Chapter 8

Electric current, potential difference and resistance

LEARNING INTENTIONS

In this chapter you will learn how to:

- understand of the nature of electric current
- understand the term charge and recognise its unit, the coulomb
- understand that charge is quantised
- solve problems using the equation $Q = It$
- solve problems using the formula $I = nAve$
- solve problems involving the mean drift velocity of charge carriers
- understand the terms potential difference, e.m.f. and the volt
- use energy considerations to distinguish between p.d. and e.m.f.
- define resistance and recognise its unit, the ohm
- solve problems using the formula $V = IR$
- solve problems concerning energy and power in electric circuits.

BEFORE YOU START

- Write down what you understand by the terms current, charge, potential difference, e.m.f. and resistance.
- Can you set up a simple circuit to measure the current in a lamp and the potential difference across it? Sketch the circuit and swap it with a classmate to check.

DEVELOPING IDEAS

Electricity plays a vital part in our lives. We use electricity as a way of transferring energy from place to place – for heating, lighting and making things move. For people in a developing nation, the arrival of a reliable electricity supply marks a great leap forward. In Kenya, a micro-hydroelectric scheme has been built on Kabiri Falls, on the slopes of Mount Kenya. Although this produces just 14 kW of power, it has given work to a number of people, as shown in Figures 8.1, 8.2 and 8.3.

Figure 8.1: An operator controls the water inlet at the Kabiri Falls power plant. The generator is on the right.

Figure 8.2: A metal workshop uses electrical welding equipment. This allows rapid repairs to farmers' machinery.

Figure 8.3: A hairdresser can now work in the evenings, thanks to electrical lighting.

With an increasing need for small generating facilities at a town or even village level, can you suggest what types of generator would be suitable in your neighbourhood? What are the advantages and disadvantages of each type of generator?

8.1 Circuit symbols and diagrams

Before we go on to study electricity we need to introduce the concept of circuit diagrams. It is impossible to draw anything but the simplest circuits as a detailed drawing. To make it possible to draw complex circuits, a shorthand method using standard circuit symbols is used. You will have seen many circuit components and their symbols in your previous studies. Some are shown in Table 8.1 and Figure 8.4.

The symbols in Table 8.1 are a small part of a set of internationally agreed conventional symbols for electrical components. It is essential that scientists, engineers, manufacturers and others around the world use the same symbol for a particular component. In addition, many circuits are now designed by computers and these need a universal language in which to work and to present their results.

The International Electrotechnical Commission (IEC) is the body that establishes agreements on such things as electrical symbols, as well as safety standards, working practices and so on. The circuit symbols used here form part of an international standard known as IEC 60617. Because this is a shared 'language', there is less chance that misunderstandings will arise between people working in different organisations and different countries.

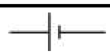
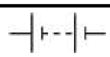
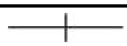
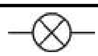
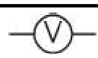
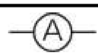
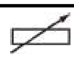
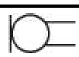
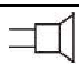
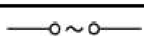
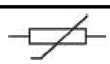
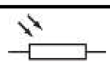
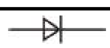
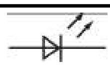
Symbol	Component name
	connecting lead
	cell
	battery of cells
	fixed resistor
	power supply
	junction of conductors
	crossing conductors (no connection)
	filament lamp
	voltmeter
	ammeter
	switch
	variable resistor
	microphone
	loudspeaker
	fuse
	earth
	alternating signal
	capacitor
	thermistor
	light-dependent resistor (LDR)
	semiconductor diode
	light-emitting diode (LED)

Table 8.1: Electrical components and their circuit symbols.

What’s in a word?

Electricity is a rather tricky word. In everyday life, its meaning may be rather vague – sometimes we use it to mean electric current; at other times, it may mean electrical energy or electrical power. In this chapter and the ones that follow, we will avoid using the word electricity and try to develop the correct usage of these more precise scientific terms.

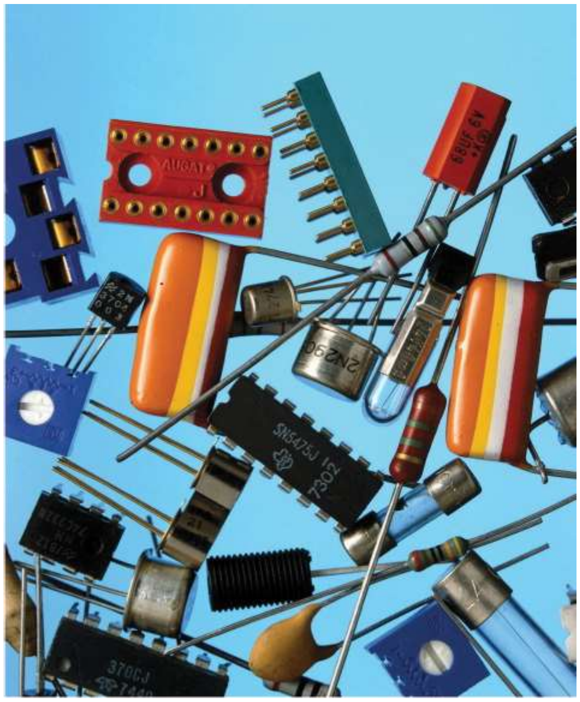


Figure 8.4: A selection of electrical components, including resistors, fuses, capacitors and microchips.

8.2 Electric current

You will have carried out many practical activities involving electric current. For example, if you connect a wire to a cell (Figure 8.5), there will be current in the wire. And of course you make use of electric currents every day of your life – when you switch on a lamp or a computer, for example.

In the circuit of Figure 8.5, the direction of the current is from the positive terminal of the cell, around the circuit to the negative terminal. This is a scientific convention: the direction of current is from positive to negative, and hence the current may be referred to as **conventional current**. But what is going on inside the wire?

A wire is made of metal. Inside a metal, there are negatively charged electrons that are free to move about. We call these **conduction** or **free** electrons, because they are the particles that allow a metal to conduct an electric current. The atoms of a metal bind tightly together; they usually form a regular array, as shown in Figure 8.6. In a typical metal, such as copper or silver, one or more electrons from each atom breaks free to become conduction electrons. The atom remains as a positively charged ion. Since there are equal numbers of free electrons (negative) and ions (positive), the metal has no overall charge – it is neutral.

