Chapter 7

Matter and materials

LEARNING INTENTIONS

In this chapter you will learn how to:

- define and use density
- define and use pressure and calculate the pressure in a fluid
- derive and use the equation $\Delta p = \rho g \Delta h$
- use a difference in hydrostatic pressure to explain and calculate upthrust
- explain how tensile and compressive forces cause deformation
- describe the behaviour of springs and use Hooke's law
- distinguish between elastic and plastic deformation, limit of proportionality and the elastic limit
- define and use stress, strain and the Young modulus
- describe an experiment to measure the Young modulus
- calculate the energy stored in a deformed material.

BEFORE YOU START

- Write down some notes to answer these questions: What are physical properties of materials? What properties make some materials really useful?
- Have you ever stretched a spring, rubber band or a small strip of plastic? Try to describe what you notice when these materials are stretched.

SPRINGY STUFF

In everyday life, we make great use of elastic materials. The term 'elastic' means springy; that is, the material deforms when a force is applied and returns to its original shape when the force is removed. Rubber is an elastic material. This is obviously important for a bungee jumper (Figure 7.1). The bungee rope must have the correct degree of elasticity. The jumper must be brought gently to a halt. What happens if the rope is too stiff or too springy? Discuss these problems with others – particularly if you have had experience of a bungee jump.

In this chapter, we will look at how forces can change the shape of an object. Before that, we will look at two important quantities, density and pressure.

Figure 7.1: The stiffness and elasticity of rubber are crucial factors in bungee jumping.

7.1 Density

Density is a property of matter. It tells us about how concentrated the matter is in a particular material. Density is a constant for a given material under specific conditions.

Density is defined as the mass per unit volume of a substance:

$$\begin{aligned} \text{density} &= \frac{\text{mass}}{\text{volume}} \\ \rho &= \frac{m}{V} \end{aligned}$$

The symbol used here for density, ρ, is the Greek letter rho.

The standard unit for density in the SI system is kg m^{-3}, but you may also find values quoted in g cm^{-3}. It is useful to remember that these units are related by:

$$1000\ \text{kg m}^{-3} = 1\ \text{g cm}^{-3}$$

and that the density of water is approximately 1000 kg m^{-3}.

KEY EQUATION

$$\begin{aligned} \text{density} &= \frac{\text{mass}}{\text{volume}} \\ \rho &= \frac{m}{V} \end{aligned}$$

Questions

1 A cube of copper has a mass of 240 g. Each side of the cube is 3.0 cm long. Calculate the density of copper in g cm^{-3} and in kg m^{-3}.

2 The density of steel is 7850 kg m^{-3}. Calculate the mass of a steel sphere of radius 0.15 m. (First, calculate the volume of the sphere using the formula $V = \frac{4}{3}\pi r^3$ and then use the density equation.)

7.2 Pressure

A fluid (liquid or gas) exerts **pressure** on the walls of its container, or on any surface with which it is in contact. Solids can also exert pressure on a surface with which it is in contact.

The pressure in a gas or liquid produces a force perpendicular to any surface.

The force the fluid pressure produces on the walls of a container can be in any direction, because the walls of the container may be horizontal, vertical or at any angle. A big force on a small area produces a high pressure.

Pressure is defined as the normal force acting per unit cross-sectional area.

We can write this as a word equation:

$$\begin{aligned} \text{pressure} &= \frac{\text{normal force}}{\text{cross-sectional area}} \\ p &= \frac{F}{A} \end{aligned}$$

The word ‘normal’ in this context means at right angles to the surface.

KEY EQUATION

$$\begin{aligned} \text{pressure} &= \frac{\text{normal force}}{\text{cross-sectional area}} \\ p &= \frac{F}{A} \end{aligned}$$

Force is measured in newtons and area is measured in square metres. The units of pressure are thus newtons per square metre (N m^{-2}), which are given the special name of pascals (Pa).

$$1 \text{ Pa} = 1 \text{ N m}^{-2}$$

Questions

3 A chair stands on four feet, each of area 10 cm^2. The chair weighs 80 N. Calculate the pressure it exerts on the floor.

4 Estimate the pressure you exert on the floor when you stand on both feet. (You could draw a rough rectangle around both your feet placed together to find the area in contact with the floor. You will also need to calculate your weight from your mass.)

Pressure in a fluid

The pressure in a fluid (a liquid or gas) increases with depth. Divers know this – the further they dive down, the greater the water pressure acting on them. The pressure acts at right angles to every part of their body and acts to crush them. Pilots know this – the higher they fly, the lower is the pressure of the atmosphere. The atmospheric pressure we experience on the surface of the Earth is due to the weight of the atmosphere above us, pressing downwards on the surface of the Earth or at right angles to every surface of our bodies.

The pressure in a fluid depends on three factors:

- the depth h below the surface
- the density ρ of the fluid
- the acceleration due to gravity, g.

In fact, change in pressure p is proportional to each of these and we have:

$$\begin{aligned} \text{change in pressure} &= \text{density} \times \text{acceleration due to gravity} \times \text{depth} \\ \Delta p &= \rho g h \end{aligned}$$

KEY EQUATION

$$\begin{aligned} \text{change in pressure} &= \text{density} \times \text{acceleration due to gravity} \times \text{depth} \\ \Delta p &= \rho g h \end{aligned}$$

You must learn how to derive this equation.

We can derive this relationship using Figure 7.2.

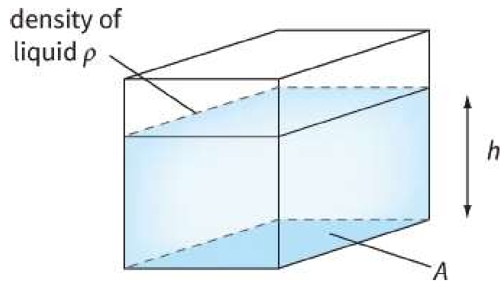

Figure 7.2: The weight of water in a tank exerts pressure on its base.

The force acting on the shaded area A on the bottom of the tank is caused by the weight of water above it, pressing downwards. We can calculate this force and hence the pressure as follows:

$$\begin{aligned}
\text{volume of water} &= A \times h \\
\text{mass of water} &= \text{density} \times \text{volume} = \rho \times A \times h \\
\text{weight of water} &= \text{mass} \times g = \rho \times A \times h \times g \\
\text{change in pressure} &= \tfrac{\text{force}}{\text{area}} \\
&= \rho \times A \times h \times \tfrac{g}{A} \\
&= \rho \times g \times h
\end{aligned}$$

The equation is written as $\Delta p = \rho g h$ because this formula calculates the *difference* in pressure between the top and bottom of the water in the tank. There is, of course, atmospheric pressure acting on the water at the top of the tank. The total pressure at the bottom of the tank is atmospheric pressure + Δp.

WORKED EXAMPLE

1 Figure 7.3 shows a manometer used to measure the pressure of a gas supply. Calculate the pressure difference between the gas inside the pipe and atmospheric pressure.

