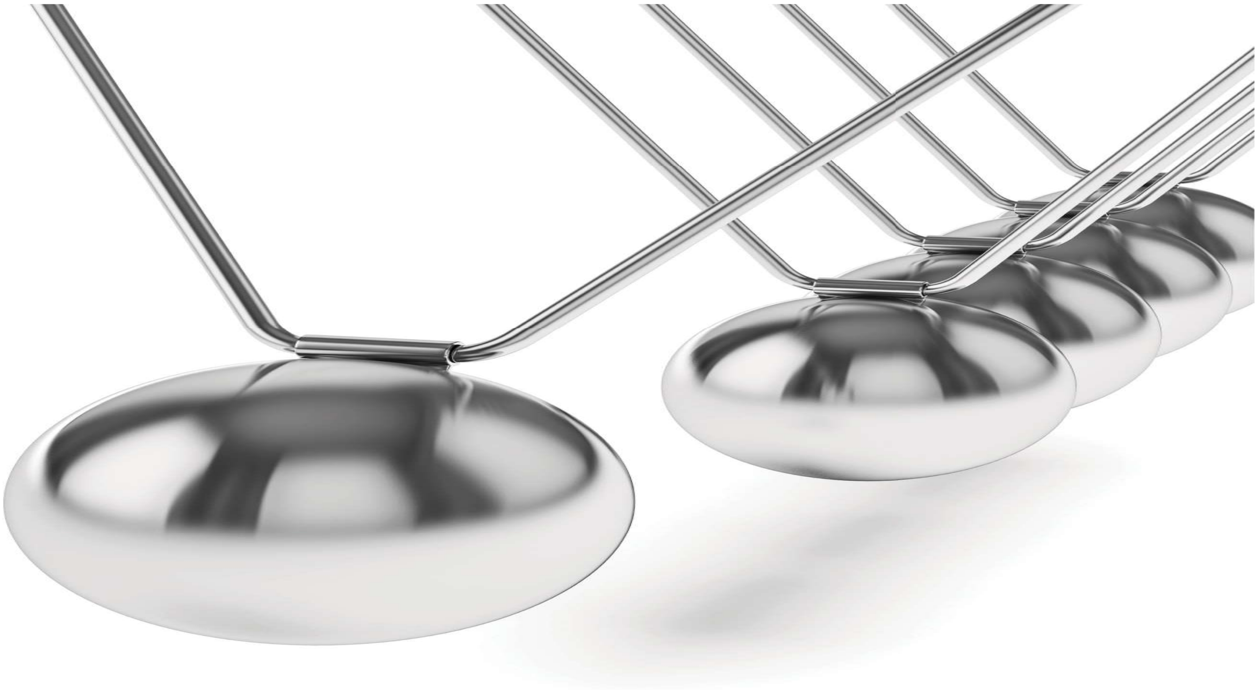


Chapter 6

Momentum

LEARNING INTENTIONS

In this chapter you will learn how to:

- define and use linear momentum
- state and apply the principle of conservation of momentum to collisions in one and two dimensions
- relate force to the rate of change of momentum and state Newton's second law of motion
- recall that, for a perfectly elastic collision, the relative speed of approach is equal to the relative speed of separation
- discuss energy changes in perfectly elastic and inelastic collisions.

BEFORE YOU START

- What do you understand about Newton's laws? Write down all three of them in your own words. Define any of the quantities mentioned in the laws.
- If you blow up a balloon and then let it go without tying the end, why does the balloon fly around in the air?

UNDERSTANDING COLLISIONS

To improve the safety of cars, the motion of a car during a crash must be understood and the forces on the driver minimised (Figure 6.1). In this way, safer cars have been developed and many lives have been saved. Find out about as many safety features of cars as you can and discuss with someone else why these features improve safety in a crash.

In this chapter, we will explore how the idea of momentum can allow us to predict how objects move after colliding (interacting) with each other. We will also see how Newton's laws of motion can be expressed in terms of momentum.

Figure 6.1: A high-speed photograph of a crash test. The cars collide head-on at 15 m s^{-1} with dummies as drivers.

6.1 The idea of momentum

Snooker players can perform some amazing moves on the table, without necessarily knowing Newton's laws of motion – see Figure 6.2.

Figure 6.2: If you play snooker often enough, you will be able to predict how the balls will move on the table. Alternatively, you can use the laws of physics to predict their motion.

However, the laws of physics can help us to understand what happens when two snooker balls collide or when one bounces off the side cushion of the table.

Here are some examples of situations involving collisions:

- Two cars collide head-on.
- A fast-moving car runs into the back of a slower car in front.
- A footballer runs into an opponent.
- A hockey stick strikes a ball.
- A comet or an asteroid collides with a planet as it orbits the Sun.
- The atoms of the air collide constantly with each other, and with the walls of their surroundings.
- Electrons that form an electric current collide with the vibrating ions that make up a metal wire.
- Two distant galaxies collide over millions of years.

From these examples, we can see that collisions are happening all around us, all the time. They happen on the microscopic scale of atoms and electrons, they happen in our everyday world, and they also happen on the cosmic scale of our Universe.

6.2 Modelling collisions

Springy collisions

Figure 6.3a shows what happens when one snooker ball collides head-on with a second, stationary ball. The result can seem surprising. The moving ball stops dead. The ball initially at rest moves off with the same velocity as that of the original ball. To achieve this, a snooker player must observe two conditions:

- The collision must be head-on. (If one ball strikes a glancing blow on the side of the other, they will both move off at different angles.)
- The moving ball must not be given any spin. (Spin is an added complication that we will ignore in our present study, although it plays a vital part in the games of pool and snooker.)

You can mimic the collision of two snooker balls in the laboratory using two identical trolleys, as shown in Figure 6.3b. The moving trolley has its spring-load released, so that the collision is springy. As one trolley runs into the other, the spring is at first compressed, and then it pushes out again to set the second trolley moving. The first trolley comes to a complete halt. The 'motion' of one trolley has been transferred to the other.

You can see another interesting result if two moving identical trolleys collide head-on. If the collision is springy, both trolleys bounce backwards. If a fast-moving trolley collides with a slower one, the fast trolley bounces back at the speed of the slow one, and the slow one bounces back at the speed of the fast one. In this collision, it is as if the velocities of the trolleys have been swapped.

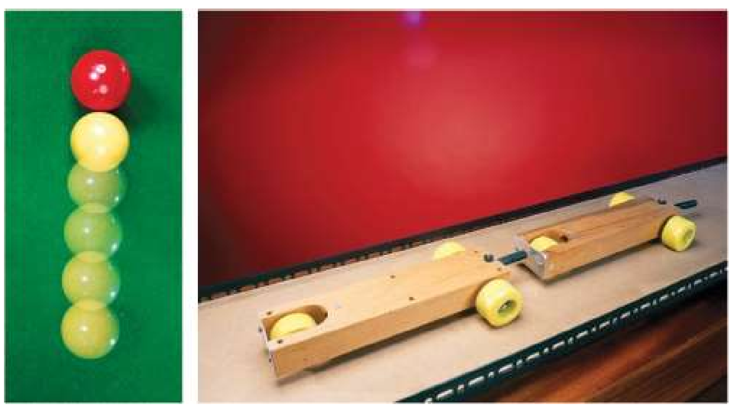


Figure 6.3: **a** The red snooker ball, coming from the left, has hit the yellow ball head-on. **b** You can do the same thing with two trolleys in the laboratory.

