Chapter 5

Work, energy and power

LEARNING INTENTIONS

In this chapter you will learn how to:

- use the concept of work and energy
- recall and apply the principle of conservation of energy
- recall and understand that the efficiency of a system is the ratio of useful energy output from the system to the total energy input
- use the concept of efficiency to solve problems
- define and use the equation for power using $P = \frac{W}{t}$ and derive $P = Fv$
- derive and use the formulae for kinetic energy and gravitational potential energy.

BEFORE YOU START

- Write down definitions for energy, work and power.
- Write down all that you know about these topics and share your ideas with someone else. Be prepared to discuss your answers with the rest of the class.

THE IDEA OF ENERGY

The Industrial Revolution started in the late 18th century in Britain. Today, many other countries have become or are becoming industrialised (Figure 5.1). Industrialisation is the development of new machines capable of doing the work of hundreds of craftsmen and labourers. At first, people used water and wind to power machines. Water stored behind a dam was used to turn a wheel, which turned many machines. Steam engines were developed, initially for pumping water out of mines. Steam engines use a fuel such as coal; there is much more energy stored in 1 kg of coal than in 1 kg of water held behind a dam.

Nowadays, most factories rely on electrical power, generated by burning coal or gas at a power station.

High-pressure steam is generated, and this turns a turbine that turns a generator. Even in the most efficient coal-fired power station, only about 40% of the energy from the fuel is transferred to the electrical energy that the station supplies to the electricity grid.

Figure 5.1: Anshan steel works, China.

Engineers worked hard to develop machines that made the most efficient use of the energy supplied to them. At the same time, scientists were working out the basic ideas of energy transfer and energy transformations. The idea of energy itself had to be developed; it was not obvious at first that heat, light, electrical energy were all forms of the same thing: energy. What is the history of your country in developing the use of machines, generating electrical power and increasing efficiency?

The earliest steam engines had very low efficiencies–many converted less than 1% of the energy supplied to them into useful work. The understanding of the relationship between work and energy led to many clever ways of making the most of the energy supplied by fuel.

Figure 5.2: The jet engines of this aircraft are designed to make efficient use of their fuel. If they were less efficient, their thrust might only be sufficient to lift the empty aircraft and the passengers would have to be left behind.

5.1 Doing work, transferring energy

The weight-lifter shown in Figure 5.3 has powerful muscles. They can provide the force needed to lift a large weight above her head – about 2 m above the ground. The force exerted by the weight-lifter transfers energy from her to the weights. We know that the weights have gained energy because, when the athlete releases them, they come crashing down to the ground.

As the athlete lifts the weights and transfers energy to them, we say that her lifting force is doing work. 'Doing work' is a way of transferring energy from one object to another. In fact, if you want to know the scientific meaning of the word 'energy', we have to say it is 'that which is transferred when a force moves through a distance'. So, work and energy are two closely linked concepts.

In physics, we often use an everyday word but with a special meaning. **Work** is an example of this.

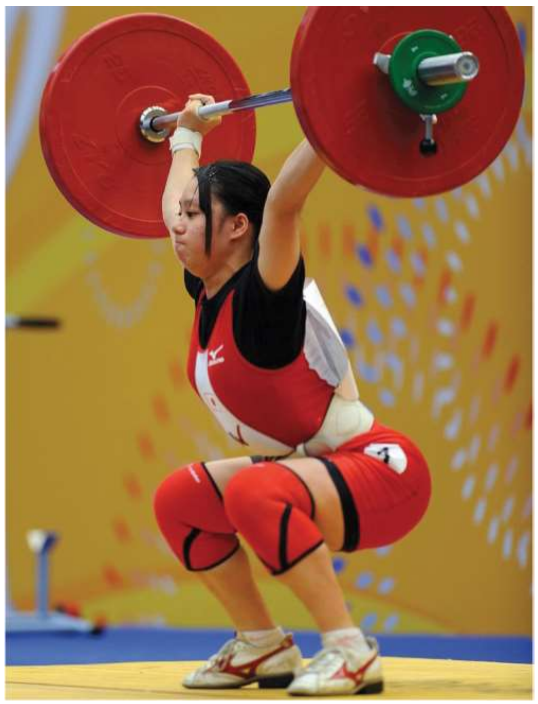


Figure 5.3: It is hard work being a weight-lifter.

Doing work	Not doing work
Pushing a car to start it moving: your force transfers energy to the car. The car's kinetic energy (that is, 'movement energy') increases.	Pushing a car but it does not budge: no energy is transferred, because your force does not move it. The car's kinetic energy does not change.
Lifting weights: you are doing work as the weights move upwards. The gravitational potential energy of the weights increases.	Holding weights above your head: you are not doing work on the weights (even though you may find it tiring) because the force you apply is not moving them. The gravitational potential energy of the weights is not changing.
A falling stone: the force of gravity is doing work. The stone's kinetic energy is increasing.	The Moon orbiting the Earth: the force of gravity is not doing work. The Moon's kinetic energy is not changing.
Writing an essay: you are doing work because you need a force to move your pen across the page, or to press the keys on the keyboard.	Reading an essay: this may seem like 'hard work', but no force is involved, so you are not doing any work.

Table 5.1: The meaning of 'doing work' in physics.

Table 5.1 describes some situations that illustrate the meaning of **doing work** in physics.

It is important to understand that our bodies sometimes mislead us. If you hold a heavy weight above your head for some time, your muscles will get tired. However, you are not doing any work **on the weights**, because you are not transferring energy to the weights once they are above your head. Your muscles get tired because they are constantly relaxing and contracting, and this uses energy, but none of the energy is being transferred to the weights.

Calculating work done

Because **doing work** defines what we mean by **energy**, we start this chapter by considering how to calculate **work done**.

There is no doubt that you do work if you push a car along the road. A force transfers energy from you to the car. But how much work do you do? Figure 5.4 shows the two factors involved:

- the size of the force F – the bigger the force, the greater the amount of work you do
- the distance s you push the car – the further you push it, the greater the amount of work done.

So, the bigger the force, and the further it moves, the greater the amount of work done.

KEY IDEA

Work is done on a body when a force moves (displaces) the body in the direction of the force. Energy is then transferred from one body to another

The work done by a force is defined as the product of the force and the distance moved in the direction of the force:

$$W = F \times s$$

where s is the distance moved in the direction of the force.

