Chapter 4

Forces: vectors and moments

LEARNING INTENTIONS

In this chapter you will learn how to:

- use a vector triangle to represent coplanar forces in equilibrium and add two or more coplanar forces
- resolve a force into perpendicular components
- represent the weight of a body as acting at a single point known as its centre of gravity
- define and apply the moment of a force and the torque of a couple
- state and apply the principle of moments
- use the idea that, when there is no resultant force and no resultant torque, a system is in equilibrium.

BEFORE YOU START

- Write down what a *vector* is. List some examples.
- Is force is a vector? Discuss with a partner.

SAILING AHEAD

Force is a vector quantity. Sailors know a lot about the vector nature of forces. For example, they can sail 'into the wind'. The sails of a yacht can be angled to provide a 'component' of force (in other words, an effect of the force in the forward direction) and the boat can then sail at almost 45° to the wind. The boat tends to 'heel over' and the crew sit on the side of the boat to provide a turning effect in the opposite direction (Figure 4.1). If the wind has an effect forwards, what stops the boat from moving sideways due to the 'component' of the wind sideways? (Hint: find out about the shape of the bottom of the boat.)

Figure 4.1: Sailing into the wind.

4.1 Combining forces

You will have learned that a vector quantity has both magnitude and direction. An object may have two or more forces acting on it and, since these are vectors, we must use vector addition (Chapter 1) to find their combined effect (their resultant).

There are several forces acting on the car (Figure 4.2) as it struggles up the steep hill. They are:

- its weight W (= mg)
- the normal contact force N of the road
- air resistance D
- the forward force F caused by friction between the car tyres and the road.

If we knew the magnitude and direction of each of these **forces**, we could work out their combined effect on the car. Will it accelerate up the hill? Or will it slide backwards down the hill?

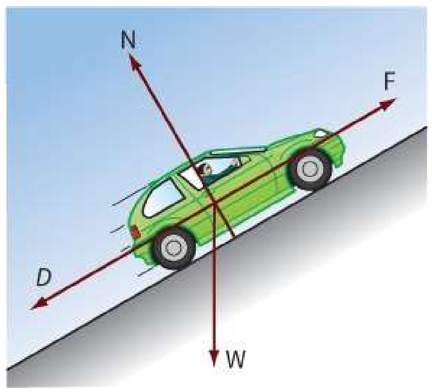

Figure 4.2: Four forces act on this car as it moves uphill.

The combined effect of several forces is known as the **resultant force**. To see how to work out the resultant of two or more forces, we will start with a relatively simple example.

Two forces in a straight line

We saw some examples in Chapter 3 of two forces acting in a straight line. For example, a falling tennis ball may be acted on by two forces: its weight mg, downwards, and air resistance D, upwards (Figure 4.3). The resultant force is then:

resultant force = $mg - D = 1.0 - 0.2 = 0.8$ N

When adding two or more forces that act in a straight line, we have to take account of their directions. A force may be positive or negative; we adopt a **sign convention** to help us decide which is which. In setting up the sign convention you decide for yourself which direction is positive. In Figure 4.3, for example, we have taken the direction downwards as positive so the weight is +1.0 N, a positive force, and the force upwards is −0.2 N, a negative force. The resultant is +0.8 N, which tells us the resultant is downwards.

You might choose the upwards direction as positive, but if you apply a sign convention correctly, the sign of your final answer will tell you the direction of the resultant force (and hence acceleration).

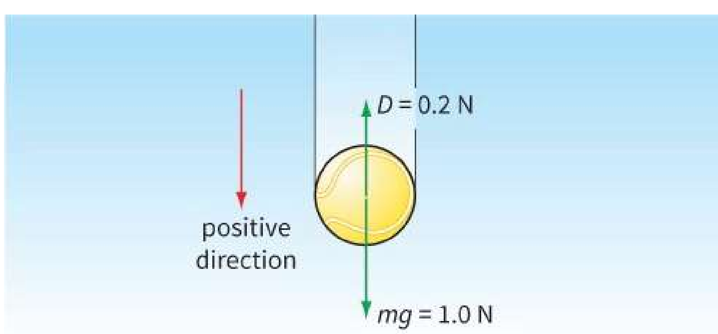

Figure 4.3: Two forces on a falling tennis ball.

Two forces at right angles

Figure 4.4 shows a shuttlecock falling on a windy day. There are two forces acting on the shuttlecock: its weight vertically downwards, and the horizontal push of the wind. (It helps if you draw the force arrows of different lengths, to show which force is greater.) We must add these two forces together to find the resultant force acting on the shuttlecock.

We add the forces by drawing two arrows, head-to-tail, as shown on the right of Figure 4.4.

- First, draw a horizontal arrow to represent the 6.0 N push of the wind.

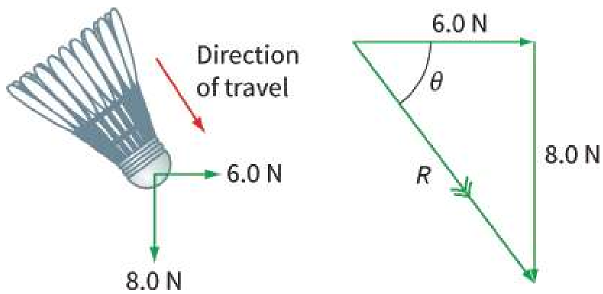

Figure 4.4: Two forces act on this shuttlecock as it travels through the air; the vector triangle shows how to find the resultant force.

- Next, starting from the end of this arrow, draw a second arrow, downwards, representing the weight of 8.0 N.
- Now, draw a line from the start of the first arrow to the end of the second arrow. This arrow represents the resultant force R, in both magnitude and direction.

The arrows are added by drawing them end-to-end; the end of the first arrow is the start of the second arrow. Now we can find the resultant force either by scale drawing or by calculation. In this case, we have a 3–4–5 right-angled triangle, so calculation is simple:

$$\begin{aligned} R^2 &= 6.0^2 + 8.0^2 = 36 + 64 \\ &= 100 \\ R &= \sqrt{100} \\ &= 10\text{ N} \\ \tan\theta &= \frac{\text{opp}}{\text{adj}} = \frac{8.0}{6.0} = \frac{4}{3} \\ \theta &= \tan^{-1}\frac{4}{3} \approx 53^\circ \end{aligned}$$

So the resultant force is 10 N, at an angle of 53° below the horizontal. This is a reasonable answer; the weight is pulling the shuttlecock downwards and the wind is pushing it to the right. The angle is greater than 45° because the downward force is greater than the horizontal force.