Sticky collisions

Figure 6.4 shows another type of collision. In this case, the trolleys have adhesive pads so that they stick together when they collide. A sticky collision like this is the opposite of a springy collision like the ones described previously.

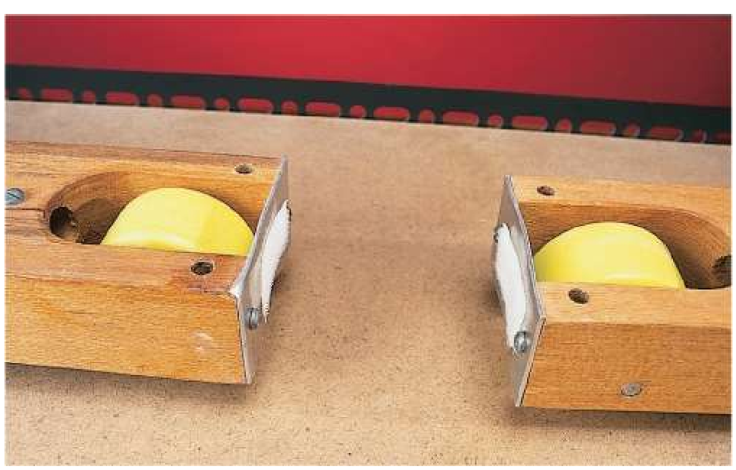


Figure 6.4: If a moving trolley sticks to a stationary trolley, they both move off together.

If a single moving trolley collides with an identical stationary one, they both move off together. After the collision, the speed of the combined trolleys is half that of the original trolley. It is as if the 'motion' of the original trolley has been shared between the two. If a single moving trolley collides with a stationary double trolley (twice the mass), they move off with one-third of the original velocity.

From these examples of sticky collisions, you can see that, when the mass of the trolley increases as a result of a collision, its velocity decreases. Doubling the mass halves the velocity, and so on.

Question

1 **a** Ball A, moving towards the right, collides with stationary ball B. Ball A bounces back; ball B moves off slowly to the right. Which has the greater mass, ball A or ball B?

b Trolley A, moving towards the right, collides with stationary trolley B. They stick together, and move off at less than half A's original speed. Which has the greater mass, trolley A or trolley B?

Defining linear momentum

From the examples discussed earlier, we can see that two quantities are important in understanding collisions:

- the mass *m* of the object
- the velocity *v* of the object.

These are combined to give a single quantity, called the **linear momentum** (or simply momentum) *p* of an object.

KEY EQUATION

$$\text{momentum} = \text{mass} \times \text{velocity}$$
$$p = mv$$

The momentum of an object is defined as the product of the mass of the object and its velocity. Hence:

$$\text{momentum} = \text{mass} \times \text{velocity}$$
$$p = mv$$

The SI unit of momentum is kg m s^{-1}. There is no special name for this unit in the SI system. The newton second (N s) can also be used as a unit of momentum (see topic 6.7).

Momentum is a vector quantity because it is a product of a vector quantity (velocity) and a scalar quantity (mass). Momentum has both magnitude and direction. Its direction is the same as the direction of the object's velocity.

In the earlier examples, we described how the 'motion' of one trolley appeared to be transferred to a second trolley, or shared with it. It is more correct to say that it is the trolley's momentum that is transferred or shared. (More precisely, we should refer to linear momentum, because there is another quantity called angular momentum that is possessed by spinning objects.)

As with energy, we find that momentum is also conserved. We have to consider objects that form a **closed system**–that is, no resultant external force acts on them. The principle of **conservation of momentum** states that, within a closed system, the total momentum in any direction is constant.

The principle of conservation of momentum can also be expressed as follows:

For a closed system where no resultant external force acts, in any direction:

total momentum of objects before collision = total momentum of objects after collision

A group of colliding objects always has as much momentum after the collision as it had before the collision. This principle is illustrated in Worked example 1.

WORKED EXAMPLE

1 In Figure 6.5, trolley A of mass 0.80 kg travelling at a velocity of 3.0 m s^{-1} collides head-on with a stationary trolley B. Trolley B has twice the mass of trolley A. The trolleys stick together and have a common velocity of 1.0 m s^{-1} after the collision. Show that momentum is conserved in this collision.

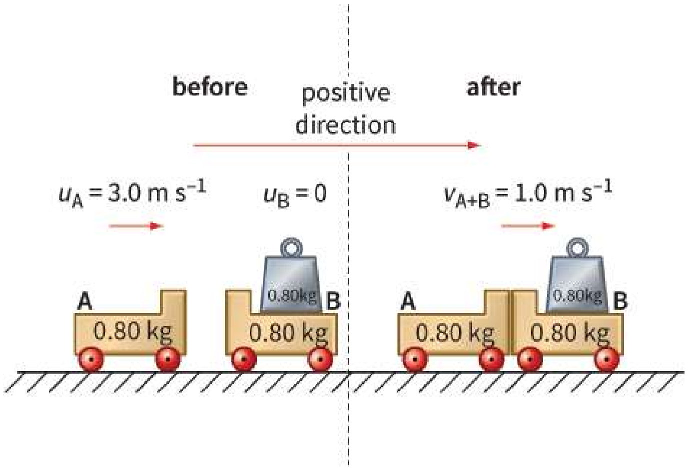


Figure 6.5: The state of trolleys A and B, before and after the collision.

Step 1 Make a sketch using the information given in the question. Notice that we need two diagrams to show the situations, one before and one after the collision. Similarly, we need two calculations – one for the momentum of the trolleys before the collision and one for their momentum after the collision.

Step 2 Calculate the momentum before the collision:

momentum of trolleys before collision

$= m_A \times u_A + m_B \times u_B$

$= (0.80 \times 3.0) + 0$

$= 2.4$ kg m s^{-1}

Trolley B has no momentum before the collision, because it is not moving.

Step 3 Calculate the momentum after the collision:

momentum of trolleys after collision

$= (m_A + m_B) \times v_{A+B}$

$= (0.80 + 1.60) \times 1.0$

$= 2.4$ kg m s^{-1}

So, both before and after the collision, the trolleys have a combined momentum of 2.4 kg m s^{-1}. Momentum has been conserved.

Questions

2 Calculate the momentum of each of the following objects:

- **a** a 0.50 kg stone travelling at a velocity of 20 m s^{-1}
- **b** a 25 000 kg bus travelling at 20 m s^{-1} on a road
- **c** an electron travelling at 2.0×10^7 m s^{-1}.
  (The mass of the electron is 9.1×10^{-31} kg.)

3 Two balls, each of mass 0.50 kg, collide as shown in Figure 6.6. Show that their total momentum before the collision is equal to their total momentum after the collision.

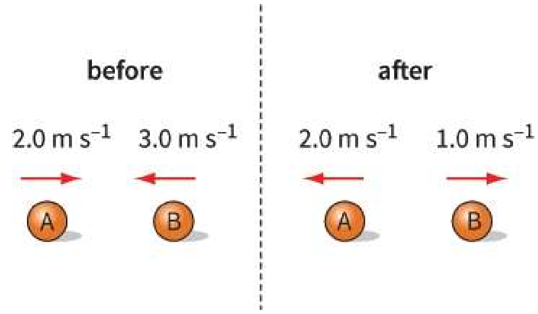


Figure 6.6: For Question 3.