In the example shown in Figure 5.4, $F = 300$ N and $s = 5.0$ m, so:

$$\text{work done } W = F \times s = 300 \times 5.0 = 1500 \text{ J}$$

KEY EQUATION

$$\begin{aligned} \text{work done} &= \text{force} \times \text{distance} \\ W &= Fs \end{aligned}$$

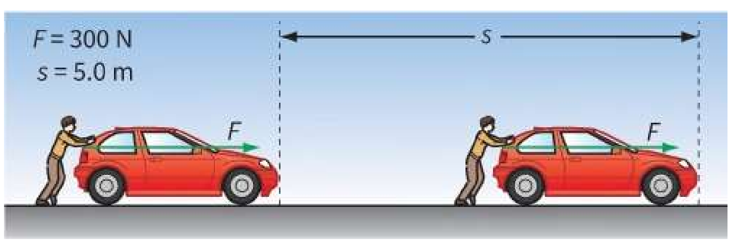


Figure 5.4: You have to do work to start the car moving.

Energy transferred

Doing work is a way of transferring energy. For both energy and work the correct SI unit is the joule (J).

The amount of work done, calculated using $W = F \times s$, shows the amount of energy transferred:

$$\text{work done} = \text{energy transferred}$$

KEY IDEA

work done = energy transferred

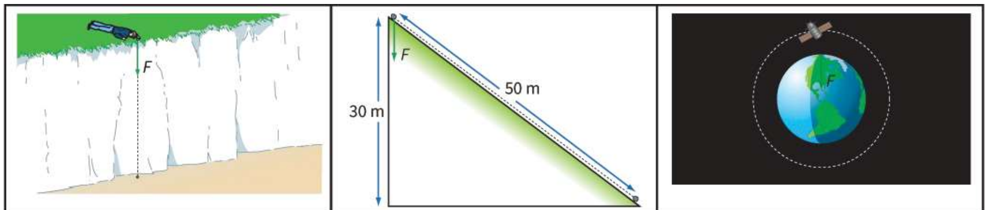
Newtons, metres and joules

From the equation $W = Fs$ we can see how the unit of force (the newton), the unit of distance (the metre) and the unit of work or energy (the joule) are related.

The joule is defined as the amount of work done when a force of 1 newton moves a distance of 1 metre in the direction of the force. Since **work done = energy transferred**, it follows that a joule is also the amount of energy transferred when a force of 1 newton moves a distance of 1 metre in the direction of the force.

Force, distance and direction

It is important to understand that, for a force to do work, there must be movement in the direction of the force. Both the force F and the distance s moved in the direction of the force are vector quantities, so you should know that their directions are likely to be important. To illustrate this, we will consider three examples involving gravity (Figure 5.5). In the equation for work done, $W = F \times s$, the distance moved s is the displacement in the direction of the force.



1 A stone weighing 5.0 N is dropped from the top of a 50 m high cliff.	**2** A stone weighing 5.0 N rolls 50 m down a slope.	**3** A satellite orbits the Earth at a constant height and at a constant speed. The weight of the satellite at this height is 500 N.
What is the work done by the force of gravity?	What is the work done by the force of gravity?	What is the work done by the force of gravity?
force on stone F = pull of gravity = weight of stone = 5.0 N vertically downwards	force on stone F = pull of gravity = weight of stone = 5.0 N vertically downwards	force on satellite F = pull of gravity = weight of satellite = 500 N towards centre of Earth
distance moved by stone in direction of force s = 50 m vertically downwards	distance moved by stone down slope = 50 m, **but** distance moved in direction of force s = 30 m	distance moved by satellite towards centre of Earth (that is, in the direction of force) s = 0. The satellite remains at a constant distance from the Earth. It does not move in the direction of F.
work done = $F \times s$ = 5.0×50 = 250 J	work done = $F \times s$ = 5.0×30 = 150 J	work done = $F \times s$ = 500×0 = 0 J

Figure 5.5: Three examples involving gravity.

Questions

1 In each of the following examples, explain whether or not any work is done by the force mentioned.

a You pull a heavy sack along rough ground.

b The force of gravity pulls you downwards when you fall off a wall.

c The tension in a string pulls on a stone when you whirl it around in a circle at a steady speed.

d The contact force of the bedroom floor stops you from falling into the room below.

2 A man of mass 70 kg climbs stairs of vertical height 2.5 m. Calculate the work done against the force of gravity. (Take g = 9.81 m s^{-2}.)

3 A stone of weight 10 N falls from the top of a 250 m high cliff.

a Calculate how much work is done by the force of gravity in pulling the stone to the foot of the cliff.

b How much energy is transferred to the stone if air resistance is ignored?

Suppose that the force F moves through a distance s that is at an angle θ to F, as shown in Figure 5.6. To determine the work done by the force, it is simplest to determine the component of F in the direction of s. This component is $F \cos \theta$, and so we have:

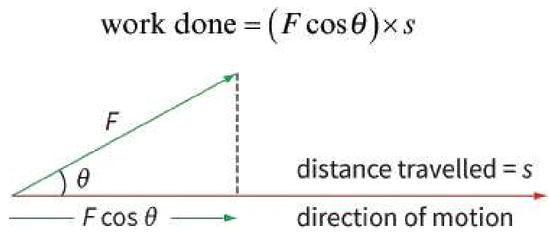


Figure 5.6: The work done by a force depends on the angle between the force and the distance it moves.

or simply:

$$\text{work done} = Fs\cos\theta$$

Worked example 1 shows how to use this.

KEY EQUATION

$$\text{work done} = Fs\cos\theta$$

WORKED EXAMPLE

1 A man pulls a box along horizontal ground using a rope (Figure 5.7). The force provided by the rope is 200 N, at an angle of 30° to the horizontal.

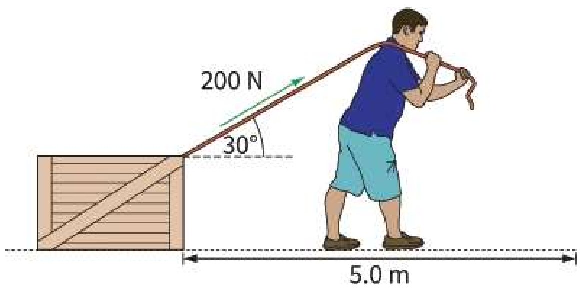


Figure 5.7: For Worked example 1.

Calculate the work done if the box moves 5.0 m along the ground.