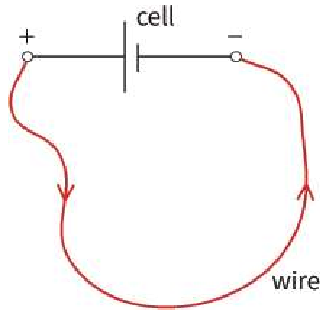


Figure 8.5: There is current in the wire when it is connected to a cell.

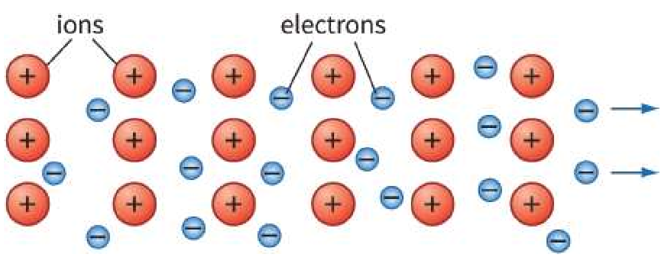


Figure 8.6: In a metal, conduction electrons are free to move among the fixed positive ions. A cell connected across the ends of the metal causes the electrons to drift towards its positive terminal.

When the cell is connected to the wire, it exerts an electrical force on the conduction electrons that makes them travel along the length of the wire. Since electrons are negatively charged, they flow away from the negative terminal of the cell and towards the positive terminal. This is in the opposite direction to conventional current. This may seem a bit strange; it happens because the direction of conventional current was chosen long before anyone had any idea what was going on inside a piece of metal carrying a current. If the names positive and negative had originally been allocated the other way round, we would now label electrons as positively charged, and conventional current and electron flow would be in the same direction.

Note that there is a current at all points in the circuit as soon as the circuit is completed. We do not have to wait for charge to travel around from the cell. This is because the charged electrons are already present throughout the metal before the cell is connected.

We can use the idea of an electric field to explain why charge flows almost instantly. Connect the terminals of a cell to the two ends of a wire and we have a complete circuit. The cell produces an electric field in the wire; the field lines are along the wire, from the positive terminal to the negative. This means that there is a force on each electron in the wire, so each electron starts to move and the current exists almost instantly.

Charge carriers

Sometimes a current is a flow of positive charges–for example, a beam of protons produced in a particle accelerator. The current is in the same direction as the particles. Sometimes a current is due to both positive and negative charges – for example, when charged particles flow through a solution. A solution that conducts is called an electrolyte and it contains both positive and negative ions. These move in opposite directions when the solution is connected to a cell (Figure 8.7). These charged particles are known as **charge carriers**. If you consider the structure of charged particles you will appreciate that charge comes in definite sized 'bits'; the smallest bit being the charge on an electron or on a single proton. This 'bittiness' is what is meant when charge is described as being **quantised**.

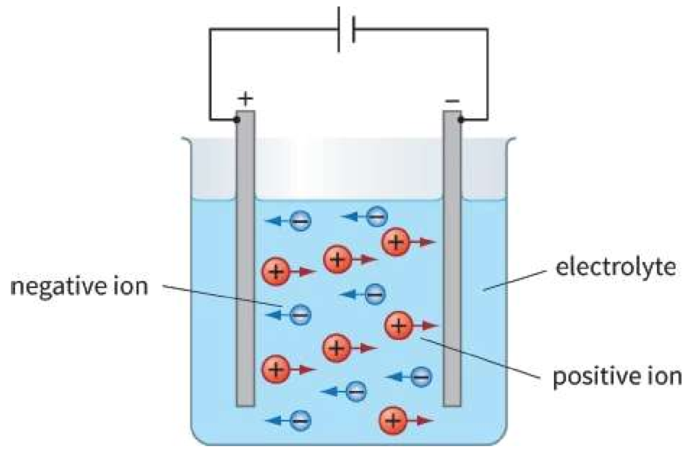


Figure 8.7: Both positive and negative charges are free to move in a solution. Both contribute to the electric current.

Questions

1 Look at Figure 8.7 and state the direction of the conventional current in the electrolyte (towards the left, towards the right or in both directions at the same time?).

2 Figure 8.8 shows a circuit with a conducting solution having both positive and negative ions.

- **a** Copy the diagram and draw in a cell between points A and B. Clearly indicate the positive and negative terminals of the cell.
- **b** Add an arrow to show the direction of the conventional current in the solution.
- **c** Add arrows to show the direction of the conventional current in the connecting wires.

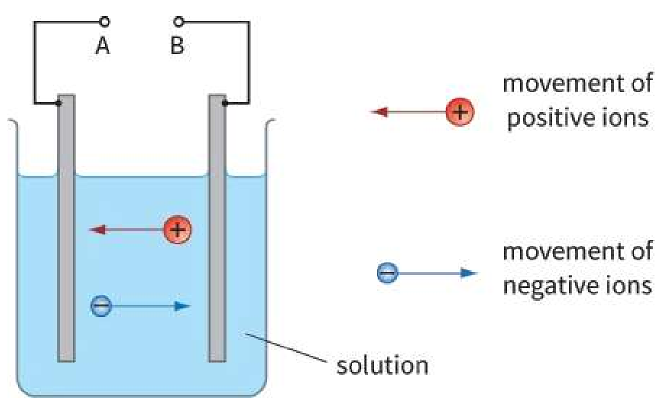


Figure 8.8: For Question 2.

Current and charge

When charged particles flow past a point in a circuit, we say that there is a **current** in the circuit. Electrical current is measured in **amperes** (A). So how much charge is moving when there is a current of 1 A? Charge is measured in **coulombs** (C). For a current of 1 A, the rate at which charge passes a point in a circuit is 1 C in a time of 1 s. Similarly, a current of 2 A gives a charge of 2 C in a time of 1 s. A current of 3 A gives a charge of 6 C in a time of 2 s, and so on. The relationship between charge, current and time may be written as the following word equation:

$$\text{current} = \frac{\text{charge}}{\text{time}}$$

This equation explains what we mean by electric current.

The equation for current can be rearranged to give an equation for charge:

$$\text{charge} = \text{current} \times \text{time}$$

KEY EQUATION

$$\text{charge} = \text{current} \times \text{time}$$
$$\Delta Q = I\Delta t$$

The unit of charge is the coulomb.

