 1 a** Choose which of the following definitions for density is correct.
- A** mass/volume
 - B** mass \times volume
 - C** volume/mass
 - D** weight/area
- [1]
- b** Calculate
- i** the mass of 5 m^3 of cement of density 3000 kg/m^3 [3]
 - ii** the mass of air in a room measuring $10 \text{ m} \times 5.0 \text{ m} \times 2.0 \text{ m}$ if the density of air is 1.3 kg/m^3 . [3]
- [Total: 7]
- 2 a** Describe how you could determine the density of a liquid. [4]
- b** An empty beaker is weighed and found to have a mass of 130 g. A measuring cylinder contains 50 cm^3 of an unknown liquid. All the liquid is poured into the beaker which is again weighed and found to have a mass of 170 g. Calculate the density of the liquid. [4]
- [Total: 8]
- 3 a** A block of wood has dimensions of $10 \text{ cm} \times 8 \text{ cm} \times 20 \text{ cm}$.
- i** Calculate the volume of the block in cubic metres. [2]
 - ii** The block is placed on a balance and found to weigh 1.2 kg. Calculate the density of the block in kg/m^3 . [3]
- b** When a golf ball is lowered into a measuring cylinder of water, the water level rises by 30 cm^3 when the ball is completely submerged. If the ball weighs 33 g in air, calculate its density in kg/m^3 . [3]
- [Total: 8]

1.5

Forces

1.5.1 Balanced and unbalanced forces

FOCUS POINTS

- ★ Understand that the size, shape and velocity of objects can be altered by forces.
- ★ Identify different types of force and use free-body diagrams to show the magnitude and direction of all the forces that act on an object.
- ★ Become familiar with load–extension graphs for an elastic solid and describe an experiment to show how a spring behaves when it is stretched.
- ★ Understand that when several forces act simultaneously on an object that a resultant can be determined.
- ★ Know that, unless acted upon by a resultant force, an object will remain at rest or will continue moving with a constant speed in a straight line.
- ★ Define the spring constant and the limit of proportionality on a load–extension graph.
- ★ Apply the equation $F = ma$ to calculate force and acceleration.
- ★ State Newton's third law of motion.

A gravitational force causes a freely falling object to accelerate and keeps a satellite moving in a circular path. Clearly a force can change the speed or direction of travel of an object. A force can also change the shape or size of an object. If you stand on an empty paper carton it will change its shape and if you pull on a spiral spring it will stretch. Several forces may act on an object at once and it is useful to calculate a resultant force to predict their combined effect; both the size and direction of the forces are needed for this. You have already learnt how to quantify some of these changes and in this topic you will learn more ways to do so.

Types of force

A **force** is a push or a pull. There are different types of forces. You have already found that weight is a gravitational force and will learn later that there are magnetic forces between magnets (Topic 4.1) and electrostatic forces between charges (Topic 4.2).

These forces do not require contact between objects. A force can cause an object at rest to move, or if the body is already moving it can change its speed or direction of motion.



▲ **Figure 1.5.1** A weightlifter in action exerts first a pull and then a push.

A force can also change a body's shape or size. For example, a spring (or wire) will stretch when loaded with a weight.

Contact forces occur between objects that are touching each other. These include **drag** and **air resistance** which are resistive forces caused by the motion of an object through a fluid such as a liquid or air. They act against the direction of motion of the object to slow it down as does friction, which occurs between two solid surfaces

in relative motion. Other contact forces are the force experienced by an object at rest on a surface (Figure 1.5.4 on p. 33) and the **tension** that occurs in a string or spring (Figure 1.5.2) being stretched (elastic force).

Thrust is a sudden force (driving force) in a particular direction.

The forces on an object can be represented in a **free-body diagram** (Figure 1.5.4, p. 33).

Elastic deformation



Practical work

Stretching a spring

Safety

- Eye protection must be worn (in case the spring snaps).

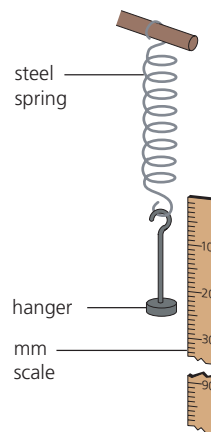
Arrange a steel spring as in Figure 1.5.2. Read the scale opposite the bottom of the hanger. Add 100 g masses one at a time (thereby increasing the **load** by steps of 1 N) and take readings from the scale after each one. Enter the readings in a table for masses up to 500 g.

Note that at the head of columns (or rows) in data tables it is usual to give the name of the quantity or its symbol followed by / and the unit.

Load/N	Scale reading/mm	Total extension/mm

Sometimes it is easier to discover laws by displaying the results on a graph. Do this on graph paper by plotting total **extension** along the *x*-axis (horizontal axis) and load along the *y*-axis (vertical axis) in a load–extension graph. Every pair of readings will give a point; mark

them by small crosses and draw a smooth line through them.



▲ Figure 1.5.2

- 1 What is the shape of the graph you plotted?
- 2 Do the results suggest any rule about how the spring behaves when it is stretched?
- 3 What precautions could you take to improve the accuracy of the results of this experiment?
- 4 How could you test if the extension of the spring is proportional to the stretching force?

Extension in springs

Springs were investigated by Robert Hooke just over 350 years ago. He found that the extension was proportional to the stretching force provided the spring was not permanently stretched. This means that doubling the force doubles the extension, trebling the force trebles the extension, and so on.

Using the sign for proportionality, \propto , we can write
extension \propto stretching force

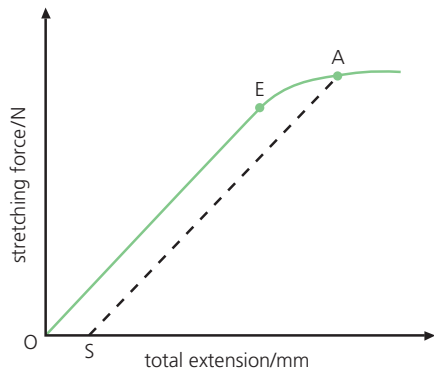
It is true only if the **limit of proportionality** of the spring is not exceeded.

Key definition

Limit of proportionality the point at which the load–extension graph becomes non-linear

1.5 FORCES

The graph of Figure 1.5.3 is for a spring stretched beyond its limit of proportionality, E. OE is a straight line passing through the origin O and is graphical proof that the extension is directly proportional to the stretching force over this range. If the force for point A on the graph is applied to the spring, the proportionality limit is passed and on removing the force some of the extension (OS) remains.



▲ Figure 1.5.3

Test yourself

- 1 In Figure 1.5.3, over which part of the graph does a spring balance work?

Spring constant

The **spring constant**, k , is defined as force per unit extension. It is the force which must be applied to a spring to cause an extension of 1 m.

If a force F produces extension x then

$$k = \frac{F}{x}$$

or

$$\text{spring constant} = \frac{\text{force}}{\text{extension}}$$

Rearranging the equation gives

$$F = kx$$

Key definition

Spring constant force per unit extension

Proportionality also holds when a force is applied to an elastic solid such as a straight metal wire, provided it is not permanently stretched.

Load–extension graphs similar to Figure 1.5.3 are obtained. You should label each axis of your graph with the name of the quantity or its symbol followed by / and the unit, as shown in Figure 1.5.3.

The limit of proportionality can be defined as the point at which the load–extension graph becomes non-linear because the extension is no longer proportional to the stretching force.

? Worked example

A spring is stretched 10 mm (0.01 m) by a weight of 2.0 N. Calculate

- a the spring constant k
- b the weight W of an object that causes an extension of 80 mm (0.08 m).

$$\text{a } k = \frac{F}{x} = \frac{2.0 \text{ N}}{0.01 \text{ m}} = 200 \text{ N/m}$$

$$\begin{aligned} \text{b } W &= \text{stretching force } F \\ &= k \times x \\ &= 200 \text{ N/m} \times 0.08 \text{ m} \\ &= 16 \text{ N} \end{aligned}$$

Now put this into practice

- 1 Calculate the spring constant of a spring which is stretched 2 mm by a force of 4 N.
- 2 A 2 N force is applied to a spring which has a spring constant of 250 N/m. Calculate the extension of the spring in mm.

Test yourself

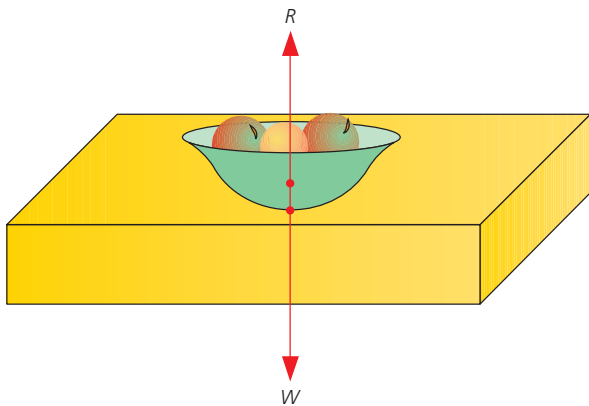
- 2 State two effects which a force may have on an object.
- 3 Make a sketch of a load–extension graph for a spring and indicate the region over which the extension is proportional to the stretching force.
- 4 Calculate the spring constant of a spring which is stretched 4 cm by a mass of 200 g.
- 5 Define the limit of proportionality for a stretched spring.

Forces and resultants

Force has both magnitude (size) and direction. It is represented in free-body diagrams by a straight line with an arrow to show its direction of action.

Usually more than one force acts on an object. As a simple example, an object resting on a table is pulled downwards by its weight W and pushed upwards by a contact force R due to the table supporting it (Figure 1.5.4). Since the object is

at rest, there is no resultant force. We say the forces are **balanced**, i.e. $R = W$.

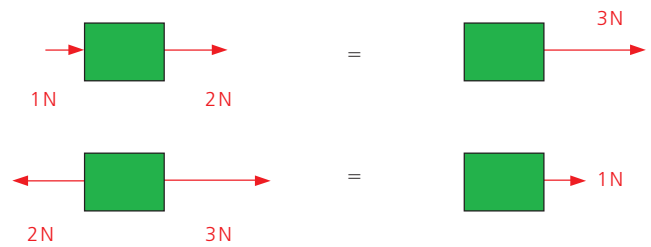


▲ **Figure 1.5.4**

In structures such as a giant oil platform (Figure 1.5.5), two or more forces may act at the same point. It is then often useful for the design engineer to know the value of the single force, i.e. the resultant force, which has exactly the same effect as these forces. If the forces act in the same straight line, the resultant is found by simple addition or subtraction as shown in Figure 1.5.6. If the forces act in different directions, the vectors are added by taking account of their direction. This was described in Topic 1.1 for the resultant of two forces at right angles.



▲ **Figure 1.5.5** The design of an offshore oil platform requires an understanding of the combination of many forces.



▲ **Figure 1.5.6** The resultant of forces acting in the same straight line is found by addition or subtraction.

If the resultant of two or more forces is not zero, we say the forces are **unbalanced**.

➔ Going further



Practical work

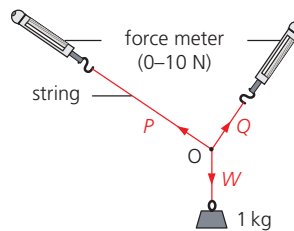
Parallelogram law

Safety

- Take care when using the mass in case it drops.

Arrange the apparatus as in Figure 1.5.7a with a sheet of paper behind it on a vertical board. We have to find the resultant of forces P and Q .

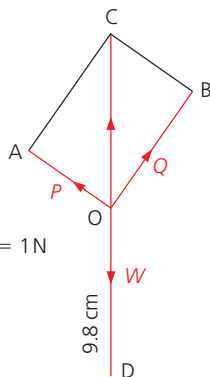
Read the values of P and Q from the spring balances. Mark on the paper the directions of P , Q and W as shown by the strings. Remove the paper and, using a scale of 1 cm to represent 1 N, draw OA , OB and OD to represent the three forces P , Q and W which act at O , as in Figure 1.5.7b. (W = weight of the 1 kg mass = 9.8 N; therefore $OD = 9.8$ cm.)



▲ **Figure 1.5.7a**

P and Q together are balanced by W and so their resultant must be a force equal and opposite to W .

Complete the parallelogram OACB. Measure the diagonal OC; if it is equal in size (i.e. 9.8 cm) and opposite in direction to W then it represents the resultant of P and Q .



Use the scale 1 cm = 1 N

▲ **Figure 1.5.7b** Finding a resultant by the parallelogram law

The parallelogram law for adding two forces is:

If two forces acting at a point are represented in size and direction by the sides of a parallelogram drawn from the point, their resultant is represented in size and direction by the diagonal of the parallelogram drawn from the point.

- 5 List the equipment you would need for this experiment.
- 6 What quantity would you vary to test the law under different conditions?

Test yourself

- 6 Jo, Daniel and Helen are pulling a metal ring. Jo pulls with a force of 100 N in one direction and Daniel with a force of 140 N in the opposite direction. If the ring does not move, what force does Helen exert if she pulls in the same direction as Jo?
- 7 A boy drags a suitcase along the ground with a force of 100 N. If the frictional force opposing the motion of the suitcase is 50 N, what is the resultant forward force on the suitcase?
- 8 A picture is supported by two vertical strings. If the weight of the picture is 50 N, what is the force exerted by each string?
- 9 Using a scale of 1 cm to represent 10 N, find the size and direction of the resultant of forces of 30 N and 40 N acting at right angles to each other.

Newton's first law

Friction and air resistance cause a car to come to rest when the engine is switched off. If these forces were absent, we believe that an object, once set in motion, would go on moving forever with a constant speed in a straight line. That is, force is not needed to keep a body moving with **uniform velocity** provided that no opposing forces act on it.

This idea was proposed by Galileo and is summed up in **Newton's first law of motion**:

An object stays at rest, or continues to move in a straight line at constant speed, unless acted on by a **resultant force**.

It seems that the question we should ask about a moving body is not what keeps it moving but what changes or stops its motion.

The smaller the external forces opposing a moving body, the smaller is the force needed to keep it moving with constant velocity. A hover scooter, which is supported by a cushion of air (Figure 1.5.8), can skim across the ground with little frictional opposition, so that relatively little power is needed to maintain motion.

A resultant force may change the velocity of an object by changing its direction of motion or speed.

Key definitions

Newton's first law of motion an object either remains at rest or continues to move in a straight line at constant speed unless acted on by a resultant force

Resultant force may change the velocity of an object by changing its direction of motion or its speed



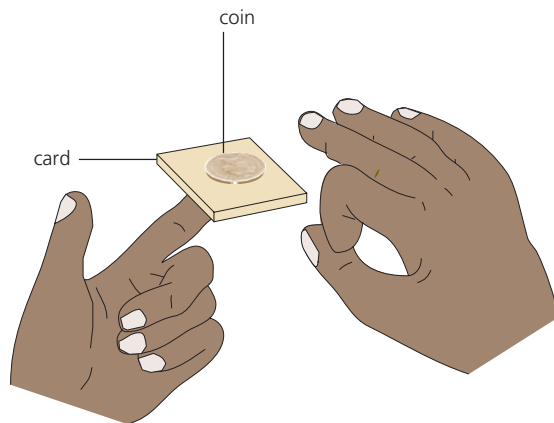
▲ **Figure 1.5.8** Friction is much reduced for a hover scooter.

Mass and inertia

Newton's first law is another way of saying that all matter has a built-in opposition to being moved if it is at rest or, if it is moving, to having its motion changed. This property of matter is called **inertia** (from the Latin word for laziness), see Topic 1.3.

Its effect is evident on the occupants of a car that stops suddenly: they lurch forwards in an attempt to continue moving, and this is why seat

belts are needed. The reluctance of a stationary object to move can be shown by placing a large coin on a piece of card on your finger (Figure 1.5.9). If the card is flicked *sharply* the coin stays where it is while the card flies off.



▲ **Figure 1.5.9** Flick the card sharply

The larger the mass of a body, the greater is its inertia, i.e. the more difficult it is to move it when at rest and to stop it when in motion. Because of this we consider that *the mass of a body measures its inertia*.



Practical work

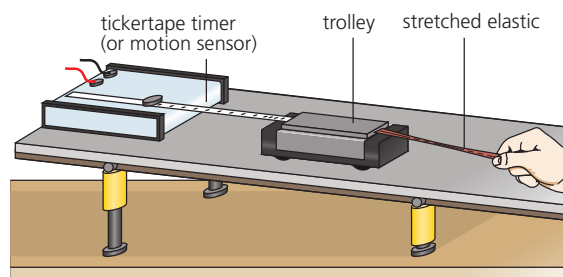
Effect of force and mass on acceleration

Safety

- Take care when rolling the trolley down the ramp. Ensure it is clear at the bottom of the ramp and use a side barrier to prevent the trolley from falling onto the floor.

The apparatus consists of a trolley to which a force is applied by a stretched length of elastic (Figure 1.5.10). The velocity of the trolley is found from a tickertape timer or a motion sensor, datalogger and computer.

First compensate the runway for friction: raise one end until the trolley runs down with constant velocity when given a push. The dots on the tickertape should be equally spaced, or a horizontal trace obtained on a speed–time graph. There is now no resultant force on the trolley and any acceleration produced later will be due only to the force caused by the stretched elastic.



▲ **Figure 1.5.10**

(a) Force and acceleration (mass constant)

Fix one end of a short length of elastic to the rod at the back of the trolley and stretch it until the other end is level with the front of the trolley. Practise pulling the trolley down the runway, keeping the same stretch on the elastic. After a few trials you should be able to produce a steady accelerating force.



1.5 FORCES

Repeat using first two and then three *identical* pieces of elastic, stretched side by side by the same amount, to give two and three units of force.

If you are using tickertape, make a tape chart for each force and use it to find the acceleration produced in cm/ten-tick². Ignore the start of the tape (where the dots are too close) and the end (where the force may not be steady). If you use a motion sensor and computer to plot a speed–time graph, the acceleration can be obtained in m/s² from the slope of the graph (Topic 1.2).

Put the results in a table.

Force (F)/(no. of pieces of elastic)	1	2	3
Acceleration (a)/cm/ten-tick ² or m/s ²			

(b) Mass and acceleration (force constant)

Do the experiment as in part (a) using two pieces of elastic (i.e. constant F) to accelerate

first one trolley, then two (stacked one above the other) and finally three. Check the friction compensation of the runway each time.

Find the accelerations from the tape charts or computer plots and tabulate the results.

Mass (m)/(no. of trolleys)	1	2	3
Acceleration (a)/cm/ten-tick ² or m/s ²			

- 7 For part (a), does a steady force cause a steady acceleration?
- 8 Do your results in part (a) suggest any relationship between acceleration a and force F ?
- 9 Do your results for part (b) suggest any relationship between a and m ?
- 10 Name the two independent variable quantities in experiments (a) and (b).
- 11 How could you use the results to verify the equation $F = ma$?

Newton's second law

The previous experiment should show roughly that the acceleration a is

- (i) directly proportional to the applied force F for a fixed mass, i.e. $a \propto F$, and
- (ii) **inversely proportional** to the mass m for a fixed force, i.e. $a \propto 1/m$.

Combining the results into one equation, we get

$$a \propto \frac{F}{m}$$

or

$$F \propto ma$$

Therefore

$$F = kma$$

where k is the **constant of proportionality**.

One newton is defined as the force which gives a mass of 1 kg an acceleration of 1 m/s², i.e. 1 N = 1 kg m/s², so if $m = 1$ kg and $a = 1$ m/s², then $F = 1$ N.