Step 1 Calculate the component of the force in the direction in which the box moves. This is the horizontal component of the force:

horizontal component of force = 200 cos 30° ≈ 173 N

Hint: *F cos θ is the component of the force at an angle θto the direction of motion.*

Step 2 Now calculate the work done:

$$\begin{aligned}\text{work done} &= \text{force} \times \text{distance moved}\\ &= 173 \times 5.0 = 865\text{ J}\end{aligned}$$

Hint: *Note that we could have used the equation work done = Fs cos θto combine the two steps into one.*

Questions

4 The crane shown in Figure 5.8 lifts its 500 N load to the top of the building from A to B. Distances are as shown on the diagram. Calculate how much work is done by the crane.

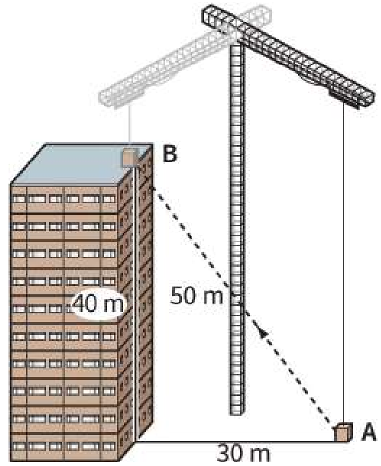


Figure 5.8: For Question 4. The dotted line shows the track of the load as it is lifted by the crane.

5 Figure 5.9 shows the forces acting on a box that is being pushed up a slope. Calculate the work done by each force if the box moves 0.50 m up the slope.

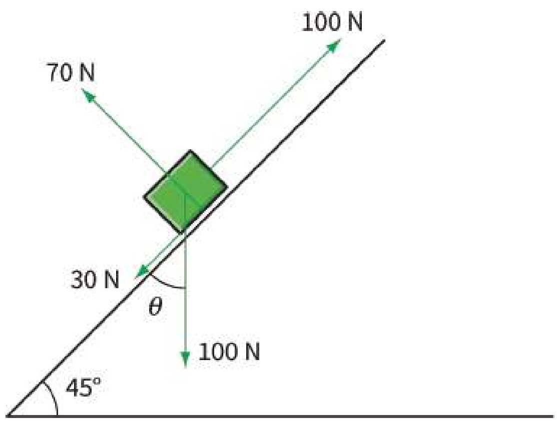


Figure 5.9: For Question 5.

5.2 Gravitational potential energy

If you lift a heavy object, you do work. You are providing an upwards force to overcome the downwards force of gravity on the object. The force moves the object upwards, so the force is doing work.

In this way, energy is transferred from you to the object. You lose energy, and the object gains energy. We say that the **gravitational potential energy, E_p** of the object has increased.

Worked example 2 shows how to calculate a change in gravitational potential energy (g.p.e.).

WORKED EXAMPLE

2 A weight-lifter raises weights with a mass of 200 kg from the ground to a height of 1.5 m. Calculate how much work he does. By how much does the g.p.e. of the weights increase?

Step 1 As shown in Figure 5.10, the downward force on the weights is their weight, $W = mg$. An equal, upward force F is required to lift them.

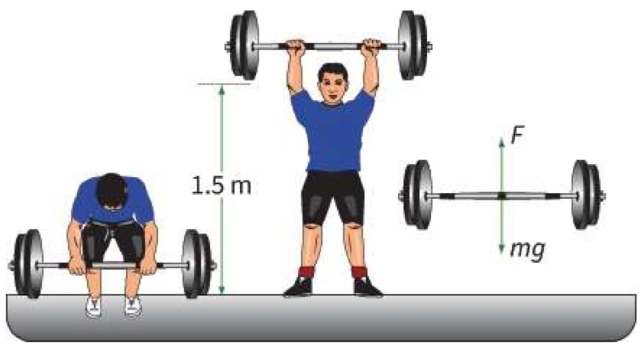


Figure 5.10: For Worked example 2.

$W = F = mg = 200 \times 9.81 = 1962$ N

Hint: *It helps to draw a diagram of the situation.*

Step 2 Now we can calculate the work done by the force F:

$$\begin{aligned} \text{work done} &= \text{force} \times \text{distance moved} \\ &= 1962 \times 1.5 \approx 2940 \text{ J} \end{aligned}$$

Note that the distance moved is in the same direction as the force. So the work done on the weights is about 2940 J. This is also the value of the increase in their g.p.e.

An equation for gravitational potential energy

The change (Δ) in the gravitational potential energy (g.p.e.) of an object, E_p, depends on the change in its height, h. We can calculate E_p using this equation:

$$\begin{aligned} \text{change in g.p.e} &= \text{weigth} \times \text{change in height} \\ \Delta E_p &= (m \times g) \times \Delta h \\ \Delta E_p &= mg\Delta h \end{aligned}$$

It should be clear where this equation comes from. The force needed to lift an object is equal to its weight mg, where m is the mass of the object and g is the acceleration of free fall or the gravitational field strength on the Earth's surface. The work done by this force is given by force × distance moved, or weight × change in height. You might feel that it takes a force greater than the weight of the object being raised to lift it upwards, but this is not so. Provided the force is equal to the weight, the object will move upwards at a steady speed.

KEY EQUATION

$$\text{change in g.p.e} = \text{weight} \times \text{change in height}$$
$$\Delta E_p = mg\Delta h$$

You must learn how to derive this equation.

Note that h stands for the vertical height through which the object moves. Note also that we can only use the equation $\Delta E_p = mg\Delta h$ for relatively small changes in height. It would not work, for example, in the case of a satellite orbiting the Earth. Satellites orbit at a height of at least 200 km and g has a smaller value at this height.

Other forms of potential energy

Potential energy is the energy an object has because of its position or shape. So, for example, an object's gravitational potential energy changes when it moves through a gravitational field. (There is much more about gravitational fields in Chapter 17.)

We can identify other forms of potential energy. An electrically charged object has electric potential energy when it is placed in an electric field (see Chapter 21). An object may have elastic potential energy when it is stretched, squashed or twisted–if it is released it goes back to its original shape (see Chapter 7).

Questions

6 Calculate how much gravitational potential energy is gained if you climb a flight of stairs. Assume that you have a mass of 52 kg and that the height you lift yourself is 2.5 m.

7 A climber of mass 100 kg (including the equipment she is carrying) ascends from sea level to the top of a mountain 5500 m high. Calculate the change in her gravitational potential energy.

8 **a** A toy car works by means of a stretched rubber band. What form of potential energy does the car store when the band is stretched?

b A bar magnet is lying with its north pole next to the south pole of another bar magnet. A student pulls them apart. Why do we say that the magnets' potential energy has increased? Where has this energy come from?

