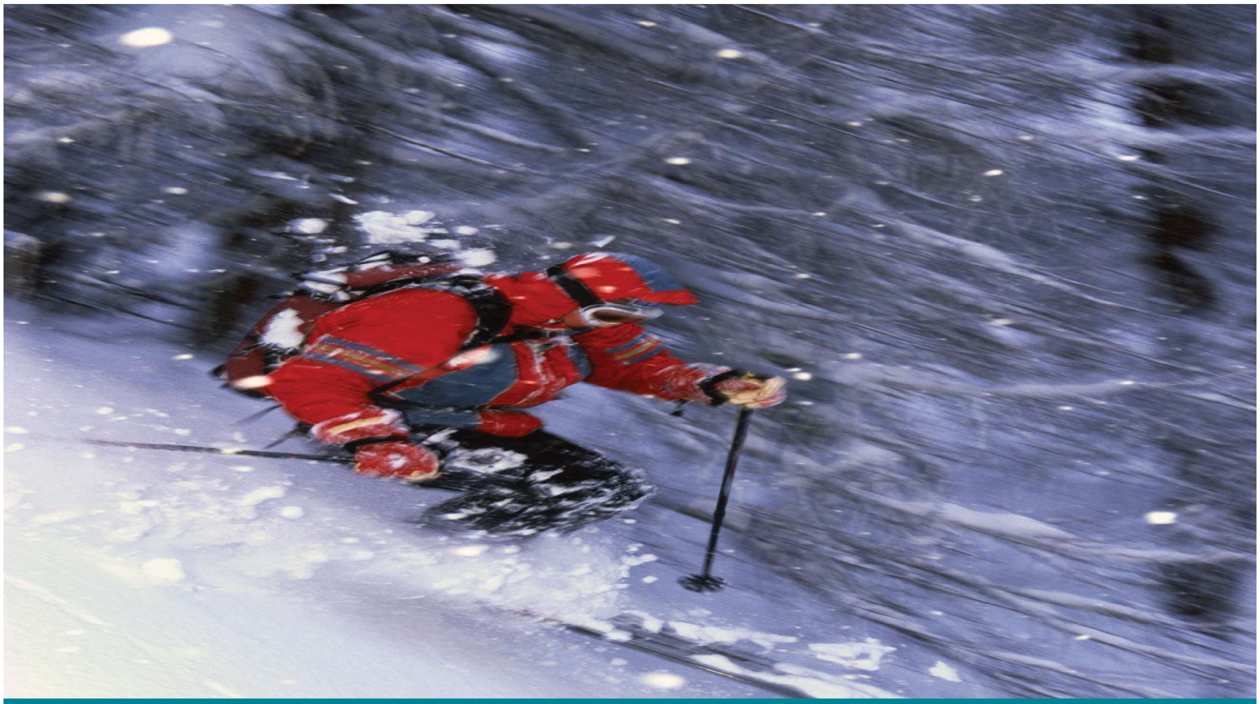


Chapter 3

Dynamics: explaining motion

LEARNING INTENTIONS

In this chapter you will learn how to:

- recognise that mass is a property of an object that resists change in motion
- identify the forces acting on a body in different situations
- describe how the motion of a body is affected by the forces acting on it
- recall $F = ma$ and solve problems using it, understanding that acceleration and resultant force are always in the same direction
- state and apply Newton's first and third laws of motion
- recall that the weight of a body is equal to the product of its mass and the acceleration of free fall
- relate derived units to base units in the SI system and use base units to check the homogeneity of an equation
- recall and use a range of prefixes.

BEFORE YOU START

- Make a list of all the different types of force that you know about. Do you have the same list as someone else? Discuss any differences and describe the types of force to each other.
- What prefixes do you know that may be placed before a unit? For example, the 'c' in cm is the prefix 'centi' and means times 10^{-2}. Write down those that you know and what they mean then see if you are correct.