In symbols, the charge flowing past a point is given by the relationship:

$$\Delta Q = I\Delta t$$

where ΔQ is the charge that flows during a time Δt, and I is the current.

Note that the ampere and the coulomb are both SI units; the ampere is a base unit while the coulomb is a derived unit (see Chapter 3).

Questions

3 The current in a circuit is 0.40 A. Calculate the charge that passes a point in the circuit in a period of 15 s.

4 Calculate the current that gives a charge flow of 150 C in a time of 30 s.

5 In a circuit, a charge of 50 C passes a point in 20 s. Calculate the current in the circuit.

6 A car battery is labelled '50 A h'. This means that it can supply a current of 50 A for one hour.

- **a** For how long could the battery supply a continuous current of 200 A needed to start the car?
- **b** Calculate the charge that flows past a point in the circuit in this time.

Charged particles

As we have seen, current is the flow of charged particles called charge carriers. But how much charge does each particle carry?

Electrons each carry a tiny negative charge of approximately -1.6×10^{-19} C. This charge is represented by $-e$. The magnitude of the charge is known as the **elementary charge**. This charge is so tiny that you would need about six million million million electrons – that's 6 000 000 000 000 000 000 of them – to have a charge equivalent to one coulomb.

Protons are positively charged, with a charge $+e$. This is equal and opposite to that of an electron. Ions carry charges that are multiples of $+e$ and $-e$.

WORKED EXAMPLES

1 There is a current of 10 A through a lamp for 1.0 hour. Calculate how much charge flows through the lamp in this time.

Step 1 We need to find the time t in seconds:

$\Delta t = 60 \times 60 = 3600$ s

Step 2 We know the current $I = 10$ A, so the charge that flows is:

$\Delta Q = I\Delta t = 10 \times 3600 = 36\,000$ C $= 3.6 \times 10^4$ C

2 Calculate the current in a circuit when a charge of 180 C passes a point in a circuit in 2.0 minutes.

Step 1 Rearranging $\Delta Q = I\Delta t$ gives:

$$I = \frac{\Delta Q}{\Delta t} = \frac{\text{charge}}{\text{time}}$$

Step 2 With time in seconds, we then have:

$$\text{current } I = \frac{180}{120} = 1.5 \text{ A}$$

Because electric charge is carried by particles, it must come in amounts that are multiples of e. So, for example, 3.2×10^{-19} C is possible, because this is $+2e$, but 2.5×10^{-19} C is impossible, because this is not an integer multiple of e.

This reinforces the idea that charge is quantised; it means that it can only come in amounts that are integer multiples of the elementary charge. If you are studying chemistry, you will know that ions have charges of $\pm e$, $\pm 2e$, etc. The only exception is in the case of the fundamental particles called quarks, which are the building blocks from which particles such as protons and neutrons are made. These have charges of $\pm {}^1\!/_3 e$ or $\pm {}^2\!/_3 e$. However, quarks always appear in twos or threes in such a way that their combined charge is zero or a multiple of e.

Questions

7 Calculate the number of protons that would have a charge of one coulomb. (Proton charge = $+1.6 \times 10^{-19}$ C.)

8 Which of the following quantities of electric charge is possible? Explain how you know.

6.0×10^{-19} C, 8.0×10^{-19} C, 10.0×10^{-19} C

8.3 An equation for current

Copper, silver and gold are good conductors of electric current. There are large numbers of conduction electrons in a copper wire – as many conduction electrons as there are atoms. The number of conduction electrons per unit volume (for example, in 1 m^3 of the metal) is called the **number density** and has the symbol *n*. For copper, the value of *n* is about 10^{29} m^{-3}.

Figure 8.9 shows a length of wire, with cross-sectional area *A*, along which there is a current *I*.

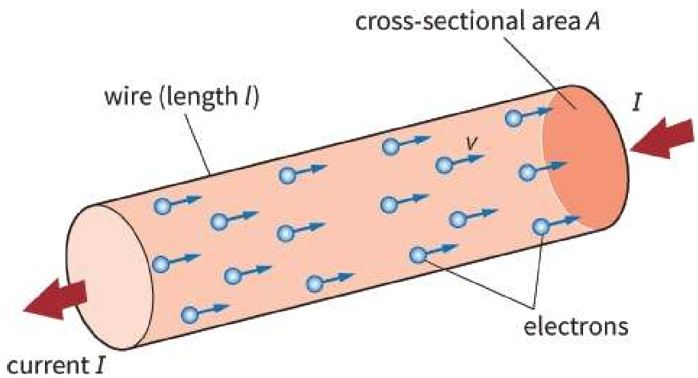


Figure 8.9: A current *I* in a wire of cross-sectional area *A*. The charge carriers are mobile conduction electrons with **mean drift velocity** *v*.

How fast do the electrons in Figure 8.9 have to travel? The following equation allows us to answer this question:

$$I = nAvq$$

where *n* = the number density, *A* = cross sectional area of the conductor, *v* = mean drift velocity of the charge carriers, *q* = the charge on each charge carrier.

KEY EQUATION

Electric current:

$$I = nAvq$$

The length of the wire in Figure 8.9 is *l*. We imagine that all of the electrons shown travel at the same speed *v* along the wire.

Now imagine that you are timing the electrons to determine their speed. You start timing when the first electron emerges from the right-hand end of the wire. You stop timing when the last of the electrons shown in the diagram emerges. (This is the electron shown at the left-hand end of the wire in the diagram.) Your timer shows that this electron has taken time *t* to travel the distance *l*.

In the time *t*, all of the electrons in the length *l* of wire have emerged from the wire. We can calculate how many electrons this is, and hence the charge that has flowed in time *t*:

$$\begin{aligned} \text{number of electrons} &= \text{number density} \times \text{volume of wire} \\ &= n \times A \times l \end{aligned}$$

$$\begin{aligned} \text{charge of electrons} &= \text{number} \times \text{electron charge} \\ &= n \times A \times l \times e \end{aligned}$$

We can find the current *I* because we know that this is the charge that flows in time *t*, and current $= \frac{\text{charge}}{\text{time}}$:

$$I = n \times A \times l \times \frac{e}{t}$$

Substituting *v* for $\frac{1}{t}$ gives:

$$I = nAve$$

The moving charge carriers that make up a current are not always electrons. They might, for example, be ions (positive or negative) whose charge q is a multiple of e. Hence we can write a more general version of the equation as:

$$I = nAvq$$

Worked example 3 shows how to use this equation to calculate a typical value of v.