5.3 Kinetic energy

As well as lifting an object, a force can make it accelerate. Again, work is done by the force and energy is transferred to the object. In this case, we say that it has gained kinetic energy, E_k. The faster an object is moving, the greater its kinetic energy (k.e.).

For an object of mass m travelling at a speed v, we have:

$$\begin{aligned} \text{kinetic energy} &= \tfrac{1}{2} \times \text{mass} \times \text{speed}^2 \\ E_\text{k} &= \tfrac{1}{2}mv^2 \end{aligned}$$

Deriving the formula for kinetic energy

KEY EQUATION

$$\begin{aligned} \text{kinetic energy} &= \tfrac{1}{2} \times \text{mass} \times \text{speed}^2 \\ E_\text{k} &= \tfrac{1}{2}mv^2 \end{aligned}$$

You must learn how to derive this equation.

The equation for kinetic energy, $E_\text{k} = \frac{1}{2}mv^2$, is related to one of the equations of motion. We imagine a car being accelerated from rest ($u = 0$) to velocity v. To give it acceleration a, it is pushed by a force F for a distance s. Since $u = 0$, we can write the equation $v^2 = u^2 + 2as$ as:

$$v^2 = 2as$$

Multiplying both sides by $\frac{1}{2}m$ gives:

$$\tfrac{1}{2}mv^2 = mas$$

Now, ma is the force F accelerating the car, and mas is the force × the distance it moves (that is, the work done by the force). So we have:

$$\tfrac{1}{2}mv^2 = \text{work done by force } F$$

This is the energy transferred to the car, and hence its kinetic energy.

WORKED EXAMPLE

3 Calculate the increase in kinetic energy of a car of mass 800 kg when it accelerates from 20 m s^{-1} to 30 m s^{-1}.

Step 1 Calculate the initial k.e. of the car:

$$\begin{aligned} E_\text{k} &= \tfrac{1}{2}mv^2 \\ &= \tfrac{1}{2} \times 800 \times (20)^2 \\ &= 160\,000 \text{ J} \equiv 160\text{kJ} \end{aligned}$$

Step 2 Calculate the final k.e. of the car:

$$\begin{aligned} E_\text{k} &= \tfrac{1}{2}mv^2 \\ &= \tfrac{1}{2} \times 800 \times (30)^2 \\ &= 360\,000 \text{ J} \equiv 360\text{kJ} \end{aligned}$$

Step 3 Calculate the change in the car's k.e.:

change in k.e. = 360 − 160 = 200 kJ

Hint: *Take care! You can't calculate the change in k.e. by squaring the change in speed. In this example, the change in speed is 10 m s^{-1}, and this would give an incorrect value for the change in k.e.*

Questions

9 Which has more k.e., a car of mass 500 kg travelling at 15 m s^{-1} or a motorcycle of mass 250 kg travelling at 30 m s^{-1}?

10 Calculate the change in kinetic energy of a ball of mass 200 g when it bounces. Assume that it hits the ground with a speed of 15.8 m s^{-1} and leaves it at 12.2 m s^{-1}.

5.4 Gravitational potential to kinetic energy transformations

A motor drags the roller-coaster car to the top of the first hill. The car runs down the other side, picking up speed as it goes (see Figure 5.11). It is moving just fast enough to reach the top of the second hill, slightly lower than the first. It accelerates downhill again. Everybody screams!

Figure 5.11: The roller-coaster car accelerates as it comes downhill. It's even more exciting if it runs through water.

The motor provides a force to pull the roller-coaster car to the top of the hill. It transfers energy to the car. But where is this energy when the car is waiting at the top of the hill? The car now has gravitational potential energy; as soon as it is given a small push to set it moving, it accelerates. It gains kinetic energy and at the same time it loses g.p.e.

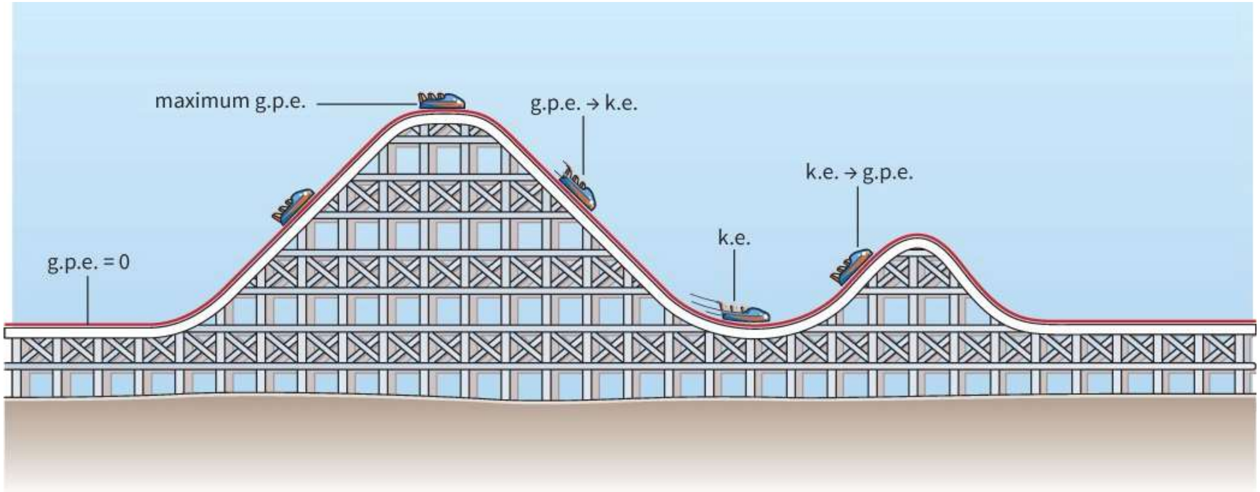


Figure 5.12: Energy changes along a roller-coaster.

As the car runs along the roller-coaster track (Figure 5.12), its energy changes.

1 At the top of the first hill, it has the most g.p.e.

2 As it runs downhill, its g.p.e. decreases and its k.e. increases.

3 At the bottom of the hill, all of its g.p.e. has been changed to k.e. and heat and sound energy.

4 As it runs back uphill, the force of gravity slows it down. k.e. is being changed to g.p.e.

Inevitably, some energy is lost by the car. There is friction with the track and air resistance. So, the car cannot return to its original height. That is why the second hill must be slightly lower than the first. It is fun if the car runs through a trough of water, but that takes even more energy, and the car cannot rise so high. There are many situations where an object's energy changes between gravitational potential energy and kinetic energy. For example:

- a high diver falling towards the water – g.p.e. changes to k.e.
- a ball is thrown upwards – k.e. changes to g.p.e.
- a child on a swing – energy changes back and forth between g.p.e. and k.e.