6.3 Understanding collisions

The cars in Figure 6.7 have been badly damaged by a collision. The front of a car is designed to absorb the impact of the crash. It has a ‘crumple zone’, which collapses on impact. This absorbs most of the kinetic energy that the car had before the collision. It is better that the car’s kinetic energy should be transferred to the crumple zone than to the driver and passengers.

Motor manufacturers make use of test labs to investigate how their cars respond to impacts. When a car is designed, the manufacturers combine soft, compressible materials that absorb energy with rigid structures that protect the people in the car. Old-fashioned cars had much more rigid structures. In a collision, they were more likely to bounce back and the violent forces involved were much more likely to prove fatal.

Figure 6.7: The front of each car has crumpled in, as a result of a head-on collision.

Two types of collision

When two objects collide, they may crumple and deform. Their kinetic energy may also disappear completely as they come to a halt. This is an example of an **inelastic collision**. Alternatively, they may spring apart, retaining all of their kinetic energy. This is a **perfectly elastic collision**. In practice, in most collisions, some kinetic energy is transformed into other forms (such as heat or sound) and the collision is inelastic. Previously we described the collisions as being ‘springy’ or ‘sticky’. We should now use the correct scientific terms, perfectly elastic and inelastic.

We will look at examples of these two types of collision and consider what happens to linear momentum and kinetic energy in each.

A perfectly elastic collision

Two identical objects, A and B, moving at the same speed but in opposite directions, have a head-on collision, as shown in Figure 6.8. Each object bounces back with its velocity reversed. This is a perfectly elastic collision.

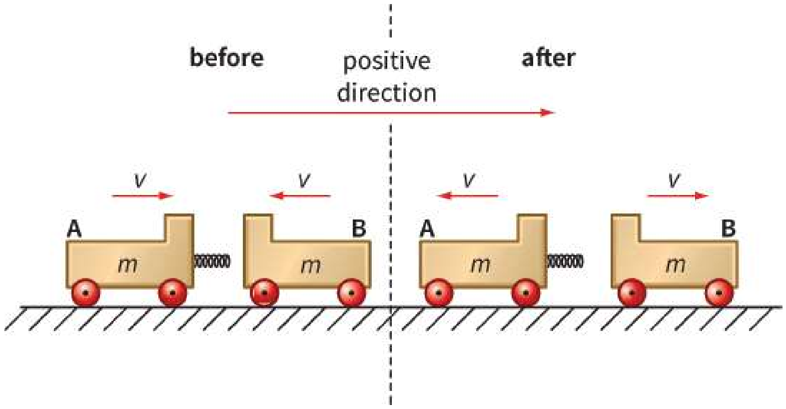


Figure 6.8: Two objects may collide in different ways: this is an elastic collision. An inelastic collision of the same two objects is shown in Figure 6.9.

You should be able to see that, in this collision, both momentum and kinetic energy are conserved. Before the collision, object A of mass m is moving to the right at speed v and object B of mass m is moving to the left at speed v. Afterwards, we still have two masses m moving with speed v, but now object A is moving to the left and object B is moving to the right. We can express this mathematically as follows.

Before the collision

Object	**Mass**	**Velocity**	**Momentum**
A	m	v	mv
B	m	$-v$	$-mv$

Object B has negative velocity and momentum because it is travelling in the opposite direction to object A. Therefore we have:

total momentum before collision

= momentum of A + momentum of B

= $mv + (-mv) = 0$

total kinetic energy before collision

= k.e. of A + k.e. of B

$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$

The magnitude of the momentum of each object is the same. Momentum is a vector quantity and we have to consider the directions in which the objects travel. The combined momentum is zero. On the other hand, kinetic energy is a scalar quantity and direction of travel is irrelevant. Both objects have the same kinetic energy and therefore the combined kinetic energy is twice the kinetic energy of a single object.

After the collision

Both objects have their velocities reversed, and we have:

$$\text{total momentum after collision} = (-mv) + mv = 0$$

$$\text{total kinetic energy after collision} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$$

So the total momentum and the total kinetic energy are unchanged. They are both conserved in a perfectly elastic collision such as this.

In this collision, the objects have a relative speed of $2v$ before the collision. After their collision, their velocities are reversed so their relative speed is $2v$ again. This is a feature of perfectly elastic collisions.

The relative speed of approach is the speed of one object measured relative to another. If two objects are travelling directly towards each other with speed v, as measured by someone stationary on the ground, then each object 'sees' the other one approaching with a speed of $2v$. Thus, if objects are travelling in opposite directions we add their speeds to find the relative speed. If the objects are travelling in the same direction then we subtract their speeds to find the relative speed.

To find the relative speed of two objects you subtract the velocity of one from the velocity of the other. This is the same as adding on a velocity in the opposite direction; so, if two objects approach each other in exactly opposite directions with velocities of v_1 and $-v_2$, their relative speed = $v_1 - (-v_2) = v_1 + v_2$.

KEY IDEA

In a perfectly elastic collision of two bodies, the relative speed of the body's approach is equal to the relative speed of their separation.

An inelastic collision

KEY IDEA

During an inelastic collision, the total kinetic energy of the bodies becomes smaller.

In Figure 6.9, the same two objects collide, but this time they stick together after the collision and come to a halt. Clearly, the total momentum and the total kinetic energy are both zero after the collision, since neither mass is moving. We have:

	Before collision	After collision
momentum	0	0
kinetic energy	$\frac{1}{2}mv^2$	0

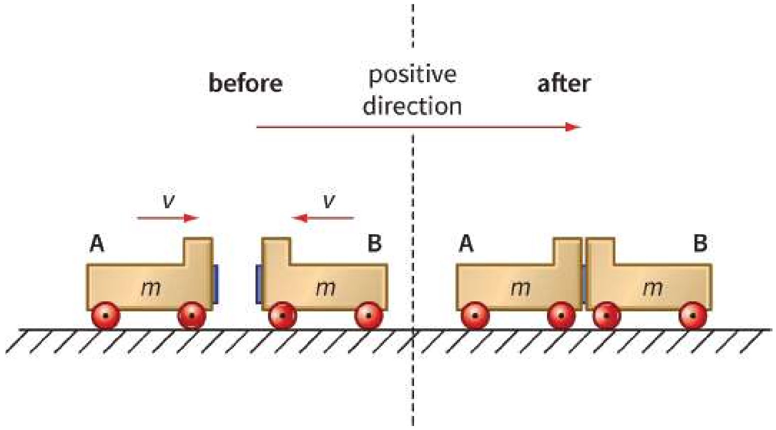


Figure 6.9: An inelastic collision between two identical objects. The trolleys are stationary after the collision.

Again we see that momentum is conserved. However, kinetic energy is not conserved. It is lost because work is done in deforming the two objects.

In fact, **momentum is always conserved in all collisions**. There is nothing else into which momentum can be converted. Kinetic energy is usually not conserved in a collision, because it can be transformed into other forms of energy – sound energy if the collision is noisy, and the energy involved in deforming the objects (which usually ends up as internal energy – they get warmer). Of course, the total amount of energy remains constant, as stated in the principle of conservation of energy.

Question

4 Copy this table, choosing the correct words from each pair.

Type of collision	perfectly elastic	inelastic
Momentum	conserved / not conserved	conserved / not conserved
Kinetic energy	conserved / not conserved	conserved / not conserved
Total energy	conserved / not conserved	conserved / not conserved

Solving collision problems

We can use the idea of conservation of momentum to solve numerical problems, as shown in Worked example 2.