WORKED EXAMPLE

3 Calculate the mean drift velocity of the electrons in a copper wire of cross-sectional area 5.0×10^{-6} m^2 carrying a current of 1.0 A. The electron number density for copper is 8.5×10^{28} m^{-3}.

Step 1 Rearrange the equation $I = nAve$ to make v the subject:

$$V = \frac{I}{nAe}$$

Step 2 Substitute values and calculate v:

$$\begin{aligned} v &= \frac{1.0}{8.5\times10^{28}\times5.0\times10^{-6}\times1.6\times10^{-19}} \\ &= 1.47\times10^{-5}\,\text{m s}^{-1} \\ &= 0.015\ \text{mm s}^{-1} \end{aligned}$$

You do not need to know how to derive $I = nAvq$ but it is interesting to recognise that the units are **homogeneous**.

The unit of current (I) is the ampere (A).

The unit of the number of charge carriers per unit volume (n) is m^{-3}.

The unit of area (A) is m^2.

The unit of the drift velocity v is m s^{-1}.

The unit of charge (q) is the coulomb (C).

All these are in base units except the coulomb and 1 coulomb is 1 ampere second (A s).

Putting the units into the right-hand side of the equation:

$$\text{m}^{-3} \times \text{m}^2 \times \text{m s}^{-1} \times \text{A s} = \text{A}$$

This is the same as the left-hand side of the equation. Although this does not prove the equation to be correct, it does give strong evidence for it.

This technique is often used for checking the validity of an expression and also to predict a possible formula.

WORKED EXAMPLE

4 A student knows that the power transfer in a resistor depends on two variables: the current and the resistance of the resistor, but is unsure of the precise nature of the relationships.

Suggest the form of the equation.

Step 1 Identify the units of the terms:

power–watt (W)

current–ampere (A)

resistance–ohms (Ω)

Step 2 Break these down into base units:

1 W = 1 J s^{-1}, 1 J = 1 N m, 1 N = 1 kg m s^{-2}

hence 1 W = 1 kg m s^{-2} m s^{-1} = 1 kg m^2 s^{-3}

1 Ω = 1 V A^{-1}, 1 V = 1 J C^{-1}, 1 J = 1 kg m^2 s^{-2}, 1 C = 1 A s

thus 1 Ω = 1 kg m^2 s^{-2} [A s]$^{-1}$ A^{-1} = 1 kg m^2 s^{-2} A^{-2} s^{-1} = 1 kg m^2 s^{-3} A^{-2}

The unit for current, the ampere is already a base unit.

Step 3 Write a possible equation linking power, current and resistance

$P = K\,I^p R^q$ where p and q are pure numbers and K is a dimensionless constant.

The units on the right-hand side of this equation are $A^p \times [1\ kg\ m^2\ s^{-3}\ A^{-2}]^q$

The units on the left-hand side of the equation are $1\ kg\ m^2\ s^{-3}$

By inspection, we can see that the amperes do not figure in the left-hand side of the equation, thus p must equal 2 to cancel the amperes on the right-hand side and q must equal 1. This would leave both sides of the equation as $1\ kg\ m^2\ s^{-3}$.

Therefore we know that the equation is of the form:

$P = K\,I^2 R$

Note that this method does not give any information of the value of the constant K, although in this case, from our choice of units, K = 1 and the equation becomes the familiar $P = I^2R$.

Slow flow

It may surprise you to find that, as suggested by the result of Worked example 3, electrons in a copper wire drift at a fraction of a millimetre per second. To understand this result fully, we need to closely examine how electrons behave in a metal. The conduction electrons are free to move around inside the metal. When the wire is connected to a battery or an external power supply, each electron within the metal experiences an electrical force that causes it to move towards the positive end of the battery. The electrons randomly collide with the fixed but vibrating metal ions. Their journey along the metal is very haphazard. The actual velocity of an electron between collisions is of the order of magnitude $10^5\ m\ s^{-1}$, but its haphazard journey causes it to have a drift velocity towards the positive end of the battery. Since there are billions of electrons, we use the term mean drift velocity v of the electrons.

Figure 8.10 shows how the mean drift velocity of electrons varies in different situations.

We can understand this using the equation:

$$v = \frac{I}{nAe}$$

- If the current increases, the drift velocity v must increase. That is:

$$v \propto I$$

- If the wire is thinner, the electrons move more quickly for a given current. That is:

$$v \propto \frac{I}{A}$$

- There are fewer electrons in a thinner piece of wire, so an individual electron must travel more quickly.
- In a material with a lower density of electrons (smaller n), the mean drift velocity must be greater for a given current. That is:

$$v \propto \frac{1}{n}$$

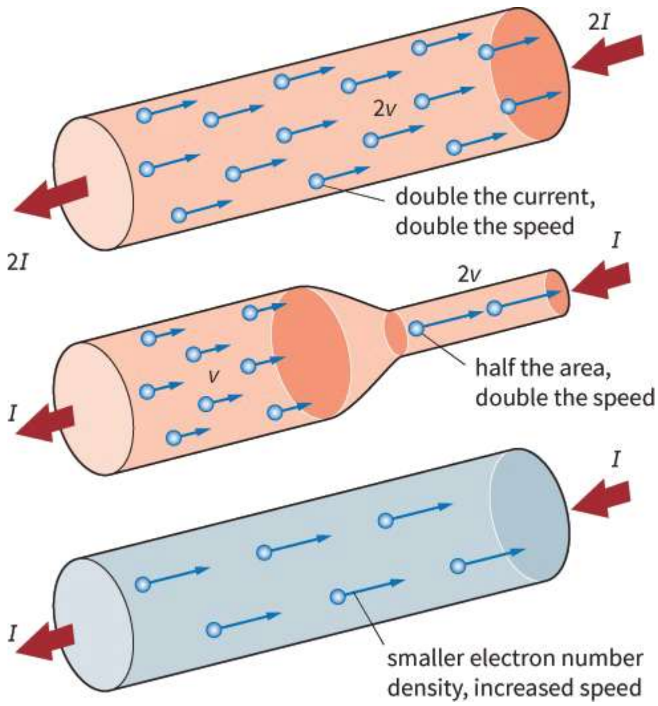


Figure 8.10: The mean drift velocity of electrons depends on the current, the cross-sectional area and the electron density of the material.