Substituting in $F = kma$, we get $k = 1$ and so we can write

$$F = ma$$

or

$$\text{resultant force} = \text{mass} \times \text{acceleration}$$

This is **Newton's second law of motion**. When using it, two points should be noted. First, F is the resultant (or unbalanced) force causing the acceleration a in the same direction as F . Second, F must be in newtons, m in kilograms and a in metres per second squared, otherwise k is not 1. The law shows that a will be largest when F is large and m small.

Key definition

Newton's second law of motion

resultant force = mass \times acceleration ($F = ma$)

You should now appreciate that when the forces acting on a body do not balance there is a net (resultant) force which causes a change of motion, i.e. the body accelerates or decelerates. The force and the acceleration are in the same direction. If the forces balance, there is no change in the motion of the body. However, there may be a change of shape, in which case internal forces in the body (i.e. forces between neighbouring atoms) balance the external forces.

? Worked example

A block of mass 2 kg has a constant velocity when it is pushed along a table by a force of 5 N. When the push is increased to 9 N what is

- the resultant force
- the acceleration?

When the block moves with constant velocity the forces acting on it are balanced. The force of friction opposing its motion must therefore be 5 N.

- When the push is increased to 9 N the resultant (unbalanced) force F on the block is $(9 - 5) \text{ N} = 4 \text{ N}$ (since the frictional force is still 5 N).
- The acceleration a is obtained from $F = ma$ where $F = 4 \text{ N}$ and $m = 2 \text{ kg}$.

$$a = \frac{F}{m} = \frac{4 \text{ N}}{2 \text{ kg}} = \frac{4 \text{ kg m/s}^2}{2 \text{ kg}} = 2 \text{ m/s}^2$$

Now put this into practice

- A box of mass 5 kg has a constant velocity when it is pushed along a table by a force of 8 N. When the push is increased to 10 N calculate
 - the resultant force
 - the acceleration.
- A force F produces a constant acceleration in a straight line of 0.5 m/s^2 on a block of mass 7 kg. Calculate the value of F .

- A block of mass 500 g is pulled from rest on a horizontal frictionless bench by a steady force F and reaches a speed of 8 m/s in 2 s. Calculate
 - the acceleration
 - the value of F .

Newton's third law

If a body A exerts a force on body B, then body B exerts an equal but opposite force on body A.

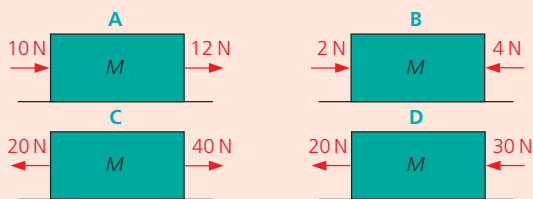
This is **Newton's third law of motion** and states that forces never occur singly but always in pairs as a result of the action between two bodies. For example, when you step forwards from rest your foot pushes backwards on the Earth, and the Earth exerts an equal and opposite force forward on you. Two bodies and two forces are involved. The small force you exert on the large mass of the Earth gives no noticeable acceleration to the Earth but the equal force it exerts on your very much smaller mass causes you to accelerate.

Key definition

Newton's third law of motion when object A exerts a force on object B, then object B exerts an equal and opposite force on object A

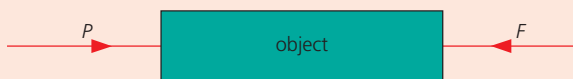
Test yourself

- Which one of the diagrams in Figure 1.5.11 shows the arrangement of forces that gives the block of mass M the greatest acceleration?



▲ Figure 1.5.11

- In Figure 1.5.12 if P is a force of 20 N and the object moves with constant velocity, what is the value of the opposing force F ?



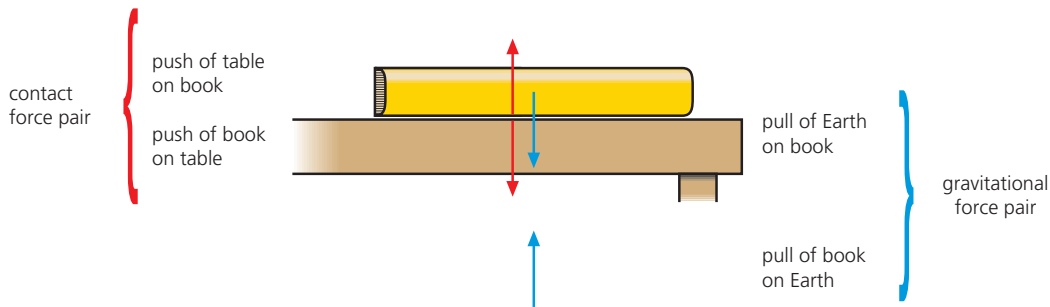
▲ Figure 1.5.12

- What resultant force produces an acceleration of 5 m/s^2 in a car of mass 1000 kg?
 - What acceleration is produced in a mass of 2 kg by a resultant force of 30 N?

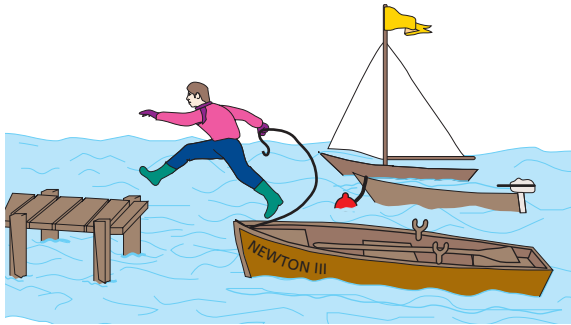
Note that the pair of equal and opposite forces *do not act on the same body*; if they did, there could never be any resultant forces and acceleration would be impossible. For a book resting on a table, the book exerts a downward force on the table and the table exerts an equal and opposite upward force on the book; this pair of forces act on different objects and are represented by the red arrows in Figure 1.5.13. The weight of the book (blue arrow) does not form a pair with the upward force on the book (although they are equal numerically) as these two forces act on the same body.

An appreciation of the third law and the effect of friction is desirable when stepping from a rowing boat (Figure 1.5.14). You push backwards on the boat and, although the boat pushes you forwards with an equal force, it is itself now moving backwards (because friction with the water is slight). This reduces your forwards motion by the same amount – so you may fall in!

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▲ **Figure 1.5.13** Forces between book and table



▲ **Figure 1.5.14** The boat moves backwards when you step forwards!

1.5.2 Friction

FOCUS POINTS

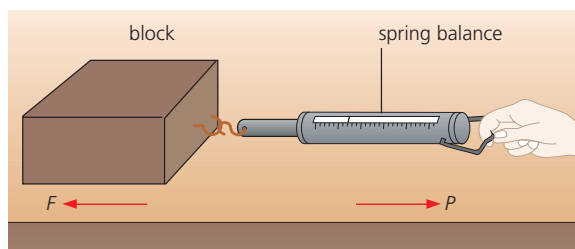
- ★ Understand that friction between surfaces acts to slow an object and produces heating.
- ★ Describe the motion of objects falling with and without air resistance or drag.
- ★ Explain how an object reaches terminal velocity.
- ★ Describe how the overall stopping distance of a car is affected by its speed and by factors that affect the friction forces or factors that affect the driver's reaction time.

Friction between a moving object and its surroundings is important as it acts to reduce the speed of the object. A steady braking force applied to a car produces a uniform deceleration. The car slows down and stops. The distance the car moves while it is braking depends on the size of the resultant force, which depends on the force applied by the brakes and the size of the friction force.

A falling object accelerates due to the pull of gravity, however drag force caused by air resistance reduces the acceleration. If the size of the drag force increases so that it is balanced by the object's weight, then the resultant force is zero and the object will move at constant speed.

Friction is the force that opposes one surface moving, or trying to move, over another. It can be a help or a hindrance. We could not walk if there was no friction between the soles of our shoes and the ground. Our feet would slip backwards, as they

tend to when we walk on ice. On the other hand, engineers try to reduce friction to a minimum in the moving parts of machinery by using lubricating oils and ball-bearings.



▲ **Figure 1.5.15** Friction opposes motion between surfaces in contact.

When a gradually increasing force P is applied through a spring balance to a block on a table (Figure 1.5.15), the block does not move at first. This is because an equally increasing but opposing frictional force F acts where the block and table touch. At any instant P and F are equal and opposite.

If P is increased further, the block eventually moves; as it does so F has its maximum value, called **starting** or **static friction**. When the block is moving at a steady speed, the balance reading is slightly less than that for starting friction. **Sliding** or **dynamic friction** is therefore less than starting or static friction.

Placing a mass on the block increases the force pressing the surfaces together and increases friction.

When work is done against friction, the temperatures of the bodies in contact rise (as you can test by rubbing your hands together); kinetic energy is transferred to thermal energy by mechanical working (see Topic 1.7).

Solid friction can be described as the force between two surfaces that may impede motion and produce heating.

Friction (drag) acts on an object such as a vehicle or falling leaf, moving through gas (air resistance), which opposes the motion of the object. Similarly, friction (drag) acts on an object moving through a liquid. Drag increases as the speed of the object increases, and acts to reduce acceleration and slow the object down.

Test yourself

- 14 a** Explain the conditions under which friction occurs.
- b** Name two effects resulting from solid friction.
- 15** A car is moving at a constant speed along a straight road. Describe how the forces acting on the car influence the speed of the car. How is a constant speed achieved?

Driving and car safety

In order to bring a moving car to rest, the brakes must be applied over a certain distance known as the **braking distance**. The **thinking distance** is the distance travelled while the driver is reacting before applying the brakes. For a driver travelling at a constant speed, the thinking distance is proportional to the reaction time t (distance = vt); the longer the reaction time, the further the car travels before the brakes are applied. The thinking distance has to be added to the braking distance to obtain the overall **stopping distance**, in other words

$$\text{stopping distance} = \text{thinking distance} + \text{braking distance}$$

Key definitions

Braking distance distance over which brakes applied before vehicle brought to rest

Thinking distance distance travelled during reaction time of driver (before brakes applied)

Stopping distance total distance travelled in the time it takes to stop a vehicle; equals thinking distance + braking distance

Typical values are given in Table 1.5.1 for different speeds. The greater the speed, the greater the stopping distance for a given braking force. The data shows that the total stopping distance for a car travelling at 90 km/h is twice as long as for a car travelling at 60 km/h if the braking force is the same. (To stop the car in a given distance, a greater braking force is needed for higher speeds.)

▼ **Table 1.5.1**

Speed/km/h	30	60	90	120
Thinking distance/metres	6	12	18	24
Braking distance/metres	6	24	54	96
Total stopping distance/metres	12	36	72	120

Thinking distance depends on the driver's **reaction time** – this will vary with factors such as the driver's degree of tiredness, use of alcohol or drugs, eyesight and the visibility of the hazard. Braking distance varies with both the road conditions and the state of the car; it is longer when the road is wet or icy, so when friction between the tyres and the road is low, than when conditions are dry. Efficient brakes and deep tyre tread help to reduce

1.5 FORCES

the braking distance. The braking distance becomes longer when the load carried by the vehicle is increased.

Air resistance: terminal velocity

In the absence of air resistance, a falling object has a constant acceleration as shown in the distance–time graph of Figure 1.2.9 on p. 19. However, we cannot usually ignore the effect of air resistance. As the object accelerates, the air resistance opposing its motion *increases as its speed rises*. This reduces its acceleration; the acceleration is no longer constant. Eventually, air resistance acting upwards equals the weight of the object acting downwards. The resultant force on the object is then zero since the gravitational force balances the frictional force. The object falls at a constant velocity, called its **terminal velocity**, whose value depends on the size, shape and weight of the object. Figure 1.2.3c on p. 14 shows the speed–time graph for an object that is decreasing its acceleration and approaching a terminal velocity.

A small dense object, such as a steel ball-bearing, has a high terminal velocity and falls a considerable distance with a constant acceleration of 9.8 m/s^2 before air resistance equals its weight. A light

object, like a raindrop, or an object with a large surface area, such as a parachute, has a low terminal velocity and only accelerates over a comparatively short distance before air resistance equals its weight. A skydiver (Figure 1.5.16) has a terminal velocity of more than 50 m/s (180 km/h) before the parachute is opened.

Objects falling in liquids behave similarly to those falling in air.



▲ Figure 1.5.16 Synchronised skydivers

1.5.3 Circular motion

FOCUS POINT

★ Describe motion in a circular path and understand the effect on force if speed, radius or mass change.

There are many examples of bodies moving in circular paths: rides at a funfair, clothes being spun dry in a washing machine, the planets going round the Sun, and the Moon circling the Earth. When a car turns a corner, it may follow an arc of a circle. ‘Throwing the hammer’ is a sport practised at Highland Games in Scotland (Figure 1.5.17), in which the hammer is whirled round and round before it is released.

To keep an object moving in a circular path requires a force to act towards the centre of the circle. In the case of a satellite orbiting the Earth, that force is provided by the Earth’s gravitational attraction;

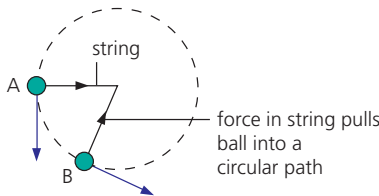
for a whirling hammer, it is the force exerted on the handle of the hammer by the athlete. The size of the force depends on a number of factors.



▲ Figure 1.5.17 'Throwing the hammer'

Centripetal force

In Figure 1.5.18 a ball attached to a string is being whirled round in a horizontal circle. Its direction of motion is constantly changing. At A, it is along the tangent at A; shortly afterwards, at B, it is along the tangent at B; and so on. It can be seen that motion in a circular path is due to a force perpendicular to the motion.



▲ Figure 1.5.18

Velocity has both size and direction; speed has only size. Velocity is speed in a stated direction and if the direction of a moving body changes, even if

its speed does not, then its velocity has changed. A change of velocity is an acceleration, and so during its whirling motion the ball is accelerating.

It follows from Newton's first law of motion that if we consider a body moving in a circle to be accelerating, then there must be a force acting on it to cause the acceleration. In the case of the whirling ball it is reasonable to say the force is provided by the string pulling inwards on the ball. Like the acceleration, the force acts towards the centre of the circle and keeps the body at a fixed distance from the centre.

A larger force is needed if

- the speed v of the ball is increased, with mass and radius constant
- the radius r of the circle is decreased, with mass and speed constant
- the mass m of the ball is increased, with speed and radius constant.

This force, which acts *towards the centre* and keeps a body moving in a circular path, is called the **centripetal force** (centre-seeking force).

Should the force be greater than the string can bear, the string breaks and the ball flies off with steady speed in a straight line *along the tangent*, i.e. in the direction of travel when the string broke (as Newton's first law of motion predicts). It is not thrown outwards.

Whenever an object moves in a circle (or circular arc) there must be a centripetal force acting on it. In throwing the hammer it is the pull of the athlete's arms acting on the hammer towards the centre of the whirling path. When a car rounds a bend, a frictional force is exerted inwards by the road on the car's tyres.

➔ Going further

Satellites

For a satellite of mass m orbiting the Earth at radius r with orbital speed v , the centripetal force, F , is the Earth's gravitational force on the mass.

To put an artificial satellite in orbit at a certain height above the Earth it must enter the orbit at the correct speed. If it does not, the force of gravity, which decreases as height above the Earth increases, will not be equal to the centripetal force needed for the orbit.

Communication satellites

Communication satellites circle the Earth in orbits above the equator. Geostationary satellites have an orbit high above the equator (36 000 km); they travel with the same speed as the Earth rotates, so appear to be stationary at a particular point above the Earth's surface – their orbital period is 24 hours. They are used for transmitting television, intercontinental telephone and data signals. Geostationary satellites need to be well separated so that they do not interfere with each other; there is room for about 400.

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Mobile phone networks use many satellites in much lower equatorial orbits; they are slowed by the Earth's atmosphere and their orbit has to be regularly adjusted by firing a rocket engine. Eventually they run out of fuel and burn up in the atmosphere as they fall to Earth.

Monitoring satellites

Monitoring satellites circle the Earth rapidly in low polar orbits, i.e. passing over both poles; at a height of 850 km the orbital period is only 100 minutes. The Earth rotates below them so they scan the whole surface at short range in a 24-hour period and can be used to map or monitor regions of the Earth's surface which may be inaccessible by other means. They are widely used in weather forecasting as they continuously transmit infrared pictures of cloud patterns down to Earth (Figure 1.5.19), which are picked up in turn by receiving stations around the world.



▲ **Figure 1.5.19** Satellite image of cloud over Europe

Test yourself

- 16 An apple is whirled round in a horizontal circle on the end of a string which is tied to the stalk. It is whirled faster and faster and at a certain speed the apple is torn from the stalk. Explain why this happens.
- 17 Is the gravitational force on a satellite greater or less when it is in a high orbit than when it is in a low orbit?

1.5.4 Turning effect of forces

FOCUS POINTS

- ★ Describe and give everyday examples of the turning effect of a force (its moment) and use the appropriate equation to calculate the moment of a force.
- ★ Apply the principle of moments to different situations.
- ★ Recall the conditions for an object being in equilibrium.
- ★ Be familiar with an experiment showing that an object in equilibrium has no resultant moment.

A seesaw in a children's playground can be balanced if the two children have similar weights or if the lighter child sits further from the pivot than the heavier child. Each child exerts a turning effect on the seesaw, either clockwise or anticlockwise, which depends not only on their weight but also on their distance from the pivot. Forces act in different ways depending on their orientation. In this topic you will discover that the turning effect of a force (its moment) depends on both its magnitude and the perpendicular distance from the pivot point. This means that a small force at a large distance can balance a much larger force applied closer to the pivot. When the combination of all the forces acting on a body is such that there is no net force or turning effect, the body is in equilibrium (the seesaw is level) and will not move unless additional forces are applied.

Moment of a force

The handle on a door is at the outside edge so that it opens and closes easily. A much larger force would be needed if the handle were near the hinge. Similarly, it is easier to loosen a nut with a long spanner than with a short one.

The turning effect of a force is called the **moment of the force**. It depends on both the size of the force and how far it is applied from the pivot. It is measured by multiplying the force by the perpendicular distance of the line of action of the force from the pivot. The unit is the newton metre (N m).

$$\text{moment of a force} = \text{force} \times \text{perpendicular distance from the pivot}$$

Key definition

Moment of a force moment = force \times perpendicular distance from pivot

In Figure 1.5.20a, a force F acts on a gate at its edge, and in Figure 1.5.20b it acts at the centre.

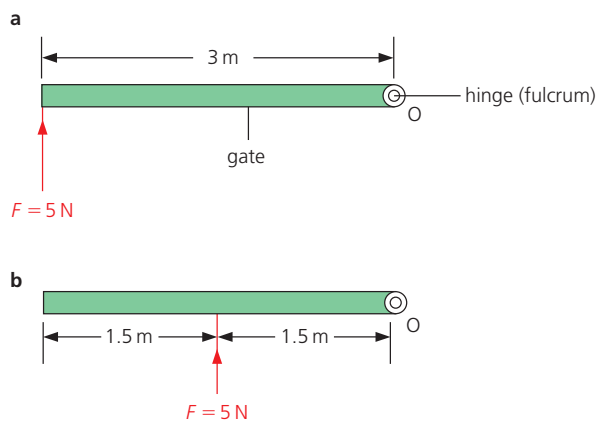
In Figure 1.5.20a:

$$\text{moment of } F \text{ about } O = 5 \text{ N} \times 3 \text{ m} = 15 \text{ N m}$$

In Figure 1.5.20b:

$$\text{moment of } F \text{ about } O = 5 \text{ N} \times 1.5 \text{ m} = 7.5 \text{ N m}$$

The turning effect of F is greater in the first case. This agrees with the fact that a gate opens most easily when pushed or pulled at the edge furthest from the hinge.