WORKED EXAMPLE

2 In the game of bowls, a player rolls a large ball towards a smaller, stationary ball. A large ball of mass 5.0 kg moving at 10.0 m s^{-1} strikes a stationary ball of mass 1.0 kg. The smaller ball flies off at 10.0 m s^{-1}.

a Determine the final velocity of the large ball after the impact.

b Calculate the kinetic energy 'lost' in the impact.

Step 1 Draw two diagrams, showing the situations before and after the collision. Figure 6.10 shows the values of masses and velocities; since we don't know the velocity of the large ball after the collision, this is shown as *v*. The direction from left to right has been assigned the 'positive' direction.

Step 2 Using the principle of conservation of momentum, set up an equation and solve for the value of *v*:

$$
\begin{aligned}
\text{total momentum before collison} &= \text{total momentum after collison} \\
(5.0 \times 10) + (1.0 \times 0) &= (5.0 \times v) + (1.0 \times 10) \\
50 + 0 &= 5.0v + 10 \\
v &= \frac{40}{5.0} \\
v &= 8.0\ m\ s^{-1}
\end{aligned}
$$

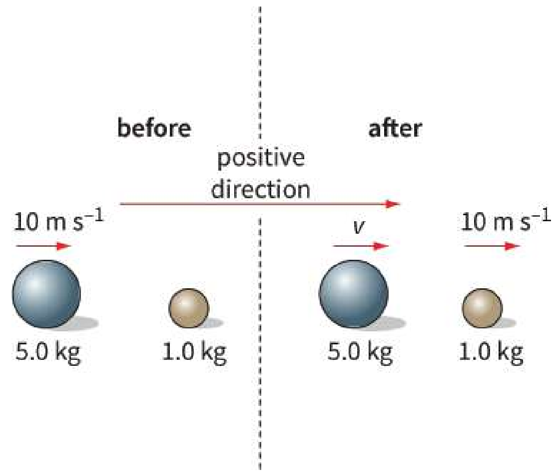


Figure 6.10: When solving problems involving collisions, it is useful to draw diagrams showing the situations before and after the collision. Include the values of all the quantities that you know.

So the speed of the large ball decreases to 8.0 m s^{-1} after the collision. Its direction of motion is unchanged – the velocity remains positive.

Step 3 Knowing the large ball's final velocity, calculate the change in kinetic energy during the collision:

$$
\begin{aligned}
\text{total k.e. before collision} &= \tfrac{1}{2} \times 5.0 \times 10^2 + 0 \\
&= 250\text{ J} \\
\text{total k.e. after collision} &= \tfrac{1}{2} \times 5.0 \times 8.0^2 + \tfrac{1}{2} \times 1.0 \times 10^2 \\
&= 210\text{ J} \\
\text{total k.e. 'lost' in the collision} &= 250\text{ J} - 210\text{ J} \\
&= 40\text{ J}
\end{aligned}
$$

This 'lost' kinetic energy will appear as internal energy (the two balls get warmer) and as sound energy (we hear the collision between the balls).

Questions

5 Figure 6.11 shows two identical balls A and B about to make a head-on collision. After the collision, ball A rebounds at a speed of 1.5 m s^{-1} and ball B rebounds at a speed of 2.5 m s^{-1}. The mass of each ball is 4.0 kg.

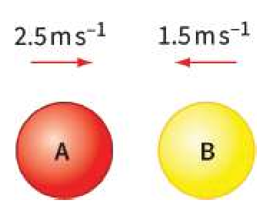


Figure 6.11: For Question 5.

a Calculate the momentum of each ball before the collision.

b Calculate the momentum of each ball after the collision.

c Is the momentum conserved in the collision?

d Show that the total kinetic energy of the two balls is conserved in the collision.

e Show that the relative speed of the balls is the same before and after the collision.

6 A trolley of mass 1.0 kg is moving at 2.0 m s^{-1}. It collides with a stationary trolley of mass 2.0 kg. This second trolley moves off at 1.2 m s^{-1}.

a Draw 'before' and 'after' diagrams to show the situation.

b Use the principle of conservation of momentum to calculate the speed of the first trolley after the collision. In what direction does it move?

6.4 Explosions and crash-landings

There are situations where it may appear that momentum is being created out of nothing, or that it is disappearing without trace. Do these contradict the principle of conservation of momentum?

The rockets shown in Figure 6.12 rise high into the sky. As they start to fall, they send out showers of chemical packages, each of which explodes to produce a brilliant sphere of burning chemicals. Material flies out in all directions to create a spectacular effect.

Does an explosion create momentum out of nothing? The important point to note here is that the burning material spreads out equally in all directions. Each tiny spark has momentum, but for every spark, there is another moving in the opposite direction, i.e., with opposite momentum. Since momentum is a vector quantity, the total amount of momentum created is zero.

Figure 6.12: These exploding rockets produce a spectacular display of bright sparks in the night sky.

At the same time, kinetic energy is created in an explosion. Burning material flies outwards; its kinetic energy has come from the chemical potential energy stored in the chemical materials before they burn.

More fireworks

Roman candles are a type of firework that fire a jet of burning material into the sky. This is another type of explosion, but it doesn't send material in all directions. The firework tube directs the material upwards. Has momentum been created out of nothing here?

Again, the answer is no. The chemicals have momentum upwards, but at the same time, the roman candle pushes downwards on the Earth. An equal amount of downwards momentum is given to the Earth. Of course, the Earth is massive, and we don't notice the tiny change in its velocity that results.

Down to Earth

If you push a large rock over a cliff, its speed increases as it falls. Where does its momentum come from? And when it lands, where does its momentum disappear to?

The rock falls because of the pull of the Earth's gravity on it. This force is its weight and it makes the rock accelerate towards the Earth. Its weight does work and the rock gains kinetic energy. It gains momentum downwards. Something must be gaining an equal amount of momentum in the opposite (upward) direction. It is the Earth, which starts to move upwards as the rock falls downwards. The mass of the Earth is so great that its change in velocity – far too small to be noticeable.

When the rock hits the ground, its momentum becomes zero. At the same instant, the Earth also stops moving upwards. The rock's momentum cancels out the Earth's momentum. At all times during the rock's fall and crash-landing, momentum has been conserved.

If a rock of mass 60 kg is falling towards the Earth at a speed of 20 m s^{-1}, how fast is the Earth moving towards it? Figure 6.13 shows the situation. The mass of the Earth is 6.0×10^{24} kg. We have:

$$\text{total momentum of Earth and rock} = 0$$

Therefore:

$$(60 \times 20) + (6.0 \times 10^{24} \times v) = 0$$

$$v = -2.0 \times 10^{-22} \text{ m s}^{-1}$$

The minus sign shows that the Earth's velocity is in the opposite direction to that of the rock. The Earth moves very slowly indeed. In the time of the rock's fall, it will move much less than the diameter of the nucleus of an atom!