Step 1 Determine the difference in height h of the water on the two sides of the manometer.

$$h = 60 - 30 = 30 \text{ cm}$$

Step 2 Because the level of water on the side of the tube next to the gas pipe is lower than on the side open to the atmosphere, the pressure in the gas pipe is above atmospheric pressure.

pressure difference = $\rho \times g \times h$

= $1000 \times 9.81 \times 0.30 = 2940$ Pa

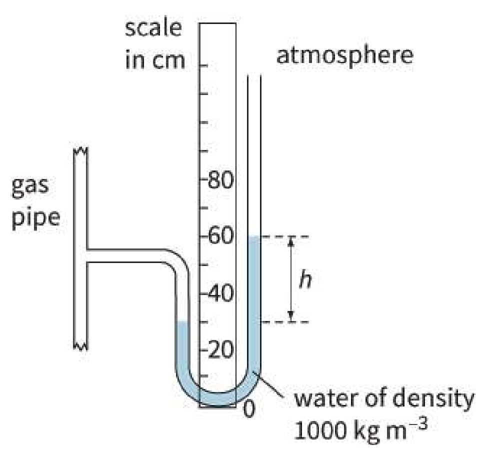

Figure 7.3: For Worked example 1.

Questions

5 Calculate the pressure due to the water on the bottom of a swimming pool if the depth of water in the pool varies between 0.8 m and 2.4 m. (Density of water = 1000 kg m^{-3}.) If atmospheric pressure is 1.01×10^5 Pa, calculate the maximum total pressure at the bottom of the swimming pool.

6 Estimate the height of the atmosphere if atmospheric density at the Earth's surface is 1.29 kg m^{-3}. (Atmospheric pressure = 101 kPa.)

7.3 Archimedes' principle

The variation of pressure with depth can be used to explain **Archimedes' principle**.

Archimedes' principle states that the upthrust acting on a body is equal to the weight of the liquid or gas that it displaces.

KEY EQUATION

$$\begin{aligned} \text{upthrust} &= \rho g V \\ &= \text{Weight of liquid displaced} \end{aligned}$$

When the object is placed in a liquid, it **displaces** *some* of the liquid. In other words, it takes up some of the space of the liquid. The volume of the liquid displaced is equal to the volume of the liquid taken up by the object. If the object floats, the volume displaced is equal to the volume of the part of the object that is *under* the surface of the liquid.

Consider a rectangular shaped object immersed in a liquid (Figure 7.4). There is a larger pressure on the bottom surface than there is on the top surface because the bottom surface is deeper in the liquid.

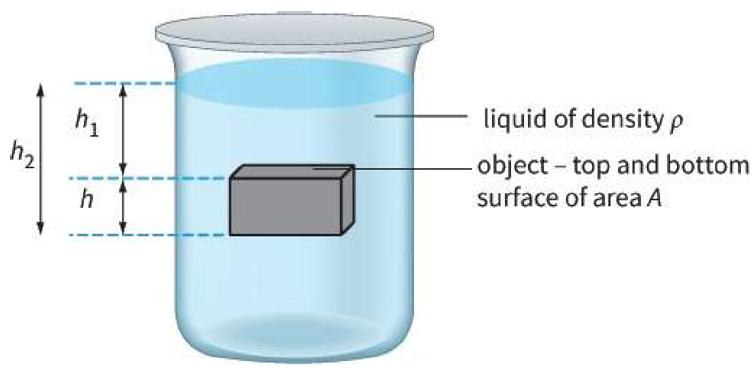

Figure 7.4: To explain Archimedes' principle.

The pressure on the top surface produces a force downwards on the top. It may seem surprising, but the pressure on the bottom surface actually produces a force upwards on the object. This is because pressure can act in any direction and always acts at right angles to a surface in a liquid. You may also be surprised to know that pressure is a scalar quantity even though it is defined in terms of force (which is a vector). Since pressure acts in all directions at a point it is not possible to define a single direction for it!

Because the pressure is larger on the bottom surface, the force acting upwards on the bottom surface is larger than the force acting downwards on the top surface. This is the cause of the upthrust, which you experience when you swim. Because your density is less than that of water, when you are underwater, the weight of water you displace is greater than your own weight. The upthrust is, therefore, greater than your own weight and there is a resultant force upwards to bring you to the surface.

To calculate this upthrust:

The force due to water on the top surface $F_1 = \rho \times g \times h_1 \times A$

Similarly, the force due to the water on the bottom surface is $F_2 = \rho \times g \times h_2 \times A$

$$\begin{aligned} \text{upthrust} &= F_2 - F_1 = \rho \times g \times (h_2 - h_1) \times A = \rho \times g \times h \times A \\ &= \rho \times g \times V \end{aligned}$$

where the volume of the object $V = h \times A$

= the weight of the liquid displaced

WORKED EXAMPLES

2 A cube of side 0.20 m floats in water with 0.15 m below the surface of the water. The density of water is 1000 kg m^{-3}. Calculate the pressure due to the water that acts upwards on the bottom surface of the cube and the force upwards on the cube caused by this pressure. (This force is the upthrust on the cube.)

Step 1 Use the equation for pressure:

$$p = \rho \times g \times h = 1000 \times 9.81 \times 0.15$$
$$= 1470 \text{ Pa}$$

Step 2 Calculate the area of the base of the cube, and use this area in the equation for force.

area of base of cube = $0.2 \times 0.2 = 0.04$ m^2

$$\text{force} = \text{pressure} \times \text{area}$$
$$= 1470 \times 0.04 = 58.8 \text{ N}$$

3 A metal block of mass 0.60 kg has dimensions 0.050 m × 0.040 m × 0.030 m.

The block is hung from a newton-meter. What is the reading on the newton-meter when the block is fully submerged in liquid of density 1200 kg m^{-3}?

Step 1 Calculate the weight of the block. This is the reading on the meter when the block is in the air, before it is placed in the liquid.

weight = mg = 0.60 × 9.81 = 5.886 = 5.9 N

Step 2 Calculate the upthrust.

The volume of liquid displaced = 0.05 × 0.04 × 0.03 = 6.0×10^{-5} m^3

mass of liquid displaced = density × volume = 1200 × 6.0×10^{-5} = 7.2×10^{-2} kg

upthrust = weight of liquid displaced = $7.2 \times 10^{-2} \times 9.81$ = 0.71 N

Step 3 Calculate the final reading

The upthrust must be subtracted from the weight of the object, so the newton-meter reads 5.89 − 0.71 = 5.2 N.

Questions

7 **a** Why is it difficult to hold an inflated plastic ball underwater?

b A submarine floats at rest under the water. To rise to the surface compressed air is used to push water out of its 'ballast' tanks into the sea. Why does this cause the submarine to rise?