KEY IDEA

When you draw a scale drawing you should:

- state the scale used
- draw a large diagram to reduce the uncertainty.

Three or more forces

The spider shown in Figure 4.5 is hanging by a thread. It is blown sideways by the wind. The diagram shows the three forces acting on it:

- weight acting downwards
- the tension in the thread
- the push of the wind.

The diagram also shows how these can be added together. In this case, we arrive at an interesting result.

Arrows are drawn to represent each of the three forces, end-to-end. The end of the third arrow coincides with the start of the first arrow, so the three arrows form a closed triangle. This tells us that the resultant force R on the spider is zero, that is, $R = 0$. The closed triangle in Figure 4.5 is known as a **triangle of forces**.

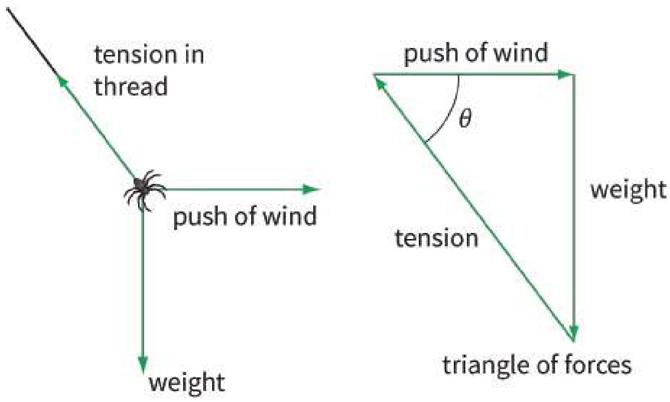

Figure 4.5: Blowing in the wind–this spider is hanging in equilibrium.

So there is no resultant force. The forces on the spider balance each other out, and we say that the spider is in **equilibrium**. If the wind blew a little harder, there would be an unbalanced force on the spider, and it would move off to the right.

We can use this idea in two ways:

- If we work out the resultant force on an object and find that it is zero, this tells us that the object is in equilibrium.
- If we know that an object is in equilibrium, we know that the forces on it must add up to zero. We can use this to work out the values of one or more unknown forces.

Questions

1 A parachutist weighs 1000 N. When she opens her parachute, it pulls upwards on her with a force of 2000 N.

a Draw a diagram to show the forces acting on the parachutist.

b Calculate the resultant force acting on her.

c What effect will this force have on her?

2 The ship shown in Figure 4.6 is travelling at a constant velocity.

a Is the ship in equilibrium (in other words, is the resultant force on the ship equal to zero)? How do you know?

b What is the upthrust U of the water?

c What is the drag D of the water?

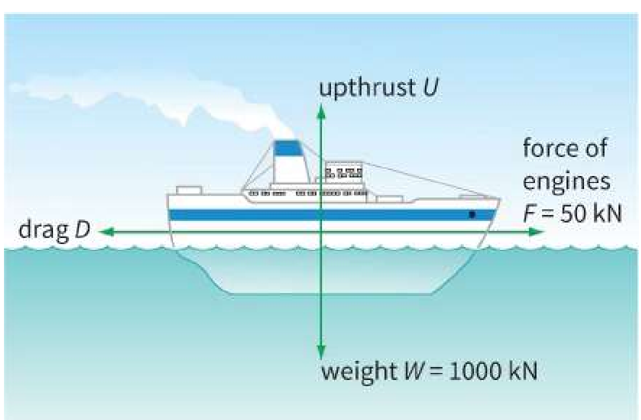

Figure 4.6: For Question 2. The force D is the frictional drag of the water on the boat. Like air resistance, drag is always in the opposite direction to the object’s motion.

3 A stone is dropped into a fast-flowing stream. It does not fall vertically because of the sideways push of the water (Figure 4.7).

a Calculate the resultant force on the stone.

b Is the stone in equilibrium?

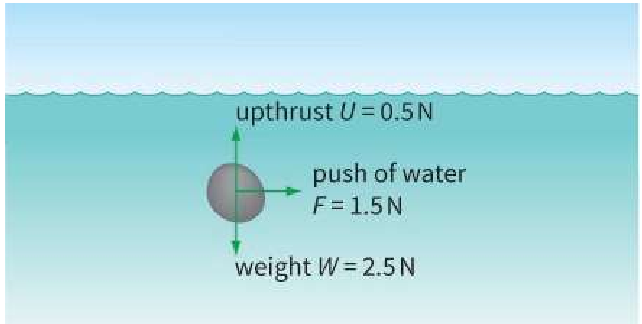

Figure 4.7: For Question 3.

4.2 Components of vectors

Look back to Figure 4.5. The spider is in equilibrium, even though three forces are acting on it. We can think of the tension in the thread as having two effects. It is:

- pulling upwards, to counteract the downward effect of gravity
- pulling to the left, to counteract the effect of the wind.

We can say that this force has two effects or **components**: an upwards (vertical) component and a sideways (horizontal) component. It is often useful to split up a vector quantity into components like this, just as we did with velocity in Chapter 2. The components are in two directions at right angles to each other, often horizontal and vertical. The process is called **resolving** the vector.

Then we can think about the effects of each component separately; we say that the perpendicular components are independent of one another. Because the two components are at 90° to each other, a change in one will have no effect on the other. Figure 4.8 shows how to resolve a force F into its horizontal and vertical components. These are:

$$\text{horizontal component of } F,\ F_x = F\cos\theta$$

$$\text{vertical component of } F,\ F_y = F\sin\theta$$

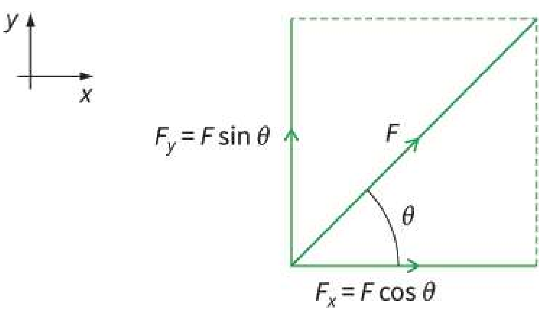

Figure 4.8: Resolving a vector into two components at right angles.

Making use of components

When the trolley shown in Figure 4.9 is released, it accelerates down the ramp. This happens because of the weight of the trolley. The weight acts vertically downwards, although this by itself does not determine the resulting motion. However, the weight has a component that acts down the slope. By calculating the component of the trolley's weight down the slope, we can determine its acceleration.

Figure 4.9: This student is investigating the acceleration of a trolley down a sloping ramp.

Figure 4.10 shows the forces acting on the trolley. To simplify the situation, we will assume there is no friction. The forces are:

- the weight of the trolley, W, which acts vertically downwards
- the contact force of the ramp, N, which acts at right angles to the ramp.