▲ Figure 1.5.20

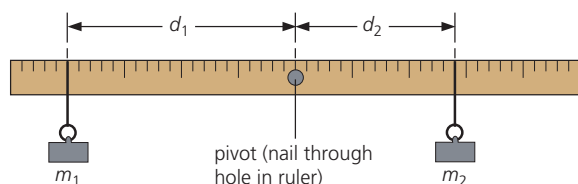
Balancing a beam

To balance a **beam** about a pivot, like the ruler in Figure 1.5.21, the weights must be moved so that the clockwise turning effect equals the anticlockwise turning effect and the net moment on the beam becomes zero. If the beam tends to swing clockwise, m_1 can be moved further from the pivot to increase its turning effect; alternatively, m_2 can be moved nearer to the pivot to reduce its turning effect. What adjustment would you make to the position of m_2 to balance the beam if it is tending to swing anticlockwise?



Practical work

Principle of moments



▲ Figure 1.5.21

Balance a half-metre ruler at its centre, adding Plasticine to one side or the other until it is horizontal.

Hang unequal loads m_1 and m_2 from either side of the pivot and alter their distances d_1 and d_2 from the centre until the ruler is again balanced (Figure 1.5.21). Forces F_1 and F_2 are exerted by gravity on m_1 and m_2 and so on the ruler; the force on 100 g is 0.98 N. Record the results in a table and repeat for other loads and distances.

m_1/g	F_1/N	d_1/cm	$F_1 \times d_1/\text{N cm}$	m_2/g	F_2/N	d_2/cm	$F_2 \times d_2/\text{N cm}$

1.5 FORCES

F_1 is trying to turn the ruler anticlockwise and $F_1 \times d_1$ is its moment. F_2 is trying to cause clockwise turning and its moment is $F_2 \times d_2$. When the ruler is balanced or, as we say, *in equilibrium*, the results should show that the anticlockwise moment $F_1 \times d_1$ equals the clockwise moment $F_2 \times d_2$.

- 12 Name the variables you will need to measure in this experiment.
- 13 Calculate the moments of a force of 5 N acting at a perpendicular distance from the pivot of
- 10 cm
 - 15 cm
 - 30 cm.

Principle of moments

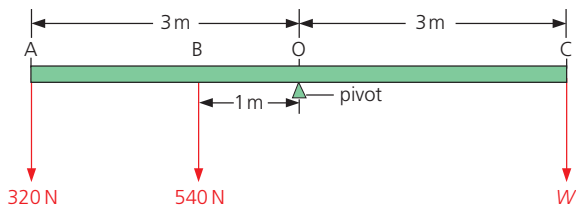
The **principle of moments** (also called the law of the lever) is stated as follows:

When a body is in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point. There is no resultant moment on an object in equilibrium.

Another way of saying this is that the sum of the moments is zero when the body is in equilibrium.

? Worked example

The seesaw in Figure 1.5.22 balances when Shani of weight 320 N is at A, Tom of weight 540 N is at B and Harry of weight W is at C. Find W .



▲ Figure 1.5.22

Taking moments about the pivot, O:

$$\begin{aligned} \text{anticlockwise moment} &= (320 \text{ N} \times 3 \text{ m}) + (540 \text{ N} \times 1 \text{ m}) \\ &= 960 \text{ Nm} + 540 \text{ Nm} \\ &= 1500 \text{ Nm} \end{aligned}$$

$$\text{clockwise moment} = W \times 3 \text{ m}$$

By the law of moments,

$$\text{clockwise moments} = \text{anticlockwise moments}$$

$$\therefore W \times 3 \text{ m} = 1500 \text{ Nm}$$

$$\therefore W = \frac{1500 \text{ Nm}}{3 \text{ m}} = 500 \text{ N}$$

Key definition

Principle of moments when a body is in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about the same point

Test yourself

- 18 A seesaw has a weight of 40 N placed 1 m from the pivot and a weight of 20 N is placed on the opposite side of the pivot at a distance of 2 m from the pivot. Is the seesaw balanced?
- 19 A half-metre ruler is pivoted at its mid-point and balances when a weight of 20 N is placed at the 10 cm mark and a weight W is placed at the 45 cm mark on the ruler. Calculate the weight W .

Lever

A lever is any device which can turn about a pivot. In a working lever a force called the **effort** is used to overcome a resisting force called the **load**. The pivotal point is called the **fulcrum**.

If we use a crowbar to move a heavy boulder (Figure 1.5.23), our hands apply the effort at one end of the bar and the load is the force exerted by the boulder on the other end. If distances from the fulcrum O are as shown and the load is 1000 N (i.e. the part of the weight of the boulder supported by the crowbar), the effort can be calculated from the law of moments. As the boulder just begins to move, we can say, taking moments about O, that

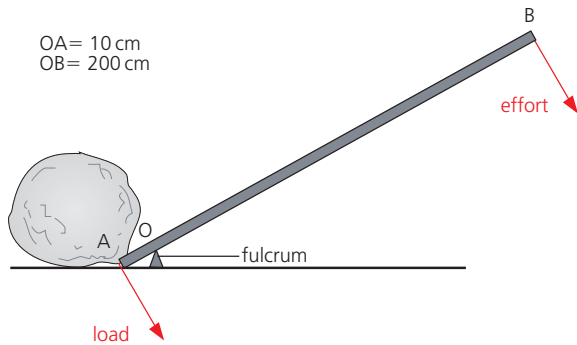
$$\text{clockwise moment} = \text{anticlockwise moment}$$

$$\text{effort} \times 200 \text{ cm} = 1000 \text{ N} \times 10 \text{ cm}$$

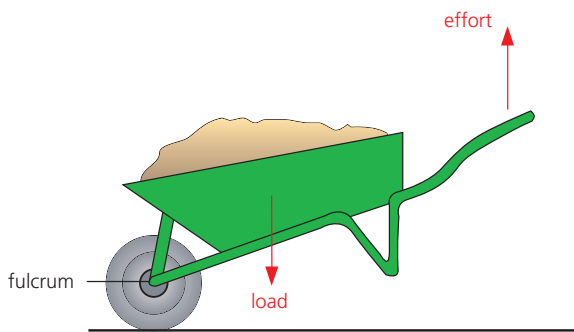
$$\text{effort} = \frac{10000 \text{ N cm}}{200 \text{ cm}} = 50 \text{ N}$$

Examples of other levers are shown in Figure 1.5.24.

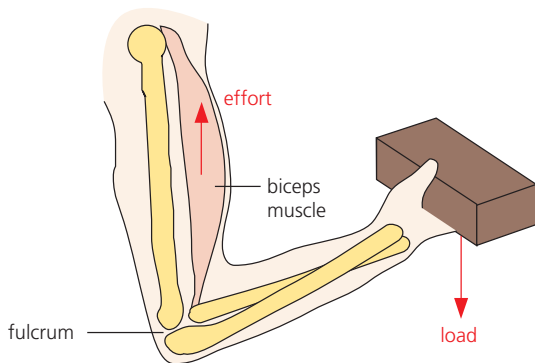
How does the effort compare with the load for scissors and a spanner in Figures 15.24c and d?



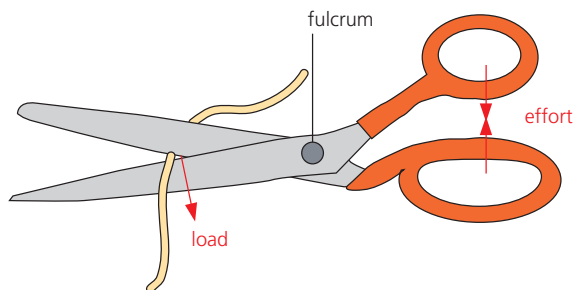
▲ Figure 1.5.23 Crowbar



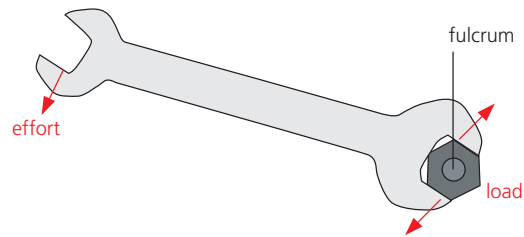
▲ Figure 1.5.24a Wheelbarrow



▲ Figure 1.5.24b Forearm



▲ Figure 1.5.24c Scissors



▲ Figure 1.5.24d Spanner

Conditions for equilibrium

Sometimes a number of parallel forces act on an object so that it is in **equilibrium**. We can then say:

- (i) The sum of the forces in one direction equals the sum of the forces in the opposite direction.
- (ii) The law of moments must apply.

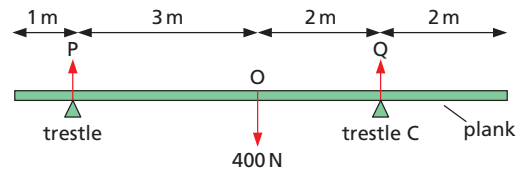
When there is no resultant force and no resultant moment, an object is in equilibrium.

Key definition

Equilibrium when there is no resultant force and no resultant moment

As an example, consider a heavy plank resting on two trestles, as in Figure 1.5.25. In Topic 1.5.5 we will see that the whole weight of the plank (400 N) may be taken to act vertically downwards at its centre, O. If P and Q are the upward forces exerted by the trestles on the plank contact forces, then we have from (i) above:

$$P + Q = 400 \text{ N} \quad (1)$$



▲ Figure 1.5.25

Moments can be taken about any point but if we take them about C, the moment due to force Q is zero.

$$\text{clockwise moment} = P \times 5 \text{ m}$$

$$\text{anticlockwise moment} = 400 \text{ N} \times 2 \text{ m} = 800 \text{ N m}$$

Since the plank is in equilibrium we have from (ii) above:

$$P \times 5 \text{ m} = 800 \text{ N m}$$

$$\therefore P = \frac{800 \text{ N m}}{5 \text{ m}} = 160 \text{ N}$$

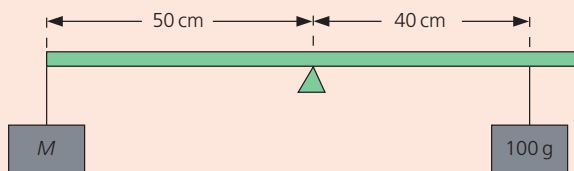
From equation (1)

$$Q = 240 \text{ N}$$

Test yourself

20 The metre ruler in Figure 1.5.26 is pivoted at its centre. If it balances, which of the following equations gives the mass of M ?

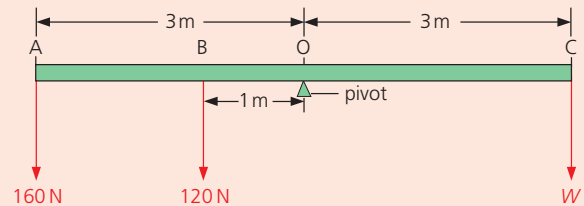
- A $M + 50 = 40 + 100$
- B $M \times 40 = 100 \times 50$
- C $M \times 50 = 100 \times 40$
- D $M/50 = 40/100$



▲ Figure 1.5.26

21 A seesaw has a weight of 60 N placed 0.5 m from the pivot and a weight of 20 N is placed on the opposite side of the pivot at a distance of 1.5 m from the pivot. Is the seesaw balanced? Justify your answer.

22 The beam shown in Figure 1.5.27 is balanced with weights of 160 N, 120 N and W in the positions shown. Calculate the value of W .



▲ Figure 1.5.27

1.5.5 Centre of gravity

FOCUS POINTS

- ★ Define centre of gravity and the effect its position has on the stability of an object.
- ★ Be familiar with an experiment determining the position of the centre of gravity of an irregularly shaped plane lamina.

Why are tall vehicles more likely to topple over on a slope than less tall ones? The answer lies in the position of the centre of gravity. In the presence of gravity an object behaves as if its entire mass is concentrated at a single point, the centre of gravity. The object's weight appears to act at this point. For a symmetrical object, such as a ball, the centre of gravity will be at its centre. In this topic, you will learn that when an object is suspended so that it can swing freely, it comes to rest with its centre of gravity vertically below the point of suspension. This enables the centre of gravity of unsymmetrical objects to be located. You will discover that it is the position of the centre of gravity that controls stability against toppling. If the centre of gravity remains within the footprint of the base of the object, it remains stable.

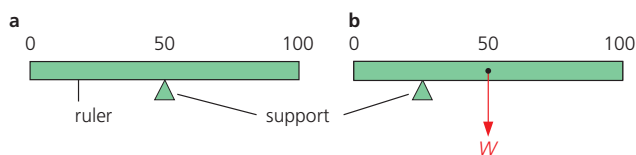
Centre of gravity

An object behaves as if its whole mass were concentrated at one point, called its **centre of gravity** even though the Earth attracts every part of it. The object's weight can be considered to act at this point. The centre of gravity of a uniform ruler is at its centre and when supported there it can be balanced, as in Figure 1.5.28a. If it is supported at

any other point it topples because the moment of its weight W about the point of support is not zero, as in Figure 1.5.28b. The centre of gravity is sometimes also termed the centre of mass.

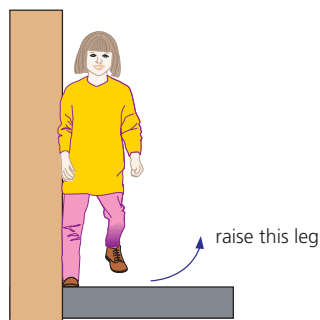
Key definition

Centre of gravity the point through which all of an object's weight can be considered to act



▲ **Figure 1.5.28**

Your centre of gravity is near the centre of your body and the vertical line from it to the floor must be within the area enclosed by your feet or you will fall over. You can test this by standing with one arm and the side of one foot pressed against a wall (Figure 1.5.29). Now try to raise the other leg sideways.



▲ **Figure 1.5.29** Can you do this without falling over?

A tightrope walker has to keep their centre of gravity exactly above the rope. Some carry a long pole to help them to balance (Figure 1.5.30). The combined weight of the walker and pole is then spread out more and if the walker begins to topple to one side, they move the pole to the other side.



▲ **Figure 1.5.30** A tightrope walker using a long pole

The centre of gravity of a regularly shaped object that has the same density throughout is at its centre. In other cases, it can be found by experiment.

Practical work

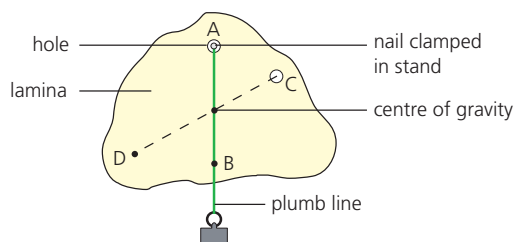
Centre of gravity of an irregularly shaped lamina

Suppose we have to find the centre of gravity of an irregularly shaped lamina (a thin sheet) of cardboard.

Make a hole A in the lamina and hang it so that it can swing *freely* on a nail clamped in a stand. It will come to rest with its centre of gravity vertically below A . To locate the vertical line through A , tie a plumb line (a thread and a weight) to the nail (Figure 1.5.31), and mark its position AB on the lamina. The centre of gravity lies somewhere on AB .

Hang the lamina from another position, C , and mark the plumb line position CD . The centre of gravity lies on CD and must be at the point of intersection of AB and CD . Check this by hanging

the lamina from a third hole. Also try balancing it at its centre of gravity on the tip of your forefinger.



▲ **Figure 1.5.31**

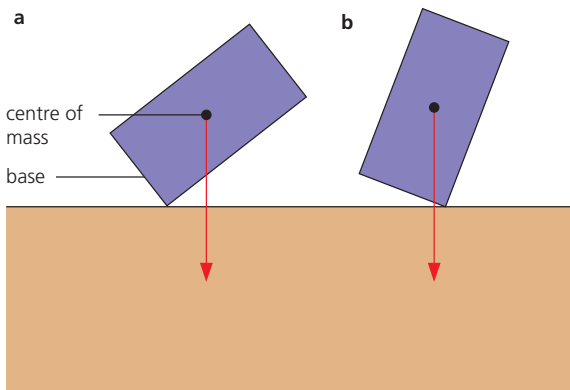
- 14 a How could you make a plumb line?
- b Explain the purpose and use of a plumb line.
- 15 When an object is suspended and allowed to swing freely, where does its centre of gravity lie when it comes to rest?

1.5 FORCES

Stability

The position of the centre of gravity of an object affects whether or not it will be stable when tilted. This is important in the design of such things as tall vehicles (which tend to overturn when rounding a corner), racing cars, reading lamps and even drinking glasses.

An object topples (falls over) when the vertical line through its centre of gravity falls outside its base, as in Figure 1.5.32a. Otherwise it remains stable, as in Figure 1.5.32b, where the object will not topple.



▲ **Figure 1.5.32**

Toppling can be investigated by placing an empty can on a plank (with a rough surface to prevent slipping) which is slowly tilted. The angle of tilt is noted when the can falls over. This is repeated with a mass of 1 kg in the can. How does this affect the position of the centre of gravity? The same procedure is followed with a second can of the same height as the first but of greater width. It will be found that the second can with the mass in it can be tilted through the greater angle.

The stability of a body is therefore increased by
(i) lowering its centre of gravity, and
(ii) increasing the area of its base.

In Figure 1.5.33a the centre of gravity of a fire truck is being found. It is necessary to do this when testing a new design since fire trucks are often driven over sloping surfaces and any tendency to overturn must be discovered.

The stability of a coach is being tested in Figure 1.5.33b. When the top deck only is fully laden with passengers (represented by sand bags in the test), it must not topple if tilted through an angle of 28° .

Racing cars have a low centre of gravity and a wide wheelbase for maximum stability.



▲ **Figure 1.5.33a** A fire truck under test to find its centre of gravity



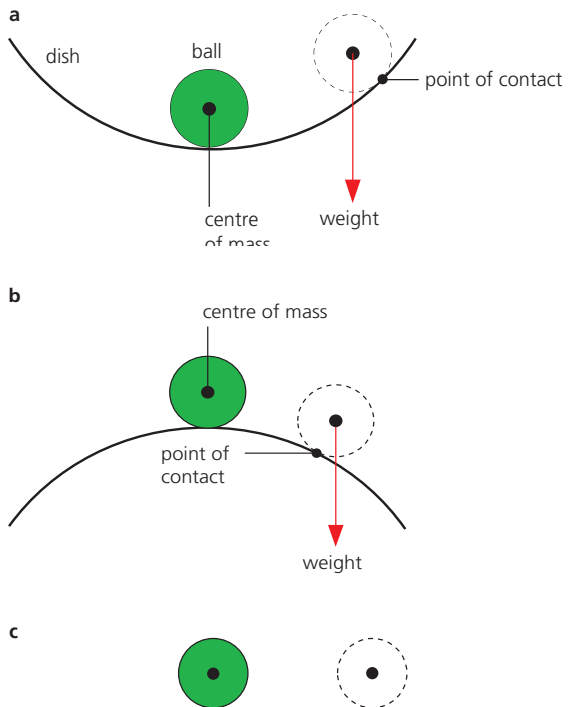
▲ **Figure 1.5.33b** A coach being tilted to test its stability

➔ Going further

Three terms are used in connection with stability.

Stable equilibrium

An object is in stable equilibrium if when slightly displaced and then released it returns to its previous position. The ball at the bottom of the dish in Figure 1.5.34a is an example. Its centre of gravity rises when it is displaced. It rolls back because its weight has a moment about the point of contact that acts to reduce the displacement.