5.5 Down, up, down: energy changes

When an object falls, it speeds up. Its g.p.e. decreases and its k.e. increases. Energy is being transformed from gravitational potential energy to kinetic energy. Some energy is likely to be lost, usually as heat because of air resistance. However, if no energy is lost in the process, we have:

decrease in g.p.e. = gain in k.e.

We can use this idea to solve a variety of problems, as illustrated by Worked example 4.

WORKED EXAMPLE

4 A pendulum consists of a brass sphere of mass 5.0 kg hanging from a long string (see Figure 5.13).

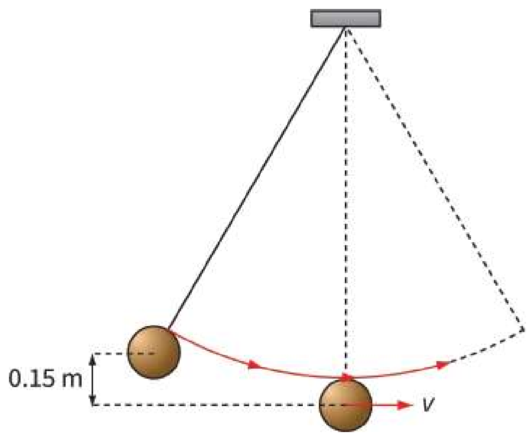


Figure 5.13: For Worked example 4.

The sphere is pulled to the side so that it is 0.15 m above its lowest position. It is then released. How fast will it be moving when it passes through the lowest point along its path?

Step 1 Calculate the loss in g.p.e. as the sphere falls from its highest position:

$E_p = mgh = 5.0 \times 9.81 \times 0.15 = 7.36$ J

Step 2 The gain in the sphere's k.e. is 7.36 J. We can use this to calculate the sphere's speed. First, calculate v^2, then v:

$$\frac{1}{2}mv^2 = 7.36$$
$$\frac{1}{2} \times 5.0 \times v^2 = 7.36$$
$$v^2 = 2 \times \frac{7.36}{5.0}$$
$$v^2 = 2.944$$
$$v = \sqrt{2.944} \approx 1.72 \text{ m s}^{-1} \approx 1.7 \text{ m s}^{-1}$$

Note that we would obtain the same result in Worked example 4 no matter what the mass of the sphere. This is because both k.e. and g.p.e. depend on mass m. If we write:

$$\text{change in g.p.e} = \text{change in k.e}$$
$$mgh = \frac{1}{2}mv^2$$

we can cancel m from both sides. Hence:

$$gh = \frac{v^2}{2}$$
$$v^2 = 2gh$$
$$v = \sqrt{2gh}$$

The final speed v only depends on g and h. The mass m of the object is irrelevant. This is not surprising; we could use the same equation to calculate the speed of an object falling from height h. An object of small mass gains the same speed as an object of large mass, provided air resistance has no effect.

Questions

11 Re-work Worked example 4 for a brass sphere of mass 10 kg, and show that you get the same result. Repeat with any other value of mass.

12 Calculate how much gravitational potential energy is lost by an aircraft of mass 80 000 kg if it descends from an altitude of 10 000 m to an altitude of 1000 m. What happens to this energy if the pilot keeps the aircraft's speed constant?

13 A high diver (see Figure 5.14) reaches the highest point in her jump with her centre of gravity 10 m above the water.

Figure 5.14: A high dive is an example of converting (transforming) gravitational potential energy to kinetic energy.

Assuming that all her gravitational potential energy becomes kinetic energy during the dive, calculate her speed just before she enters the water.

5.6 Energy transfers

Climbing bars

If you are going to climb a mountain, you will need a supply of energy. This is because your gravitational potential energy is greater at the top of the mountain than at the base. A good supply of energy would be some bars of chocolate. Each bar supplies 1200 kJ. Suppose your weight is 600 N and you climb a 2000 m high mountain. The work done by your muscles is:

$$\text{work done} = Fs = 600 \times 2000 = 1200 \text{ kJ}$$

So, one bar of chocolate should provide enough energy. Of course, in reality, it would not. Your body is inefficient. It cannot convert 100% of the energy from food into gravitational potential energy. A lot of energy is wasted as your muscles warm up, you perspire and your body rises and falls as you walk along the path. Your body is perhaps only 5% efficient as far as climbing is concerned, and you will need to eat 20 chocolate bars to get you to the top of the mountain. And you will need to eat more to get you back down again.

KEY EQUATION

$$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100\%$$

Many energy transfers are inefficient. That is, only part of the energy is transferred to where it is wanted. The rest is wasted, and appears in some form that is not wanted (such as waste heat) or in the wrong place. You can determine the efficiency of any device or system using the following equation:

$$\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100\%$$

A car engine is more efficient than a human body, but not much more. Figure 5.15 shows how this can be represented by a Sankey diagram. The width of the arrow represents the fraction of the energy which is transformed to each new form. In the case of a car engine, we want it to provide kinetic energy to turn the wheels. In practice, 80% of the energy is transformed into heat: the engine gets hot, and heat escapes into the surroundings. So the car engine is only 20% efficient.

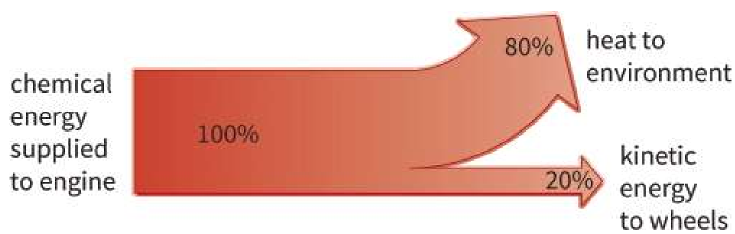


Figure 5.15: We want a car engine to supply kinetic energy. This Sankey diagram shows that only 20% of the energy supplied to the engine ends up as kinetic energy – it is 20% efficient.

We have previously considered situations where an object is falling, and all of its gravitational potential energy changes to kinetic energy.

In Worked example 5, we will look at a similar situation, but in this case the energy change is not 100% efficient.

WORKED EXAMPLE

5 Figure 5.16 shows a dam that stores water. The outlet of the dam is 20 m below the surface of the water in the reservoir. Water leaving the dam is moving at 16 m s^{-1}. Calculate the percentage of the gravitational potential energy that is lost when converted into kinetic energy.