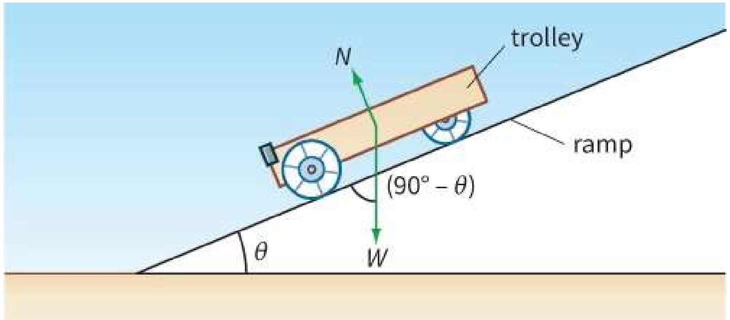

Figure 4.10: A force diagram for a trolley on a ramp.

You can see at once from Figure 4.10 that the forces cannot be balanced, since they do not act in the same straight line.

To find the component of W down the slope, we need to know the angle between W and the slope. The slope makes an angle θ with the horizontal, and from the diagram we can see that the angle between the weight and the ramp is $(90° - \theta)$. Using the rule for calculating the component of a vector given previously, we have:

$$\text{component of } W \text{ down the slope} = W \cos(90° - \theta) = W \sin\theta$$

(It is helpful to recall that $\cos(90° - \theta) = \sin\theta$; you can see this from Figure 4.10.)

Does the contact force N help to accelerate the trolley down the ramp? To answer this, we must calculate its component down the slope. The angle between N and the slope is 90°. So:

$$\text{component of } N \text{ down the slope} = N \cos 90° = 0$$

The cosine of 90° is zero, and so N has no component down the slope. This shows why it is useful to think in terms of the components of forces; we don't know the value of N, but, since it has no effect down the slope, we can ignore it.

(There's no surprise about this result. The trolley runs down the slope because of the influence of its weight, not because it is pushed by the contact force N.)

Changing the slope

If the students in Figure 4.9 increase the slope of their ramp, the trolley will move down the ramp with greater acceleration. They have increased θ, and so the component of W down the slope will have increased.

Now we can work out the trolley's acceleration. If the trolley's mass is m, its weight is mg. So the force F making it accelerate down the slope is:

$$F = mg \sin\theta$$

Since from Newton's second law for constant mass we have $a = \frac{F}{m}$, the trolley's acceleration a is given by:

$$a = \frac{mg\sin\theta}{m} = g\sin\theta$$

We could have arrived at this result simply by saying that the trolley's acceleration would be the component of g down the slope (Figure 4.11). The steeper the slope, the greater the value of $\sin\theta$, and hence the greater the trolley's acceleration.

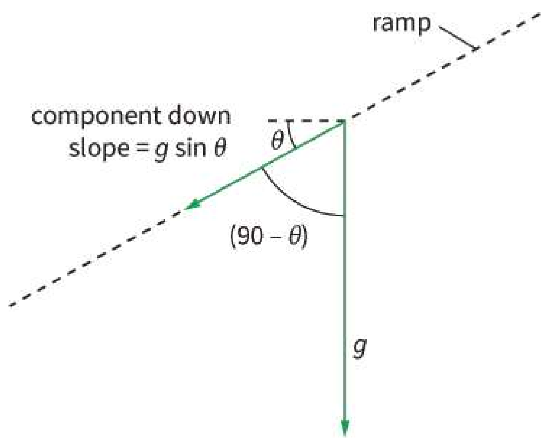

Figure 4.11: Resolving g down the ramp.

Questions

4 The person in Figure 4.12 is pulling a large box using a rope. Use the idea of components of a force to explain why they are more likely to get the box to move if the rope is horizontal (as in **a**) than if it is sloping upwards (as in **b**).

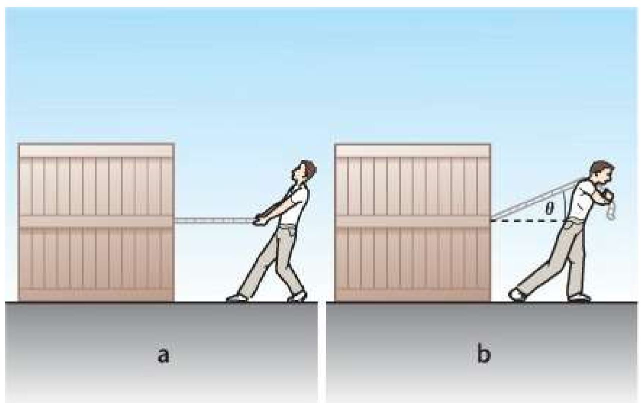

Figure 4.12: Why is it easier to move the box with the rope horizontal? For Question 4.

5 A crate is sliding down a slope. The weight of the crate is 500 N. The slope makes an angle of 30° with the horizontal.

- **a** Draw a diagram to show the situation. Include arrows to represent the weight of the crate and the contact force of the slope acting on the crate.
- **b** Calculate the component of the weight down the slope.
- **c** Explain why the contact force of the slope has no component down the slope.
- **d** What third force might act to oppose the motion? In which direction would it act?

Solving problems by resolving forces

A force can be resolved into two components at right angles to each other; these can then be treated independently of one another. This idea can be used to solve problems, as illustrated in Worked example 1.

WORKED EXAMPLE

1 A boy of mass 40 kg is on a waterslide that slopes at 30° to the horizontal. The frictional force up the slope is 120 N. Calculate the boy's acceleration down the slope. Take the acceleration of free fall g to be 9.81 m s^{-2}.

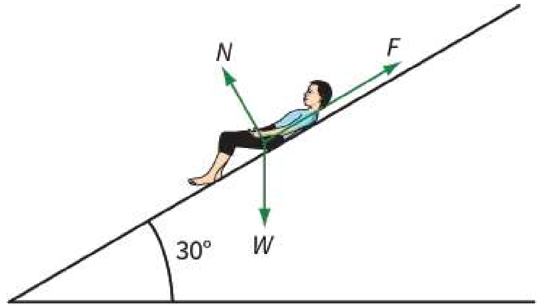

Figure 4.13: For Worked example 1.

Step 1 Draw a labelled diagram showing all the forces acting on the object of interest (Figure 4.13). This is known as a **free-body force diagram**. The forces are:

the boy's weight $W = 40 \times 9.81 = 392$ N

the frictional force up the slope $F = 120$ N

the contact force N at 90° to the slope.