8 A boat has a uniform cross-sectional area at the water line of 750 m^2. Fifteen cars of average mass 1200 kg are driven on board. Calculate the extra depth that the boat sinks in water of density 1000 kg m^{-3}.

9 Describe how to use a newton-meter, a micrometer screw gauge, a metal cube of side approximately 1.0 cm and a beaker of water to show *experimentally* that Archimedes' principle is correct. The density of water is known to be 1000 kg m^{-3}.

10 A balloon of volume 3000 m^{-3} is filled with hydrogen of density 0.090 kg m^{-3}. The mass of the fabric of the balloon is 100 kg. Calculate the greatest mass that the balloon can lift in air of density 1.2 kg m^{-3}.

7.4 Compressive and tensile forces

A pair of forces is needed to change the shape of a spring. If the spring is being squashed and shortened, we say that the forces are **compressive**. More usually, we are concerned with stretching a spring, in which case the forces are described as **tensile** (Figure 7.5).

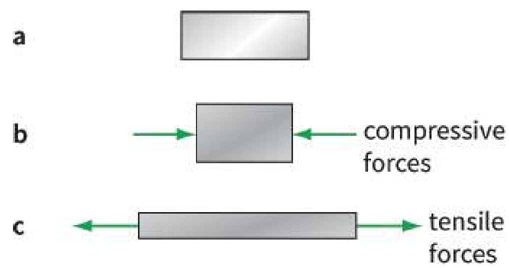

Figure 7.5: The effects of compressive and tensile forces.

When a wire is bent, some parts become longer and are in tension while other parts become shorter and are in compression. Figure 7.6 shows that the line AA becomes longer when the wire is bent and the line BB becomes shorter. The thicker the wire, the greater the compression and tension forces along its edges.

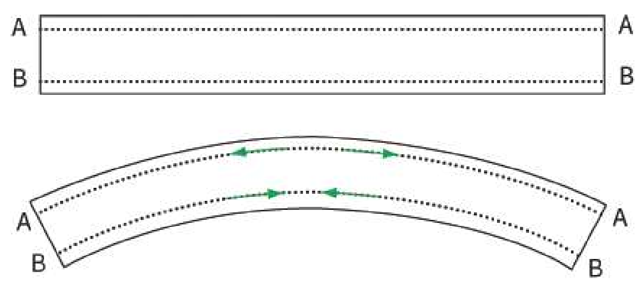

Figure 7.6: Bending a straight wire or beam results in tensile forces along the upper surface (the outside of the bend) and compressive forces on the inside of the bend.

It is simple to investigate how the length of a helical spring is affected by the applied force or load. The spring hangs freely with the top end clamped firmly (Figure 7.7). A load is added and gradually increased. For each value of the load, the extension of the spring is measured. Note that it is important to determine the increase in length of the spring, which we call the **extension**.

We can plot a graph of *force* against *extension* to find the stiffness of the spring, as shown in Figure 7.8.

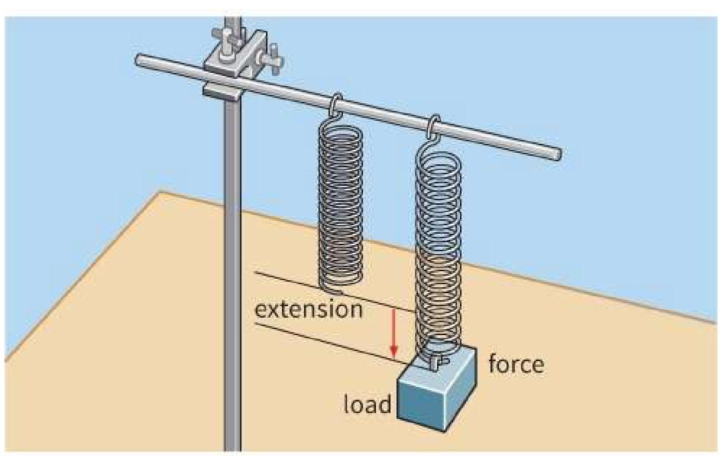

Figure 7.7: Stretching a spring.

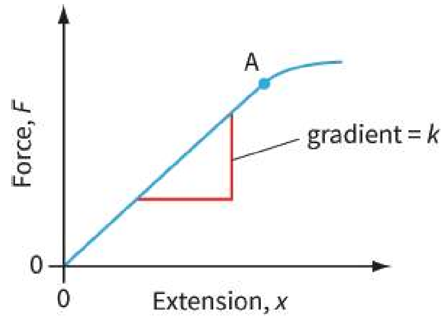

Figure 7.8: Force–extension graph for a spring.

Hooke's law

The usual way of plotting the results would be to have the force along the horizontal axis and the extension along the vertical axis. This is because we are changing the force (the independent variable) and this results in a change in the extension (the dependent variable). The graph shown in Figure 7.7 has extension on the horizontal axis and force on the vertical axis. This is a departure from the convention because the gradient of the straight section of this graph turns out to be an important quantity, known as the **spring constant**.

For a typical spring, the first section of this graph OA is a straight line passing through the origin. The extension x is directly proportional to the applied force (load) F. The behaviour of the spring in the linear region OA of the graph can be expressed by the following equation:

$$x \propto F \text{ or } F = kx$$

where k is the spring constant (sometimes called the stiffness or force constant of the spring). The spring constant is the force per unit extension, given by:

$$k = \frac{F}{x}$$

KEY EQUATION

$$\text{spring constant} = \frac{\text{force}}{\text{extension}}$$
$$k = \frac{F}{x}$$

The SI unit for the force constant is newtons per metre or N m^{-1}. We can find the force constant k from the gradient of section OA of the graph:

$$k = \text{gradient}$$

A stiffer spring will have a larger value for the force constant k. Beyond point A, the graph is no longer a straight line; its gradient changes and we can no longer use the equation $F = kx$.

If a spring or anything else responds to a pair of tensile forces in the way shown in section OA of Figure 7.7, we say that it obeys **Hooke's law**. A material obeys Hooke's law if the extension produced in it is proportional to the applied force (load).

The point A is known as the **limit of proportionality**. This is the point beyond which the extension is no longer proportional to the force.

If you apply a small force to a spring and then release it, it will return to its original length (this is **elastic deformation**.). However, if you apply a large force, the spring may not return to its original length; the spring has become permanently deformed (this is **plastic deformation**.). The force beyond which the spring becomes permanently deformed is known as the **elastic limit**.

The elastic limit is not necessarily the same point as the limit of proportionality, although they are likely to be close to each other.

This use of the word 'elastic' in elastic limit is slightly different from the idea of an elastic collision covered in Chapter 6. But the two ideas are related.