Questions

9 Calculate the current in a gold wire of cross-sectional area 2.0 mm^2 when the mean drift velocity of the electrons in the wire is 0.10 mm s^{-1}. The electron number density for gold is 5.9×10^{28} m^{-3}.

10 Calculate the mean drift velocity of electrons in a copper wire of diameter 1.0 mm with a current of 5.0 A. The electron number density for copper is 8.5×10^{28} m^{-3}.

11 A length of copper wire is joined in series to a length of silver wire of the same diameter. Both wires have a current in them when connected to a battery. Explain how the mean drift velocity of the electrons will change as they travel from the copper into the silver. Electron number densities:

copper $n = 8.5 \times 10^{28}$ m^{-3}

silver $n = 5.9 \times 10^{28}$ m^{-3}.

It may help you to picture how the drift velocity of electrons changes by thinking about the flow of water in a river. For a high rate of flow, the water moves fast – this corresponds to a greater current I. If the course of the river narrows, it speeds up – this corresponds to a smaller cross-sectional area A.

Metals have a high electron number density–typically of the order of 10^{28} or 10^{29} m^{-3}. Semiconductors, such as silicon and germanium, have much lower values of n–perhaps 10^{23} m^{-3}. In a semiconductor, electron mean drift velocities are typically a million times greater than those in metals for the same current. Electrical insulators, such as rubber and plastic, have very few conduction electrons per unit volume to act as charge carriers.

8.4 The meaning of voltage

The term **voltage** is often used in a rather casual way. In everyday life, the word is used in a less scientific and often incorrect sense – for example, 'A big voltage can go through you and kill you.' In this topic, we will consider a bit more carefully just what we mean by voltage and potential difference in relation to electric circuits.

Look at the simple circuit in Figure 8.11. Assume the power supply has negligible internal resistance. (We look at internal resistance later in Chapter 10). The three voltmeters are measuring three voltages or potential differences. With the switch open, the voltmeter placed across the supply measures 12 V. With the switch closed, the voltmeter across the power supply still measures 12 V and the voltmeters placed across the resistors measure 8 V and 4 V. You will not be surprised to see that the voltage across the power supply is equal to the sum of the voltages across the resistors.

Earlier in this chapter we saw that electric current is the rate of flow of electric charge. Figure 8.12 shows the same circuit as in Figure 8.11, but here we are looking at the movement of one coulomb (1 C) of charge round the circuit. Electrical energy is transferred to the charge by the power supply. The charge flows round the circuit, transferring some of its electrical energy to internal energy in the first resistor, and the rest to internal energy in the second resistor.

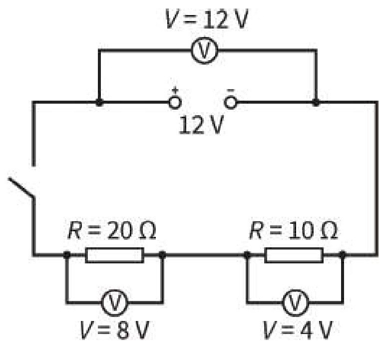


Figure 8.11: Measuring voltages in a circuit. Note that each voltmeter is connected across the component.

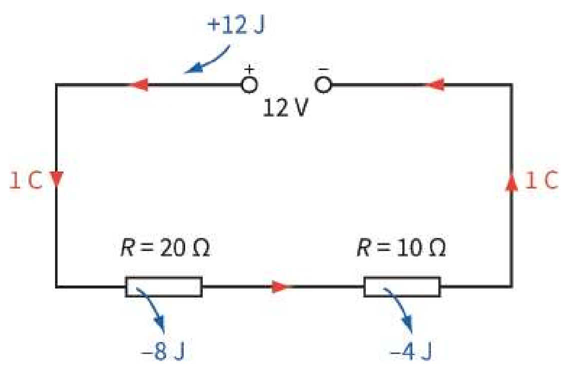


Figure 8.12: Energy transfers as 1 C of charge flows round a circuit. This circuit is the same as that shown in Figure 8.11.

The voltmeter readings indicate the energy transferred to the component by each unit of charge. The voltmeter placed across the power supply measures the e.m.f. of the supply, whereas the voltmeters placed across the resistors measure the potential difference (p.d.) across these components. The terms e.m.f. and potential difference have different meanings, so you have to be very vigilant.

The term **potential difference** is used when charges **lose** energy by transferring electrical energy to other forms of energy in a component, such as thermal energy or kinetic energy. Potential difference, V, is defined as the energy transferred per unit charge.

The potential difference between two points, A and B, is the energy transferred per unit charge as it moves from point A to point B.

$$\text{potential difference} = \frac{\text{energy transferred}}{\text{charge}} \equiv V = \frac{\Delta W}{\Delta Q}$$

This equation can be rearranged to calculate the energy transferred in a component:

$$\Delta W = V\,\Delta Q$$

KEY EQUATION

$$\begin{aligned}\text{energy transferred} &= \text{potential difference} \times \text{charge} \\ \Delta W &= V\Delta Q\end{aligned}$$

A power supply or a battery transfers energy to electrical charges in a circuit. The electromotive force (**e.m.f.**), *E*, of the supply is also defined as the energy transferred per unit charge. However, this refers to the energy given to the charge by the supply. The e.m.f. of a source is the energy transferred per unit charge in driving charge around a complete circuit.

Note that e.m.f. stands for electromotive *force*. This is a misleading term. It has nothing at all to do with force. This term is a legacy from the past and we are stuck with it! It is best to forget where it comes from and simply use the term e.m.f.

8.5 Electrical resistance

If you connect a lamp to a battery, a current in the lamp causes it to glow. But what determines the size of the current? This depends on two factors:

- the potential difference or voltage V across the lamp – the greater the potential difference, the greater the current for a given lamp
- the resistance R of the lamp – the greater the resistance, the smaller the current for a given potential difference.

Now we need to think about the meaning of **electrical resistance**. The resistance of any component is defined as the ratio of the potential difference to the current.

This is written as:

$$\text{resistance} = \frac{\text{potential difference}}{\text{current}} = R = \frac{V}{I}$$

where R is the resistance of the component, V is the potential difference across the component and I is the current in the component.

KEY EQUATION

$$\text{resistance} = \frac{\text{potential difference}}{\text{current}}$$

$$R = \frac{V}{I}$$

You can rearrange the equation to give:

$$I = \frac{V}{R} \text{ or } V = IR$$

Table 8.2 summarises these quantities and their units.