Step 2 We are trying to find the resultant force on the boy that makes him accelerate down the slope. We resolve the forces down the slope, i.e., we find their components in that direction.

component of W down the slope = 392 × sin 30° = 196 N

component of F down the slope = −120 N (negative because F is directed up the slope)

component of N down the slope = 0 (because it is at 90° to the slope)

It is convenient that N has no component down the slope, since we do not know the value of N.

Step 3 Calculate the resultant force on the boy:

resultant force = 196 − 120 = 76 N

Step 4 Calculate his acceleration:

$$\begin{aligned} \text{acceleration} &= \frac{\text{resultant force}}{\text{mass}} \\ &= \frac{76}{40} \\ &= 1.9 \text{ m s}^{-2} \end{aligned}$$

So the boy's acceleration down the slope is 1.9 m s^{-2}. We could have arrived at the same result by resolving vertically and horizontally, but that would have led to two simultaneous equations from which we would have had to eliminate the unknown force N. It often helps to resolve forces at 90° to an unknown force.

Question

6 A child of mass 40 kg is on a water slide. The slide slopes down at 25° to the horizontal. The acceleration of free fall is 9.81 m s^{-2}. Calculate the child's acceleration down the slope:

a when there is no friction and the only force acting on the child is his weight

b if a frictional force of 80 N acts up the slope.

4.3 Centre of gravity

We have weight because of the force of gravity of the Earth on us. Each part of our body – arms, legs, head, for example – experiences a force, caused by the force of gravity. However, it is much simpler to picture the overall effect of gravity as acting at a single point. This is our **centre of gravity** – the point where all the weight of the object may be considered to act.

For a person standing upright, the centre of gravity is roughly in the middle of the body, behind the navel. For a sphere, it is at the centre. It is much easier to solve problems if we simply indicate an object's weight by a single force acting at the centre of gravity, rather than a large number of forces acting on each part of the object. Figure 4.14 illustrates this point. The athlete performs a complicated manoeuvre. However, we can see that his centre of gravity follows a smooth, parabolic path through the air, just like the paths of projectiles we discussed in Chapter 2.

Figure 4.14: The dashed line indicates the path of the athlete's centre of gravity, which follows a smooth trajectory through the air. With his body curved like this, the athlete's centre of gravity is actually outside his body, just below the small of his back. At no time is the whole of his body above the bar.

PRACTICAL ACTIVITY 4.1

Finding the centre of gravity

The centre of gravity of a thin sheet, or lamina, of cardboard or metal can be found by suspending it freely from two or three points (Figure 4.15).

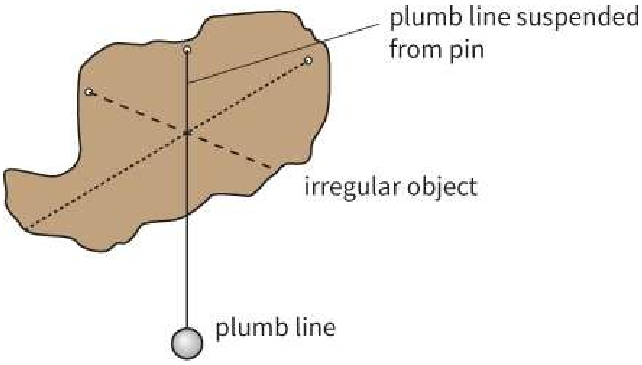

Figure 4.15: The centre of gravity is located at the intersection of the lines.

Small holes are made round the edge of the irregularly shaped object. A pin is put through one of the holes and held firmly in a clamp and stand so the object can swing freely. A length of string is attached to the pin. The other end of the string has a heavy mass attached to it. This arrangement is called a *plumb line*.

The object will stop swinging when its centre of gravity is vertically below the point of suspension. A line is drawn on the object along the vertical string of the plumb line. The centre of gravity must lie on this line. To find the position of the centre of gravity, the process is repeated with the object suspended from different holes. The centre of gravity will be at the point of intersection of the lines drawn on the object.

4.4 The turning effect of a force

Forces can make things accelerate. They can do something else as well: they can make an object turn round. We say that they can have a turning effect. Figure 4.16 shows how to use a spanner to turn a nut (a fastener with a threaded hole).

To maximise the turning effect of his force, the operator pulls close to the end of the spanner, as far as possible from the pivot (the centre of the nut) and at 90° to the spanner.

Figure 4.16: A mechanic turns a nut.

Moment of a force

The quantity that tells us about the turning effect of a force is its **moment**. The moment of a force depends on two quantities, the:

- magnitude of the force (the bigger the force, the greater its moment)
- perpendicular distance of the force from the pivot (the further the force acts from the pivot, the greater its moment).

The moment of a force = force × perpendicular distance of the pivot from the line of action of the force.

Figure 4.17a shows these quantities. The force F_1 is pushing down on the lever, at a perpendicular distance x_1 from the pivot. The moment of the force F_1 about the pivot is then given by:

$$\begin{aligned} \text{moment} &= \text{force} \times \text{distance from pivot} \\ &= F_1 \times x_1 \end{aligned}$$

The unit of moment is the newton metre (N m). This is a unit that does not have a special name. You can also determine the moment of a force in N cm.

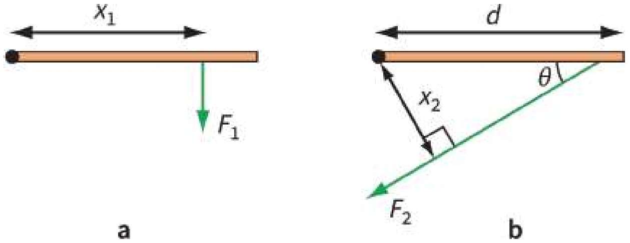

Figure 4.17: The quantities involved in calculating the moment of a force.

Figure 4.17b shows a slightly more complicated situation. F_2 is pushing at an angle θ to the lever, rather than at 90°. This makes it have less turning effect. There are two ways to calculate the moment of the force.

Method 1

Draw a perpendicular line from the pivot to the line of the force.

Find the distance x_2. Calculate the moment of the force, $F_2 \times x_2$. From the right-angled triangle, we can see that:

$$x_2 = d \sin \theta$$

Hence:

moment of force $= F_2 \times d \sin \theta = F_2 d \sin \theta$

Method 2

Calculate the component of F_2 that is at 90° to the lever.

This is $F_2 \sin \theta$. Multiply this by d.

$$\text{moment} = F_2 \sin \theta \times d$$

We get the same result as Method 1:

$$\text{moment of force} = F_2 d \sin \theta$$

Note that any force (such as the component $F_2 \cos \theta$) that passes through the pivot has no turning effect, because the distance from the pivot to the line of the force is zero.