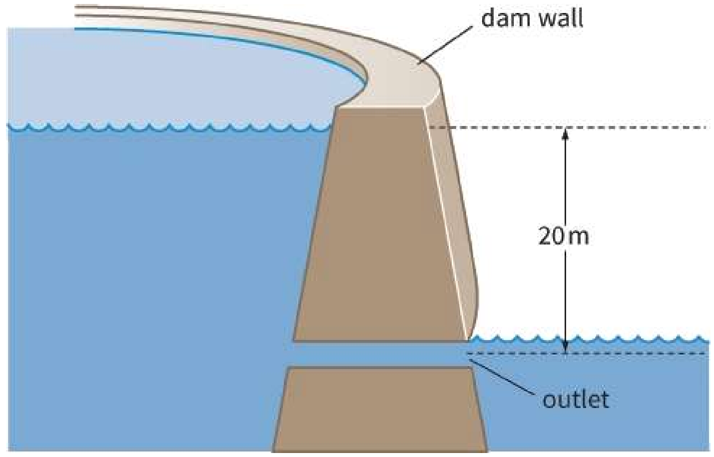


Figure 5.16: Water stored behind the dam has gravitational potential energy; the fast-flowing water leaving the foot of the dam has kinetic energy.

Step 1 We will picture 1 kg of water, starting at the surface of the lake (where it has g.p.e., but no k.e.) and flowing downwards and out at the foot (where it has k.e., but less g.p.e.). Then:

change in g.p.e. of water between surface and outflow = $mgh = 1 \times 9.81 \times 20 = 196$ J

Step 2 Calculate the k.e. of 1 kg of water as it leaves the dam:

$$\begin{aligned}\text{k.e. of water leaving dam} &= \tfrac{1}{2}mv^2 \\ &= \tfrac{1}{2} \times 1 \times (16)^2 \\ &= 128\ \text{J}\end{aligned}$$

Step 3 For each kilogram of water flowing out of the dam, the loss of energy is:

loss = 196 − 128 = 68 J

$$\begin{aligned}\text{percentage loss} &= \tfrac{68}{196} \times 100\% \\ &= 34.69\% \approx 35\%\end{aligned}$$

If you wanted to use this moving water to generate electricity, you would have already lost more than a third of the energy that it stores when it is behind the dam.

Conservation of energy

Where does the lost energy from the water in the reservoir go? Most of it ends up warming the water, or warming the pipes that the water flows through. The outflow of water is probably noisy, so some sound is produced.

Here, we are assuming that all of the energy ends up somewhere. None of it disappears. We assume the same thing when we draw a Sankey diagram. The total thickness of the arrow remains constant. We could not have an arrow which got thinner (energy disappearing) or thicker (energy appearing out of nowhere).

We are assuming that energy is conserved. This is a principle, known as the **principle of conservation of energy**, which we expect to apply in all situations.

Energy cannot be created or destroyed. It can only be converted from one form to another.

We should always be able to add up the total amount of energy at the beginning, and be able to account for it all at the end. We cannot be sure that this is always the case, but we expect it to hold true.

We have to think about energy changes within a closed system; that is, we have to draw an imaginary boundary around all of the interacting objects that are involved in an energy transfer.

Sometimes, applying the principle of conservation of energy can seem like a scientific fiddle. When physicists were investigating radioactive decay involving beta particles, they found that the particles after the decay had less energy in total than the particles before. They guessed that there was another, invisible particle that was carrying away the missing energy. This particle, named the neutrino, was proposed by the theoretical physicist Wolfgang Pauli in 1931. The neutrino was not detected by experimenters until 25 years later.

Although we cannot prove that energy is always conserved, this example shows that the principle of conservation of energy can be a powerful tool in helping us to understand what is going on in nature, and that it can help us to make fruitful predictions about future experiments.

Question

14 A stone falls from the top of a cliff, 80 m high. When it reaches the foot of the cliff, its speed is 38 m s^{-1}.

a Calculate the proportion of the stone's initial g.p.e. that is converted to k.e.

b What happens to the rest of the stone's initial energy?

5.7 Power

The word **power** has several different meanings – such as political power, powers of ten or electrical power from power stations. In physics, it has a specific meaning related to these other meanings. Figure 5.17 illustrates what we mean by power in physics.

Figure 5.17: A lift needs a powerful motor to raise the car when it has a full load of people. The motor does many thousands of joules of work each second.

The lift shown in Figure 5.17 can lift a heavy load of people. The motor at the top of the building provides a force to raise the lift car, and this force does work against the force of gravity. The motor transfers energy to the lift car. The **power** *P* of the motor is the rate at which it does work over a unit of time.

Power is defined as the rate of work done per unit of time. As a word equation, power is given by:

$$\text{power} = \frac{\text{work done}}{\text{time taken}}$$
$$P = \frac{W}{t}$$

where W is the work done in a time t.

KEY EQUATION

$$\text{power} = \frac{\text{work done}}{\text{time taken}} \equiv P = \frac{W}{t}$$

Units of power: the watt

Power is measured in watts, named after James Watt, the Scottish engineer famous for his development of the steam engine in the second half of the 18th century. The **watt** is defined as a rate of working of 1 joule per second. Hence:

1 watt = 1 joule per second

or

$$1\ \text{W} = 1\ \text{J s}^{-1}$$

In practice, we also use kilowatts (kW) and megawatts (MW).

$$1000 \text{ watts} = 1 \text{ kilowatt } (1 \text{ kW})$$

$$1\,000\,000 \text{ watts} = 1 \text{ megawatt } (1 \text{ MW})$$

The labels on light bulbs display their power in watts; for example, 60 W or 10 W. The values of power on the labels tell you about the energy transferred by an electrical current, rather than by a force doing work.

WORKED EXAMPLE

6 The motor of the lift shown in Figure 5.18 provides a force of 20 kN; this force is enough to raise the lift by 18 m in 10 s. Calculate the output power of the motor.

Step 1 First, we must calculate the work done:

$$\begin{aligned} \text{work done} &= \text{force} \times \text{distance moved} \\ &= 20 \times 18 = 360 \text{ kJ} \end{aligned}$$

Step 2 Now we can calculate the motor's output power:

$$\begin{aligned} \text{power} &= \frac{\text{work done}}{\text{time taken}} \\ &= \frac{360 \times 10^3}{10} \\ &= 36 \text{ kW} \end{aligned}$$

Hint: *Take care not to confuse the two uses of the letter 'W':*

W = *watt (a unit)*

W = work done (a quantity)

So the lift motor's power is 36 kW. Note that this is its mechanical power output. The motor cannot be 100% efficient since some energy is bound to be wasted as heat due to friction, so the electrical power input must be more than 36 kW.