Defining the ohm

The unit of resistance, the **ohm** (Ω), can be determined from the equation that defines resistance:

$$\text{resistance} = \frac{\text{potential difference}}{\text{current}}$$

The ohm is equivalent to 1 volt per ampere:

$$1\ \Omega = 1\ \mathrm{V\,A^{-1}}$$

Quantity	Symbol for quantity	Unit	Symbol for unit
current	I	ampere (amp)	A
voltage (p.d., e.m.f.)	V	volt	V
resistance	R	ohm	Ω

Table 8.2: Basic electrical quantities, their symbols and SI units. Take care to understand the difference between V (in italics) meaning the quantity voltage and V meaning the unit volt.

Questions

12 A car headlamp bulb has a resistance of 36 Ω. Calculate the current in the lamp when connected to a '12 V' battery.

13 You can buy lamps of different brightness to fit in light fittings at home (Figure 8.13). A '100 watt' lamp glows more brightly than a '60 watt' lamp. Explain which of the lamps has the higher resistance.

14 **a** Calculate the potential difference across a motor carrying a current of 1.0 A and having a resistance of 50 Ω.

b Calculate the potential difference across the same motor when the current is doubled. Assume its resistance remains constant.

15 Calculate the resistance of a lamp carrying a current of 0.40 A when connected to a 230 V supply.

Figure 8.13: Both of these lamps work from the 230 V mains supply, but one has a higher resistance than the other. For Question 13.

WORKED EXAMPLE

5 Calculate the current in a lamp given that its resistance is 15 Ω and the potential difference across its ends is 3.0 V.

Step 1 Here we have $V = 3.0$ V and $R = 15$ Ω.

Step 2 Substituting in $I = \frac{V}{R}$ gives:

current $I = \frac{3.0}{15} = 0.20$ A

So the current in the lamp is 0.20 A.

PRACTICAL ACTIVITY 8.1

Determining resistance

As we have seen, the equation for resistance is:

$$R = \frac{V}{I}$$

To determine the resistance of a component, we therefore need to measure both the potential difference V across it and the current I through it. To measure the current, we need an ammeter. To measure the potential difference, we need a voltmeter. Figure 8.14 shows how these meters should be connected to determine the resistance of a metallic conductor, such as a length of wire.

- The ammeter is connected **in series** with the conductor, so that there is the same current in both.
- The voltmeter is connected across (**in parallel** with) the conductor, to measure the potential difference across it.

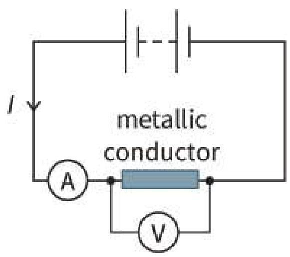


Figure 8.14: Connecting an ammeter and a voltmeter to determine the resistance of a metallic conductor in a circuit.

Question

16 In Figure 8.14 the reading on the ammeter is 2.4 A and the reading on the voltmeter is 6.0 V. Calculate the resistance of the metallic conductor.

8.6 Electrical power

The rate at which energy is transferred is known as power. Power P is measured in watts (W). (If you are not sure about this, refer back to Chapter 5, where we looked at the concept of power in relation to forces and work done.)

$$\text{power} = \frac{\text{energy transferred}}{\text{time taken}} \equiv P = \frac{\Delta W}{\Delta t}$$

where P is the power and ΔW is the energy transferred in a time Δt.

Take care not to confuse W for energy transferred or work done with W for watts.

Refer back to the equation derived from the definition of potential difference:

$$V = \frac{\Delta W}{\Delta Q}$$

This can be rearranged as:

$$\Delta W = V\Delta Q$$

Thus:

$$P = \frac{W}{\Delta t} = \frac{V\Delta Q}{\Delta t} = V\left(\frac{\Delta Q}{\Delta t}\right)$$

The ratio of charge to time, $\frac{\Delta Q}{\Delta t}$, is the current I in the component. Therefore:

$$P = VI$$

By substituting from the resistance equation $V = IR$, we get the alternative equations for power:

$$P = I^2R \text{ and } P = \frac{V^2}{R}$$

KEY EQUATIONS

Equations for power:

$$P = VI$$
$$P = I^2R$$
$$P = \frac{V^2}{R}$$

WORKED EXAMPLE

6 Calculate the rate at which energy is transferred by a 230 V mains supply that provides a current of 8.0 A to an electric heater.

Step 1 Use the equation for power:

$P = VI$

with $V = 230$ V and $I = 8.0$ A.

Step 2 Substitute values:

$P = 8 \times 230 = 1840$ W (1.84 kW)

7 **a** A power station produces 20 MW of power at a voltage of 200 kV. Calculate the current supplied to the grid cables.

Step 1 Here we have P and V and we have to find I, so we can use $P = VI$.

Step 2 Rearranging the equation and substituting the values we know gives:

$$\text{current } I = \frac{P}{V} = \frac{20\times10^6}{200\times10^3} = 100 \text{ A}$$

Hint: *Remember to convert megawatts into watts and kilovolts into volts.*

So, the power station supplies a current of 100 A.

b The grid cables are 15 km long, with a resistance per unit length of 0.20 Ω km^{-1}. How much power is wasted as heat in these cables?

Step 1 First, we must calculate the resistance of the cables:

resistance R = 15 km × 0.20 Ω km^{-1} = 3.0 Ω

Step 2 Now we know I and R and we want to find P. We can use $P = I^2R$:

power wasted as heat,

$$\begin{aligned} P &= I^2R = (100)^2 \times 3.0 \\ &= 3.0 \times 10^4 \mathrm{W} \\ &= 30 \text{ k W} \end{aligned}$$

Hence, of the 20 MW of power produced by the power station, 30 kW is wasted – just 0.15%.

8 A bathroom heater, when connected to a 230 V supply has an output power of 1.0 kW. Calculate the resistance of the heater.

Step 1 We have P and V and have to find R, so we can use $P = \frac{V^2}{R}$

Step 2 Rearrange the equation and substitute in the known values:

resistance $R = \frac{V^2}{P} = \frac{230}{1000}^2 = 53\Omega$

Note: *The kilowatts were converted to watts in a similar way to the previous example.*

Questions

17 Calculate the current in a 60 W light bulb when it is connected to a 230 V power supply.

18 A power station supplies electrical energy to the grid at a voltage of 25 kV. Calculate the output power of the station when the current it supplies is 40 kA.