Note also that we can calculate the moment of a force about any point, not just the pivot. However, in solving problems, it is often most convenient to take moments about the pivot as there is often an unknown force acting through the pivot (its contact force on the object).

Balanced or unbalanced?

We can use the idea of the moment of a force to solve two sorts of problem. We can:

- check whether an object will remain balanced or start to rotate
- calculate an unknown force or distance if we know that an object is balanced.

We can use the **principle of moments** to solve problems. The principle of moments states that, for any object that is in **equilibrium**, the sum of the clockwise moments about any point provided by the forces acting on the object equals the sum of the anticlockwise moments about that same point.

Note that, for an object to be in equilibrium, we also require that no resultant force acts on it. Worked examples 2, 3 and 4 illustrate how we can use these ideas to determine unknown forces.

WORKED EXAMPLES

2 Is the see-saw shown in Figure 4.18 in equilibrium (balanced), or will it start to rotate?

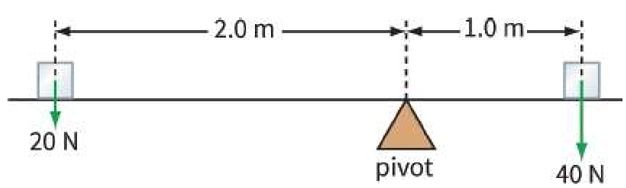

Figure 4.18: Will these forces make the see-saw rotate, or are their moments balanced?

The see-saw will remain balanced, because the 20 N force is twice as far from the pivot as the 40 N force.

To prove this, we need to think about each force individually. Which direction is each force trying to turn the see-saw, clockwise or anticlockwise? The 20 N force is tending to turn the see-saw anticlockwise, while the 40 N force is tending to turn it clockwise.

Step 1 Determine the anticlockwise moment:

moment of anticlockwise force = 20 × 2.0 = 40 N m

Step 2 Determine the clockwise moment:

moment of clockwise force = 40 × 1.0 = 40 N m

Step 3 We can see that:

clockwise moment = anticlockwise moment

So the see-saw is balanced and therefore does not rotate. The see-saw is in equilibrium.

3 The beam shown in Figure 4.19 is in equilibrium. Determine the force X.

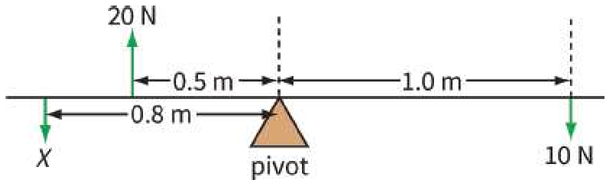

Figure 4.19: For Worked example 3.

The unknown force X is tending to turn the beam anticlockwise. The other two forces (10 N and 20 N) are tending to turn the beam clockwise. We will start by calculating their moments and adding them together.

Step 1 Determine the clockwise moments:

$$\begin{aligned}\text{sum of moments of clockwise forces} &= (10 \times 1.0) + (20 \times 0.5)\\ &= 10 + 10\\ &= 20\text{ N m}\end{aligned}$$

Step 2 Determine the anticlockwise moment:

moment of anticlockwise force = X × 0.8

Step 3 Since we know that the beam must be balanced, we can write:

sum of clockwise moments = sum of anticlockwise moments

$$\begin{aligned}20 &= X \times 0.8\\ X &= \tfrac{20}{0.8}\\ &= 25\text{ N}\end{aligned}$$

So a force of 25 N at a distance of 0.8 m from the pivot will keep the beam still and prevent it from rotating (keep it balanced).

4 Figure 4.20 shows the internal structure of a human arm holding an object. The biceps is a muscle attached to one of the bones of the forearm. This muscle provides an upwards force.

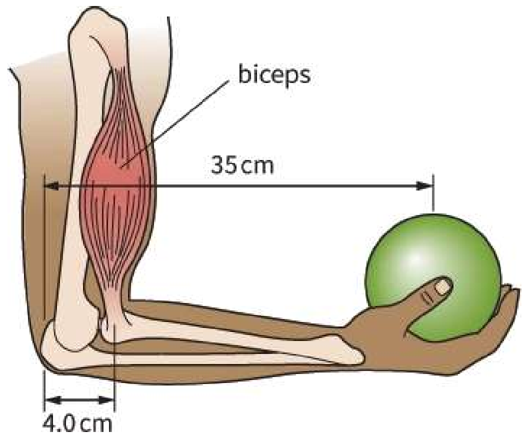

Figure 4.20: The human arm. For Worked example 4.

An object of weight 50 N is held in the hand with the forearm at right angles to the upper arm. Use the principle of moments to determine the muscular force F provided by the biceps, given the following data:

weight of forearm = 15 N

distance of biceps from elbow = 4.0 cm

distance of centre of gravity of forearm from elbow = 16 cm

distance of object in the hand from elbow = 35 cm

Step 1 There is a lot of information in this question. It is best to draw a simplified diagram of the forearm that shows all the forces and the relevant distances (Figure 4.21). All distances must be from the pivot, which in this case is the elbow.

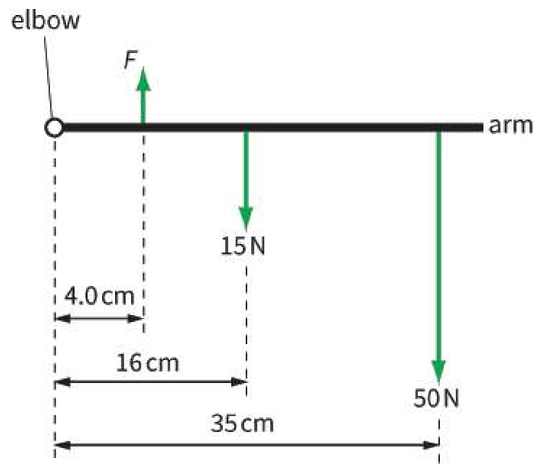

Figure 4.21: Simplified diagram showing forces on the forearm. For Worked example 4. Note that another force acts on the arm at the elbow; we do not know the size or direction of this force but we can ignore it by taking moments about the elbow.

Step 2 Determine the clockwise moments:

$$\begin{aligned} \text{sum of moments of clockwise forces} &= (15 + 0.16) + (50 \times 0.35) \\ &= 2.4 + 17.5 \\ &= 19.9 \text{ N m} \end{aligned}$$

Step 3 Determine the anticlockwise moment:

moment of anticlockwise force = $F \times 0.04$

Step 4 Since the arm is in balance, according to the principle of moments we have:

sum of clockwise moments = sum of anticlockwise moments

$$\begin{aligned} 19.9 &= 0.04F \\ F &= \tfrac{19.9}{0.04} \\ &= 497.5 \text{ N} \approx 500 \text{ N} \end{aligned}$$

The biceps provides a force of 500 N–a force large enough to lift 500 apples!