DYNAMIC AEROPLANES

Figure 3.1 shows a modern aeroplane. To decrease cost and the effect on the environment, such an aircraft must *reduce* air resistance and weight, yet be able to *use* air resistance and other forces to stop when landing. If you have ever flown in an aeroplane you will know how the back of the seat pushes you forwards when the aeroplane accelerates down the runway. The pilot must control many forces on the aeroplane in take-off, flying and landing.

In Chapters 1 and 2 we saw how motion can be described in terms of displacement, velocity, acceleration and so on. Now we are going to look at how we can explain how an object moves in terms of the forces that change its motion.

Apart from air resistance, see how many other forces you can discover that act on an aeroplane. Compare your list with someone else. What causes all these forces?

Figure 3.1: A modern aircraft flying over the ocean.

3.1 Force, mass and acceleration

Figure 3.2a shows how we represent the force that the motors on a train provide to cause it to accelerate. The resultant force is represented by a green arrow. The direction of the arrow shows the direction of the resultant force. The magnitude (size) of the resultant force of 20 000 N is also shown.

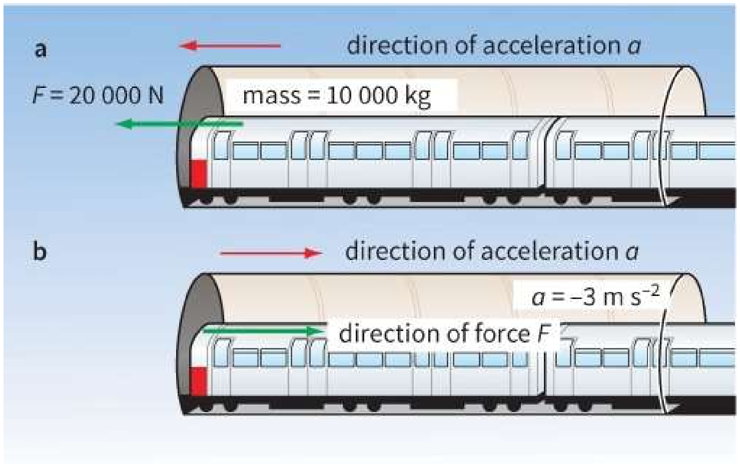


Figure 3.2: A force is needed to make the train *a* accelerate, and *b* decelerate.

To calculate the acceleration *a* of the train produced by the resultant force *F*, we must also know the train's mass *m* (Table 3.1). These quantities are related by:

$$a = \frac{F}{m} \text{ or } F = ma$$

KEY EQUATION

$$\text{resultant force} = \text{mass} \times \text{acceleration}$$
$$F = ma$$

Quantity	Symbol	Unit
resultant force	F	N (newtons)
mass	m	kg (kilograms)
acceleration	a	m s^{-2} (metres per second squared)

Table 3.1: The quantities related by $F = ma$.

In this example, we have F = 20 000 N and m = 10 000 kg, and so:

$$a = \frac{F}{m} = \frac{20\,000}{10\,000} = 2 \text{ m s}^{-2}$$

In Figure 3.2b, the train is decelerating as it comes into a station. Its acceleration is −3.0 m s^{-2}. What force must be provided by the braking system of the train?

$$F = ma = 10\,000 \times -3 = -30\,000 \text{ N}$$

The minus sign shows that the force must act towards the right in the diagram, in the opposite direction to the motion of the train.

Newton's second law of motion

The equation we used, $F = ma$, is a simplified version of **Newton's second law of motion**: For a body of constant mass, its acceleration is directly proportional to the resultant force applied to it.

An alternative form of Newton's second law is given in Chapter 6, when you have studied momentum.

Since Newton's second law holds for objects that have a constant mass, this equation can be applied to a train whose mass remains constant during its journey.