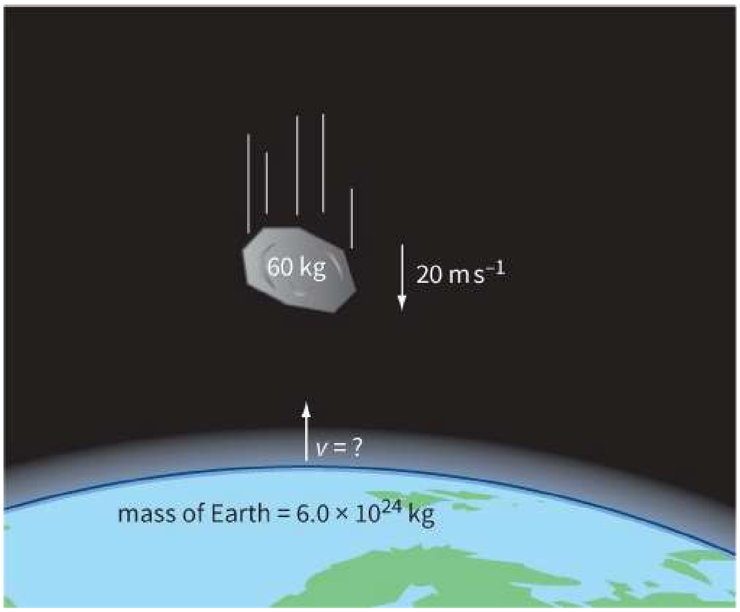


Figure 6.13: The rock and Earth gain momentum in opposite directions.

Questions

7 Discuss whether momentum is conserved in each of the following situations.

- **a** A star explodes in all directions – a supernova.
- **b** You jump up from a trampoline. As you go up, your speed decreases; as you come down again, your speed increases.

8 A ball of mass 0.40 kg is thrown at a wall. It strikes the wall with a speed of 1.5 m s^{-1} perpendicular to the wall and bounces off the wall with a speed of 1.2 m s^{-1}. Explain the changes in momentum and energy that happen in the collision between the ball and the wall. Give numerical values where possible.

6.5 Collisions in two dimensions

It is rare that collisions happen in a straight line–in one dimension. Figure 6.14 shows a two-dimensional collision between two snooker balls. From the multiple images, we can see how the velocities of the two balls change:

- At first, the white ball is moving straight forwards. When it hits the red ball, it moves off to the right. Its speed decreases; we can see this because the images get closer together.
- The red ball moves off to the left. It moves off at a bigger angle than the white ball, but more slowly – the images are even closer together.

How can we understand what happens in this collision, using the ideas of momentum and kinetic energy?

At first, only the white ball has momentum, and this is in the forward direction. During the collision, this momentum is shared between the two balls. We can see this because each has a component of velocity in the forward direction.

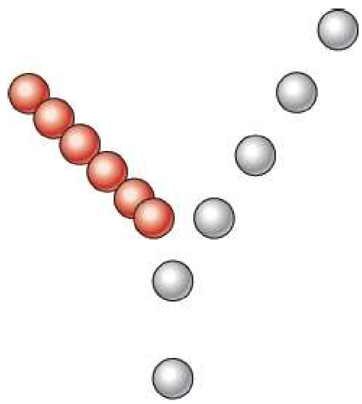


Figure 6.14: The white ball strikes the red ball a glancing blow. The two balls move off in different directions.

At the same time, each ball gains momentum in the sideways direction, because each has a sideways component of velocity – the white ball to the right, and the red ball to the left. These must be equal in magnitude and opposite in direction, otherwise we would conclude that momentum had been created out of nothing. The red ball moves at a greater angle, but its velocity is less than that of the white ball, so that the component of its velocity at right angles to the original track is the same as the white ball's.

Figure 6.15a shows the momentum of each ball before and after the collision. We can draw a vector triangle to represent the changes of momentum in this collision (Figure 6.15b). The two momentum vectors after the collision add up to equal the momentum of the white ball before the collision. The vectors form a closed triangle because momentum is conserved in this two-dimensional collision.

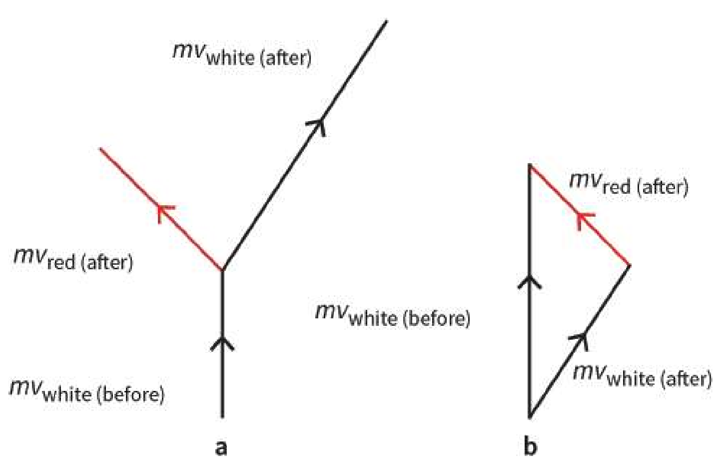


Figure 6.15: **a** These vectors represent the momenta of the colliding balls shown in Figure 6.14. **b** The closed vector triangle shows that momentum is conserved in the collision.

Components of momentum

Momentum is a vector quantity and so we can split it into components in order to solve problems.

Worked example 3 shows how to find an unknown velocity.

Worked example 4 shows how to demonstrate that momentum has been conserved in a two-dimensional collision.

WORKED EXAMPLES

3 A white ball of mass m = 1.0 kg and moving with initial speed u = 0.5 m s^{-1} collides with a stationary red ball of the same mass. They move off so that each has the same speed and the angle between their paths is 90°. What is their speed?

Step 1 Draw a diagram to show the velocity vectors of the two balls, before and after the collision (Figure 6.16). We will show the white ball initially travelling along the y-direction.

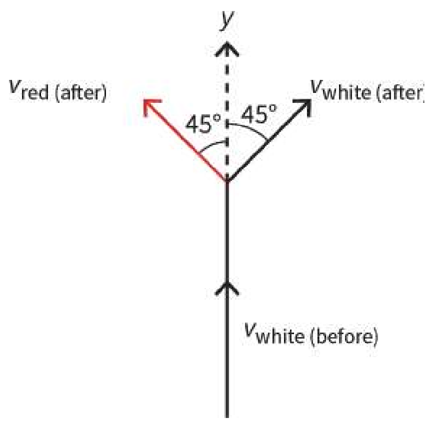


Figure 6.16: Velocity vectors for the white and red balls.

Because we know that the two balls have the same final speed v, their paths must be symmetrical about the y-direction. Since their paths are at 90° to one other, each must be at 45° to the y-direction.

Step 2 We know that momentum is conserved in the y-direction. Hence we can say:

initial momentum of white ball in y-direction

= final component of momentum of white ball in y-direction

+ final component of momentum of red ball in y-direction

This is easier to understand using symbols:

$$mu = mv_y + mv_y$$

where v_y is the component of v in the y-direction. The right-hand side of this equation has two identical terms, one for the white ball and one for the red. We can simplify the equation to give:

$$mu = 2mv_y$$

Step 3 The component of v in the y-direction is v cos 45°. Substituting this, and including values of m and u, gives

$$0.5 = 2v \cos 45°$$

and hence

$v = \frac{0.5}{2\cos 45^\circ} \approx 0.354 \text{ m s}^{-1}$

So each ball moves off at 0.354 m s^{-1} at an angle of 45° to the initial direction of the white ball.

4 Figure 6.17 shows the momentum vectors for particles 1 and 2, before and after a collision. Show that momentum is conserved in this collision.