Questions

7 A wheelbarrow is loaded as shown in Figure 4.22.

a Calculate the force that the person needs to exert to hold the wheelbarrow's legs off the ground.

b Calculate the force exerted by the ground on the legs of the wheelbarrow (taken both together) when the gardener is not holding the handles.

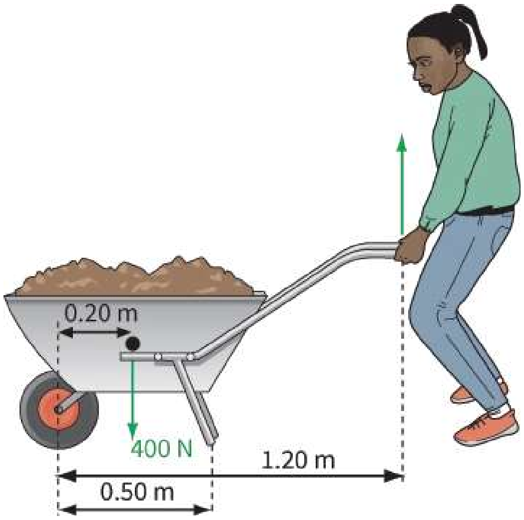

Figure 4.22: For Question 7.

8 A traditional pair of scales uses sliding masses of 10 g and 100 g to achieve a balance. A diagram of the arrangement is shown in Figure 4.23. The bar itself is supported with its centre of gravity at the pivot.

a Calculate the value of the mass *M*, attached at X.

b State **one** advantage of this method of measuring mass.

c Determine the upwards force of the pivot on the bar.

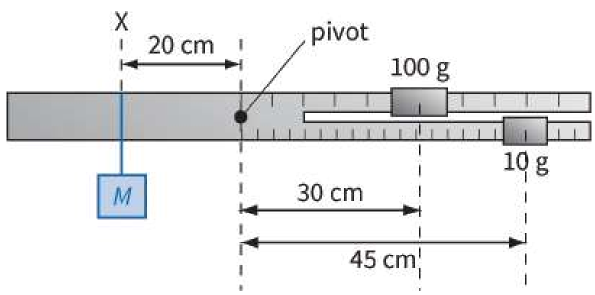

Figure 4.23: For Question 8.

9 Figure 4.24 shows a beam with four forces acting on it.

a For each force, calculate the moment of the force about point P.

b State whether each moment is clockwise or anticlockwise.

c State whether or not the moments of the forces are balanced.

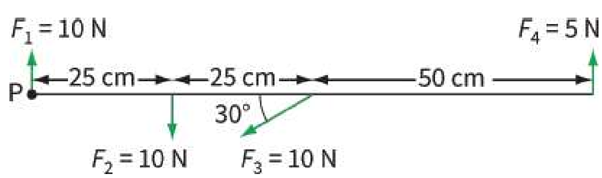

Figure 4.24: For Question 9.

4.5 The torque of a couple

Figure 4.25 shows the forces needed to turn a car's steering wheel. The two forces balance up and down (15 N up and 15 N down), so the wheel will not move up, down or sideways. However, the wheel is not in equilibrium. The pair of forces will cause it to rotate.

Figure 4.25: Two forces act on this steering wheel to make it turn.

A pair of forces like that in Figure 4.25 is known as a **couple**.

A couple has a turning effect, but does not cause an object to accelerate. To form a couple, the two forces must be:

- equal in magnitude
- parallel, but opposite in direction
- separated by a distance d.

The turning effect or moment of a couple is known as its **torque**.

We can calculate the torque of the couple in Figure 4.25 by adding the moments of each force about the centre of the wheel:

$$\begin{aligned} \text{torque of couple} &= (15 \times 0.20) + (15 \times 0.20) \\ &= 6.0 \text{ N m} \end{aligned}$$

We could have found the same result by multiplying one of the forces by the perpendicular distance between them:

$$\text{torque of a couple} = 15 \times 0.4 = 6.0 \text{ N m}$$

The torque of a couple is defined as follows:

$$\text{torque of a couple} = \text{one of the forces} \times \text{perpendicular distance between the forces}$$

Question

10 The driving wheel of a car travelling at a constant velocity has a torque of 137 N m applied to it by the axle that drives the car (Figure 4.26). The radius of the tyre is 0.18 m. Calculate the driving force provided by this wheel.

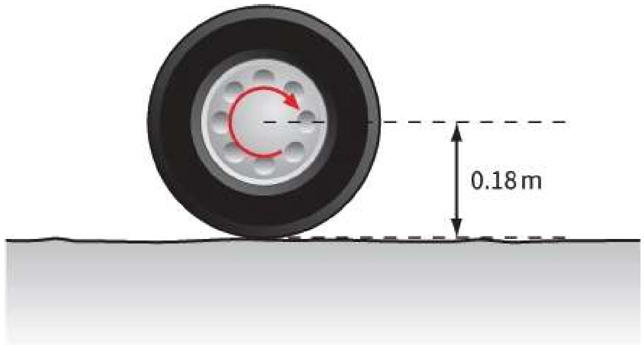

Figure 4.26: For Question 10.

Pure turning effect

When we calculate the moment of a single force, the result depends on the point or pivot about which the moment acts. The further the force is from the pivot, the greater the moment. A couple is different; the moment of a couple does not depend on the point about which it acts, only on the perpendicular distance between the two forces. A single force acting on an object will tend to make the object accelerate (unless there is another force to balance it). A couple, however, is a pair of equal and opposite forces, so it will not make the object accelerate. This means we can think of a couple as a pure ‘turning effect’, the size of which is given by its torque.

For an object to be in equilibrium, two conditions must be met at the same time:

- The resultant force acting on the object is zero.
- The resultant moment is zero.

KEY IDEA

If a body is in equilibrium, there is no resultant force and no resultant torque or moment about any point.

REFLECTION

Are there any things that you need more help with to fully understand vectors and moments?

Work out a simple way for yourself to remember which component is which. Check your method by explaining it to someone else with lots of examples.

SUMMARY

Forces are vector quantities that can be added by means of a vector triangle. Their resultant can be determined using trigonometry or by scale drawing.

Forces can be resolved into components. Components at right angles to one another can be treated independently of one another. For a force F at an angle θ to the x-direction, the components are:

$$x\text{-direction: } F\cos\theta$$

$$y\text{-direction: } F\sin\theta$$

The moment of a force = force × perpendicular distance of the pivot from the line of action of the force.