Power and resistance

A current I in a resistor of resistance R transfers energy to it. The resistor dissipates energy heating the resistor and the surroundings.. The p.d. V across the resistor is given by $V = IR$. Combining this with the equation for power, $P = VI$, gives us two further forms of the equation for power dissipated in the resistor:

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

Which form of the equation we use in any particular situation depends on the information we have available to us. This is illustrated in Worked examples 7a and 7b, which relate to a power station and to the grid cables that lead from it (Figure 8.15).

Figure 8.15: A power station and electrical transmission lines. How much electrical power is lost as heat in these cables? (See Worked examples 7a and 7b.)

Questions

19 A calculator is powered by a 3.0 V battery. The calculator's resistance is 20 kΩ. Calculate the power transferred to the calculator.

20 An energy-efficient light bulb is labelled '230 V, 15 W'. This means that when connected to the 230 V mains supply it is fully lit and changes electrical energy to heat and light at the rate of 15 W. Calculate:

- **a** the current in the bulb when fully lit
- **b** its resistance when fully lit.

21 Calculate the resistance of a 100 W light bulb that draws a current of 0.43 A from a power supply.

Calculating energy

We can use the relationship for power as energy transferred per unit time and the equation for electrical power to find the energy transferred in a circuit.

Since:

$$\text{power} = \text{current} \times \text{voltage}$$

and:

$$\text{energy} = \text{power} \times \text{time}$$

we have:

$$\begin{aligned} \text{energy transferred} &= \text{current} \times \text{voltage} \times \text{time} \\ W &= IV\Delta t \end{aligned}$$

Working in SI units, this gives energy transferred in joules.

Questions

22 A 12 V car battery can supply a current of 10 A for 5.0 hours. Calculate how many joules of energy the battery transfers in this time.

23 A lamp is operated for 20 s. The current in the lamp is 10 A. In this time, it transfers 400 J of energy to the lamp. Calculate:

- **a** how much charge flows through the lamp
- **b** how much energy each coulomb of charge transfers to the lamp
- **c** the p.d. across the lamp.

REFLECTION

Without referring back to your textbook, explain to a classmate the difference between potential difference and electromotive force.

A common error is to think that the higher the resistance between two points, the greater the power output. Explain to someone, without using mathematics, why this is incorrect.

As you look at this activity, what is one thing you would like to change?

SUMMARY

Electric current is the rate of flow of charge. In a metal, the charge is electrons; in an electrolyte, it is both positive and negative ions.
The direction of conventional current is from positive to negative; because electrons are negative, they move in the opposite direction.
The SI unit of charge is the coulomb (C). One coulomb is the charge passing a point when there is a current of one ampere at that point for one second: $$\text{charge} = \text{current} \times \text{time}\,(\Delta Q = I\Delta t)$$
The current I in a conductor of cross-sectional area A depends on the mean drift velocity (v) of the charge carriers and the number density (n): $$I = nAvQ$$
The term potential difference is used when charge transfers energy to the component or the surroundings. It is defined as energy transferred per unit charge: $$V = \frac{\Delta W}{\Delta Q} \text{ or } \Delta W = V\Delta Q$$
The term electromotive force is used when describing the maximum energy per unit charge that a source can provide: $$E = \frac{\Delta W}{\Delta Q} \text{ or } \Delta W = E\Delta Q$$
A volt is a joule per coulomb (1 J C^{-1}).
Power is the energy transferred per unit time. There are three formulae to calculate power used according to the quantities that are given: $$P = VI \text{ or } P = I^2R \text{ or } P = \frac{V^2}{R}$$
Resistance is the ratio of voltage to current: $$R = \frac{V}{I}$$
The resistance of a component is 1 ohm when the voltage of 1 V produces a current of 1 ampere.
Energy transferred in the circuit in a time Δt is given by the equation: $$PV = \frac{\Delta W}{\Delta Q} \text{ or } \Delta W = V\Delta Q$$ $$W = IV\Delta t$$

EXAM-STYLE QUESTIONS

1 A small immersion heater is connected to a power supply of e.m.f. of 12 V for a time of 150 s. The output power of the heater is 100 W.

What charge passes through the heater? **[1]**

A 1.4 C

B 8.0 C

C 1250 C

D 1800 C

2 Which statement defines e.m.f.? **[1]**

A The e.m.f. of a source is the energy transferred when charge is driven through a resistor.

B The e.m.f. of a source is the energy transferred when charge is driven round a complete circuit.

C The e.m.f. of a source is the energy transferred when unit charge is driven round a complete circuit.

D The e.m.f. of a source is the energy transferred when unit charge is driven through a resistor.

3 Calculate the charge that passes through a lamp when there is a current of 150 mA for 40 minutes. **[3]**

4 A generator produces a current of 40 A. Calculate how long will it take for a total of 2000 C to flow through the output. **[2]**

5 In a lightning strike there is an average current of 30 kA, which lasts for 2000 μs. Calculate the charge that is transferred in this process. **[3]**

6 **a** A lamp of resistance 15 Ω is connected to a battery of e.m.f. 4.5 V. Calculate the current in the lamp. **[2]**

b Calculate the resistance of the filament of an electric heater that takes a current of 6.5 A when it is connected across a mains supply of 230 V. **[2]**

c Calculate the voltage that is required to drive a current of 2.4 A through a wire of resistance 3.5 Ω. **[2]**

[Total: 6]

7 A battery of e.m.f. 6 V produces a steady current of 2.4 A for 10 minutes. Calculate:

a the charge that it supplied **[2]**

b the energy that it transferred. **[2]**

[Total: 4]

8 Calculate the energy gained by an electron when it is accelerated through a potential difference of 50 kV. (Charge on the electron = -1.6×10^{-19} C.) **[2]**

9 A woman has available 1 A, 3 A, 5 A, 10 A and 13 A fuses. Explain which fuse she should use for a 120 V, 450 W hairdryer. **[3]**