Question

11 Figure 7.9 shows the force–extension graph for a wire that is stretched and then released.

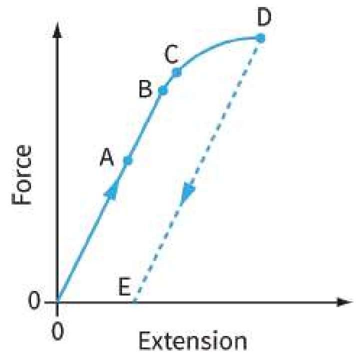

Figure 7.9: Force–extension graph for a wire.

a Which point shows the limit of proportionality?

b Which point shows the elastic limit?

PRACTICAL ACTIVITY 7.1

Investigating springs

Springs can be combined in different ways (Figure 7.10): end-to-end (in series) and side-by-side (in parallel). Using identical springs, you can measure the force constant of a single spring, and of springs in series and in parallel. Before you do this, predict the outcome of such an experiment. If the force constant of a single spring is k, what will be the equivalent force constant of:

- two springs in series?
- two springs in parallel?

This approach can be applied to combinations of three or more springs.

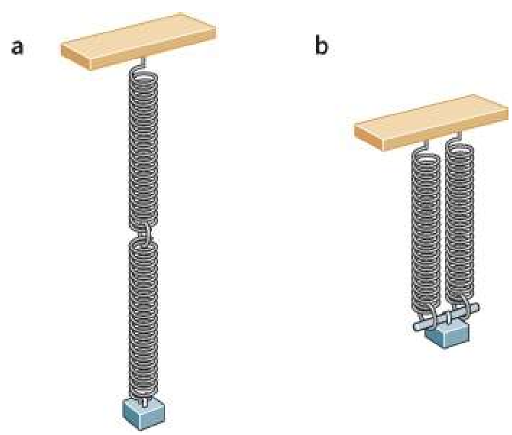

Figure 7.10: Two ways to combine a pair of springs: *a* in series; *b* in parallel.

12 Figure 7.11 shows the force–extension graphs for four springs, A, B, C and D.

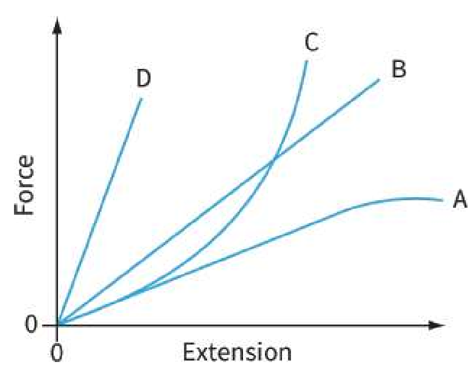

Figure 7.11: Force–extension graphs for four different springs.

- **a** State which spring has the greatest value of force constant.
- **b** State which is the least stiff.
- **c** State which of the four springs does not obey Hooke's law.

7.5 Stretching materials

When we determine the force constant of a spring, we are only finding out about the stiffness of that particular spring. However, we can compare the stiffness of different materials. For example, steel is stiffer than copper, but copper is stiffer than lead.

Stress and strain

Figure 7.12 shows a simple way of assessing the stiffness of a wire in the laboratory. As the long wire is stretched, the position of the sticky tape pointer can be read from the scale on the bench.

Why do we use a long wire? Obviously, this is because a short wire would not stretch as much as a long one.

We need to take account of this in our calculations, and we do this by calculating the strain produced by the load. The **strain** is defined as the increase in length of a wire (its extension) divided by its the original length.

That is:

$$\text{strain} = \frac{\text{extension}}{\text{original length}}$$
$$\varepsilon = \frac{x}{L}$$

where ε is the strain, x is the extension of the wire and L is its original length.

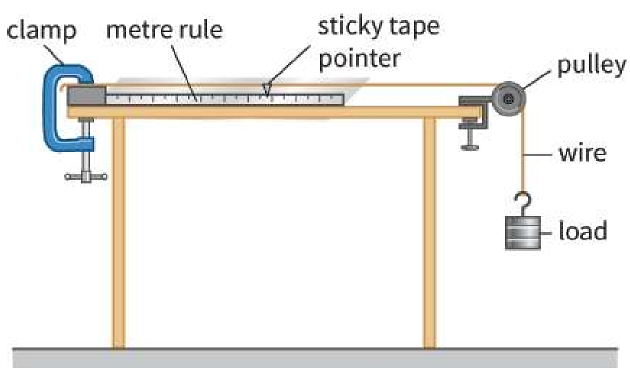

Figure 7.12: Stretching a wire in the laboratory. Wear eye protection and be careful not to overload the wire.

KEY EQUATION

$$\text{strain} = \frac{\text{extension}}{\text{original length}}$$
$$\varepsilon = \frac{x}{L}$$

For example, if a wire of length 1.500 m is stretched and the length becomes 1.518 m, the extension is 0.018 m and the strain $= \frac{0.018}{1.5000} = 0.012$.

Note that both extension and original length must be in the same units, and so strain is a ratio, without units. Sometimes, strain is given as a percentage. For example, a strain of 0.012 is equivalent to 1.2%.

Why do we use a thin wire? This is because a thick wire would not stretch as much for the same force. Again, we need to take account of this in our calculations, and we do this by calculating the **stress** produced by the load.

The stress is defined as the force applied per unit cross-sectional area of the wire. That is:

$$\text{stress} = \frac{\text{normal force}}{\text{cross-sectional area}}$$
$$\sigma = \frac{F}{A}$$

where σ is the stress, F is the applied force that acts normally (at right angles) on a wire of cross-sectional area A.

KEY EQUATION

$$\text{stress} = \frac{\text{normal force}}{\text{cross-sectional area}}$$
$$\sigma = \frac{F}{A}$$

The units of stress are newtons per square metre (N m^{-2}) or pascals (Pa), the same as the units of pressure:

$$1 \text{ Pa} = 1 \text{ N m}^{-2}$$

The Young modulus

We can now find the stiffness of the material we are stretching. Rather than calculating the ratio of force to extension as we would for a spring or a wire, we calculate the ratio of stress to strain. This ratio is a constant for a particular material and does not depend on its shape or size. The ratio of stress to strain is called the **Young modulus** of the material. That is:

$$\text{Young modulus} = \frac{\text{stress}}{\text{strain}}$$
$$E = \frac{\sigma}{\varepsilon}$$

where E is the Young modulus of the material, σ is the stress and ε is the strain.

The unit of the Young modulus is the same as that for stress, N m^{-2} or Pa. In practice, values may be quoted in MPa or GPa. These units are related as:

$$1 \text{ MPa} = 10^6 \text{ Pa}$$

$$1 \text{ GPa} = 10^9 \text{ Pa}$$

Usually, we plot a graph with stress on the vertical axis and strain on the horizontal axis (Figure 7.13).

It is drawn like this so that the gradient is the Young modulus of the material. It is important to consider only the first, linear section of the graph. In the linear section stress is proportional to strain and the wire under test obeys Hooke's law.

Table 7.1 gives some values of the Young modulus for different materials.