▲ **Figure 1.5.34** States of equilibrium

Unstable equilibrium

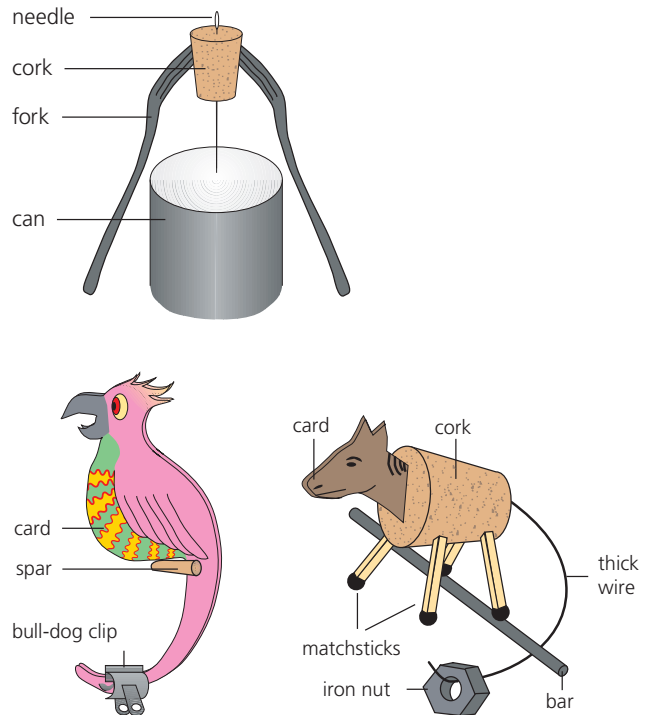
An object is in unstable equilibrium if it moves further away from its previous position when slightly displaced and released. The ball in Figure 1.5.34b behaves in this way. Its centre of gravity falls when it is displaced slightly because there is a moment which increases the displacement. Similarly, in Figure 1.5.28a the balanced ruler is in unstable equilibrium.

Neutral equilibrium

An object is in neutral equilibrium if it stays in its new position when displaced (Figure 1.5.34c). Its centre of gravity does not rise or fall because there is no moment to increase or decrease the displacement.

Balancing tricks and toys

Some tricks that you can try or toys you can make are shown in Figure 1.5.35. In each case the centre of gravity is vertically below the point of support and equilibrium is stable.



▲ **Figure 1.5.35** Balancing tricks

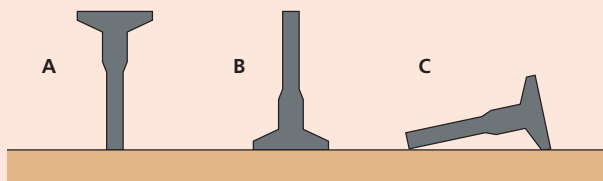
A self-righting toy (Figure 1.5.36) has a heavy base and, when tilted, the weight acting through the centre of gravity has a moment about the point of contact. This restores it to the upright position.



▲ **Figure 1.5.36** A self-righting toy

Test yourself

- 23 Where does the centre of gravity lie for a
 a uniform ruler
 b sphere of uniform density?
- 24 a When does an object topple?
 b How can the stability of an object be increased?
- 25 Figure 1.5.37 shows a Bunsen burner in three different positions. State the type of equilibrium when it is in position
 i A
 ii B
 iii C.



▲ Figure 1.5.37

Revision checklist

After studying Topic 1.5 you should know and understand:

- ✓ the significance of the term limit of proportionality
- ✓ Newton's first law of motion
- ✓ friction as the force between two surfaces that impedes motion and results in heating and that friction also acts on an object moving through the air or a liquid
- ✓ that Newton's third law of motion describes pairs of forces of the same type acting on different objects
- ✓ the conditions for equilibrium
- ✓ that an object's weight acts through the centre of gravity.

After studying Topic 1.5 you should be able to:

- ✓ recall that a force can cause a change in the motion, size or shape of a body
- ✓ identify and show different types of force on free-body diagrams
- ✓ describe an experiment to study the relation between force and extension for springs; plot and draw conclusions from load–extension graphs
- ✓ define the spring constant and use the equation $\text{spring constant} = \text{force}/\text{extension}$ ($k = F/x$) to solve problems

- ✓ combine forces acting along the same straight line to find their resultant
- ✓ recall the equation resultant force = mass \times acceleration ($F = ma$) and use it to solve problems
- ✓ define the thinking distance, braking distance and stopping distance for a vehicle when a driver applies the brakes and explain the factors that affect thinking and braking distance
- ✓ explain how an object falling in a fluid reaches terminal velocity
- ✓ describe qualitatively motion in a circular path due to a perpendicular force and recall that the force required to maintain circular motion changes when the speed, radius of orbit or mass changes
- ✓ define the moment of a force about a pivot and give everyday examples; recall the principle of moments and use it to solve problems, including the balancing of a beam
- ✓ recall that an object behaves as if its whole mass acts through its centre of gravity
- ✓ describe an experiment to find the centre of gravity of an object and connect the stability of an object to the position of its centre of gravity.

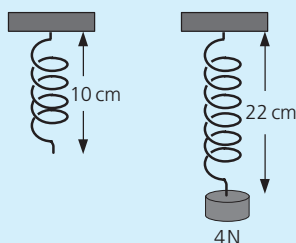
Exam-style questions

- 1 a Describe how you would investigate the variation of the extension of a spring when different loads are applied. Mention two precautions you would take to obtain accurate results. [6]
- b The table below shows the results obtained in an experiment to study the stretching of a spring. Copy the table and fill in the missing values. What can you say about the relationship between the extension of the spring and the stretching force? [4]

Mass/g	Stretching force/N	Scale reading/mm	Extension/mm
0		20.0	0
100		20.2	
200		20.4	
300		20.6	
400		20.8	
500		21.0	

[Total: 10]

- 2 The spring in Figure 1.5.38 stretches from 10 cm to 22 cm when a force of 4 N is applied.
- a Calculate the spring constant of the spring. [3]
- b If the extension is proportional to the stretching force when a force of 6 N is applied, calculate
- the new extension length of the spring [2]
 - the final length in cm of the spring. [1]



▲ Figure 1.5.38

[Total: 6]

- 3 Two forces of 5 N and 12 N act at a point.
- The two forces first act in opposite directions.
 - Make a sketch showing the direction of the forces. [2]
 - Calculate the resultant of the forces and mark its direction on your sketch. [2]
 - The two forces then act at 90° to each other. Calculate the magnitude and direction of the resultant force by calculation. [6]
- [Total: 10]
- 4 Starting from rest on a level road, a girl can reach a speed of 5 m/s in 10 s on her bicycle.
- Calculate the acceleration. [2]
 - Calculate the average speed during the 10 s. [2]
 - Calculate the distance she travels in 10 s. [2]
 - Eventually, even though she still pedals as fast as she can, she stops accelerating and her speed reaches a maximum value. Explain in terms of the forces acting why this happens. [2]
- [Total: 8]
- 5 Explain the following using $F = ma$.
- A racing car has a powerful engine and is made of strong but lightweight material. [3]
 - A car with a small engine can still accelerate rapidly. [3]
- [Total: 6]
- 6 A rocket has a mass of 500 kg.
- Calculate the weight of the rocket on Earth where $g = 9.8 \text{ N/kg}$. [1]
 - At lift-off the rocket engine exerts an upward force of 25 000 N.
 - Calculate the resultant force on the rocket. [2]
 - Calculate the initial acceleration of the rocket. [3]
- [Total: 6]

1.5 FORCES

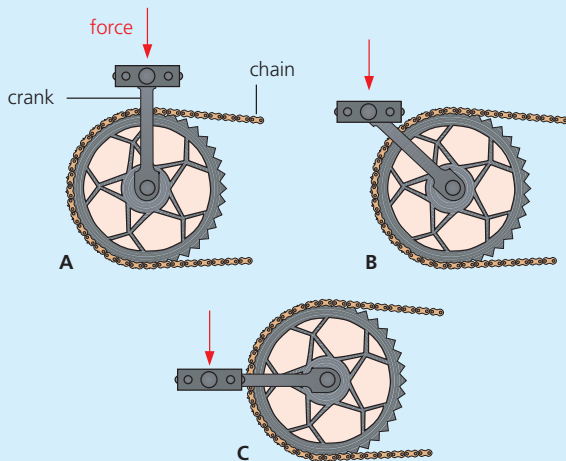
- 7 A car rounding a bend travels in an arc of a circle.
- What provides the force to keep the car travelling in a circle? [2]
 - Is a larger or a smaller force required if
 - the car travels faster [1]
 - the bend is less curved [1]
 - the car has more passengers? [1]
 - Explain why racing cars are fitted with tyres called 'slicks', which have no tread pattern, for dry tracks and with 'tread' tyres for wet tracks. [2]

[Total: 7]

- 8 Figure 1.5.39 shows three positions of the pedal on a bicycle which has a crank 0.20 m long. The cyclist exerts the same vertically downward push of 25 N with his foot. Calculate the turning effect in

- A [2]
- B [2]
- C. [2]

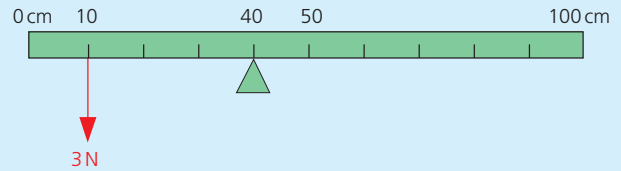
[Total: 6]



▲ Figure 1.5.39

- 9 The weight of the uniform bar in Figure 1.5.40 is 10 N and it is 100 cm long.
- Calculate the clockwise moment about the pivot. [3]
 - Calculate the anticlockwise moment about the pivot. [3]
 - Does the beam balance, tip to the right or tip to the left? [2]

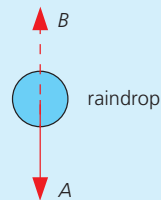
[Total: 8]



▲ Figure 1.5.40

- 10 a Describe how you could find the centre of gravity of an irregular lamina. [5]
- b A heavy box with a square base and a height twice the length of a side is to be transported by a lorry. Explain how the stability of
- the lorry [2]
 - the box [2]
- will be affected if the box lies on its side in the van rather than its base. [Total: 9]

- 11 Figure 1.5.41 shows the forces acting on a raindrop which is falling to the ground.



▲ Figure 1.5.41

- A is the force which causes the raindrop to fall. Give the name of this force. [1]
 - B is the total force opposing the motion of the drop. State *one* possible cause of this force. [1]
- What happens to the drop when force $A =$ force B ? [2]

[Total: 4]

- 12 A car is travelling at a constant speed of 20 m/s. The reaction time of the driver is 0.7 s and there is a road block 50 m ahead.
- Calculate the driver's thinking distance. [2]
 - The car comes to a halt at the road block. Calculate the braking distance. [1]
 - State **two** factors that can affect
 - thinking distance [2]
 - braking distance. [2]

[Total: 7]

Alternative to Practical

13 A physics class is asked to investigate the extension of a stretched spring. You will be supplied with a spring, a clamp stand, a half-metre ruler, a set square and a hanger with 100 g weights and sticky tape.

- a** Describe how you would carry out the experiment. [5]
b Suggest any precautions you would take to achieve good results. [3]

[Total: 8]

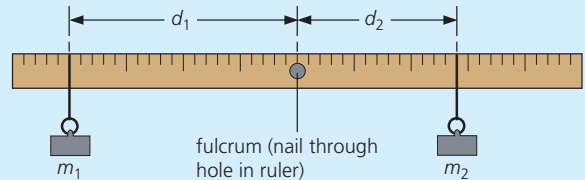
14 a The table below shows the extension of a spring for increasing stretching forces.

Stretching force/N	0	1	2	3	4	5
Extension/mm	0	2	4	6	8.5	12

- i** Plot a graph with extension/mm along the x -axis and stretching force/N on the y -axis. [4]
ii Draw the best line through the points; mark the region over which proportionality holds. [2]
iii Indicate the limit of proportionality. [1]
b Calculate the gradient of the graph. [2]
c Determine the spring constant k . [1]

[Total: 10]

15 In an experiment to investigate the law of moments, a half-metre ruler is balanced at its centre as shown in Figure 1.5.42.



▲ Figure 1.5.42

Masses of 50 g, 100 g and 150 g are placed in turn at the positions given in the table below.

- a** Complete the table, filling in values for
i the units at the head of each column [1]
ii force (F) [2]
iii distance from pivot (d) [2]
iv moment about pivot ($F \times d$). [2]
b State which combinations of two different masses could be used to balance the beam. [3]

Mass/g	Force/	Ruler reading/cm	$d/$	$F \times d/$	
50		5			A
50		10			B
50		15			C
50		20			D
100		30			E
100		35			F
100		40			G
150		20			H
150		35			I

[Total: 10]

1.6

Momentum

FOCUS POINTS

- ★ Define momentum, impulse and resultant force and use the correct equations to calculate them.
- ★ Solve simple one-dimensional problems using the principle of the conservation of momentum.

When a tennis ball is struck by a racket or a gas molecule rebounds from the side of its container, their behaviour can be understood by introducing the concept of momentum. Momentum is defined as the product of mass and velocity. In a collision, momentum is conserved unless there are external forces acting such as friction. You can demonstrate conservation of momentum with a Newton's cradle (Figure 1.7.10, p. 65); the last ball in the line moves off with the same velocity as the first. Collisions generally occur over a very short interval of time; the shorter the time interval, the greater the force on the bodies involved in the collision. Crumple zones at the front and rear of a car help to prolong the collision time and reduce the force of an impact.

Momentum is a useful quantity to consider when bodies are involved in collisions and explosions. It is defined as the mass of the body multiplied by its velocity and is measured in kilogram metre per second (kg m/s) or newton second (Ns).

$$\text{momentum} = \text{mass} \times \text{velocity}$$

In symbols, momentum

$$p = mv$$

and the change in momentum

$$\Delta p = \Delta(mv)$$

A 2 kg mass moving at 10 m/s has momentum 20 kg m/s, the same as the momentum of a 5 kg mass moving at 4 m/s.

Key definition

Momentum mass \times velocity



Practical work

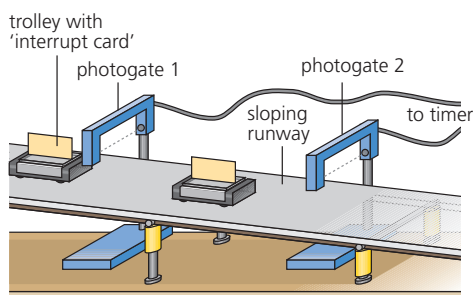
Collisions and momentum

Safety

- Take care when rolling the trolley down the ramp. Ensure it is clear at the bottom of the ramp and use a side barrier to prevent the trolley from falling onto the floor.

Figure 1.6.1 shows an arrangement which can be used to find the velocity of a trolley before and after a collision. If a trolley of length l takes time t to pass through a photogate, then its velocity = distance/time = l/t .

Two photogates are needed, placed each side of the collision point, to find the velocities before and after the collision. Set them up so that they will record the time taken for the passage of a trolley.



▲ Figure 1.6.1

A tickertape timer or motion sensor, placed at the top end of the runway, could be used instead of the photogates if preferred.

Attach a strip of Velcro to each trolley so that they adhere to each other on collision and compensate the runway for friction (see Topic 1.5.1).

Place one trolley at rest halfway down the runway and another at the top; give the top trolley a push. It will move forwards with uniform velocity and should hit the second trolley so that they travel on as one. Using the times recorded by the photogate timer, calculate the velocity of the moving trolley before the collision and the common velocity of both trolleys after the collision.

Repeat the experiment with another trolley stacked on top of the one to be pushed so that two are moving before the collision and three after.

Copy and complete the tables of results.

Before collision (m_2 at rest)		
Mass m_1 (no. of trolleys)	Velocity $v/m/s$	Momentum m_1v
1		
2		

After collision (m_1 and m_2 together)		
Mass $m_1 + m_2$ (no. of trolleys)	Velocity $v_1/m/s$	Momentum $(m_1 + m_2)v_1$
2		
3		

- Do the results suggest any connection between the momentum before the collision and after it in each case?
- Why is it necessary to tilt the runway slightly before taking measurements?
- Calculate the momentum of a 2 kg trolley moving with a velocity of
 - 0.2 m/s
 - 0.8 m/s
 - 5 cm/s.
 - Calculate the momentum of a trolley moving at 3 m/s and having a mass of
 - 200 g
 - 500 g
 - 1 kg.

Conservation of momentum

When two or more bodies act on one another, as in a collision, the total momentum of the bodies remains constant, provided no external forces act (e.g. friction).

This statement is called the **principle of conservation of momentum**. Experiments like those

in the *Practical work* section show that it is true for all types of collisions.

Key definition

Principle of conservation of momentum when two or more bodies act on one another, the total momentum of the bodies remains constant, provided no external forces act

? Worked example

Suppose a truck of mass 60 kg moving with velocity 3 m/s collides and joins with a stationary truck of mass 30 kg (Figure 1.6.2a). The two move off together with the same velocity v which we can find as follows (Figure 1.6.2b).

Total momentum before collision is

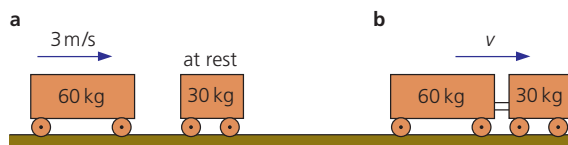
$$(60 \text{ kg} \times 3 \text{ m/s}) + (30 \text{ kg} \times 0 \text{ m/s}) = 180 \text{ kg m/s}$$

Total momentum after collision is

$$(60 \text{ kg} + 30 \text{ kg}) \times v = 90 \text{ kg} \times v$$

Since momentum is not lost

$$90 \text{ kg} \times v = 180 \text{ kg m/s} \text{ or } v = 2 \text{ m/s}$$



▲ Figure 1.6.2

Now put this into practice

- A trolley of mass 3 kg moving with velocity 5 m/s collides and joins with a stationary trolley of mass 2 kg and the two move off together with the same velocity v . Assuming momentum is not lost in the collision, calculate the value of v .
- A trolley of mass 5 kg moving with velocity 5 m/s collides with a stationary trolley of mass 2 kg. The 5 kg trolley stops and the 2 kg trolley moves off with velocity v . Assuming momentum is not lost in the collision, calculate the value of v .

Explosions

Momentum, like velocity, is a vector since it has both magnitude and direction. Vectors cannot be added by ordinary addition unless they act in the same direction. If they act in exactly opposite directions, such as east and west, the smaller subtracts from the greater or, if they are the same, they cancel out.

Momentum is conserved in an explosion such as occurs when a rifle is fired. Before firing, the total momentum is zero since both rifle and bullet are at rest. During the firing the rifle and bullet receive *equal but opposite* amounts of momentum so that the *total* momentum after firing is zero. For example, if a rifle fires a bullet of mass 0.01 kg with a velocity of 300 m/s,

$$\begin{aligned}\text{forward momentum of bullet} &= 0.01 \text{ kg} \times 300 \text{ m/s} \\ &= 3 \text{ kg m/s}\end{aligned}$$

$$\therefore \text{backward momentum of rifle} = 3 \text{ kg m/s}$$

If the rifle has mass m , it recoils (kicks back) with a velocity v such that

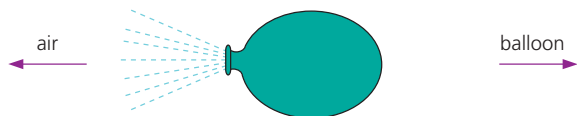
$$mv = 3 \text{ kg m/s}$$

Taking $m = 6 \text{ kg}$ gives $v = 3/6 \text{ m/s} = 0.5 \text{ m/s}$.

Rockets and jets

If you release an inflated balloon with its neck open, it flies off in the opposite direction to that of the escaping air. In Figure 1.6.3 the air has momentum to the left and the balloon moves to the right with equal momentum.

This is the principle of rockets and jet engines. In both, a high-velocity stream of hot gas is produced by burning fuel and leaves the exhaust with large momentum. The rocket or jet engine itself acquires an equal forward momentum. Space rockets carry their own oxygen supply; jet engines use the surrounding air.