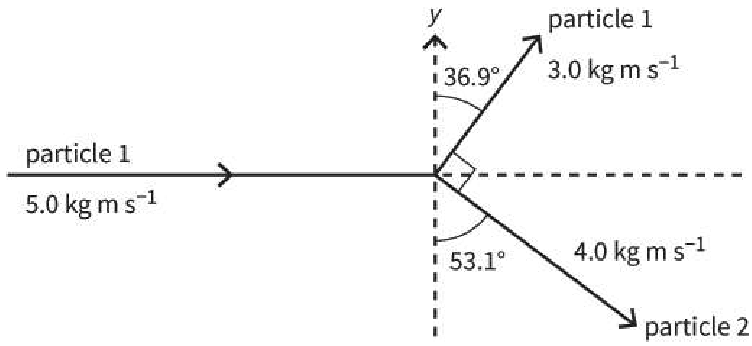


Figure 6.17: Momentum vectors: particle 1 has come from the left and collided with particle 2.

Step 1 Consider momentum changes in the *y*-direction.

Before collision:

$$\text{momentum} = 0$$

(because particle 1 is moving in the *x*-direction and particle 2 is stationary).

After collision:

component of momentum of particle 1

$= 3.0 \cos 36.9° \approx 2.40$ kg m s^{-1} upwards

component of momentum of particle 2

$= 4.0 \cos 53.1° \approx 2.40$ kg m s^{-1} downwards

These components are equal and opposite, and hence their sum is zero. Hence, momentum is conserved in the *y*-direction.

Step 2 Consider momentum changes in the *x*-direction.

Before collision:
momentum = 5.0 kg m s^{-1} to the right

After collision:

component of momentum of particle 1

$= 3.0 \cos 53.1° \approx 1.80$ kg m s^{-1} to the right

component of momentum of particle 2

$= 4.0 \cos 36.9° \approx 3.20$ kg m s^{-1} to the right

total momentum to the right = 5.0 kg m s^{-1}

Hence, momentum is conserved in the *x*-direction.

Step 3 An alternative approach would be to draw a vector triangle similar to Figure 6.15b. In this case, the numbers have been chosen to make this easy; the vectors form a 3–4–5 right-angled triangle.

Because the vectors form a closed triangle, we can conclude that:

momentum before collision = momentum after collision

(in other words, momentum is conserved)

Questions

9 A snooker ball strikes a stationary ball. The second ball moves off sideways at 60° to the initial path of the first ball.

Use the idea of conservation of momentum to explain why the first ball cannot travel in its initial direction after the collision. Illustrate your answer with a diagram.

10 Look back to Worked example 4. Draw the vector triangle that shows that momentum is conserved in the collision described in the question. Show the value of each angle in the triangle.

11 Figure 6.18 shows the momentum vectors for two identical particles, 1 and 2, before and after a collision. Particle 2 was at rest before the collision. Show that momentum is conserved in this collision.

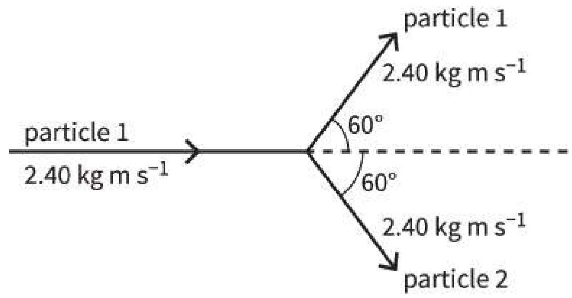


Figure 6.18: For Question 11.

12 A snooker ball collides with a second identical ball as shown in Figure 6.19.

a Determine the components of the velocity of the first ball in the *x*- and *y*-directions.

b Hence, determine the components of the velocity of the second ball in the *x*- and *y*-directions.

c Hence, determine the velocity (magnitude and direction) of the second ball.

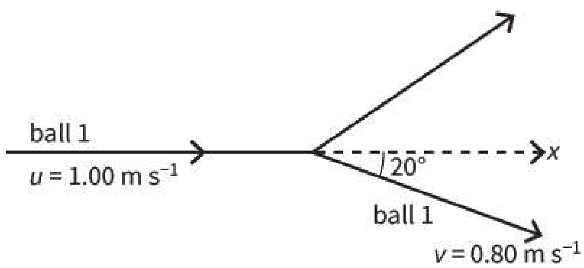
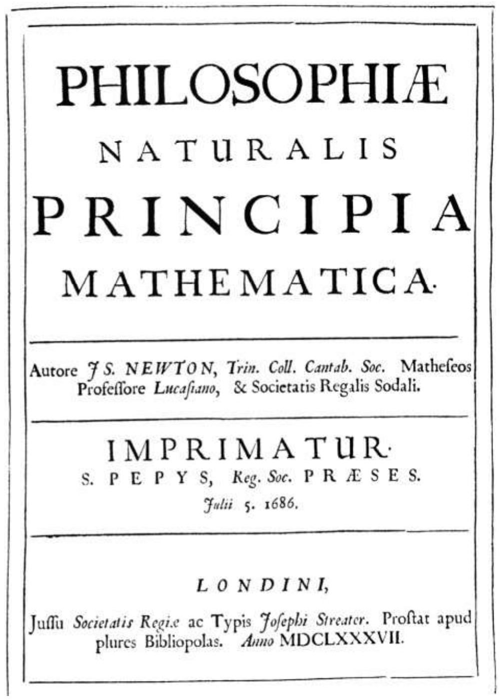


Figure 6.19: For Question 12.

6.6 Momentum and Newton's laws

The main concepts in physics are often very simple; it takes only a few words to express them and they can be applied to lots of situations. However, 'simple' does not mean 'easy'. Some concepts are quite abstract – such as force, energy and voltage. Scientists had to use their imagination to conceive such concepts. Other scientists then spent years working, experimenting, testing and refining the concepts until they finally reached the established concepts that we use today.



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

Figure 6.20: The title page of Newton's *Principia*, in which he outlined his theories of the laws that govern the motion of objects.

Isaac Newton's work on motion is a good example. Newton published his ideas in a book; the book's title translates as *Mathematical Principles of Natural Philosophy*.

Newton wanted to develop an understanding of the idea of 'force'. You may have been told in your early studies of science that 'a force is a push or a pull'. Newton's idea was that forces are interactions between bodies and that they change the motion of the body that they act on. Forces acting on an object can produce acceleration. For an object of constant mass, this acceleration is directly proportional to the resultant force acting on the object. That is much more like a scientific definition of force.

6.7 Understanding motion

In Chapter 3, we looked at Newton's laws of motion. We can get further insight into these laws by thinking about them in terms of momentum.

Newton's first law of motion

In everyday speech, we sometimes say that something has momentum when we mean that it keeps on moving on its own. An oil tanker is difficult to stop at sea, because of its momentum. We use the same word even when we're not talking about an object: 'The election campaign is gaining momentum', for example. This idea of keeping on moving is just what we discussed in connection with **Newton's first law of motion**:

An object will remain at rest or keep travelling at constant velocity unless it is acted on by a resultant force.

An object travelling at constant velocity has constant momentum. Hence, the first law is really saying that the momentum of an object remains the same unless the object experiences an external force.

Newton's second law of motion

Newton's second law of motion links the idea of the resultant force acting on an object and its momentum. A statement of Newton's second law is:

The resultant force acting on an object is directly proportional to the rate of change of the linear momentum of that object. The resultant force and the change in momentum are in the same direction.

Hence:

resultant force ∝ rate of change of momentum

This can be written as:

$$F \propto \frac{\Delta p}{\Delta t}$$

where F is the resultant force and Δp is the change in momentum taking place in a time interval of Δt. (Remember that the Greek letter delta, Δ, is a shorthand for 'change in', so Δp means 'change in momentum'.) The changes in momentum and force are both vector quantities, so these two quantities must be in the same direction.