The principle of moments states that, for any object in equilibrium, the sum of the clockwise moments about any point provided by the forces acting on the object is equal to the sum of the anticlockwise moments about that same point.

A couple is a pair of equal, parallel but opposite forces whose effect is to produce a turning effect on a body without giving it linear acceleration.

torque of a couple = one of the forces × perpendicular distance between the forces

For an object in equilibrium, the resultant force acting on the object must be zero and the resultant moment must be zero.

EXAM-STYLE QUESTIONS

1 A force F is applied at a distance d from the hinge H and an angle x to the door.

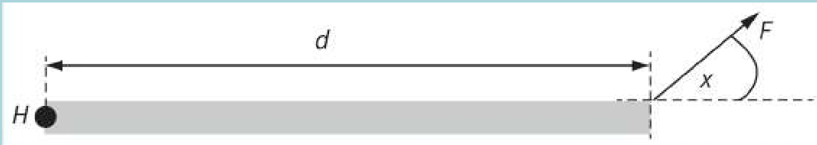

Figure 4.27

What is the moment of the force F about the point H? **[1]**

A $Fd \cos x$

B $\frac{Fd}{\cos x}$

C $Fd \sin x$

D $\frac{Fd}{\sin x}$

2 The angle between two forces, each of magnitude 5.0 N, is 120°.

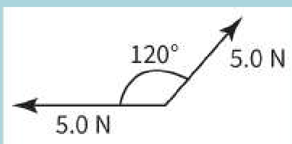

Figure 4.28

What is the magnitude of the resultant of these two forces? **[1]**

A 1.7 N

B 5.0 N

C 8.5 N

D 10 N

3 A ship is pulled at a constant speed by two small boats, A and B, as shown. The engine of the ship does not produce any force.

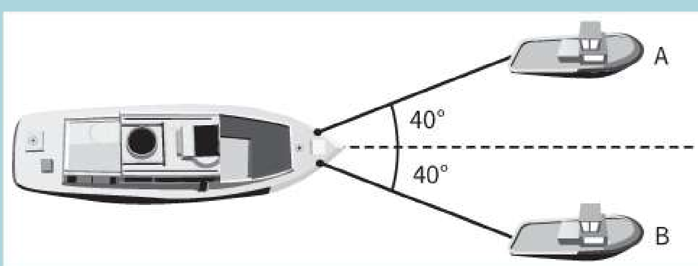

Figure 4.29

The tension in each cable between A and B and the ship is 4000 N.

a Draw a free-body diagram showing the three horizontal forces acting on the ship. **[2]**

b Draw a vector diagram to scale showing these three forces and use your diagram to find the value of the drag force on the ship. **[2]**

[Total: 4]

4 A block of mass 1.5 kg is at rest on a rough surface which is inclined at 20° to the horizontal as shown.

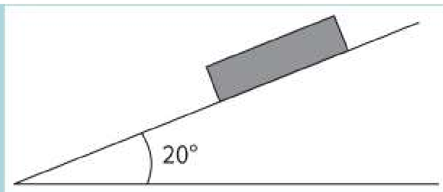

Figure 4.30

a Draw a free-body diagram showing the three forces acting on the block. **[2]**

b Calculate the component of the weight that acts down the slope. **[2]**

c Use your answer to part **b** to determine the force of friction that acts on the block. **[2]**

d If the angle of the surface is actually measured as 19° and 21° determine the absolute uncertainty in this angle and the uncertainty this produces in the value for part **b**. **[3]**

e Determine the normal contact force between the block and the surface. **[3]**

[Total: 12]

5 This free-body diagram shows three forces that act on a stone hanging at rest from two strings.

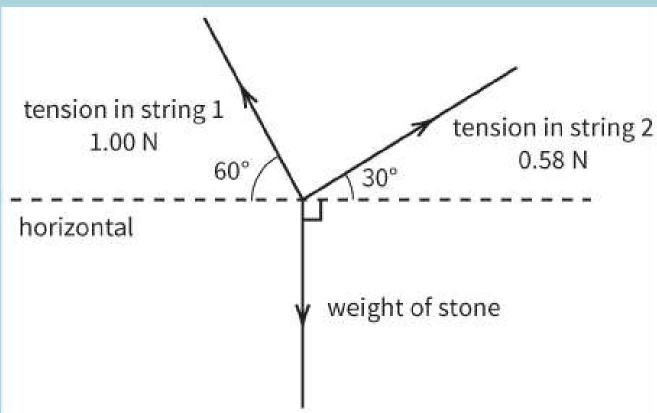

Figure 4.31

a Calculate the horizontal component of the tension in each string. State why these two components are equal in magnitude? **[5]**

b Calculate the vertical component of the tension in each string. **[4]**

c Use your answer to part **b** to calculate the weight of the stone. **[2]**

d Draw a vector diagram of the forces on the stone. This should be a triangle of forces. **[1]**

e Use your diagram in part **d** to calculate the weight of the stone. **[2]**

[Total: 14]

6 The force F shown here has a moment of 40 N m about the pivot. Calculate the magnitude of the force F. **[4]**

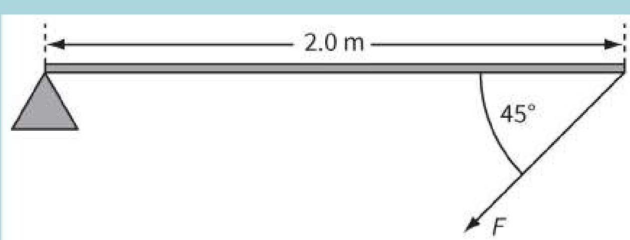

Figure 4.32

7 The asymmetric bar shown has a weight of 7.6 N and a centre of gravity that is 0.040 m from the wider end, on which there is a load of 3.3 N. It is pivoted a distance of 0.060 m from its centre of gravity. Calculate the force *P* that is needed at the far end of the bar in order to maintain equilibrium. **[4]**

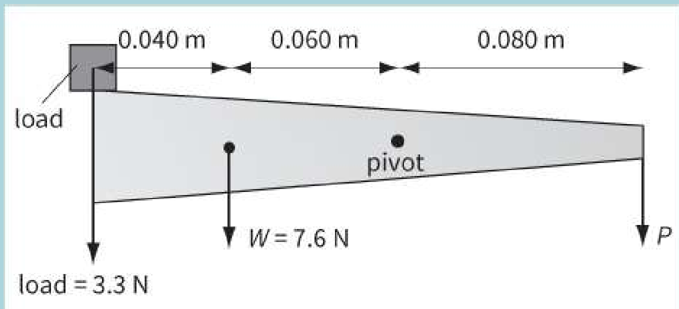

Figure 4.33

8 **a** State what is meant by:

i a couple **[1]**

ii torque. **[2]**

b The engine of a car produces a torque of 200 N m on the axle of the wheel in contact with the road. The car travels at a constant velocity towards the right:

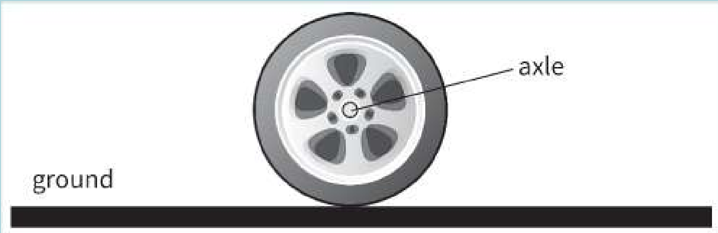

Figure 4.34

i Copy the diagram of the wheel and show the direction of rotation of the wheel, and the horizontal component of the force that the road exerts on the wheel. **[2]**

ii State the resultant torque on the wheel. Explain your answer. **[2]**

iii The diameter of the car wheel is 0.58 m. Determine the value of the horizontal component of the force of the road on the wheel. **[1]**

[Total: 8]

9 **a** Explain what is meant by the centre of gravity of an object. **[2]**

b A flagpole of mass 25 kg is held in a horizontal position by a cable as shown. The centre of gravity of the flagpole is at a distance of 1.5 m from the fixed end.

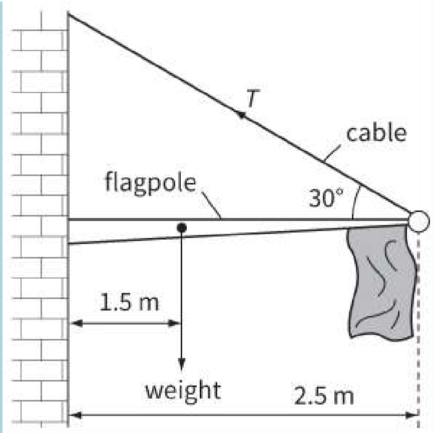

Figure 4.35

i Write an equation to represent taking moments about the left-hand end of the flagpole. Use your equation to find the tension T in the cable. **[4]**

ii Determine the vertical component of the force at the left-hand end of the flagpole. **[2]**

[Total: 8]

10 a State the two conditions necessary for an object to be in equilibrium. **[2]**

b A metal rod of length 90 cm has a disc of radius 24 cm fixed rigidly at its centre, as shown. The assembly is pivoted at its centre.

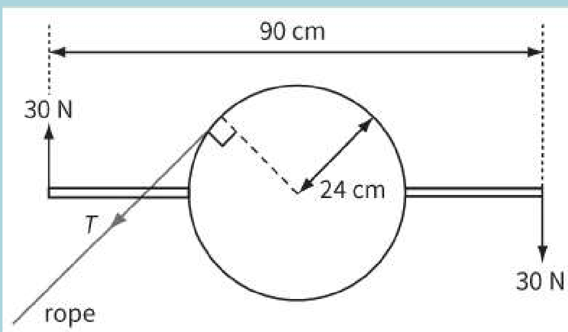

Figure 4.36

Two forces, each of magnitude 30 N, are applied normal to the rod at each end so as to produce a turning effect on the rod. A rope is attached to the edge of the disc to prevent rotation.

Calculate:

i the torque of the couple produced by the 30 N forces **[1]**

ii the tension T in the rope. **[3]**

[Total: 6]

11 a State what is meant by the torque of a couple. **[2]**

b Three strings, A, B and C, are attached to a circular ring, as shown in Figure 4.35.

The strings and the ring all lie on a smooth horizontal surface and are at rest. The tension in string A is 8.0 N. Calculate the tension in strings B and C. **[4]**

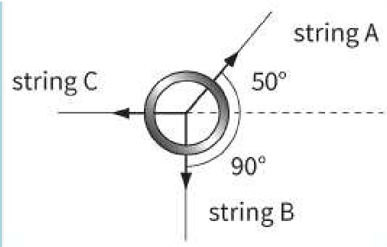

Figure 4.37

[Total: 6]

12 This diagram shows a picture hanging symmetrically by two cords from a nail fixed to a wall. The picture is in equilibrium.

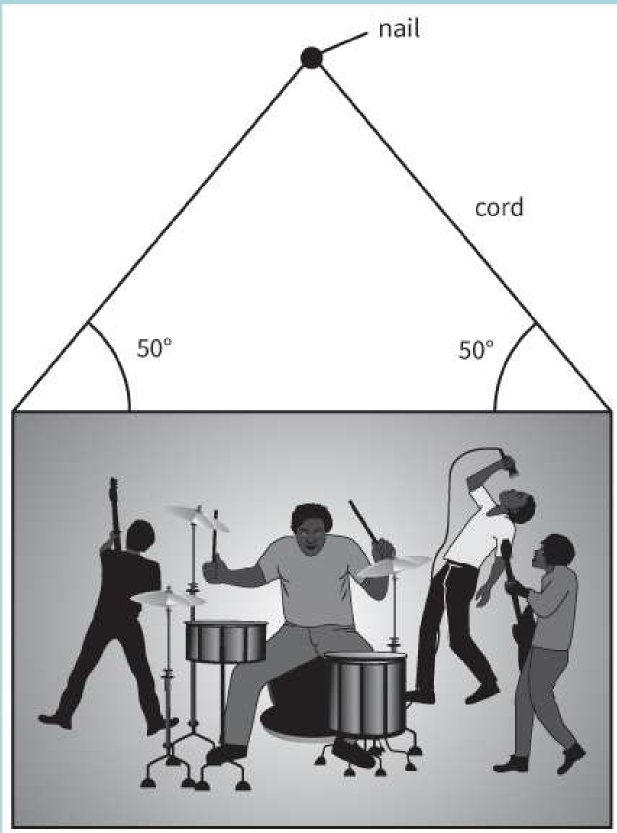

Figure 4.38

a Explain what is meant by equilibrium. **[2]**

b Draw a vector diagram to represent the three forces acting on the picture in the vertical plane. Label each force clearly with its name and show the direction of each force with an arrow. **[2]**

c The tension in the cord is 45 N and the angle that each end of the cord makes with the horizontal is 50°. Calculate:

i the vertical component of the tension in the cord **[1]**

ii the weight of the picture. **[1]**

[Total: 6]