▲ **Figure 1.6.3** A deflating balloon demonstrates the principle of a rocket or a jet engine.

Test yourself

- What is the momentum in kg m/s of a 10 kg truck travelling at
 a 5 m/s b 20 cm/s c 36 km/h?
- A ball X of mass 1 kg travelling at 2 m/s has a head-on collision with an identical ball Y at rest. X stops and Y moves off. What is Y's velocity?
- A boy with mass 50 kg running at 5 m/s jumps on to a 20 kg trolley travelling in the same direction at 1.5 m/s. What is their common velocity?
- A girl of mass 50 kg jumps out of a rowing boat of mass 300 kg on to the bank, with a horizontal velocity of 3 m/s. With what velocity does the boat begin to move backwards?

Force and momentum

If a steady force F acting on an object of mass m increases its velocity from u to v in time Δt , the acceleration a is given by

$$a = (v - u)/\Delta t$$

Substituting for a in $F = ma$,

$$F = \frac{m(v - u)}{\Delta t} = \frac{mv - mu}{\Delta t}$$

We also have

$$\text{impulse} = F\Delta t = mv - mu = \Delta(mv)$$

where mv is the final momentum, mu the initial momentum and $F\Delta t$ is called the **impulse**.

Since

$$F\Delta t = mv - mu = \Delta(mv)$$

We can write

$$F = \frac{\Delta(mv)}{\Delta t} = \frac{\Delta p}{\Delta t}$$

and define the **resultant force** F as the change in momentum per unit time:

$$\text{resultant force} = \frac{\text{change in momentum}}{\text{time taken}}$$

This is another version of Newton's second law. For some problems it is more useful than $F = ma$.

Key definitions

Impulse force \times time for which force acts

Resultant force the change in momentum per unit time

Sport: impulse and collision time

The good cricketer or tennis player ‘follows through’ with the bat or racket when striking the ball (Figure 1.6.4a). The force applied then acts for a longer time, the impulse is greater and so also is the gain of momentum (and velocity) of the ball.

When we want to stop a moving ball such as a cricket ball, however, its momentum has to be reduced to zero. An impulse is then required in the form of an opposing force acting for a certain time. While any number of combinations of force and time will give a particular impulse, the ‘sting’ can be removed from the catch by drawing back the hands as the ball is caught (Figure 1.6.4b). A smaller average force is then applied for a longer time.



▲ **Figure 1.6.4a** This cricketer is ‘following through’ after hitting the ball.



▲ **Figure 1.6.4b** To reduce the force on his hands, this cricketer should move his hands backwards as he catches the ball.

The use of sand gives a softer landing for long-jumpers (Figure 1.6.5), as a smaller stopping force is applied over a longer time. In a car crash the car’s momentum is reduced to zero in a very short time. If the time of impact can be extended by using crumple zones (see Figure 1.7.11, p. 65) and extensible seat belts, the average force needed to stop the car is reduced so the injury to passengers should also be less.



▲ **Figure 1.6.5** Sand reduces the athlete’s momentum more gently.

Test yourself

- 5 A force of 5 N is applied to a cricket ball for 0.02 s. Calculate
- the impulse on the ball
 - the change in momentum of the ball.
- 6 In a collision, a car of mass 1000 kg travelling at 24 m/s comes to rest in 1.2 s. Calculate
- the change in momentum of the car
 - the steady stopping force applied to the car.

Revision checklist

After studying Topic 1.6 you should know and understand the following:

- ✓ the relationship between force and rate of change of momentum and use it to solve problems.

After studying Topic 1.6 you should be able to:

- ✓ define momentum and apply the principle of conservation of momentum to solve problems
- ✓ recall that in a collision, impulse = $F\Delta t = \Delta(mv)$ and use the definition to explain how the time of impact affects the resultant force acting in a collision.

Exam-style questions

- 1 A truck A of mass 500 kg moving at 4 m/s collides with another truck B of mass 1500 kg moving in the same direction at 2 m/s.
- State an expression for momentum. [1]
 - Calculate the momentum of truck A before the collision. [2]
 - Calculate the momentum of truck B before the collision. [2]
 - Determine the common velocity of the trucks after the collision. [4]
- [Total: 9]
- 2 The velocity of an object of mass 10 kg increases from 4 m/s to 8 m/s when a force acts on it for 2 s. Calculate the
- initial momentum of the object [2]
 - final momentum of the object [2]
 - momentum gained in 2 s [2]
 - value of the force [2]
 - impulse of the force. [2]
- [Total: 10]
- 3 A rocket of mass 10 000 kg uses 5.0 kg of fuel and oxygen to produce exhaust gases ejected at 5000 m/s.
- Define momentum. [1]
 - Calculate the backward momentum of the ejected gas. [2]
 - Explain what is meant by the principle of conservation of momentum. [2]
 - Calculate the increase in velocity of the rocket. [3]
- [Total: 8]
- 4 A boy hits a stationary billiard ball of mass 30 g head on with a cue. The cue is in contact with the ball for a time of 0.001 s and exerts a force of 50 N on it.
- Calculate the acceleration of the ball during the time it is in contact with the cue. [2]
 - Calculate the impulse on the ball in the direction of the force. [2]
 - Calculate the velocity of the ball just after it is struck. [2]
 - Give *two* ways by which the velocity of the ball could be increased. [2]
- [Total: 8]

1.7

Energy, work and power

1.7.1 Energy

FOCUS POINTS

- ★ Identify different energy stores and describe how energy is transferred from one store to another.
- ★ Use the correct equations for kinetic energy and change in gravitational potential energy.
- ★ Apply the principle of the conservation of energy to simple examples and use it to interpret flow diagrams.
- ★ Apply the principle of the conservation of energy to complex examples represented by Sankey diagrams.

Energy is a theme that appears in all branches of science. It links a wide range of phenomena and enables us to explain them. There are different ways in which energy can be stored and, when something happens, it is likely to be due to energy being transferred from one store to another. Energy transfer is needed to enable people, computers, machines and other devices to work and to enable processes and changes to occur. For example, in Figure 1.7.1, the water skier can be pulled along by the boat only if energy is transferred from the burning petrol to its rotating propeller. Although energy can be transferred to different stores, the total energy of a system remains constant. In this topic you will learn in detail about the potential energy associated with the position of an object in a gravitational field and the kinetic energy which is associated with its motion.



▲ **Figure 1.7.1** Energy transfer in action

Energy stores

Energy can be stored in a number of different ways.

Key definition

Energy may be stored as kinetic, gravitational potential, chemical, elastic (strain), nuclear, electrostatic and internal (thermal) energy

Chemical energy

Food and fuels, like oil, gas, coal and wood, are concentrated stores of **chemical energy**. The energy of food is released by chemical reactions in our bodies, and during the transfer to other stores we are able to do useful jobs. Fuels cause **energy transfers** when they are burnt in an engine or a boiler. Batteries are compact sources of chemical energy, which can be transferred by an electric current to devices like lamps, heaters or motors (electrical working).

Gravitational potential energy

This is the energy an object has because of its position. A body above the Earth's surface, like water in a mountain reservoir, has energy stored as **gravitational potential energy**.

Elastic strain energy

This is energy an object has because of its condition. Work has to be done to compress or stretch a spring or elastic material and energy is transferred to **elastic strain energy**. If the bow string in Figure 1.7.3c on the next page were released, the strain energy would be transferred to the arrow.

Kinetic energy

Any moving object has **kinetic energy** and the faster it moves, the more kinetic energy it has. As a hammer drives a nail into a piece of wood, there is a transfer of energy from the kinetic energy of the moving hammer to other energy stores.

Electrostatic energy

Energy can be stored by charged objects (see Topic 4.2.1) as **electrostatic energy**. This energy can be transferred by an electric current.

Nuclear energy

The energy stored in the nucleus of an atom is known as **nuclear energy**. It can be transferred to other energy stores in nuclear reactions such as fission and fusion (Topic 5.1.2).

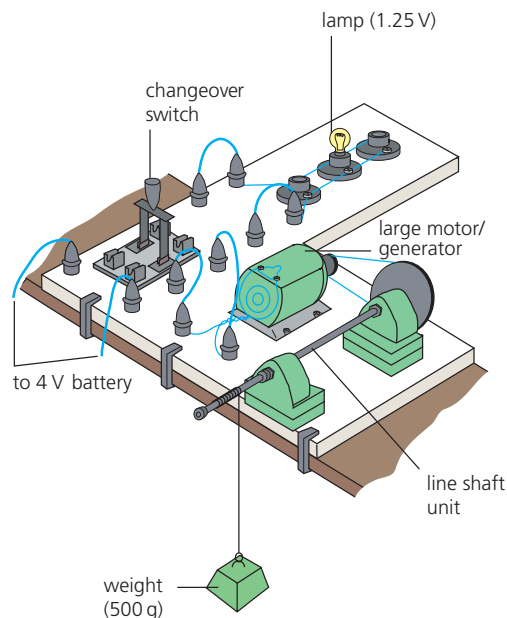
Internal energy

This is also called **thermal energy** and is the final fate of other energy stores. It is transferred by conduction, convection or radiation.

Energy transfers

Demonstration

The apparatus in Figure 1.7.2 can be used to show how energy is transferred between different energy stores. Chemical energy stored in the battery is transferred by an electric current (electrical working) to kinetic energy in the electric motor. The weight is raised when kinetic energy stored in the motor is transferred (by mechanical working) to gravitational potential energy stored in the weight. If the changeover switch is joined to the lamp and the weight allowed to fall, the motor acts as a **generator** of an electric current that transfers (by electrical working) kinetic energy stored in the rotating coil of the generator to internal energy in the lamp. Energy is transferred from the lamp to the environment (by electromagnetic waves and by heating).



▲ **Figure 1.7.2** Demonstrating energy transfers

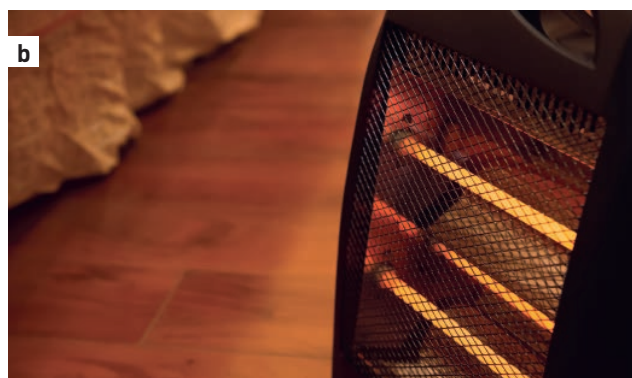
Other examples

In addition to electrical working, mechanical working, electromagnetic waves and heating, energy can be transferred between stores by other types of waves, such as sound waves. Sound waves transfer energy from a vibrating source to our eardrums or

to a microphone. Heating water in a boiler transfers chemical energy stored in a fuel to internal energy stored in the water.

In summary, energy can be transferred between stores in the following ways:

- mechanical working – by the action of a force (Topic 1.5)
- electrical working – by an electric current (Topic 4.2.2)
- waves – electromagnetic, sound and other waves (Topic 3.3)
- heating – by conduction, convection or radiation (Topic 2.3).



▲ **Figure 1.7.3** Some energy transfers

Measuring energy transfers

In an energy transfer, work is done. The work done is a measure of the amount of energy transferred. Energy, as well as work, is measured in joules (J).

For example, if you have to exert an upward force of 10 N to raise a stone steadily through a vertical distance of 1.5 m, the mechanical work done is 15 J (see Topic 1.7.2).

$$\text{work} = \text{force} \times \text{distance moved in the direction of force}$$

Some energy transfers are shown in Figures 1.7.3a to d.

- Potential energy is transferred to kinetic energy by mechanical working (action of a gravitational force).
- Thermal energy stored in an electric fire element is transferred by electromagnetic waves and by heating to the environment.
- Chemical energy (stored in muscles in the arm) is transferred to elastic energy in the bow by mechanical working.
- Gravitational potential energy stored in the water in the upper reservoir is transferred to the kinetic energy of a turbine by mechanical working.

This is also the amount of chemical energy transferred from your muscles to the gravitational potential energy of the stone.

Principle of conservation of energy

The **principle of conservation of energy** is one of the basic laws of physics.

Key definition

Principle of conservation of energy energy cannot be created or destroyed; it is always conserved

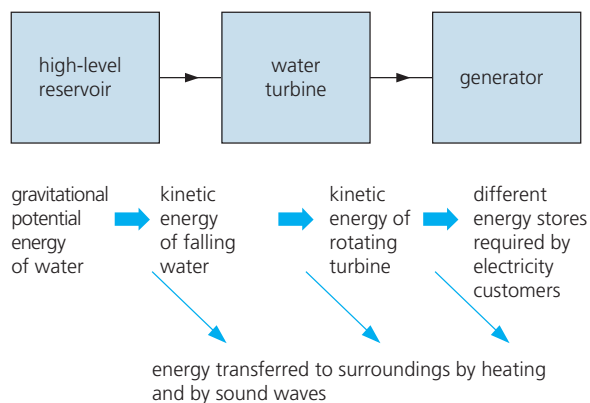
1.7 ENERGY, WORK AND POWER

However, energy is continually being transferred from one store to another. Some stores, such as those of electrostatic and chemical energy, are easily transferred; for others, such as internal energy, it is hard to arrange a useful transfer.

Ultimately all energy transfers result in the surroundings being heated (as a result of doing work against friction) and the energy is wasted, i.e. spread out and increasingly more difficult to use. For example, when a brick falls, its gravitational potential energy is transferred by mechanical working (gravitational force) to kinetic energy; when the brick hits the ground, kinetic energy is transferred to the surroundings by heating and by sound waves. If it seems in a transfer that some energy has disappeared, the 'lost' energy is often transferred into non-useful thermal energy. This appears to be the fate of all energy in the Universe and is one reason why new sources of useful energy have to be developed.

Representing energy transfers

- The flow diagram of energy transfers for a hydroelectric power station like that in Figure 1.7.3d is shown in Figure 1.7.4.



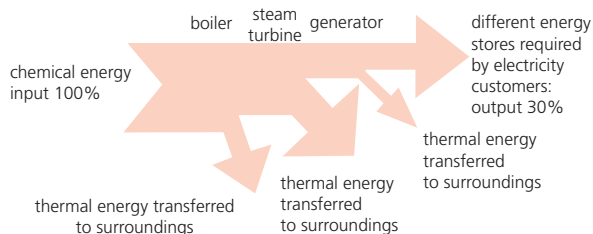
▲ **Figure 1.7.4** Energy transfers in a hydroelectric power station



▲ **Figure 1.7.6** Kinetic energy depends on the square of the velocity.

➔ Going further

- In thermal power stations, thermal energy transferred from burning fossil fuels heats the water in a boiler and turns it into steam. The steam drives turbines which in turn drive the generators that produce electricity as described in Topic 4.5. Figure 1.7.5 shows a Sankey diagram for a thermal power station, where the thickness of the bars represents the size of energy transfer at each stage.



▲ **Figure 1.7.5** Sankey diagram depicting energy transfers in a thermal power station

Kinetic energy (E_k)

Kinetic energy is the energy an object has because of its motion.

For an object of mass m travelling with velocity v ,

$$\text{kinetic energy} = E_k = \frac{1}{2}mv^2$$

If m is in kg and v in m/s, then kinetic energy is in J.

Since E_k depends on v^2 , a high-speed vehicle travelling at 1000 km/h (Figure 1.7.6), has one hundred times the kinetic energy it has at 100 km/h.

? Worked example

Calculate the kinetic energy of a football of mass 0.4 kg (400 g) moving with a speed of 20 m/s.

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 0.4 \text{ kg} \times (20 \text{ m/s})^2 \\ &= 0.2 \times 400 \text{ kg m}^2/\text{s}^2 \\ &= 80 \text{ Nm} = 80 \text{ J} \end{aligned}$$

Now put this into practice

- 1 Calculate the kinetic energy of a ball of mass 0.4 kg moving with a speed of 80 m/s.
- 2 Calculate the kinetic energy of a ball of mass 50 g moving with a speed of 40 m/s.

Gravitational potential energy (E_p)

Potential energy is the energy an object has because of its position or condition.

An object above the Earth's surface is considered to have gained an amount of gravitational potential energy equal to the work that has been done against gravity by the force used to raise it. To lift an object of mass m through a vertical height Δh at a place where the Earth's gravitational field strength is g needs a force equal and opposite to the weight mg of the body. Hence

$$\begin{aligned} \text{work done by force} &= \text{force} \times \text{vertical height} \\ &= mg \times \Delta h \end{aligned}$$

$$\therefore \text{the change in gravitational potential energy} = \Delta E_p = mg\Delta h$$

When m is in kg, g in N/kg (or m/s^2) and Δh in m, then ΔE_p is in J.

? Worked example

Taking $g = 9.8 \text{ N/kg}$, calculate the potential energy gained by a 0.1 kg (100 g) mass raised vertically by 1 m.

$$\Delta E_p = mg\Delta h = 0.1 \text{ kg} \times 9.8 \text{ N/kg} \times 1 \text{ m} = 1 \text{ Nm} = 1 \text{ J}$$

Now put this into practice

- 1 Calculate the gravitational potential energy gained by a 0.2 kg mass raised vertically by 2 m.
- 2 Calculate the gravitational potential energy lost by a 0.4 kg mass which falls vertically by 3 m.



Practical work

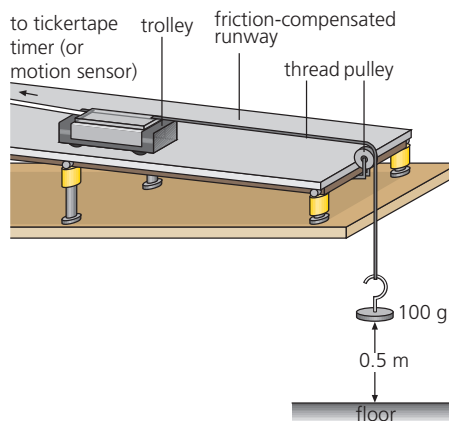
Transfer of gravitational potential energy to kinetic energy

Safety

- Place something soft on the floor to absorb the impact of the masses.
- Take care to keep feet well away from the falling masses.

Friction-compensate a runway by raising the start point slightly so that the trolley maintains a constant speed on the slope when no weight is attached. Arrange the apparatus as in Figure 1.7.7 with the bottom of the 0.1 kg (100 g) mass 0.5 m from the floor.

Start the timer and release the trolley. It will accelerate until the falling mass reaches the floor; after that it moves with *constant* velocity v .



▲ Figure 1.7.7

- 1 From your results calculate v in m/s (on the tickertape 50 ticks = 1 s). Find the mass of the trolley in kg. Work out:
Kinetic energy gained by trolley and 0.1 kg mass = ___ J
Potential energy lost by 0.1 kg mass = ___ J
Compare and comment on the results.
- 2 Explain why
 - a the runway should be friction-compensated
 - b the trolley in the experiment will move at a constant speed when the mass hits the floor.
- 3 Calculate the change in gravitational potential energy of a mass of 300 g falling through a distance of 80 cm.
- 4 Calculate the kinetic energy of a mass of 300 g travelling at a speed of 4.0 m/s.