The unit of force (the newton, N) is defined to make the constant of proportionality equal to one, so we can write the second law of motion mathematically as:

$$F = \frac{\Delta p}{\Delta t}$$

Worked example 5 shows how to use this equation. This equation also shows the newton second (N s) can be used as a unit of momentum.

If the forces acting on an object are balanced, there is no resultant force and the object's momentum will remain constant. If a resultant force acts on an object, its momentum (velocity and/or direction) will change. The equation gives us another way of stating Newton's second law of motion:

The resultant force acting on an object is equal to the rate of change of its momentum. The resultant force and the change in momentum are in the same direction.

This statement effectively defines what we mean by a force; it is an interaction that causes an object's momentum to change. So, if an object's momentum is changing, there must be a force acting on it. We can find the size and direction of the force by measuring the rate of change of the object's momentum.

KEY EQUATION

Resultant force ∝ rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t}$$

WORKED EXAMPLE

5 Calculate the average force acting on a 900 kg car when its velocity changes from 5.0 m s^{-1} to 30 m s^{-1} in a time of 12 s.

Step 1 Write down the quantities given:

m = 900 kg

initial velocity u = 5.0 m s^{-1}

Δt = 12 s

Step 2 Calculate the initial momentum and the final momentum of the car:

momentum = mass × velocity

$$\begin{aligned} \text{initial momentum} &= mu = 900 \times 5.0 \\ &= 4500\text{k g m s}^{-1} \\ \text{final momentum} &= mv = 900 \times 30 \\ &= 27\,000 \text{ kgm s}^{-1} \end{aligned}$$

Step 3 Use Newton's second law of motion to calculate the average force on the car:

$$\begin{aligned} F &= \frac{\Delta p}{\Delta t} \\ &= \frac{27000-4500}{12} \\ &= 1875 \text{ N} \approx 1900 \text{ N} \end{aligned}$$

The average force acting on the car is about 1.9 kN.

A special case of Newton's second law of motion

Imagine an object of constant mass m acted upon by a resultant force F. The force will change the momentum of the object. According to Newton's second law of motion, we have:

$$F = \frac{\Delta p}{\Delta t} = \frac{mv-mu}{t}$$

where u is the initial velocity of the object, v is the final velocity of the object and t is the time taken for the change in velocity. The mass m of the object is a constant; hence the equation can be rewritten as:

$$\begin{aligned} F &= \frac{m(v-u)}{\Delta t} \\ &= m\left(\frac{v-u}{t}\right) \end{aligned}$$

The term in brackets on the right-hand side is the acceleration a of the object. Therefore, a special case of Newton's second law is:

$$F = ma$$

We have already met this equation in Chapter 3. In Worked example 5, you could have determined the average force acting on the car using this simplified equation for Newton's second law of motion.

Remember that the equation $F = ma$ is a special case of $F = \frac{\Delta p}{\Delta t}$ that only applies when the mass of the object is constant. There are situations where the mass of an object changes as it moves, for example, a rocket that burns a phenomenal amount of chemical fuel as it accelerates upwards.

Questions

13 A car of mass 1000 kg is travelling at a velocity of +10 m s^{-1}. It accelerates for 15 s, reaching a velocity of +24 m s^{-1}. Calculate:

- **a** the change in the momentum of the car in the 15 s period
- **b** the average resultant force acting on the car as it accelerates.

14 A ball is kicked by a footballer. The average force on the ball is 240 N and the impact lasts for a time interval of 0.25 s.

- **a** Calculate the change in the ball's momentum.
- **b** State the direction of the change in momentum.

15 Water pouring from a broken pipe lands on a flat roof. The water is moving at 5.0 m s^{-1} when it strikes the roof. The water hits the roof at a rate of 10 kg s^{-1}. Calculate the force of the water hitting the roof. (Assume that the water does not bounce as it hits the roof. If it did bounce, would your answer be greater or smaller?)

16 A golf ball has a mass of 0.046 kg. The final velocity of the ball after being struck by a golf club is 50 m s^{-1}. The golf club is in contact with the ball for a time of 1.3 ms. Calculate the average force exerted by the golf club on the ball.

Newton's third law of motion

Newton's third law of motion is about interacting objects. These could be two magnets attracting or repelling each other, two electrons repelling each other, etc. Newton's third law states:

When two bodies interact, the forces they exert on each other are equal and opposite.

How can we relate this to the idea of momentum? Imagine holding two magnets, one in each hand. You gradually bring them towards each other (Figure 6.21) so that they start to attract each other. Each feels a force pulling it towards the other. The two forces are the same size, even if one magnet is stronger than the other. One magnet could even be replaced by an unmagnetised piece of steel and they would still attract each other equally.

If you release the magnets, they will gain momentum as they are pulled towards each other. One gains momentum to the left while the other gains equal momentum to the right.

Each is acted on by the same force, and for the same time. So, momentum is conserved. In fact, the law of conservation of momentum can be proved using Newton's second and third laws of motion. Consider an object of mass m_x and velocity v_x colliding with a mass m_y and velocity v_y. If the system is closed, then the force F_x and the force F_y on the twomasses are equal and opposite.

$$\begin{aligned} F_x &= -F_y \\ \frac{\Delta m_x v_x}{\Delta t} &= -\frac{\Delta m_y v_y}{\Delta t} \\ \frac{\Delta (m_x v_x + m_y v_y)}{\Delta t} &= 0 \end{aligned}$$

So, $\Delta\left(m_x v_x + m_y v_y\right) = 0$ and there has been no change in the total momentum.

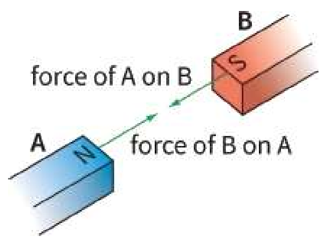


Figure 6.21: Newton's third law states that the forces these two magnets exert one each other must be equal and opposite.

REFLECTION

What did you learn about yourself as you worked through this chapter?

Which principle do you think is the most important, conservation of momentum or conservation of energy?

SUMMARY

Linear momentum is the product of mass and velocity: $p = mv$
The principle of conservation of momentum: For a closed system, the total momentum before an interaction (e.g., collision) is equal to the total momentum after the interaction.
In all interactions or collisions, momentum and total energy are conserved.
Kinetic energy is conserved in a perfectly elastic collision; relative speed is unchanged in a perfectly elastic collision.
In an inelastic collision, kinetic energy is not conserved. It is transferred into other forms of energy (such as heat or sound). Most collisions are inelastic.
Newton's first law of motion: An object will remain at rest or keep travelling at constant velocity unless it is acted on by a resultant force.
Newton's second law of motion: The resultant force acting on a body is equal to the rate of change of its momentum: resultant force = rate of change of momentum or $F = \frac{\Delta p}{\Delta t} = ma$ when mass m remains constant.
Newton's third law of motion: When two bodies interact, the forces they exert on each other are equal and opposite.

EXAM-STYLE QUESTIONS

1 Which quantity has the same unit as the rate of change of momentum? **[1]**

A acceleration

B energy

C weight

D work

2 A railway truck of mass 8000 kg travels along a level track at a velocity of 2.5 m s^{-1} and collides with a stationary truck of mass 12 000 kg. The collision takes 4.0 s and the two trucks move together at the same velocity after the collision.