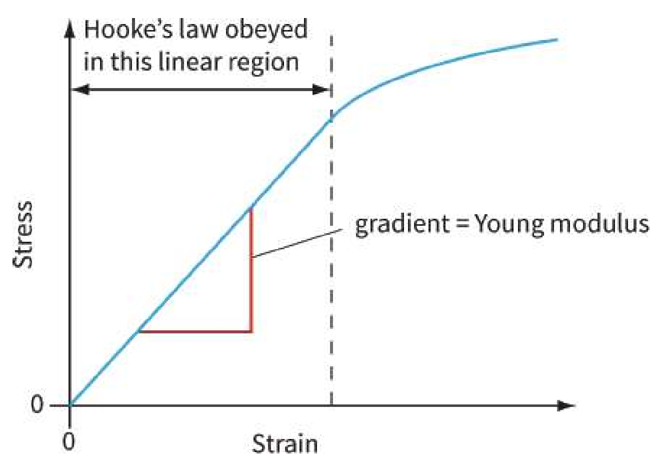

Figure 7.13: Stress–strain graph, and how to deduce the Young modulus. Note that we can only use the first, straight-line section of the graph.

Material	Young modulus / GPa
aluminium	70
brass	90–110

brick	7–20
concrete	40
copper	130
glass	70–80
iron (wrought)	200
lead	18
Perspex®	3
polystyrene	2.7–4.2
rubber	0.01
steel	210
tin	50
wood	10 approx.

Table 7.1: The Young modulus of various materials. Many of these values depend on the precise composition of the material concerned. (Remember, 1 GPa = 10^9 Pa.)

Questions

13 List the metals in Table 7.1 from stiffest to least stiff.

14 Which of the non-metals in Table 7.1 is the stiffest?

15 Figure 7.14 shows stress–strain graphs for two materials, A and B. Use the graphs to determine the Young modulus of each material.

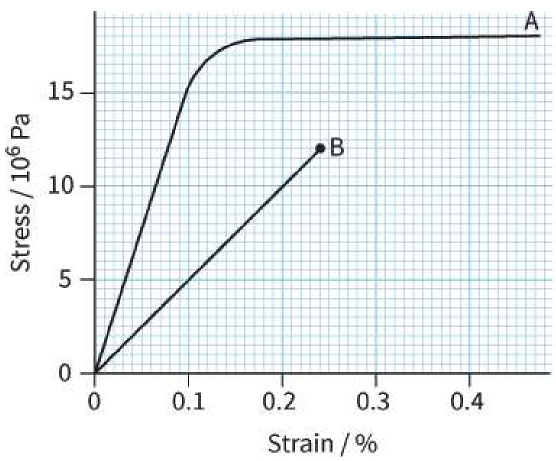

Figure 7.14: Stress–strain graphs for two different materials.

16 A piece of steel wire, 200.0 cm long and having cross-sectional area of 0.50 mm^2, is stretched by a force of 50 N. Its new length is found to be 200.1 cm. Calculate the stress and strain, and the Young modulus of steel.

17 Calculate the extension of a copper wire of length 1.00 m and diameter 1.00 mm when a tensile force of 10 N is applied to the end of the wire. (Young modulus of copper = 130 GPa.)

18 In an experiment to measure the Young modulus of glass, a student draws out a glass rod to form a fibre 0.800 m in length. Using a travelling microscope, she estimates its diameter to be 0.40 mm. Unfortunately, it proves impossible to obtain a series of readings for load and extension. The fibre snaps when a load of 1.00 N is hung on the end. The student judges that the fibre extended by no more than 1 mm before it snapped.

Use these values to obtain an estimate for the Young modulus of the glass used. Explain how the actual or accepted value for the Young modulus might differ from this estimate.

19 For each of the materials whose stress–strain graphs are shown in Figure 7.15, deduce the values of the Young modulus.

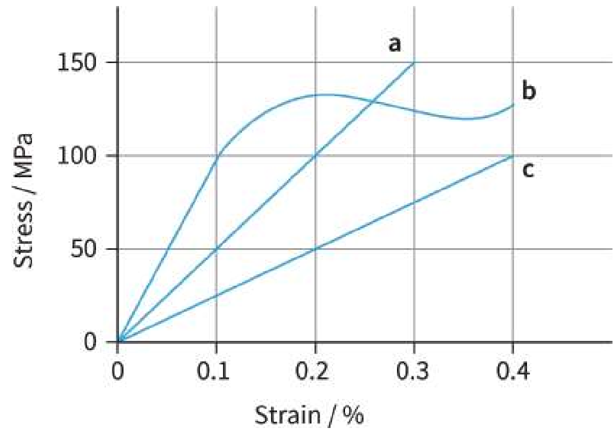

Figure 7.15: Stress–strain graphs for three materials.

PRACTICAL ACTIVITY 7.2

Determining the Young modulus

You must be able to describe this experiment in detail. Learn how to draw the diagram in Figure 7.11 and how to measure each of the quantities in:

$$E = \frac{FL}{Ax} = \left(\frac{F}{x}\right) \times \left(\frac{4L}{\pi d^2}\right)$$

Metals are not very elastic. Normally, they can only be stretched by about 0.1% of their original length. Beyond this, they become permanently or plastically deformed. As a result, some careful thought must be given to getting results that are good enough to give an accurate value of the Young modulus.

First, the wire used must be long. The increase in length is proportional to the original length, and so a longer wire gives larger and more measurable extensions. Typically, extensions up to 1 mm must be measured for a wire of length 1 m. To get suitable measurements of extension there are two possibilities: use a very long wire, or use a method that allows measurement of extensions that are a fraction of a millimetre.

The apparatus shown in Figure 7.12 can be used with a travelling microscope placed above the wire and focused on the sticky tape pointer. When the pointer moves, the microscope is adjusted to keep the pointer at the middle of the cross-hairs on the microscope. The distance that the pointer has moved can then be measured accurately from the scale on the microscope.

Second, the cross-sectional area of the wire must be known accurately. The diameter of the wire is measured using a micrometer screw gauge. This is reliable to within ±0.01 mm. Once the wire has been loaded in increasing steps, the load must be gradually decreased to ensure that there has been no permanent deformation of the wire.

A graph of F against x can be drawn and the gradient used to find an average value of $\frac{F}{X}$, where F is the weight of the load and x is the extension shown by the distance moved by the pointer.

The area A is found from $A = \frac{\pi d^2}{4}$, where d is the diameter of the wire.

The diameter should be measured at several points along the wire and the average value found. The length L is measured from the sticky pointer to the point where the wire is clamped.

Other materials, such as glass and many plastics, are also quite stiff and so it is difficult to measure their Young modulus. Rubber is not as stiff, and strains of several hundred per cent can be achieved. However, the stress–strain graph for rubber is not a straight line. This means the value of the Young modulus found is not very precise, because it only has a very small linear region on a stress–strain graph.