Conservation of energy

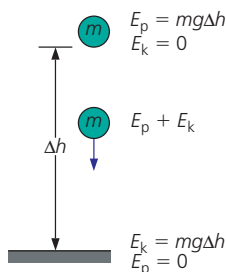
A mass m at height Δh above the ground has gravitational potential energy $= mg\Delta h$ (Figure 1.7.8). When an object falls, its speed increases and it gains kinetic energy at the expense of its gravitational potential energy. If it starts from rest and air resistance is negligible, the kinetic energy it has gained on reaching the ground equals the gravitational potential energy lost by the mass

$$E_k = \Delta E_p$$

or

$$\frac{1}{2}mv^2 = mg\Delta h$$

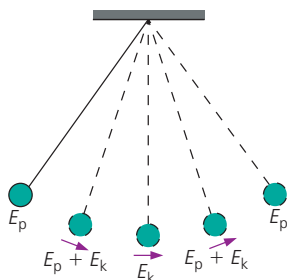
where v is the speed of the mass when it reaches the ground.



▲ **Figure 1.7.8** Loss of gravitational potential energy = gain of kinetic energy

This is an example of the principle of conservation of energy which was discussed earlier.

In the case of a pendulum (Figure 1.7.9), kinetic and gravitational potential energy are interchanged continually. The energy of the bob is all gravitational potential energy at the end of the swing and all kinetic energy as it passes through its central position. In other positions it has both gravitational potential and kinetic energy. Eventually all the energy is transferred to thermal energy as a result of overcoming air resistance.



▲ **Figure 1.7.9** Interchange of potential and kinetic energy for a simple pendulum

? Worked example

A boulder of mass 4 kg rolls over a cliff and reaches the beach below with a velocity of 20 m/s. Calculate

- the kinetic energy of the boulder as it lands
- the gravitational potential energy of the boulder when it was at the top of the cliff
- the height of the cliff.

- mass of boulder $= m = 4 \text{ kg}$
velocity of boulder as it lands $= v = 20 \text{ m/s}$

$$\begin{aligned} \therefore \text{kinetic energy of boulder as it lands} &= E_k = \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 4 \text{ kg} \times (20)^2 \text{ m}^2/\text{s}^2 \\ &= 800 \text{ kg m}^2/\text{s}^2 \\ &= 800 \text{ N m} \\ &= 800 \text{ J} \end{aligned}$$

- Applying the principle of conservation of energy (and neglecting energy lost in overcoming air resistance)
change in potential energy = kinetic energy of boulder as it lands

$$\therefore \Delta E_p = E_k = 800 \text{ J}$$

- If Δh is the height of the cliff

$$\Delta E_p = mg\Delta h$$

$$\therefore \Delta h = \frac{\Delta E_p}{mg} = \frac{800 \text{ J}}{4 \text{ kg} \times 10 \text{ m/s}^2} = \frac{800 \text{ N m}}{40 \text{ kg m/s}^2} = 20 \text{ m}$$

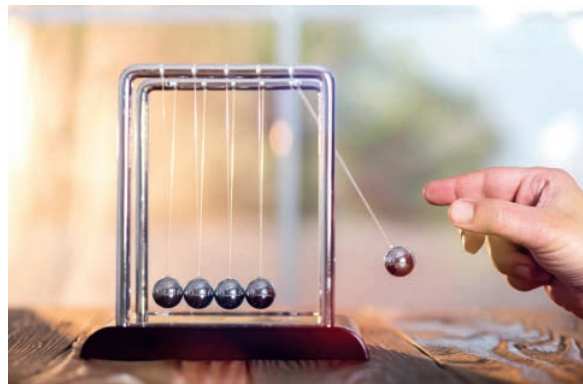
Now put this into practice

- A stone of mass 2 kg rolls off the flat roof of a building and reaches the ground with a speed of 10 m/s. Calculate
 - the kinetic energy of the stone when it reaches the ground
 - the gravitational potential energy of the stone when it was on the roof
 - the height of the roof. Neglect air resistance.
- A football of mass 0.4 kg rolls off a 30 m high cliff. Calculate the speed of the football when it lands on the beach (neglecting air resistance).

➔ Going further

Elastic and inelastic collisions

In all collisions (where no external force acts) some kinetic energy is usually transferred to thermal energy and, to a small extent, to sound. The greater the proportion of kinetic energy transferred, the less *elastic* is the collision, i.e. the more inelastic it is. In a perfectly elastic collision, kinetic energy is conserved.



▲ **Figure 1.7.10** Newton's cradle is an instructive toy for studying collisions and conservation of energy.

Car design and safety

When a car stops rapidly in a collision, large forces are produced on the car and its passengers, and their kinetic energy has to be dissipated.

Crumple zones at the front and rear collapse in such a way that the kinetic energy is absorbed gradually (Figure 1.7.11). As we saw in Topic 1.6, this extends the collision time and reduces the decelerating force and hence the potential for injury to the passengers.



▲ **Figure 1.7.11** Cars in an impact test showing the collapse of the front crumple zone

Extensible seat belts exert a backwards force (of 10000 N or so) over about 0.5 m, which is roughly the distance between the front seat occupants and the windscreen. In a car travelling at 15 m/s (54 km/h), the effect felt by anyone *not* using a seat belt is the same as that, for example, produced by jumping off a building 12 m high.

Air bags in most cars inflate and protect the driver from injury by the steering wheel.

Head restraints ensure that if the car is hit from behind, the head goes forwards with the body and not backwards over the top of the seat. This prevents damage to the top of the spine.

All these are secondary safety devices which aid *survival* in the event of an accident. Primary safety factors help to *prevent* accidents and depend on the car's roadholding, brakes, steering, handling and above all on the driver since most accidents are due to driver error.

The chance of being killed in an accident is about *five times less* if seat belts are worn and head restraints are installed.

Test yourself

- Identify the way by which energy is transferred in the following processes:
 - a battery is used to light a lamp
 - a ball is thrown upwards
 - water is heated in a boiler.
- State how energy is stored in the following:
 - fossil fuels
 - hot water
 - a rotating turbine
 - a stretched spring.
- Calculate the kinetic energy of a
 - 1 kg trolley travelling at 2 m/s
 - 2 g (0.002 kg) bullet travelling at 400 m/s
 - 500 kg car travelling at 72 km/h.
- What is the velocity of an object of mass 1 kg which has 200 J of kinetic energy?
 - Calculate the gravitational potential energy of a 5 kg mass when it is
 - 3 m and
 - 6 m, above the ground. ($g = 9.8 \text{ N/kg}$)
- It is estimated that $7 \times 10^6 \text{ kg}$ of water pours over the Niagara Falls every second. If the falls are 50 m high, and if all the energy of the falling water could be harnessed, what power would be available? ($g = 9.8 \text{ N/kg}$)

1.7.2 Work

FOCUS POINTS

- ★ Understand that when mechanical or electrical work is done, energy is transferred.
- ★ Use the correct equation to calculate mechanical work.

In science the word work has a different meaning from its everyday use. Here work is associated with the motion of a force. When you lift and move a heavy box upstairs you will have done work in either sense! In the absence of thermal energy being generated, the work done is a measure of the amount of energy transferred. When moving the heavy box, chemical energy from your muscles is transferred to gravitational potential energy. If an electric motor is used to move the box, an equal amount of electrical work will be done. In this topic you will learn how to calculate the mechanical work done in different situations.

Work

In an energy transfer, work is done. *The work done is a measure of the amount of energy transferred.*

The same amount of mechanical or electrical work is done in transferring equal amounts of energy.

Mechanical **work** is done when a force moves. No work is done in the scientific sense by someone standing still holding a heavy pile of books: an upward force is exerted, but no motion results.

If a building worker carries ten bricks up to the first floor of a building, they do more work than if they carry only one brick because they have to exert a larger force. Even more work is required if they carry the ten bricks to the second floor. The amount of work done W depends on the size of the force F applied and the distance d it moves. We therefore measure work by

$$\text{work} = \text{force} \times \text{distance moved in direction of force}$$

or

$$W = Fd = \Delta E$$

where ΔE is the energy transferred.

Key definition

work done = force \times distance moved in direction of force

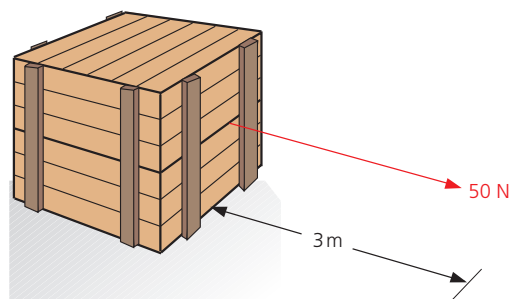
The unit of work is the **joule** (J); it is the work done when a force of 1 newton (N) moves through 1 metre (m). For example, if you have to pull with a force of 50 N to move a crate steadily 3 m in the direction of the force (Figure 1.7.12a),

the work done is $50 \text{ N} \times 3 \text{ m} = 150 \text{ Nm} = 150 \text{ J}$. That is

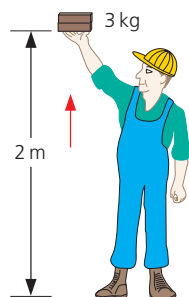
$$\text{joules} = \text{newtons} \times \text{metres}$$

If you lift a mass of 3 kg vertically through 2 m (Figure 1.7.12b), you have to exert a vertically upward force equal to the weight of the body, i.e. 30 N (approximately), and the work done is $30 \text{ N} \times 2 \text{ m} = 60 \text{ Nm} = 60 \text{ J}$.

Note that you must always take the distance in the direction in which the force acts.



▲ Figure 1.7.12a



▲ Figure 1.7.12b

Test yourself

- 6 How much work is done when a mass of 3 kg ($g = 9.8 \text{ N/kg}$) is lifted vertically through 6 m?
- 7 A hiker climbs a hill 300 m high. If she has a mass of 51 kg, calculate the work she does in lifting her body to the top of the hill.
- 8 An electric motor does 80 J of work in lifting a box vertically upwards through 5 m. Calculate the weight of the box.

1.7.3 Energy resources

FOCUS POINTS

- ★ Distinguish between renewable and non-renewable energy sources.
- ★ Understand the various ways that useful energy may be obtained or electrical power generated and give advantages and disadvantages of each method.
- ★ Understand efficiency of energy transfer.
- ★ Define efficiency and use the correct equations to calculate it.

Energy is needed to heat buildings, to make cars move, to provide artificial light, to make computers work, and so on. The list is endless. This useful energy needs to be produced in controllable energy transfers. For example, in power stations a supply of useful energy is transferred by electric currents to different energy stores required by electricity customers. The raw materials for energy production are energy sources. These may be non-renewable or renewable.

Although energy cannot be destroyed, as you learnt in the previous section, it can be transferred into non-useful stores, such as internal energy. The efficiency of a device measures the useful energy as a percentage of the total energy supplied.

You will be able to recognise many different types of **energy sources**. Such sources may be **renewable** or **non-renewable**; non-renewable sources represent previously stored energy. Sunlight is used in biological processes to store chemical energy and can be harnessed to generate electricity directly in solar cells.

Non-renewable energy sources

Once used up these cannot be replaced.

Two advantages of all non-renewable fuels are

- (i) their high **energy density** (i.e. they are concentrated sources) and the relatively small size of the energy transfer device (e.g. a furnace) which releases their energy, and
- (ii) their ready **availability** when energy demand increases suddenly or fluctuates seasonally.

Fossil fuels

Fossil fuels include coal, oil and natural gas, formed from the remains of plants and animals which lived millions of years ago and obtained energy originally from the Sun. Their energy is stored as chemical energy and at present they are our main energy source. Predictions vary as to how long they will last since this depends on what reserves are recoverable and on the future demands of a world population expected to increase from about 7700 million in 2019 to about 9700 million by the year 2050. Some estimates say oil and gas will run low early in the present century but coal should last for 200 years or so.

Burning fossil fuels in power stations and in cars pollutes the atmosphere with harmful gases such as carbon dioxide and sulfur dioxide. Carbon dioxide emission increases global warming. It is not

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immediately feasible to prevent large amounts of carbon dioxide entering the atmosphere, but less is produced by burning natural gas than by burning oil or coal; burning coal produces most carbon dioxide for each unit of energy produced. When coal and oil are burnt they also produce sulfur dioxide which causes acid rain. The sulfur dioxide can be extracted from the waste gases so it does not enter the atmosphere or the sulfur can be removed from the fuel before combustion, but these are both costly processes which increase the price of electricity produced using these measures.

Nuclear fuels

The energy released in a nuclear reactor (Topic 5.1) from the fission of uranium, found as an ore in the ground, can be used to produce electricity. **Nuclear fuels** do not pollute the atmosphere with carbon dioxide or sulfur dioxide but they do generate **radioactive** waste materials with very long half-lives (Topic 5.2); safe ways of storing this waste for perhaps thousands of years must be found. As long as a reactor is operating normally it does not pose a radiation risk, but if an accident occurs, dangerous radioactive material can leak from the reactor and spread over a large area.

Renewable energy sources

These cannot be exhausted and are generally non-polluting.

Solar energy

The energy falling on the Earth from the Sun is transferred mostly by visible light and infrared radiation and in an hour equals the total energy used by the world in a year. Unfortunately, its low energy density requires large collecting devices and its availability varies. The greatest potential use of **solar energy** is as an energy source for low-temperature water heating. The energy transferred by electromagnetic waves from the Sun is stored as internal energy in **solar panels** and can be transferred by heating to produce domestic hot water at about 70°C and to heat swimming pools.

Solar energy can also be used to produce high-temperature heating, up to 3000°C or so, if a large curved mirror (a solar furnace) focuses the Sun's rays onto a small area. The energy can then be used to turn water to steam for driving the turbine of an electric generator in a power station.

Solar cells, made from semiconducting materials, convert sunlight into electricity directly. A number of cells connected together can be used to supply electricity to homes and to the electronic equipment in communication and other satellites. They are also used for small-scale power generation in remote areas where there is no electricity supply. The energy generated by solar cells can be stored in batteries for later use. Recent developments have made large-scale generation more cost effective and large solar power plants are becoming more common (Figure 1.7.13). There are many designs for prototype light vehicles run on solar power (Figure 1.7.14).



▲ Figure 1.7.13 Solar power plant in Bahawalpur, Pakistan



▲ Figure 1.7.14 Solar-powered car

Wind energy

Infrared radiation from the Sun is also responsible for generating wind energy. Giant windmills called **wind turbines** with two or three blades each up to 30 m long drive electrical generators. Wind farms of 20 to 100 turbines spaced about 400 m apart (Figure 1.7.15) supply about 400 MW (enough

electricity for 250 000 homes) in the UK and provide a useful 'top-up' to the National Grid.

Wind turbines can be noisy and are considered unsightly by some people so there is some environmental objection to wind farms, especially as the best sites are often in coastal or upland areas of great natural beauty.



▲ **Figure 1.7.15** Wind farm turbines

Wave energy

The rise and fall of sea waves have to be transferred by some kind of **wave energy** converter into the rotary motion required to drive a generator. It is a difficult problem and the large-scale production of electricity by this means is unlikely in the near future. However, small systems are being developed to supply island communities with power.

Tidal and hydroelectric energy

The flow of water from a higher to a lower level is used to drive a water turbine (water wheel) connected to a generator. In **tidal energy**, water is held behind a tidal barrage (barrier) in an estuary. In a **hydroelectric energy** scheme, water is held behind a dam across a river.

One of the largest working tidal schemes is the La Grande I project in Canada (Figure 1.7.16). Such schemes have significant implications for the environment, as they may destroy wildlife habitats of wading birds for example, and also for shipping routes.

Over 100 years ago, India was one of the first countries to develop hydroelectric power; today such power provides about 14% of the country's electricity supply. China is the world's largest producer of hydroelectricity, generating around 20% of the country's needs. With good management, hydroelectric energy is a reliable energy source, but there are risks connected with the construction of dams, and a variety of problems may result from the

impact of a dam on the environment. Land previously used for forestry or farming may have to be flooded.



▲ **Figure 1.7.16** Tidal barrage in Canada

Geothermal energy

If cold water is pumped down a shaft into hot rocks below the Earth's surface, it may be forced up another shaft as steam. This can be used to drive a turbine and generate electricity or to heat buildings. The **geothermal energy** that heats the rocks is constantly being released by radioactive elements deep in the Earth as they decay (Topic 5.2).

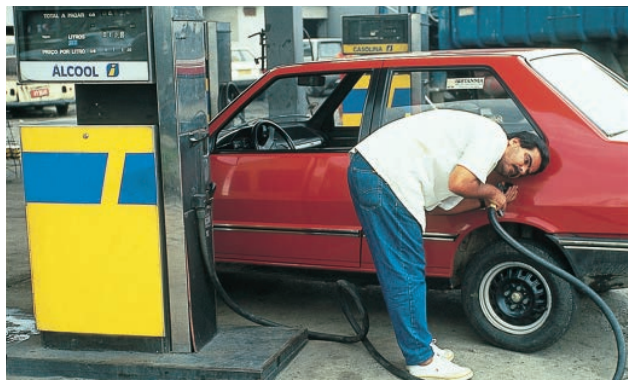
Geothermal power stations are in operation in the USA, New Zealand and Iceland. A disadvantage is that they can only be built in very specific locations where the underlying rocks are hot enough for the process to be viable.

Biofuels (vegetable fuels)

Biomass includes cultivated crops (e.g. oilseed rape), crop residues (e.g. cereal straw), natural vegetation (e.g. gorse), trees grown for their wood (e.g. spruce), animal dung and sewage. Chemical energy can be stored in **biofuels** such as alcohol (ethanol) and methane gas can be obtained from them by fermentation using enzymes or by decomposition by bacterial action in the absence of air. Liquid biofuels can replace petrol (Figure 1.7.17); although they have up to 50% less energy per litre, they are lead- and sulfur-free and so do not pollute the atmosphere with lead or sulfur dioxide when they are burned. **Biogas** is a mix of methane and carbon dioxide with an energy content about two-thirds that of natural gas. It is produced from animal and human waste in digesters (Figure 1.7.18) and used for heating and cooking. Biogas is cheap to produce on

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a small scale but not economically viable for large-scale production. It reduces landfills but due to its methane content it is unstable and may explode.



▲ **Figure 1.7.17** Filling up with biofuel in Brazil



▲ **Figure 1.7.18** Methane generator in India

➔ Going further

The Sun as an energy source

The Sun is the main source of energy for many of the energy sources described on the previous pages. The exceptions are geothermal, nuclear and tidal sources. Fossil fuels such as oil, coal and gas are derived from plants which grew millions of years ago in biological processes requiring light from the Sun. Sunlight is also needed by the plants used in biomass energy production today. Energy from the Sun drives the weather systems which enable wind and hydroelectric power to be harnessed. Solar energy is used directly in solar cells for electricity generation.

The source of the Sun's energy is nuclear fusion in the Sun. You will learn more about the fusion process which produces large amounts of energy in Topic 5.1. At present it is not possible to reproduce the fusion process on Earth for the large-scale production of electricity but much research is being directed towards that goal.

Power stations

The processes involved in the production of electricity at power stations depend on the energy source being used.

Non-renewable sources

Fossil fuels and nuclear fuels are used in **thermal power stations** to provide thermal energy that turns water into steam. The steam drives turbines which in turn drive the generators that produce electric current as described in Topic 4.5. If fossil fuels are the energy source (usually coal but natural gas is favoured in new stations), the steam is obtained from a boiler. If nuclear fuel is used, such as uranium or plutonium, the steam is produced in a heat exchanger as explained in Topic 5.1.

The action of a **steam turbine** resembles that of a water wheel but moving steam, not moving water, causes the motion. Steam enters the turbine and is directed by the **stator** or diaphragm (sets of fixed blades) onto the **rotor** (sets of blades on a shaft that can rotate) (Figure 1.7.19). The rotor revolves and drives the electrical generator. The steam expands as it passes through the turbine and the size of the blades increases along the turbine to allow for this.



▲ **Figure 1.7.19** The rotor of a steam turbine

The overall efficiency of thermal power stations is only about 30%. They require cooling towers to condense steam from the turbine to water and this is a waste of energy.