Questions

15 Calculate how much work is done by a 50 kW car engine in a time of 1.0 minute.

16 A car engine does 4200 kJ of work in one minute. Calculate its output power, in kilowatts.

17 A particular car engine provides a force of 700 N when the car is moving at its top speed of 40 m s^{-1}.

a Calculate how much work is done by the car's engine in one second.

b State the output power of the engine.

Moving power

An aircraft is kept moving forwards by the force of its engines pushing air backwards. The greater the force and the faster the aircraft is moving, the greater the power supplied by its engines.

Suppose that an aircraft is moving with velocity v. Its engines provide the force F needed to overcome the drag of the air. In time t, the aircraft moves a distance s equal to $v \times t$.

So, the work done by the engines is:

$$\begin{aligned} \text{work done} &= \text{force} \times \text{distance} \\ W &= F \times v \times t \end{aligned}$$

We know that:

$$\begin{aligned} \text{power} &= \frac{\text{work done}}{\text{time taken}} \\ P &= \frac{W}{t} \end{aligned}$$

Substituting W for gives:

$$P = \frac{F \times v \times t}{t}$$

Which can be simplified to:

$$P = F \times v$$
$$\text{power} = \text{force} \times \text{velocity}$$

KEY EQUATION

power = force × velocity ≡ $P = F \times v$

It may help to think of this equation in terms of units. The right-hand side is in N × m s^{-1}, and N m is the same as J. So the right-hand side has units of J s^{-1}, or W, the unit of power. If you look back to Question 17, you will see that, to find the power of the car engine, rather than considering the work done in 1 s, we could simply have multiplied the engine's force by the car's speed.

Human power

Our energy supply comes from our food. A typical diet supplies 2000–3000 kcal (kilocalories) per day. This is equivalent (in SI units) to about 10 MJ of energy. We need this energy for our daily requirements – keeping warm, moving about, brainwork and so on. We can determine the average power of all the activities of our body:

$$\begin{aligned}\text{average power} &= 10\ \text{MJ per day} \\ &= 10 \times \frac{10^6}{86\,400} \\ &= 116\ \text{W}\end{aligned}$$

So we dissipate energy at the rate of about 100 W. We supply roughly as much energy to our surroundings as a 100 W light bulb. Twenty people will keep a room as warm as a 2 kW electric heater.

Note that this is our average power. If you are doing some demanding physical task, your power will be greater. This is illustrated in Worked example 7.

Note also that the human body is not a perfectly efficient system; a lot of energy is wasted when, for example, we lift a heavy load. We might increase an object's g.p.e. by 1000 J when we lift it, but this might require five or ten times this amount of energy to be expended by our bodies.

WORKED EXAMPLE

7 A person who weighs 500 N runs up a flight of stairs in 5.0 s (Figure 5.18). Their gain in height is 3.0 m. Calculate the rate at which work is done against the force of gravity.

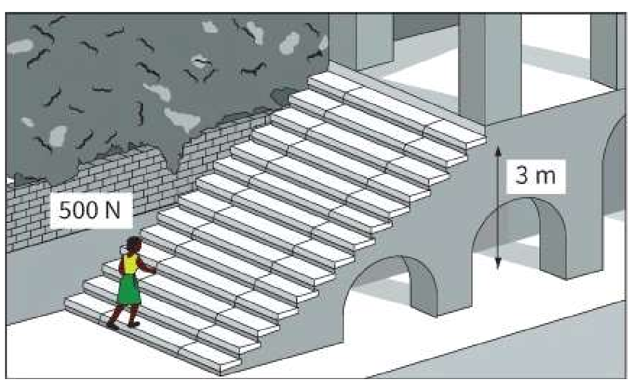


Figure 5.18: Running up stairs can require a high rate of doing work. You may have investigated your own power in this way.

Step 1 Calculate the work done against gravity:

$$\begin{aligned}\text{work done } W &= F \times s \\ &= 500 \times 3.0 \\ &= 1500\ \text{J}\end{aligned}$$

Step 2 Now calculate the power:

$$\begin{aligned} \text{power } P &= \frac{W}{t} \\ &= \frac{1500}{5.0} \\ &= 300 \text{ W} \end{aligned}$$

So, while the person is running up the stairs, they are doing work against gravity at a greater rate than their average power – perhaps three times as great. And, since our muscles are not very efficient, they need to be supplied with energy even faster, perhaps at a rate of 1 kW. This is why we cannot run up stairs all day long without greatly increasing the amount we eat. The inefficiency of our muscles also explains why we get hot when we exert ourselves.

Question

18 In an experiment to measure a student's power, she times herself running up a flight of steps. Use the data to work out her useful power.

number of steps = 28

height of each step = 20 cm

acceleration of free fall = 9.81 m s^{-2}

mass of student = 55 kg

time taken = 5.4 s

REFLECTION

How do you feel about this topic? What parts of it do you particularly like or dislike? And why?

Think about a number of important machines that you use in your house or school. Is it worthwhile increasing their efficiency and can you suggest how this might be done? Discuss this with others.

Make notes about the new things you have learnt from this chapter.

SUMMARY

The work done W when a force F moves through a displacement s in the direction of the force: $W = Fs$ or $W = Fs \cos\theta$ where θ is the angle between the force and the displacement.
A joule is defined as the work done (or energy transferred) when a force of 1 N moves a distance of 1 m in the direction of the force.
When an object of mass m rises through a height h, its gravitational potential energy E_p increases by an amount: $E_p = mgh$
The kinetic energy E_k of a body of mass m moving at speed v is: $E_\text{k} = \frac{1}{2}mv^2$
The principle of conservation of energy states that, for a closed system, energy can be transferred to other forms but the total amount of energy remains constant.
The efficiency of a device or system is determined using the equation: $\text{efficiency} = \frac{\text{useful output energy}}{\text{total input energy}} \times 100\,\%$
Power is the rate at which work is done (or energy is transferred): $P = \frac{W}{t}$ and $P = Fv$
A watt is defined as a rate of transfer of energy of one joule per second.