7.6 Elastic potential energy

Whenever you stretch a material, you are doing work. This is because you have to apply a force and the material extends in the direction of the force. You will know this if you have ever used exercise equipment with springs to develop your muscles (such as in Figure 7.16). Similarly, when you push down on the end of a springboard before diving, you are doing work. You transfer energy to the springboard, and you recover the energy when it pushes you up into the air.

We call the energy in a deformed solid the **elastic potential energy** or **strain energy**. If the material has been strained elastically (the elastic limit has not been exceeded), the energy can be recovered. If the material has been plastically deformed, some of the work done has gone into moving atoms past one another and the energy cannot be recovered.

The material warms up slightly. We can determine how much elastic potential energy is involved from a force–extension graph: see Figure 7.17. We need to use the equation that defines the amount of work done by a force. That is:

$$\text{work done} = \text{force} \times \text{distance moved in the direction of the force}$$

Figure 7.16: Using springs to help you exercise is hard work.

This equation only holds when the force is constant. When you stretch a spring the force varies; so how can you find the work done?

There are two approaches. You can:

- use the average force in the equation for work; this works well where the force–extension graph is a straight line
- add together many small extensions, in each of which the force hardly changes; adding together lots of very small extensions shows us that the work done is the area under the force–extension graph.

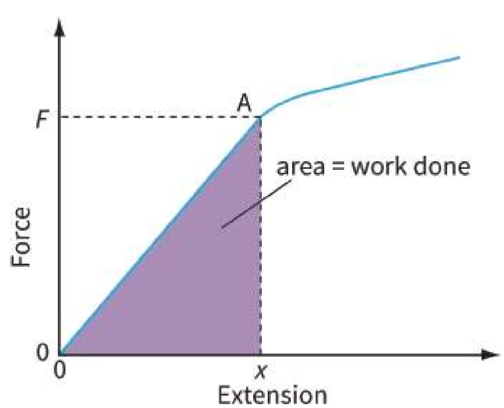

Figure 7.17: Elastic potential energy is equal to the area under the force–extension graph.

First, consider the linear region of the graph where Hooke's law is obeyed, OA. The graph in this region is a straight line through the origin. The extension x is directly proportional to the applied force F. There are two ways to find the work done.

Method 1

We can think about the average force needed to produce an extension x. The average force is half the final force F, and so we can write:

$$\begin{aligned} \text{elastic potential energy} &= \text{work done} \\ \text{elastic potential energy} &= \frac{\text{final force}}{2} \times \text{extension} \\ E &= \frac{1}{2}Fx \end{aligned}$$

Method 2

The other way to find the elastic potential energy is to recognise that we can get the same answer by finding the area under the graph. The area shaded in Figure 7.17 is a triangle whose area is given by:

$$\text{area} = \frac{1}{2} \times \text{base} \times \text{height}$$

This again gives:

$$E = \frac{1}{2}Fx$$

The **work done** in **stretching** or **compressing** a material is always equal to the area under the graph of force against extension.

This is true whatever the shape of the graph, provided we draw the graph with extension on the horizontal axis. If the graph is not a straight line, we cannot use the Fx relationship, so we have to resort to counting squares or some other technique to find the answer.

Note that the elastic potential energy relates to the elastic part of the graph (i.e. up to the elastic limit), so we can only consider the force–extension graph up to the elastic limit.

There is an alternative equation for elastic potential energy. We know that, according to Hooke's law, applied force F and extension x are related by $F = kx$, where k is the force constant. Substituting for F gives:

$$\begin{aligned} \text{elastic potential energy} &= \frac{1}{2}Fx \\ &= \frac{1}{2} \times kx \times x \\ &= \frac{1}{2}kx^2 \end{aligned}$$

KEY EQUATION

$$\text{elastic potential energy} = \frac{1}{2}Fx$$

$$E = \frac{1}{2}Fx = \frac{1}{2}kx^2$$

Questions

20 A force of 12 N extends a length of rubber band by 18 cm. Estimate the energy stored in this rubber band. Explain why your answer can only be an estimate.

21 A spring has a force constant of 4800 N m^{-1}. Calculate the elastic potential energy when it is compressed by 2.0 mm.

WORKED EXAMPLE

4 Figure 7.18 shows a simplified version of a force–extension graph for a piece of metal. Find the elastic potential energy when the metal is stretched to its limit of proportionality, and the total work that must be done to break the metal.

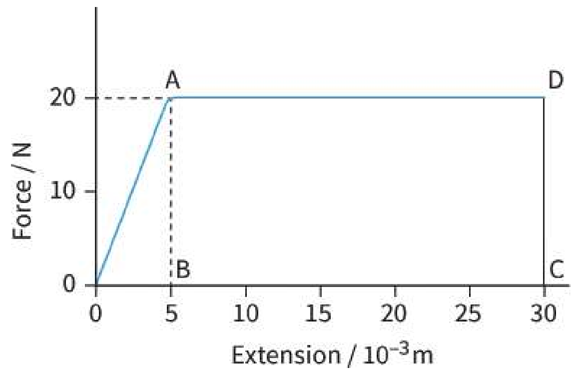

Figure 7.18: For Worked example 4.

Step 1 The elastic potential energy when the metal is stretched to its elastic limit is given by the area under the graph up to the elastic limit. We will have to take the limit of proportionality, the point A, as the elastic limit. The graph is a straight line up to x = 5.0 mm, F = 20 N, so the elastic potential energy is the area of triangle 0AB:

$$\begin{aligned} \text{elastic potential energy} &= \tfrac{1}{2}Fx \\ &= \tfrac{1}{2} \times 20 \times 5.0 \times 10^{-3} \\ &= 0.05 \text{ J} \end{aligned}$$

Step 2 To find the work done to break the metal, we need to add on the area of the rectangle ABCD:

$$\begin{aligned} \text{work done} &= \text{total area under graph} \\ &= 0.05 + \left(20 \times 25 \times 10^{-3}\right) \\ &= 0.05 + 0.50 \\ &= 0.55 \text{ J} \end{aligned}$$

22 Figure 7.19 shows force–extension graphs for two materials. For each of the following questions, make the statement required. Also explain how you deduce your answer from the graphs.

a State which polymer has the greater stiffness.

b State which polymer requires the greater force to break it.

c State which polymer requires the greater amount of work to be done in order to break it.

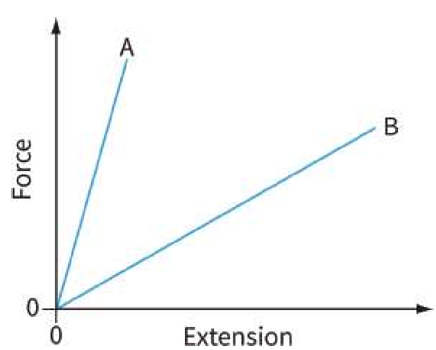

Figure 7.19: Force–extension graph for two polymers.

REFLECTION

What is the most important thing that you learned personally in this chapter?

Think of examples where having materials with high or low Young modulus is useful.