A Sankey diagram (Figure 1.7.5) shows the energy transfers that occur in a thermal power station.

In gas-fired power stations, natural gas is burnt in a **gas turbine** linked directly to an electricity generator. The hot exhaust gases from the turbine are not released into the atmosphere but used to

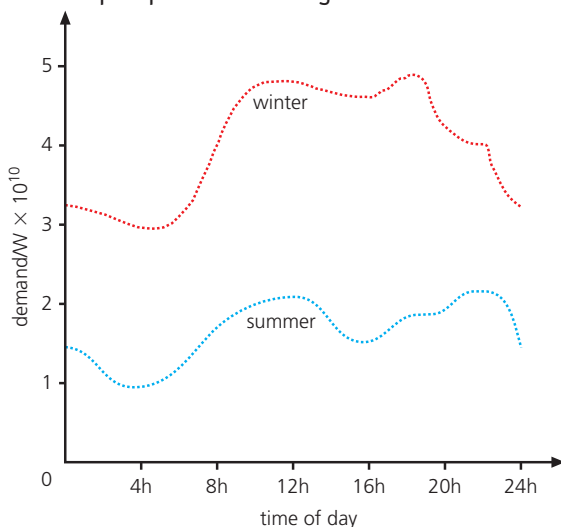
produce steam in a boiler. The steam is then used to generate more electricity from a steam turbine driving another generator. The efficiency is claimed to be over 50% without any extra fuel consumption. Furthermore, the gas turbines have a near 100% combustion efficiency, so very little harmful exhaust gas (i.e. unburnt methane) is produced, and natural gas is almost sulfur-free, so the environmental pollution caused is much less than for coal.

Renewable sources

In most cases the renewable energy source is used to drive turbines directly, as explained earlier in the cases of hydroelectric, wind, wave, tidal and geothermal schemes.

The efficiency of a large installation can be as high as 85–90% since many of the causes of loss in thermal power stations (e.g. water-cooling towers) are absent. In some cases, the generating costs are half those of thermal stations.

A feature of some hydroelectric stations is **pumped storage**. Electricity cannot be stored on a large scale but must be used as it is generated. The demand for electricity varies with the time of day and the season (Figure 1.7.20), so in a pumped-storage system electricity generated at off-peak periods is used to pump water back up from a low-level reservoir to a higher-level one. It is easier to do this than to reduce the output of the generator. At peak times the stored gravitational potential energy of the water in the high-level reservoir is transferred to kinetic energy in the turbine and used to generate electricity; three-quarters of the electricity that was used to pump the water is generated.



▲ Figure 1.7.20 Variation in power demand

Economic, environmental and social issues

When considering the large-scale generation of electricity, the economic and environmental costs of using various energy sources have to be weighed against the benefits that electricity brings to society as a clean, convenient and fairly cheap energy supply.

Environmental problems such as polluting emissions that arise with different energy sources were outlined when each was discussed previously. Apart from people using less energy, how far pollution can be reduced by, for example, installing desulfurisation processes in coal-fired power stations, is often a matter of cost.

Although there are no fuel costs associated with electricity generation from renewable energy sources such as wind power, the energy is so dilute that the capital costs of setting up the generating installation are high. Similarly, although fuel costs for nuclear power stations are relatively low, the costs of building the stations and of dismantling them at the end of their useful lives is higher than for gas- or coal-fired stations.

It has been estimated that currently it costs between 9 US ¢ and 22 US ¢ to produce a unit of electricity in a gas- or coal-fired power station in the UK. Wind energy costs vary, depending upon location, but are in the range 7 US ¢ to 16 US ¢ per unit. In the most favourable locations wind competes with coal and gas generation. The cost for a nuclear power station is in excess of 10 US ¢ per unit. After the Tohoku earthquake and tsunami disaster which led to the damage and closure of the Fukushima nuclear reactor in Japan, several countries have reduced their dependence on nuclear energy and Germany plans to phase out nuclear power completely by 2022.

The reliability of a source has also to be considered, as well as how easily production can be started up and shut down as demand for electricity varies. Natural gas power stations have a short start-up time, while coal and then oil power stations take successively longer to start up; nuclear power stations take longest. They are all reliable in that they can produce electricity at any time of day and in any season of the year as long as fuel is available. Hydroelectric power stations are also very reliable and have a very short start-up time, which means they can be switched on when the

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demand for electricity peaks. The electricity output of a tidal power station, although predictable, is not as reliable because it depends on the height of the tide which varies over daily, monthly and seasonal time scales. The wind and the Sun are even less reliable sources of energy since the output of a wind turbine changes with the strength of the wind and that of a solar cell with the intensity of light falling on it; the output may not be able to match the demand for electricity at a particular time.

Renewable sources are still only being used on a small scale globally. The contribution of the main energy sources to the world's total energy consumption at present is given in Table 1.7.2. (The use of biofuels is not well documented.) The great dependence on fossil fuels worldwide is evident. It is clear the world has an energy problem and new solutions to energy production need to be found.

▼ **Table 1.7.2** World use of energy sources

Oil	Coal	Gas	Nuclear	Hydroelectric
34%	27%	24%	4%	7%

Consumption varies from one country to another; North America and Europe are responsible for about 42% of the world's energy consumption each year. Table 1.7.3 shows approximate values for the annual consumption per head of population for different areas. These figures include the hidden consumption in the manufacturing and transporting of goods. The world average consumption is $76 \times 10^9 \text{ J}$ per head per year.

▼ **Table 1.7.3** Energy consumption per head per year/ $\text{J} \times 10^9$

N. America	240
UK	121
Japan	150
S. America	56
China	97
Africa	15
Pakistan	20
Bangladesh	9

Efficiency of energy transfers

- 1 The **efficiency** of a device is the percentage of the energy supplied to it that is usefully transferred.
- 2 Efficiency is calculated from the expression

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

For example, for the lever shown in Figure 1.5.26 (p. 46)

$$\text{efficiency} = \frac{\text{work done on load}}{\text{work done by effort}} \times 100\%$$

This will be less than 100% if there is friction in the fulcrum.

Key definition

$$\text{Efficiency (\%)} = \frac{(\text{useful energy output})}{(\text{total energy input})} (\times 100\%)$$

Table 1.7.4 lists the efficiencies of some devices.

▼ **Table 1.7.4**

Device	% efficiency
large electric motor	90
large electric generator	90
domestic gas boiler	75
compact fluorescent lamp	50
steam turbine	45
car engine	25
filament lamp	10

A device is efficient if it transfers energy mainly to useful stores and the lost energy is small.

The efficiency of a device can also be defined in terms of power output and input:

$$\text{efficiency} = \frac{(\text{useful power output})}{(\text{total power input})} \times 100\%$$

Key definition

$$\text{Efficiency (\%)} = \frac{(\text{useful power output})}{(\text{total power input})} (\times 100\%)$$

? Worked example

- a The energy input to an electric motor is 400 J when raising a load of 200 N through a vertical distance of 1.5 m. Calculate the efficiency of the motor.

$$\begin{aligned}\text{work done on load} &= \text{force} \times \text{distance} \\ &= 200 \text{ N} \times 1.5 \text{ m} \\ &= 300 \text{ N m} = 300 \text{ J}\end{aligned}$$

$$\text{useful energy output} = 300 \text{ J}$$

$$\text{total energy input} = 400 \text{ J}$$

$$\begin{aligned}\text{efficiency} &= \frac{\text{useful energy output}}{\text{total energy input}} \times 100\% \\ &= \frac{300 \text{ J}}{400 \text{ J}} \times 100\% = 75\%\end{aligned}$$

- b If the energy input to an electric drill is 300 J/s and it transfers 100 J/s of energy to thermal energy when in use, calculate its efficiency.

$$\text{power supplied to the drill} = 300 \text{ J/s}$$

$$\begin{aligned}\text{useful power output} &= (300 - 100) \text{ J/s} \\ &= 200 \text{ J/s}\end{aligned}$$

$$\begin{aligned}\text{efficiency} &= \frac{(\text{useful power output})}{(\text{total power input})} \times 100\% \\ &= \frac{200}{300} \times 100\% = 67\%\end{aligned}$$

Now put this into practice

- 1 A robot is used to lift a load. If the energy input to the robot is 8000 J in the time it takes to lift a load of 500 N through 12 m, calculate the efficiency of the robot.
- 2 If the energy input to an electric motor is 560 J/s and 170 J/s of energy is transferred to thermal energy when in use, calculate its efficiency.

Test yourself

- 9 List six properties which you think the ideal energy source should have for generating electricity in a power station.
- 10 a List six social everyday benefits for which electricity generation is responsible.
 - b Draw up two lists of suggestions for saving energy
 - i in the home, and
 - ii globally.
- 11 Calculate the efficiency of a compact fluorescent light if a power input of 20 J/s gives an output power of 9 J/s.

1.7.4 Power

FOCUS POINT

- ★ Define power and use the correct equations to calculate power in terms of the rate at which work is done or energy transferred.

To heat up a frozen dinner in a microwave oven you need to know the power of the oven, if over- or under-cooking is to be avoided. Similarly, one needs to check the power rating of a light bulb before inserting it into a socket to ensure over-heating does not occur. Most electrical appliances have their power rating marked on them, usually at the rear or base of the device. The power of a device is the rate at which it does work and so is equal to the rate at which it transfers energy to different stores.

Power

The more powerful a car is, the faster it can accelerate or climb a hill, i.e. the more rapidly it does work. The **power** of a device is the work it does per second, i.e. the rate at which it does work. This is *the same as the rate at which it transfers energy from one store to another*.

$$\text{power} = \frac{\text{work done}}{\text{time taken}} = \frac{\text{energy transferred}}{\text{time taken}}$$

$$\text{power } P = \frac{W}{t}$$

where W is the work done in time t

$$\text{also } P = \frac{\Delta E}{t}$$

where ΔE is the energy transferred in time t .

Key definition

Power the work done per unit time and the energy transferred per unit time

The unit of power is the **watt (W)** and is *a rate of working of 1 joule per second*, i.e. $1 \text{ W} = 1 \text{ J/s}$. Larger units are the **kilowatt (kW)** and the **megawatt (MW)**:

$$1 \text{ kW} = 1000 \text{ W} = 10^3 \text{ W}$$

$$1 \text{ MW} = 1\,000\,000 \text{ W} = 10^6 \text{ W}$$

If a machine does 500 J of work in 10 s, its power is $500 \text{ J}/10 \text{ s} = 50 \text{ J/s} = 50 \text{ W}$. A small car develops a maximum power of about 25 kW.

Test yourself

- 12 A boy whose weight is 600 N runs up a flight of stairs 10 m high in 12 s. What is his average power?
- 13 Calculate the power of a lamp that transfers 2400 J to thermal and light energy in 1 minute.
- 14 An escalator carries 60 people of average mass 70 kg to a height of 5 m in one minute. Calculate the power needed to do this.



Practical work

Measuring your own power

Safety

- You should only volunteer for this if you feel able to. No one should pressure you into taking part.

Get someone with a stopwatch to time you running up a flight of stairs; the more steps the better. Find your weight (in newtons). Calculate the total vertical height (in metres) you have climbed by measuring the height of one step and counting the number of steps.

The work you do (in joules) in lifting your weight to the top of the stairs is (your weight) \times (vertical height of stairs). Calculate your power (in watts).

- 5 Name the stores between which energy is transferred as you run up the stairs.
- 6 How is energy transferred when you run up the stairs?

Revision checklist

After studying Topic 1.7 you should know and understand the following:

- ✓ work is done when energy is transferred
- ✓ the different ways of harnessing solar, wind, wave, tidal, hydroelectric, geothermal and biofuel energy, and the use of fossil and nuclear fuels to generate electricity in power stations
- ✓ the difference between renewable and non-renewable energy sources
- ✓ the meaning of efficiency in energy and that power is the work done per unit time or energy transferred per unit time.

After studying Topic 1.7 you should be able to:

- ✓ recall different stores of energy and describe energy transfers in given examples
- ✓ recall the principle of conservation of energy and apply it to simple systems including the interpretation of flow diagrams
- ✓ define kinetic energy and perform calculations using $E_k = \frac{1}{2}mv^2$
- ✓ define gravitational potential energy and perform calculations using $\Delta E_p = mgh$
- ✓ apply the principle of conservation of energy to complex systems and interpret Sankey diagrams

- ✓ relate work done to the magnitude of a force and the distance moved, and recall the units of work, energy and power
- ✓ recall and use the equation
work done = force \times distance moved in the direction of the force ($W = Fd$)
to calculate energy transfer
- ✓ compare and contrast the advantages and disadvantages of using different energy sources to generate electricity

- ✓ recall and use the equations

$$\text{power} = \frac{\text{work done}}{\text{time taken}} \left(P = \frac{W}{t} \right)$$

and

$$\text{power} = \frac{\text{energy transferred}}{\text{time taken}} \left(P = \frac{E}{t} \right)$$

to calculate power

- ✓ define efficiency in relation to the transfer of energy and power
- ✓ recall and use the equations

$$\text{efficiency} = \frac{(\text{useful energy output})}{(\text{total energy input})} (\times 100\%)$$

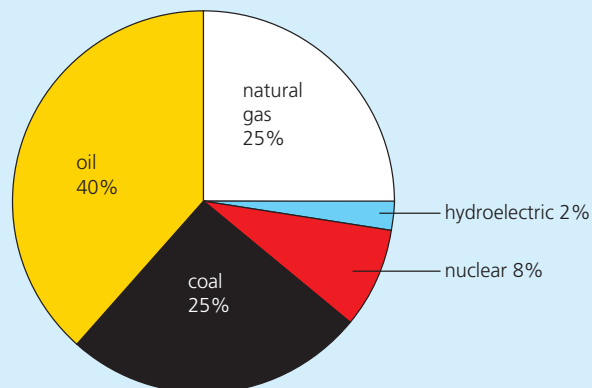
and

$$\text{efficiency} = \frac{(\text{useful power output})}{(\text{total power input})} (\times 100\%)$$



Exam-style questions

- 1 State how energy is transferred from
- a toaster [2]
 - a refrigerator [2]
 - an audio system. [2]
- [Total: 6]
- 2 A 100 g steel ball falls from a height of 1.8 m onto a metal plate and rebounds to a height of 1.25 m. Neglecting air resistance, calculate the
- gravitational potential energy of the ball before the fall ($g = 9.8 \text{ m/s}^2$) [2]
 - kinetic energy of the ball as it hits the plate [1]
 - velocity of the ball on hitting the plate [3]
 - kinetic energy of the ball as it leaves the plate on the rebound [2]
 - velocity of rebound. [3]
- [Total: 11]
- 3 A ball of mass 0.5 kg is thrown vertically upwards with a kinetic energy of 100 J. Neglecting air resistance calculate
- the initial speed of the ball [3]
 - the gravitational potential energy of the ball at its highest point [1]
 - the maximum height to which the ball rises. [3]
- [Total: 7]
- 4 In loading a lorry a man lifts boxes each of weight 100 N through a height of 1.5 m.
- Calculate the work done in lifting one box. [2]
 - Calculate how much energy is transferred when one box is lifted. [1]
 - If he lifts four boxes per minute, at what power is he working? [3]
- [Total: 6]
- 5 The pie chart in Figure 1.7.21 shows the percentages of the main energy sources used by a certain country.



▲ Figure 1.7.21

- Give the percentage supplied by hydroelectric energy. [1]
 - Identify any of the sources that are renewable. [1]
 - Explain what is meant by a renewable source. [1]
 - Give two other renewable sources. [2]
 - If energy is always conserved, explain the importance of developing renewable sources. [2]
- [Total: 7]
- 6 a Give
- two advantages and
 - two disadvantages
- of using fossil fuels in electricity generating stations. [4]
- b Give
- two advantages and
 - two disadvantages
- of using solar energy in electricity generating stations. [4]
- [Total: 8]
- 7 When the energy input to a gas-fired power station is 1000 MJ, the useful energy output is 300 MJ.
- Calculate the efficiency of the power station. [3]
 - Calculate how much energy is lost and identify the energy store to which it is moved. [2]
 - Describe where the lost energy goes. [2]
- [Total: 7]

1.8

Pressure

FOCUS POINTS

- ★ Define pressure as force per unit area, and illustrate with examples.
- ★ Describe the use of a barometer to measure atmospheric pressure.
- ★ Describe how pressure varies with depth in a liquid.
- ★ Calculate the change in pressure beneath the surface of a liquid using the correct equation.

The large flat feet of an Arabian camel prevent it sinking into the soft sand of the desert. This is because the weight of the camel is spread over the area of its four large feet. It appears that the effect of a force depends on the area over which it acts. The effect can be quantified by introducing the concept of pressure. In this topic you will learn that pressure increases as the force increases and the area over which the force acts becomes less. Pressure in a liquid is found to increase with both density and depth. This is why a dam must be thicker at the bottom than at the top. The properties of liquid pressure are utilised in applications ranging from water supply systems and dam construction to hydraulic lifts.

Pressure

To make sense of some effects in which a force acts on an object we have to consider not only the force but also the area on which it acts. For example, wearing skis prevents you sinking into soft snow because your weight is spread over a greater area. We say the **pressure** is less.

Pressure is defined as the force per unit area (i.e. 1 m^2) and is calculated from

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$
$$p = \frac{F}{A}$$

Key definition

Pressure the force per unit area

The unit of pressure is the **pascal** (Pa). It equals 1 newton per square metre (N/m^2) and is quite a small pressure. An apple in your hand exerts about 1000 Pa.

The greater the area over which a force acts, the less the pressure. This is why a tractor with wide wheels can move over soft ground. The pressure is

large when the area is small and this is why nails are made with sharp points. Walnuts can be broken in the hand by squeezing two together, rather than one alone, because the area of contact is smaller leading to a higher pressure on the shells (Figure 1.8.1).



▲ **Figure 1.8.1** Cracking walnuts by hand

? Worked example

Figure 1.8.2 shows the pressure exerted on the floor by the same box standing on end (Figure 1.8.2a) and lying flat (Figure 1.8.2b). If the box has a weight of 24 N, calculate the pressure on the floor when the box is

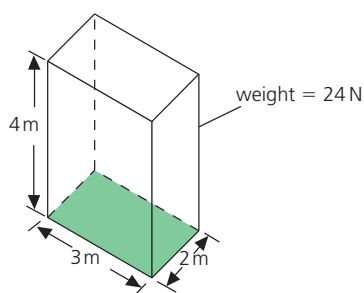
- standing on end as in Figure 1.8.2a and
- lying flat as in Figure 1.8.2b.

a area = $3\text{ m} \times 2\text{ m} = 6\text{ m}^2$

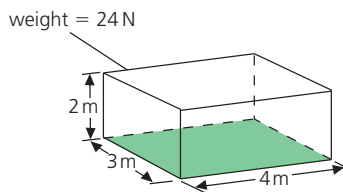
$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{24\text{ N}}{6\text{ m}^2} = 4\text{ Pa}$$

b area = $3\text{ m} \times 4\text{ m} = 12\text{ m}^2$

$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{24\text{ N}}{12\text{ m}^2} = 2\text{ Pa}$$



▲ Figure 1.8.2a



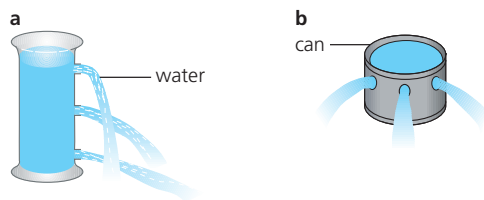
▲ Figure 1.8.2b

Now put this into practice

- A rectangular box has a width of 2 m, a height of 5 m and a depth of 2 m.
 - Calculate the area of
 - the base of the box
 - one of the sides of the box.
 - If the box has a weight of 80 N, calculate the pressure on
 - the base of the box
 - one of the sides of the box.
- Calculate the pressure on a surface when a force of 50 N acts on an area of
 - 2.0 m^2
 - 100 m^2
 - 0.50 m^2 .
 - A pressure of 10 Pa acts on an area of 3.0 m^2 . What is the force acting on the area?