What is the average force that acts on the 8000 kg truck during the collision? **[1]**

A 2000 N

B 3000 N

C 5000 N

D 12 000 N

3 An object has mass 2.0 ± 0.2 kg and a velocity of 10 ± 1 m s^{-1}.

What is the percentage uncertainty in the momentum of the object? **[1]**

A 1%

B 6%

C 10%

D 20%

4 An object is dropped and its momentum increases as it falls toward the ground. Explain how the law of conservation of momentum and Newton's third law of motion can be applied to this situation. **[2]**

5 A ball of mass 2.0 kg, moving at 3.0 m s^{-1}, strikes a wall and rebounds with almost exactly the same speed. State and explain whether there is a change in:

a the momentum of the ball **[3]**

b the kinetic energy of the ball. **[1]**

[Total: 4]

6 **a** Define linear momentum. **[1]**

b Determine the base units of linear momentum in the SI system. **[1]**

c A car of mass 900 kg starting from rest has a constant acceleration of 3.5 m s^{-2}. Calculate its momentum after it has travelled a distance of 40 m. **[2]**

d This diagram shows two identical objects about to make a head-on collision. The objects stick together during the collision. Determine the final speed of the objects. State the direction in which they move. **[3]**

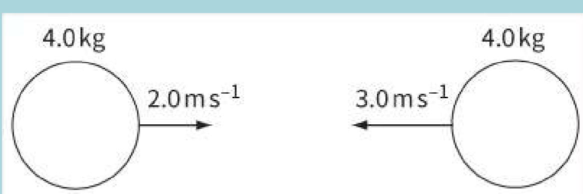


Figure 6.22

[Total: 7]

7 **a** Explain what is meant by an:

i elastic collision **[1]**

ii inelastic collision. **[1]**

b A snooker ball of mass 0.35 kg hits the side of a snooker table at right angles and bounces off also at right angles. Its speed before collision is 2.8 m s^{-1} and its speed after is 2.5 m s^{-1}. Calculate the change in the momentum of the ball. **[2]**

c Explain whether or not momentum is conserved in the situation described in part **b**. **[1]**

[Total: 5]

8 A car of mass 1100 kg is travelling at 24 m s^{-1}. The driver applies the brakes and the car decelerates uniformly and comes to rest in 20 s.

a Calculate the change in momentum of the car. **[2]**

b Calculate the braking force on the car. **[2]**

c Determine the braking distance of the car. **[2]**

[Total: 6]

9 A marble of mass 100 g is moving at a speed of 0.40 m s^{-1} in the *x*-direction.

a Calculate the marble's momentum. **[2]**

The marble strikes a second, identical marble. Each moves off at an angle of 45° to the *x*-direction.

b Use the principle of conservation of momentum to determine the speed of each marble after the collision. **[3]**

c Show that kinetic energy is conserved in this collision. **[2]**

[Total: 7]

10 A cricket bat strikes a ball of mass 0.16 kg travelling towards it. The ball initially hits the bat at a speed of 25 m s^{-1} and returns along the same path with the same speed. The time of impact is 0.0030 s. You may assume no force is exerted on the bat by the cricketer during the actual collision.

a Determine the change in momentum of the cricket ball. **[2]**

b Determine the force exerted by the bat on the ball. **[2]**

c Describe how the laws of conservation of energy and momentum apply to this impact and state whether the impact is elastic or inelastic. **[4]**

[Total: 8]

11 a State the principle of conservation of momentum and state the condition under which it is valid. **[2]**

b An arrow of mass 0.25 kg is fired horizontally towards an apple of mass 0.10 kg that is hanging on a string, as shown in Figure 6.23.

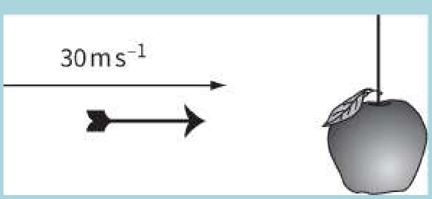


Figure 6.23

The horizontal velocity of the arrow as it enters the apple is 30 m s^{-1}. The apple was initially at rest and the arrow sticks in the apple.

i Calculate the horizontal velocity of the apple and arrow immediately after the impact. **[2]**

ii Calculate the change in momentum of the arrow during the impact. **[2]**

iii Calculate the change in total kinetic energy of the arrow and apple **[2]** during the impact.

iv A rubber-tipped arrow of mass 0.25 kg is fired at the centre of a stationary ball of mass 0.25 kg. The collision is perfectly elastic. Describe what happens and state the relative speed of separation of the arrow and the ball. **[2]**

[Total: 10]

12 a State what is meant by:

i a perfectly elastic collision **[1]**

ii a completely inelastic collision. **[1]**

b A stationary uranium nucleus disintegrates, emitting an alpha-particle of mass 6.65×10^{-27} kg and another nucleus X of mass 3.89×10^{-25} kg.

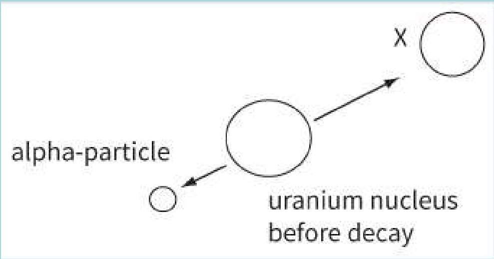


Figure 6.24

i Explain why the alpha-particle and nucleus X must be emitted in exactly opposite directions. **[2]**

ii Using the symbols v_α and v_x for velocities, write an equation for the conservation of momentum in this disintegration. **[1]**

iii Using your answer to part **b ii**, calculate the ratio $v_\alpha : v_x$ after the disintegration. **[1]**

[Total: 6]

13 a State **two** quantities that are conserved in an elastic collision. **[1]**

b A machine gun fires bullets of mass 0.014 kg at a speed of 640 m s^{-1}.

i Calculate the momentum of each bullet as it leaves the gun. **[1]**

ii Explain why a soldier holding the machine gun experiences a force when the gun is firing. **[2]**

iii The maximum steady horizontal force that a soldier can exert on the gun is 140 N. Calculate the maximum number of bullets that the gun can fire in one second. **[2]**

[Total: 6]

14 Two railway trucks are travelling in the same direction and collide. The mass of truck X is 2.0×10^4 kg and the mass of truck Y is 3.0×10^4 kg. This graph shows how the velocity of each truck varies with time.

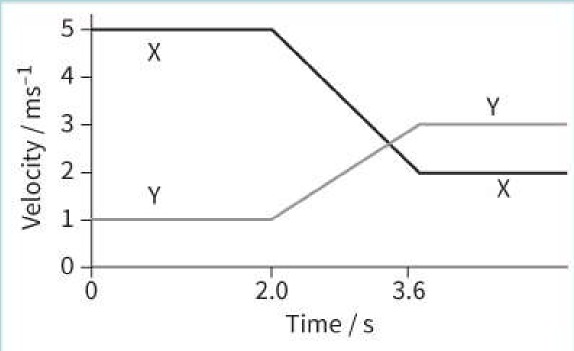


Figure 6.25

a Copy and complete the table. **[6]**

	Change in momentum / kg m s^{-1}	**Initial kinetic energy / J**	**Final kinetic energy / J**
truck X			
truck Y			

Table 6.1: For Question 14.

b State and explain whether the collision of the two trucks is an example of an elastic collision. **[2]**

c Determine the force that acts on each truck during the collision. **[2]**

[Total: 10]