The equation $a = \frac{F}{m}$ relates acceleration, resultant force and mass. In particular, it shows that the bigger the force, the greater the acceleration it produces. You will probably feel that this is an unsurprising result. For a given object, the acceleration is directly proportional to the resultant force:

$$a \propto F$$

The equation also shows that the acceleration produced by a force depends on the mass of the object. The **mass** of an object is a measure of its **inertia**, or its ability to resist any change in its motion. The greater the mass, the smaller the acceleration that results. If you push your hardest against a small car (which has a small mass), you will have a greater effect than if you push against a more massive car (Figure 3.3). So, for a constant force, the acceleration is inversely proportional to the mass:

$$a \propto \frac{1}{m}$$

The train driver knows that when the train is full during the rush hour, it has a smaller acceleration. This is because its mass is greater when it is full of people. Similarly, it is more difficult to stop the train once it is moving. The brakes must be applied earlier to avoid the train overshooting the platform at the station.

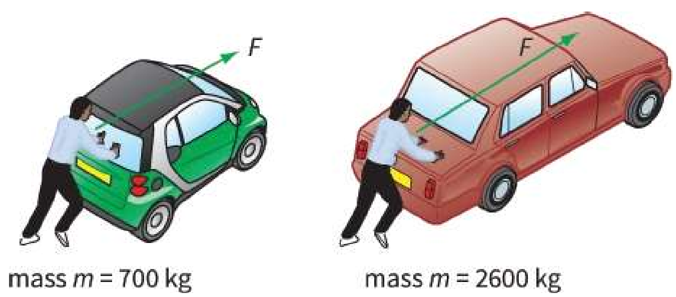


Figure 3.3: It is easier to make a small mass accelerate than a large mass.

WORKED EXAMPLES

1 A cyclist of mass 60 kg rides a bicycle of mass 20 kg. When starting off, the cyclist provides a force of 200 N. Calculate the initial acceleration.

Step 1 This is a straightforward example. First, we must calculate the combined mass m of the bicycle and its rider:

$m = 20 + 60 = 80$ kg

We are given the force F:

force causing acceleration $F = 200$ N

Step 2 Substituting these values gives:

$$a = \frac{F}{m} = \frac{200}{80} = 2.5 \text{ m s}^{-2}$$

So the cyclist's acceleration is 2.5 m s^{-2}.

2 A car of mass 500 kg is travelling at 20 m s^{-1}. The driver sees a red traffic light ahead, and slows to a halt in 10 s. Calculate the braking force provided by the car.

Step 1 In this example, we must first calculate the acceleration required. The car's final velocity is 0 m s^{-1}, so its change in velocity $\Delta v = 0 - 20 = -20 \text{ m s}^{-1}$

$$\text{acceleration } a = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta v}{\Delta t} = \frac{-20}{10} = -2\text{ m s}^{-2}$$

Step 2 To calculate the force, we use:

$F = ma = 500 \times -2 = -1000$ N

So the brakes must provide a force of 1000 N. (The minus sign shows a force decreasing the velocity of the car.)

Questions

1 Calculate the force needed to give a car of mass 800 kg an acceleration of 2.0 m s^{-2}.

2 A rocket has a mass of 5000 kg. At a particular instant, the resultant force acting on the rocket is 200 000 N. Calculate its acceleration.

3 (In this question, you will need to make use of the equations of motion that you studied in Chapter 2.) A motorcyclist of mass 60 kg rides a bike of mass 40 kg. As she sets off from the lights, the forward force on the bike is 200 N. Assuming the resultant force on the bike remains constant, calculate the bike's velocity after 5.0 s.