Make sure you know the formulae for stress, strain and Young modulus and can write them down in terms of F, A, x and L.

SUMMARY

Density is defined as the mass per unit volume of a substance:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Pressure is defined as the normal force acting per unit cross-sectional area:

$$\text{pressure} = \frac{\text{normal force}}{\text{cross-sectional area}}$$

Pressure in a fluid increases with depth: $p = \rho gh$

Upthrust on an object in a fluid is given by $F = \rho gV$ (Archimedes' principle).

Hooke's law states that the extension of a material is directly proportional to the applied force, provided the limit of proportionality is not exceeded. For a spring or a wire, $F = kx$, where k is the force constant. The force constant has units of N m^{-1}.

Stress is defined as:

$$\text{stress} = \frac{\text{force}}{\text{cross-sectional area}} \text{ or } \sigma = \frac{F}{A}$$

Strain is defined as:

$$\text{strain} = \frac{\text{extension}}{\text{original length}} \text{ or } \varepsilon = \frac{x}{L}$$

To describe the behaviour of a material under tensile and compressive forces, we have to draw a graph of stress against strain. The gradient of the initial linear section of the graph is equal to the Young modulus. The Young modulus is an indication of the stiffness of the material.

The Young modulus E is given by:

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{\varepsilon} \left(\text{unit: pascal (Pa) or N m}^{-2}\right)$$

The area under a force–extension graph is equal to the work done by the force.

For a spring or a wire obeying Hooke's law, the elastic potential energy E is given by:

$$E = \frac{1}{2}\text{Fx or E} = \frac{1}{2}kx^2$$

EXAM-STYLE QUESTIONS

1 Which force is caused by a difference in pressure? **[1]**

A drag

B friction

C upthrust

D weight

2 Two wires P and Q both obey Hooke's law. They are both stretched and have the same strain. The Young modulus of P is four times larger than that of Q. The diameter of P is twice that of Q.

What is the ratio of the tension in P to the tension in Q? **[1]**

A $\frac{1}{2}$

B 1

C 2

D 16

3 **a** **i** Define density. **[1]**

ii State the SI base units in which density is measured. **[1]**

b **i** Define pressure. **[1]**

ii State the SI base units in which pressure is measured. **[1]**

[Total: 4]

4 Sketch a force–extension graph for a spring that has a spring constant of 20 N m^{-1} and that obeys Hooke's law for forces up to 5.0 N. Your graph should cover forces between 0 and 6 N and show values on both axes. **[3]**

5 Two springs, each with a spring constant 20 N m^{-1}, are connected in series. Draw a diagram of the two springs in series and determine the total extension if a mass, with weight 2.0 N, is hung on the combined springs. **[5]**

6 A sample of fishing line is 1.0 m long and is of circular cross-section of radius 0.25 mm. When a weight is hung on the line, the extension is 50 mm and the stress is 2.0×10^8 Pa. Calculate:

a the cross-sectional area of the line **[1]**

b the weight added **[2]**

c the strain in the line **[2]**

d the Young modulus **[2]**

e the percentage and absolute uncertainties in the Young modulus if the uncertainty in the extension is ±1 mm. **[2]**

[Total: 9]

7 This is the force–extension graph for a metal wire of length 2.0 m and cross-sectional area 1.5×10^{-7} m^2.

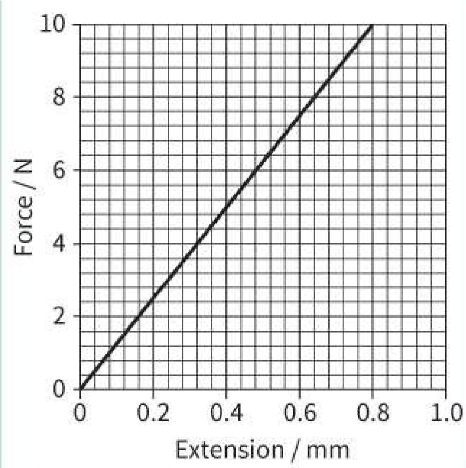

Figure 7.20

a Calculate the Young modulus. **[3]**

b Determine the energy stored in the wire when the extension is 0.8 mm. **[2]**

c Calculate the work done in stretching the wire between 0.4 mm and 0.8 mm. **[2]**

[Total: 7]

8 A piece of wax is attached to a newton-meter. In air, the reading on the newton-meter is 0.27 N and when submerged in water of density 1000 kg m^{-3} the reading is 0.16 N. Calculate:

a the upthrust on the wax when in water **[1]**

b the volume of the wax **[2]**

c the reading on the newton-meter when the wax is submerged in a liquid of density 800 kg m^{-3}. **[2]**

[Total: 5]

9 **a** These are stress–strain curves for three different materials, P, Q and R. State and explain which material has the greatest Young modulus. **[2]**

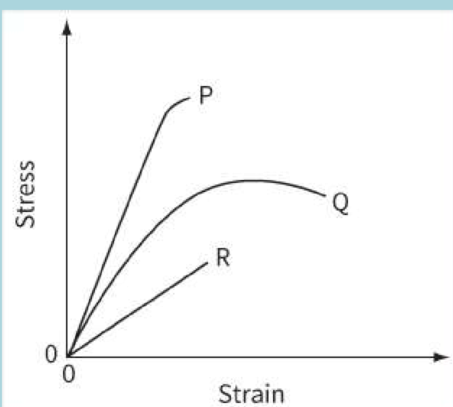

Figure 7.21

b Describe an experiment to determine the Young modulus for a material in the form of a wire. Include a labelled diagram and explain how you would make the necessary measurements. Show how you would use your measurements to calculate the Young modulus. **[7]**

[Total: 9]

10 **a** State the meaning of tensile stress and tensile strain. **[2]**

b A vertical steel wire of length 1.6 m and cross-sectional area 1.3×10^{-6} m^2 carries a weight of 60 N. The Young modulus for steel is 2.1×10^{11} Pa. Calculate:

i the stress in the wire **[2]**

ii the strain in the wire **[2]**

iii the extension produced in the wire by the weight. **[2]**

[Total: 6]

11 To allow for expansion in the summer when temperatures rise, a steel railway line laid in cold weather is pre-stressed by applying a force of 2.6×10^5 N to the rail of cross-sectional area 5.0×10^{-3} m^2.