10 This diagram shows the electrolysis of copper chloride.

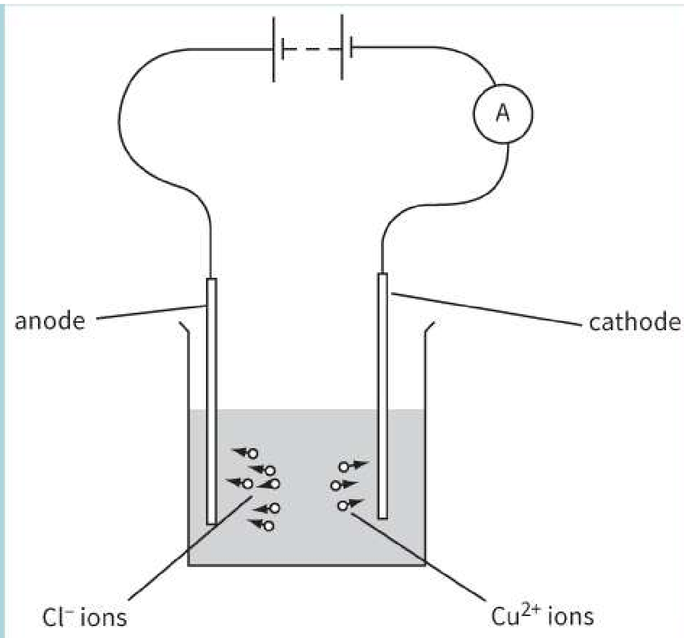


Figure 8.16

a **i** On a copy of the diagram, mark the direction of the conventional current in the electrolyte. Label it conventional current. **[1]**

ii Mark the direction of the electron flow in the connecting wires. Label this electron flow. **[1]**

b In a time period of 8 minutes, 3.6×10^{16} chloride (Cl^-) ions are neutralised and liberated at the anode and 1.8×10^{16} copper (Cu^{2+}) ions are neutralised and deposited on the cathode.

i Calculate the total charge passing through the electrolyte in this time. **[2]**

ii Calculate the current in the circuit. **[2]**

[Total: 6]

11 This diagram shows an electron tube. Electrons moving from the cathode to the anode constitute a current. The current in the ammeter is 4.5 mA.

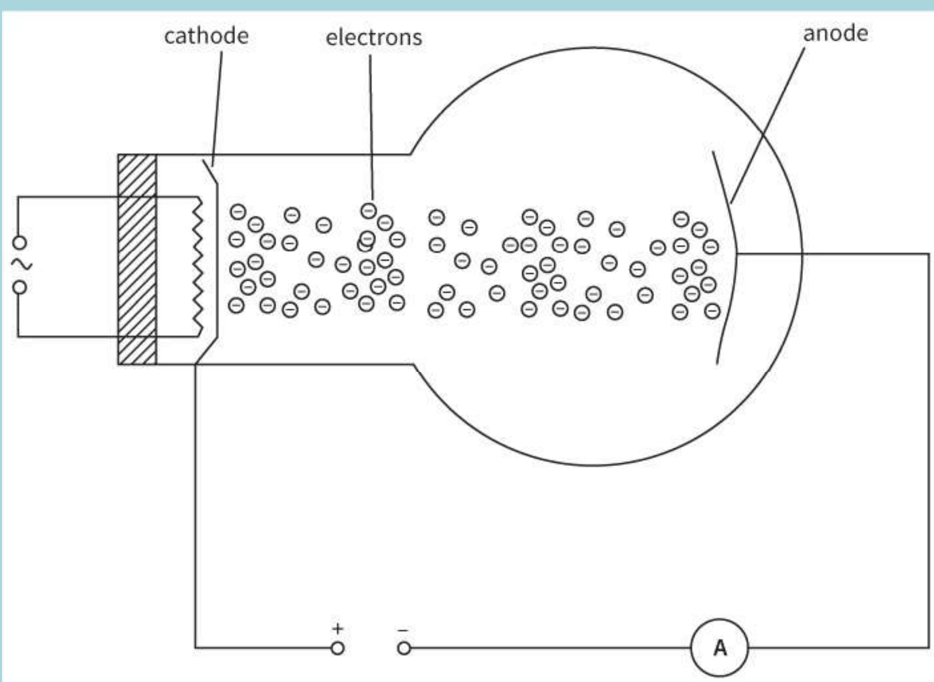


Figure 8.17

a Calculate the charge passing through the ammeter in 3 minutes. **[3]**

b Calculate the number of electrons that hit the anode in 3 minutes. **[3]**

c The potential difference between the cathode and the anode is 75 V. Calculate the energy gained by an electron as it travels from the cathode to the anode. **[2]**

[Total: 8]

12 A length of copper track on a printed circuit board has a cross-sectional area of 5.0×10^{-8} m^2. The current in the track is 3.5 mA. You are provided with some useful information about copper:

1 m^3 of copper has a mass of 8.9×10^3 kg

54 kg of copper contains 6.0×10^{26} atoms

In copper, there is roughly one electron liberated from each copper atom.

a Show that the electron number density n for copper is about 10^{29} m^{-3}. **[2]**

b Calculate the mean drift velocity of the electrons. **[3]**

[Total: 5]

13 a Explain the difference between **potential difference** and **e.m.f.** **[2]**

b A battery has negligible internal resistance, an e.m.f. of 12.0 V and a capacity of 100 A h (ampere-hours). Calculate:

i the total charge that it can supply **[2]**

ii the total energy that it can transfer. **[2]**

c The battery is connected to a 27 W lamp. Calculate the resistance of the lamp. **[2]**

[Total: 8]

14 Some electricity-generating companies use a unit called the kilowatt-hour (kW h) to calculate energy bills. 1 kWh is the energy a kilowatt appliance transfers in 1 hour.

a Show that 1 kWh is equal to 3.6 MJ. **[2]**

b An electric shower heater is rated at 230 V, 9.5 kW.

i Calculate the current it will take from the mains supply. **[2]**

ii Suggest why the shower requires a separate circuit from other appliances. **[1]**

iii Suggest a suitable current rating for the fuse in this circuit. **[1]**

c Calculate the energy transferred when a boy uses the shower for 5 minutes. **[2]**

[Total: 8]

15 A student is measuring the resistance per unit length of a resistance wire. He takes the following measurements.

Quantity	Value	Uncertainty
length of wire	80 mm	±2%
current in the wire	2.4 A	±0.1 A
potential difference across the wire	8.9 V	±5%

a Calculate the percentage uncertainty in the measurement of the current. **[1]**

b Calculate the value of the resistance per unit length of the wire. **[1]**

c Calculate the absolute uncertainty of the resistance per unit length of the wire. **[2]**

[Total: 4]