EXAM-STYLE QUESTIONS

1 How is the joule related to the base units of m, kg and s? **[1]**

A kg m^{-1} s^2

B kg m^2 s^{-2}

C kg m^2 s^{-1}

D kg s^{-2}

2 An object falls at terminal velocity in air. What overall conversion of energy is occurring? **[1]**

A gravitational potential energy to kinetic energy

B gravitational potential energy to thermal energy

C kinetic energy to gravitational potential energy

D kinetic energy to thermal energy

3 In each case a–c, describe the energy changes taking place:

a An apple falling towards the ground **[1]**

b A car decelerating when the brakes are applied **[1]**

c A space probe falling towards the surface of a planet. **[1]**

[Total: 3]

4 A 120 kg crate is dragged along the horizontal ground by a 200 N force acting at an angle of 30° to the horizontal, as shown.

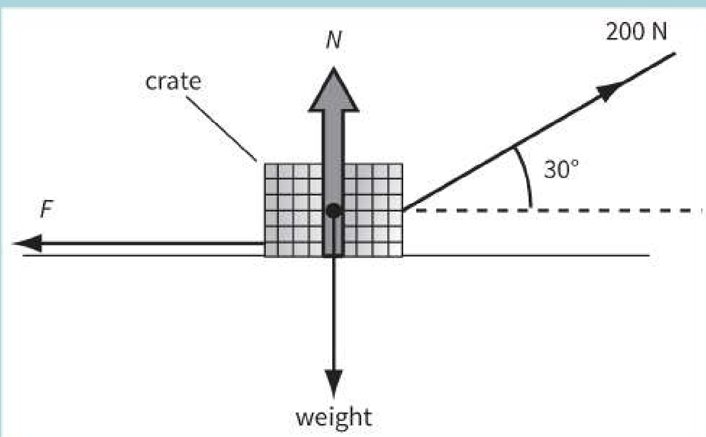


Figure 5.19

The crate moves along the surface with a constant velocity of 0.5 m s^{-1}. The 200 N force is applied for a time of 16 s.

a Calculate the work done on the crate by:

i the 200 N force **[3]**

ii the weight of the crate **[2]**

iii the normal contact force *N*. **[2]**

b Calculate the rate of work done against the frictional force *F*. **[1]**

[Total: 8]

5 Explain which of the following has greater kinetic energy?

- A 20-tonne truck travelling at a speed of 30 m s^{-1}
- A 1.2 g dust particle travelling at 150 km s^{-1} through space. **[3]**

6 A 950 kg sack of cement is lifted to the top of a building 50 m high by an electric motor.

a Calculate the increase in the gravitational potential energy of the sack of cement. **[2]**

b The output power of the motor is 4.0 kW. Calculate how long it took to raise the sack to the top of the building. **[2]**

c The electrical power transferred by the motor is 6.9 kW. In raising the sack to the top of the building, how much energy is wasted in the motor as heat? **[3]**

[Total: 7]

7 **a** Define power and state its unit. **[2]**

b Write a word equation for the kinetic energy of a moving object. **[1]**

c A car of mass 1100 kg starting from rest reaches a speed of 18 m s^{-1} in 25 s. Calculate the average power developed by the engine of the car. **[2]**

[Total: 5]

8 A cyclist pedals a long slope which is at 5.0° to the horizontal, as shown.

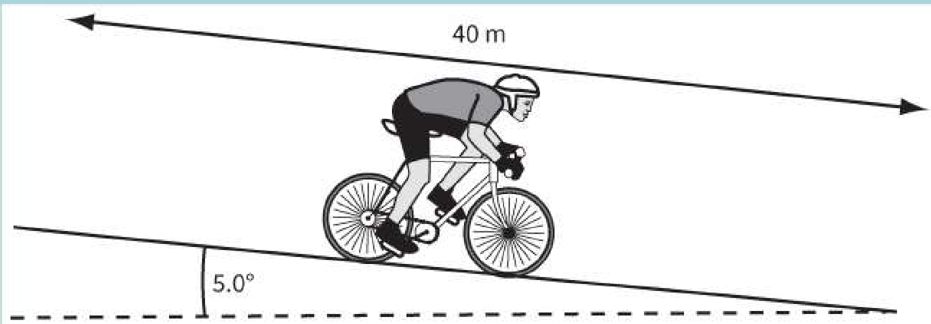


Figure 5.20

The cyclist starts from rest at the top of the slope and reaches a speed of 12 m s^{-1} after a time of 67 s, having travelled 40 m down the slope. The total mass of the cyclist and bicycle is 90 kg.

a Calculate:

i the loss in gravitational potential energy as he travels down the slope **[3]**

ii the increase in kinetic energy as he travels down the slope. **[2]**

b **i** Use your answers to a to determine the useful power output of the cyclist. **[3]**

ii Suggest one reason why the actual power output of the cyclist is larger than your value in **i**. **[2]**

[Total: 10]

9 **a** Explain what is meant by work. **[2]**

b **i** Explain how the principle of conservation of energy applies to a man sliding from rest down a vertical pole, if there is a constant force of friction acting on him. **[2]**

ii The man slides down the pole and reaches the ground after falling a distance h = 15 m. His potential energy at the top of the pole is 1000 J. Sketch a graph to show how his gravitational potential energy E_p varies with h. Add to your graph a line to show the variation of his kinetic energy E_k with h. **[3]**

[Total: 7]

10 **a** Use the equations of motion to show that the kinetic energy of an object of mass m moving with velocity v is $\frac{1}{2}mv^2$. **[2]**

b A car of mass 800 kg accelerates from rest to a speed of 20 m s^{-1} in a time of 6.0 s.

i Calculate the average power used to accelerate the car in the first 6.0 s. **[2]**

ii The power passed by the engine of the car to the wheels is constant. Explain why the acceleration of the car decreases as the car accelerates. **[2]**

[Total: 6]

11 **a** **i** Define potential energy. **[1]**

ii **Identify** differences between gravitational potential energy and elastic potential energy. **[2]**

b Seawater is trapped behind a dam at high tide and then released through turbines. The level of the water trapped by the dam falls 10.0 m until it is all at the same height as the sea.

i Calculate the mass of seawater covering an area of 1.4×10^6 m^2 and with a depth of 10.0 m. (Density of seawater = 1030 kg m^{-3}.) **[1]**

ii Calculate the maximum loss of potential energy of the seawater in **i** when passed through the turbines. **[2]**

iii The potential energy of the seawater, calculated in **ii**, is lost over a period of 6.0 hours. Estimate the average power output of the power station over this time period, given that the efficiency of the power station is 50%. **[3]**

[Total: 9]