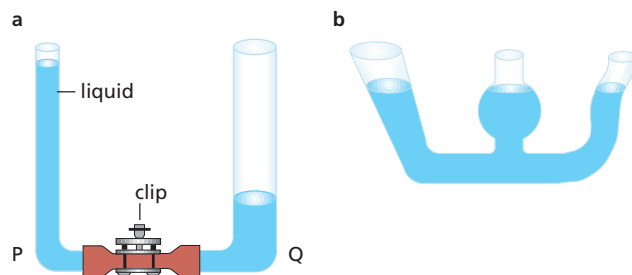
Liquid pressure

- Pressure in a liquid increases with depth.* This is because the further down you go, the greater the weight of liquid above. In Figure 1.8.3a water spurts out fastest and furthest from the lowest hole.
- Pressure at one depth acts equally in all directions.* The can of water in Figure 1.8.3b has similar holes all round it at the same level. Water comes out equally fast and spurts equally far from each hole. Hence the pressure exerted by the water at this depth is the same in all directions. The water emerges horizontally and normal to the surface of the container, showing that the pressure produces a force in a direction at right angles to the surface.



▲ Figure 1.8.3

- A liquid finds its own level.* In the U-tube of Figure 1.8.4a the liquid pressure at the foot of P is greater than at the foot of Q because the left-hand column is higher than the right-hand one. When the clip is opened, the liquid flows from P to Q until the pressure and the levels are the same, i.e. the liquid 'finds its own level'. Although the weight of liquid in Q is now greater than in P, it acts over a greater area because tube Q is wider. In Figure 1.8.4b the liquid is at the same level in each tube and confirms that the pressure at the foot of a liquid column depends only on the *vertical* depth of the liquid and not on the tube width or shape.



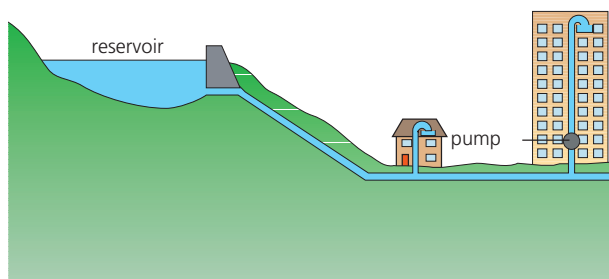
▲ Figure 1.8.4

- 4 Pressure depends on the density of the liquid. The denser the liquid, the greater the pressure at any given depth. The densities of some different liquids are listed in Table 1.4.1 in Topic 1.4.

Water supply system

A town's water supply often comes from a reservoir on high ground. Water flows from it through pipes to any tap or storage tank that is below the level of water in the reservoir (Figure 1.8.5). The lower the place supplied, the greater the water pressure. In very tall buildings it may be necessary to pump the water to a large tank in the roof.

Reservoirs for water supply or for hydroelectric power stations are often made in mountainous regions by building a dam at one end of a valley. The dam must be thicker at the bottom than at the top due to the large water pressure at the bottom.



▲ Figure 1.8.5 Water supply system

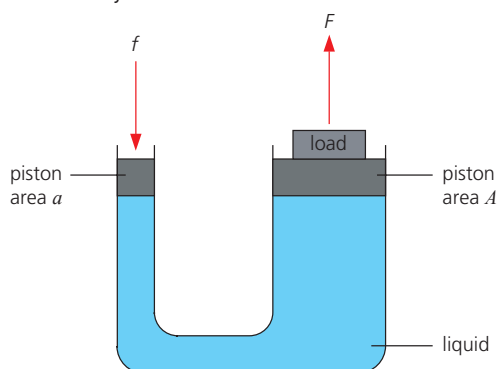
Test yourself

- 1 Why is the pump needed in the high-rise building shown in Figure 1.8.5?
- 2 Why are dam walls built to be thicker at the bottom than the top?

Going further

Hydraulic machines

Liquids are almost incompressible (i.e. their volume cannot be reduced by squeezing) and they 'pass on' any pressure applied to them. Use is made of these facts in hydraulic machines. Figure 1.8.6 shows the principle on which they work.



▲ Figure 1.8.6 The hydraulic principle

Suppose a downward force f acts on a piston of area a . The pressure transmitted through the liquid is

$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{f}{a}$$

This pressure acts on a second piston of larger area A , producing an upward force, $F = \text{pressure} \times \text{area}$:

$$F = \frac{f}{a} \times A$$

or

$$F = f \times \frac{A}{a}$$

Since A is larger than a , F must be larger than f and the hydraulic system is a force multiplier; the multiplying factor is A/a .

? Worked example

A hydraulic jack is used to lift a heavy load. A force of 1 N is applied to a piston of area 0.01 m^2 and pressure is transmitted through the liquid to a second piston of area of 0.5 m^2 . Calculate the load which can be lifted.

Taking $f = 1 \text{ N}$, $a = 0.01 \text{ m}^2$ and $A = 0.5 \text{ m}^2$ then

$$F = f \times \frac{A}{a}$$

so

$$F = 1 \text{ N} \times \frac{0.5 \text{ m}^2}{0.01 \text{ m}^2} = 50 \text{ N}$$

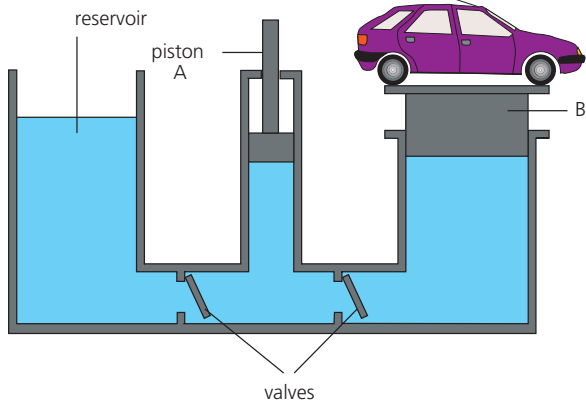
A force of 1 N could lift a load of 50 N; the hydraulic system multiplies the force 50 times.

Now put this into practice

- 1 In a hydraulic jack a force of 20 N is applied to a piston of area 0.1 m^2 . Calculate the load which can be lifted by a second piston of area 1.5 m^2 .
- 2 In a hydraulic jack a load of 70 N is required to be lifted on an area of 1.0 m^2 . Calculate the force that must be applied to a piston of area 0.1 m^2 to lift the load.
- 3 Name the property of a liquid on which a hydraulic jack relies.

1.8 PRESSURE

A hydraulic jack (Figure 1.8.7) has a platform on top of piston B and is used in garages to lift cars. Both valves open only to the right and they allow B to be raised a long way when piston A moves up and down repeatedly.



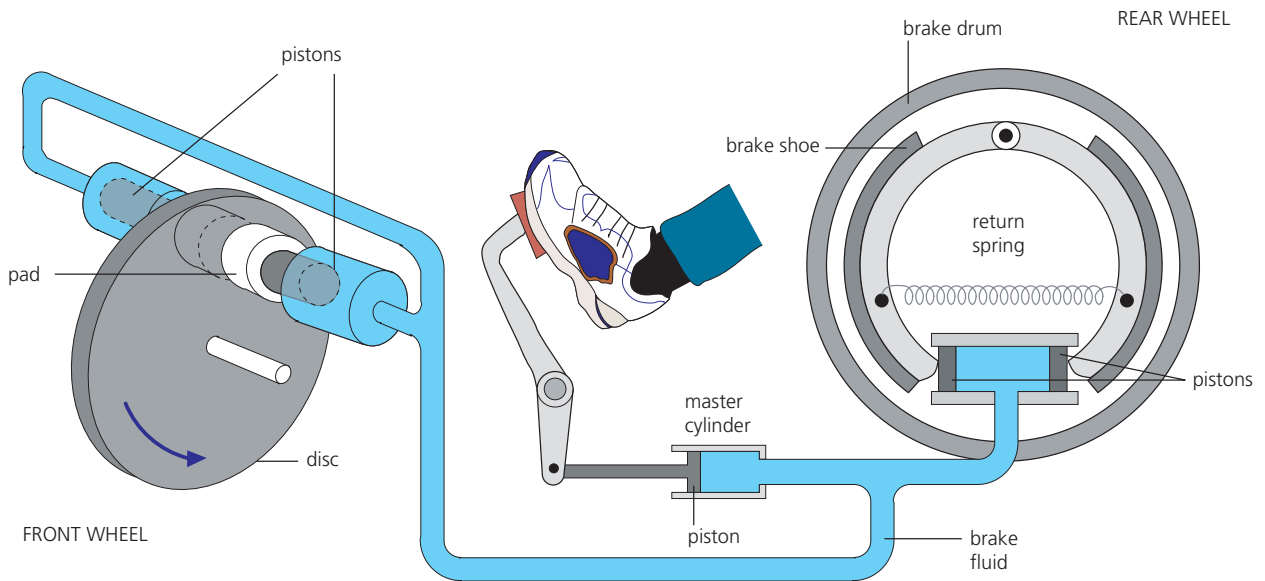
▲ **Figure 1.8.7** A hydraulic jack

Hydraulic fork-lift trucks and similar machines such as loaders (Figure 1.8.8) work in the same way.



▲ **Figure 1.8.8** A hydraulic machine in action

Hydraulic car brakes are shown in Figure 1.8.9. When the brake pedal is pushed, the piston in the master cylinder exerts a force on the brake fluid and the resulting pressure is transmitted equally to eight other pistons (four are shown). These force the brake shoes or pads against the wheels and stop the car.



▲ **Figure 1.8.9** Hydraulic car brakes

Expression for liquid pressure

In designing a dam an engineer has to calculate the pressure at various depths below the water surface. The pressure increases with depth and density.

An expression for the change in pressure Δp at a depth Δh below the surface of a liquid of density ρ can be found by considering a horizontal area A (Figure 1.8.10). The force acting vertically downwards on A equals the weight of a liquid column of height Δh and cross-sectional area A above it. Then

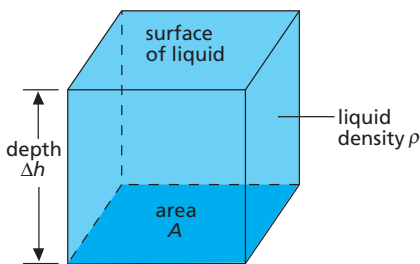
$$\text{volume of liquid column} = \Delta h A$$

Since mass = density \times volume, then

$$\text{mass of liquid column } m = \rho \Delta h A$$

$$\text{weight of liquid column} = mg = \rho \Delta h A g$$

$$\therefore \text{force on area } A = \rho \Delta h A g$$



▲ Figure 1.8.10

As

$$\text{pressure due to liquid column} = \text{force/area} \\ = \rho g \Delta h A / A$$

we can write

$$\Delta p = \rho g \Delta h$$

where Δp is the change in pressure beneath the surface of the liquid at depth Δh due to the weight of a liquid of density ρ and g is the gravitational field strength. This can also be written in words as:

$$\text{change in pressure} = \text{density} \times \text{gravitational field strength} \times \text{change in height}$$

This pressure acts equally in all directions at depth Δh and depends only on Δh and ρ . Its value will be in Pa if Δh is in m and ρ in kg/m^3 .

Test yourself

- 3 Calculate the increase in pressure at a depth of 2 m below the surface of water of density 1000 kg/m^3 .
- 4 Calculate the depth of water of density 1020 kg/m^3 where the pressure is $3.0 \times 10^6 \text{ Pa}$.

Going further

Pressure gauges

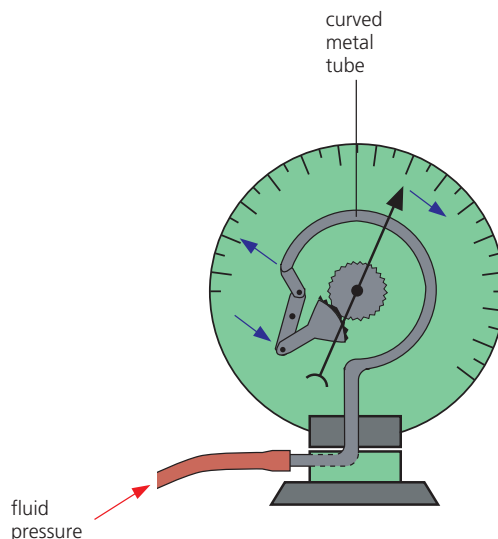
These measure the pressure exerted by a fluid, in other words by a liquid or a gas.

Bourdon gauge

This works like the toy shown in Figure 1.8.11, where the harder you blow into the paper tube, the more it uncurls. In a Bourdon gauge (Figure 1.8.12), when a fluid pressure is applied, the curved metal tube tries to straighten out and rotates a pointer over a scale. Car oil-pressure gauges and the gauges on gas cylinders are of this type.



▲ Figure 1.8.11



▲ Figure 1.8.12 A Bourdon gauge

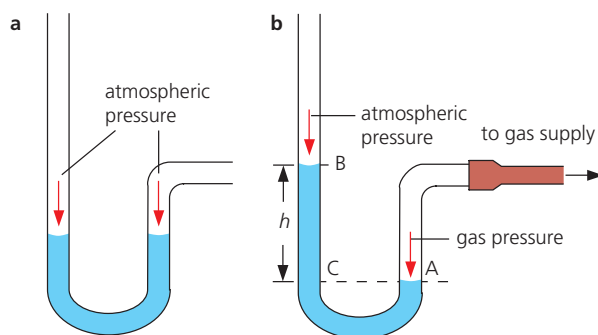
1.8 PRESSURE

U-tube manometer

In Figure 1.8.13a each surface of the liquid is acted on equally by atmospheric pressure and the levels are the same. If one side is connected to, for example, the gas supply (Figure 1.8.13b), the gas exerts a pressure on surface A and level B rises until

$$\text{pressure of gas} = \text{atmospheric pressure} + \text{pressure due to liquid column BC}$$

The pressure of the liquid column BC therefore equals the amount by which the gas pressure *exceeds* atmospheric pressure. It equals $h\rho g$ (in Pa) where h is the vertical height of BC (in m) and ρ is the density of the liquid (in kg/m^3). The height h is called the head of liquid and sometimes, instead of stating a pressure in Pa, we say that it is so many cm of water (or mercury for higher pressures).

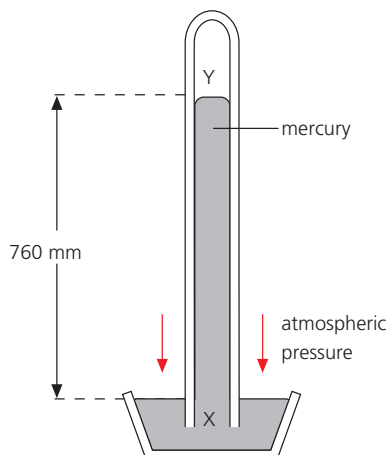


▲ Figure 1.8.13 A U-tube manometer

Mercury barometer

Atmospheric pressure is caused by the weight of air above a surface (Topic 2.1.3) and can be measured with a **barometer**. A simple barometer is shown in Figure 1.8.14. The weight of the column of mercury

(XY) is balanced by the force due to the atmospheric pressure pushing down on the mercury in the bowl.



▲ Figure 1.8.14 Mercury barometer

The difference in height between the top of the mercury column and the surface of the mercury in the bowl (Δh) is a measure of the atmospheric pressure in mm of mercury (mm Hg). Normal atmospheric pressure is about 760 mm Hg. The value in Pa equals $\rho g \Delta h$ where Δh is in metres, ρ is the density of mercury in kg/m^3 and g is the gravitational field strength. Δh increases if the atmospheric pressure increases and decreases if the atmospheric pressure decreases. Atmospheric pressure produces a vertical force on the mercury surface so Δh is the vertical height of the column; if the tube is tilted, the mercury will extend further along the tube but Δh remains the same. Would it be different with a wider tube? The space above the mercury in the tube is a vacuum (except for a little mercury vapour).

Revision checklist

After studying Topic 1.8 you should know and understand:

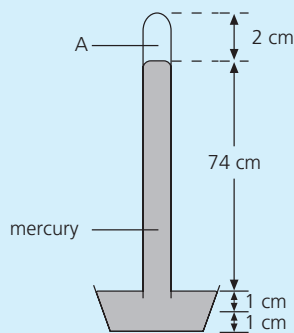
- ✓ what causes atmospheric pressure on a surface
- ✓ how pressure on a surface varies with force and area
- ✓ that the pressure beneath a liquid surface increases with depth and density and that pressure is transmitted through a liquid.

After studying Topic 1.8 you should be able to:

- ✓ describe how to show that pressure at a surface produces a force in a direction at right angles to the surface
- ✓ describe how a barometer measures atmospheric pressure
- ✓ define pressure from the equation pressure = force/area ($p = F/A$) and give everyday examples of its use; recall the units of pressure
- ✓ calculate the change in pressure below the surface of a liquid
- ✓ use the equation change in pressure = density \times gravitational field strength \times change in height ($\Delta p = \rho g \Delta h$).

Exam-style questions

- 1 The following statements relate to definitions of pressure. In each case state if the statement is *true* or *false*.
- A** Pressure is the force acting on unit area. [1]
B Pressure is calculated from force/area. [1]
C The SI unit of pressure is the pascal (Pa) which equals 1 newton per square metre (1 N/m^2). [1]
D The greater the area over which a force acts, the greater the pressure. [1]
E Force = pressure \times area. [1]
F The SI unit of pressure is the pascal (Pa) which equals 1 newton per metre (1 N/m). [1]
 [Total: 6]
- 2 **a** Calculate the pressure exerted on a wood-block floor by each of the following:
- i** A box weighing 2000 kN standing on an area of 2 m^2 . [2]
ii An elephant weighing 200 kN standing on an area of 0.2 m^2 . [2]
iii A girl of weight 0.5 kN wearing high-heeled shoes standing on an area of 0.0002 m^2 . [2]
- b** A wood-block floor can withstand a pressure of 2000 kPa (2000 kN/m^2). State which of the objects in **a** will damage the floor and explain why. [2]
 [Total: 8]
- 3 In a hydraulic press a force of 20 N is applied to a piston of area 0.20 m^2 . The area of the other piston is 2.0 m^2 . The pressure applied by the smaller piston to the liquid is transferred through the liquid and applied to the larger piston.
- a** Calculate the pressure transmitted through the liquid. [2]
b Calculate the force on the larger piston. [2]
 [Total: 4]
- 4 **a** The pressure in a liquid varies with depth and density. State whether the following statements are *true* or *false*.
- A** The pressure in a liquid increases with depth. [1]
B The pressure in a liquid increases with density. [1]
C The pressure in a liquid is greater vertically than horizontally. [1]
- b** Calculate the increase in pressure at a depth of 100 m below the surface of sea water of density 1150 kg/m^3 . [4]
 [Total: 7]
- 5 **a** State the equation which relates the change in pressure in a liquid to the depth below the liquid surface. [2]
b Identify the unit of pressure. [1]
c Calculate the depth of water of density 1030 kg/m^3 where the pressure is $7.5 \times 10^6 \text{ Pa}$. [3]
 [Total: 6]
- 6 Figure 1.8.15 shows a simple barometer.
- a** What is the region A? [1]
b What keeps the mercury in the tube? [1]
c What is the value of the atmospheric pressure being shown by the barometer? [1]
d State what would happen to this reading if the barometer were taken up a high mountain. Give a reason. [2]



▲ Figure 1.8.15

[Total: 5]