If the railway line is not pre-stressed, a strain of 1.4×10^{-5} is caused by each degree Celsius rise in temperature. The Young modulus of the steel is 2.1×10^{11} Pa.

a State and explain whether the force applied to the rail when it is laid should be tensile or compressive. **[2]**

b Calculate:

i the strain produced when the rail is laid **[3]**

ii the temperature rise when the rail becomes unstressed. **[2]**

[Total: 7]

12 This is a stress–strain graph for a metal wire.

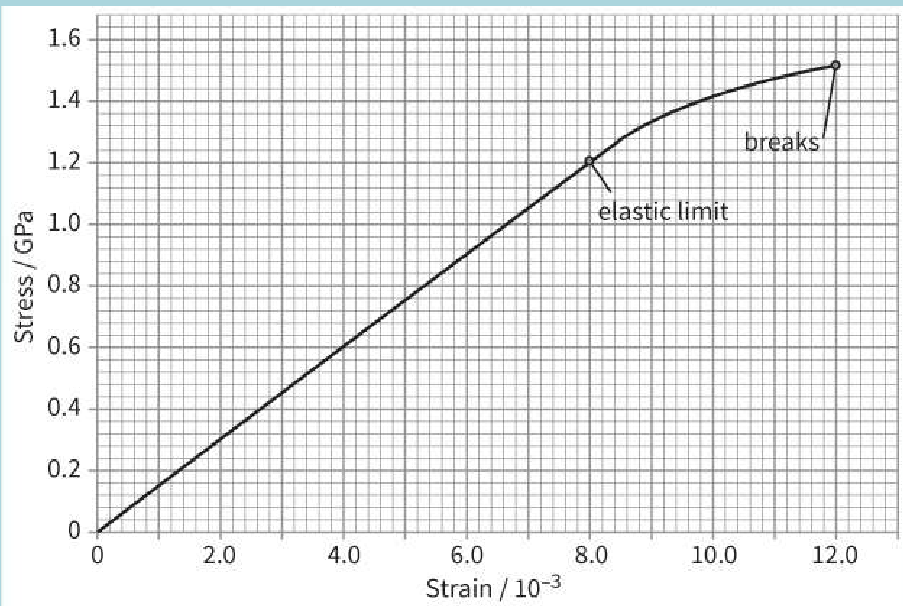

Figure 7.22

The wire has a diameter of 0.84 mm and a natural length of 3.5 m.

Use the graph to determine:

a the Young modulus of the wire **[3]**

b the extension of the wire when the stress is 0.60 GPa **[2]**

c the force that causes the wire to break, assuming that the cross-sectional area of the wire remains constant **[3]**

d the energy stored when the wire has a stress of 0.60 GPa. **[3]**

[Total: 11]

13 This is a force–extension graph for a spring.

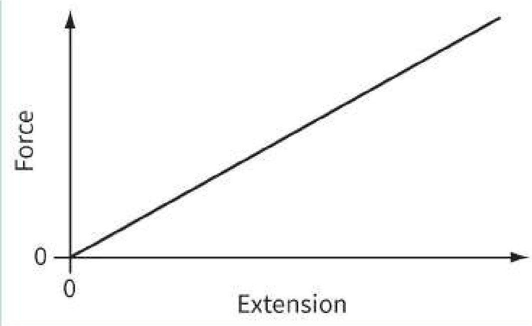

Figure 7.23

a State what is represented by:

i the gradient of the graph **[1]**

ii the area under the graph. **[1]**

b The spring has force constant k = 80 N m^{-1}. The spring is compressed by 0.060 m, within the limit of proportionality, and placed between two trolleys that run on a friction-free, horizontal track. Each trolley has a mass of 0.40 kg. When the spring is released the trolleys fly apart with equal speeds but in opposite directions.

i How much energy is stored in the spring when it is compressed by 0.060 m? **[2]**

ii Explain why the two trolleys must fly apart with equal speeds. **[2]**

iii Calculate the speed of each trolley. **[2]**

[Total: 8]

14 a Liquid of density ρ fills a cylinder of base area A and height h.

i Using the symbols provided, state the mass of liquid in the container. **[1]**

ii Using your answer to **i**, derive a formula for the pressure exerted on the base of the cylinder. **[2]**

b A boy stands on a platform of area 0.050 m^2 and a manometer measures the pressure created in a flexible plastic container by the weight W of the boy, as shown.

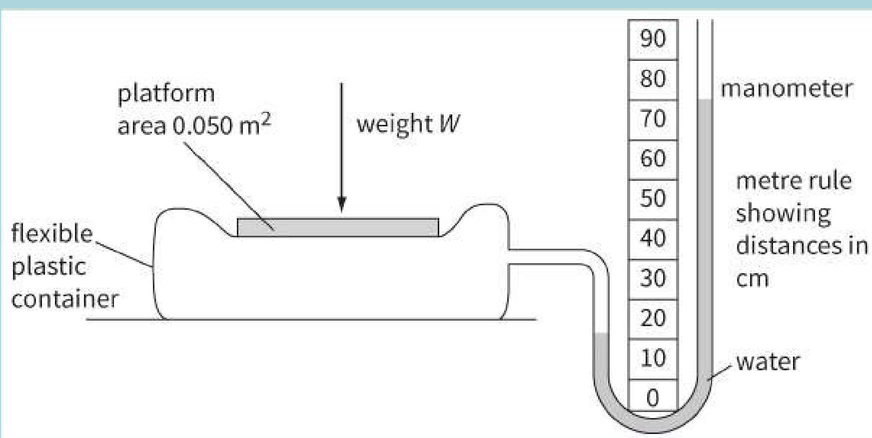

Figure 7.24

The density of water is 1000 kg m^{-3}. Determine:

i the pressure difference between the inside of the plastic container and the atmosphere outside **[2]**

ii the weight W of the boy. **[2]**

[Total: 7]

15 This diagram shows water in a container filled to a depth of 0.50 m. The density of water is 1000 kg m^{-3}.

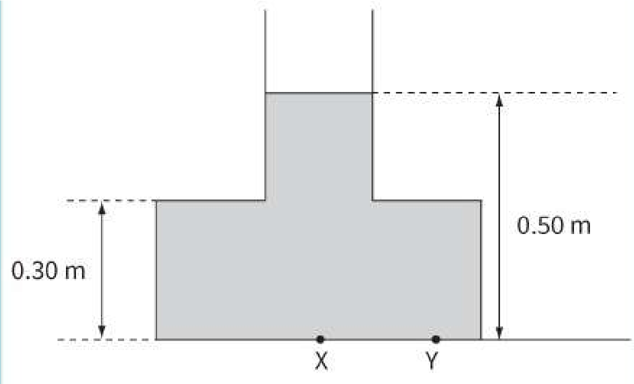

Figure 7.25

a Calculate the pressure at X on the base of the container. **[2]**

b Explain why the pressure at X must be equal to the pressure at Y. **[1]**

c Explain why the force downwards on the base of the container is larger than the weight of the liquid in the container. **[2]**

[Total: 5]

16 A light spring that obeys Hooke's law has an unstretched length of 0.250 m. When an object of mass 2.0 kg is hung from the spring the length of the spring becomes 0.280 m. When the object is fully submerged in a liquid of density 1200 kg m^{-3}, the length of the spring becomes 0.260 m.

Calculate:

a the spring constant of the spring. **[2]**

b the upthrust on the object. **[2]**

c the volume of the object. **[2]**

d the density of the material from which the object is made. **[2]**

[Total: 8]