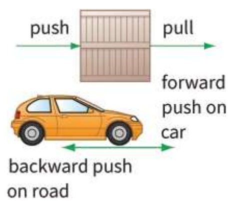
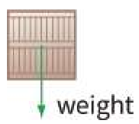
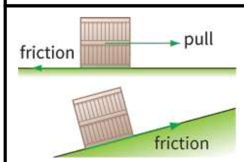
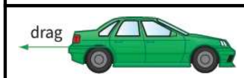
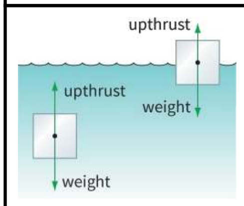
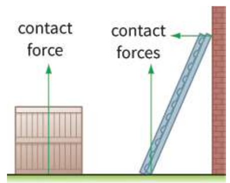
3.2 Identifying forces

It is important to be able to identify the forces which act on an object. When we know what forces are acting, we can predict how the object will move. Table 3.2 shows some important forces, how they arise and how we represent them in diagrams.

3.3 Weight, friction and gravity

Now we need to consider some specific forces – such as **weight** and **friction**.

When Isaac Newton was confined to his rural home to avoid the plague which was spreading uncontrollably in other parts of England, he is said to have noticed an apple fall to the ground. From this, he developed his theory of gravity that relates the motion of falling objects here on Earth to the motion of the Moon around the Earth, and the planets around the Sun.

Diagram	Force	Important situations
	**Pushes and pulls.** You can make an object accelerate by pushing and pulling it. Your force is shown by an arrow pushing (or pulling) the object. The engine of a car provides a force to push backwards on the road. Frictional forces from the road on the tyre push the car forwards.	• pushing and pulling • lifting • force of car engine • attraction and repulsion by magnets and by electric charges
	**Weight.** This is the force of gravity acting on the object. It is usually shown by an arrow pointing vertically downwards from the object's centre of gravity.	• any object in a gravitational field • less on the Moon
	**Friction.** This is the force that arises when two surfaces rub over one another. If an object is sliding along the ground, friction acts in the opposite direction to its motion. If an object is stationary, but tending to slide – perhaps because it is on a slope – the force of friction acts up the slope to stop it from sliding down. Friction always acts along a surface, never at an angle to it.	• pulling an object along the ground • vehicles cornering or skidding • sliding down a slope
	**Drag.** This force is similar to friction. When an object moves through air, there is friction between it and the air. Also, the object has to push aside the air as it moves along. Together, these effects make up drag. Similarly, when an object moves through a liquid, it experiences a drag force. Drag acts to oppose the motion of an object; it acts in the opposite direction to the object's velocity. It can be reduced by giving the object a streamlined shape.	• vehicles moving • aircraft flying • parachuting • objects falling through air or water • ships sailing
	**Upthrust.** Any object placed in a fluid such as water or air experiences an upwards force. This is what makes it possible for something to float in water. Upthrust arises from the pressure that a fluid exerts on an object. The deeper you go, the greater the pressure. So there is more pressure on the lower surface of an object than on the upper surface, and this tends to push it upwards. If upthrust is greater than the object's weight, it will float up to the surface.	• boats and icebergs floating • people swimming • divers surfacing • a hot air balloon rising
	**Contact force.** When you stand on the floor or sit on a chair, there is usually a force that pushes up against your weight, and which supports you so that you do not fall down. The contact force is sometimes known as the normal contact force of the floor or chair. (In this context, normal means 'perpendicular'.)	• standing on the ground • one object sitting on top of another • leaning

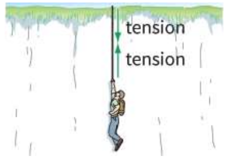
	The contact force always acts at right angles to the surface that produces it. The floor pushes straight upwards; if you lean against a wall, it pushes back against you horizontally.	against a wall • one object bouncing off another
	**Tension.** This is the force in a rope or string when it is stretched. If you pull on the ends of a string, it tends to stretch. The tension in the string pulls back against you. It tries to shorten the string. Tension can also act in springs. If you stretch a spring, the tension pulls back to try to shorten the spring. If you squash (compress) the spring, the tension acts to expand the spring.	• pulling with a rope • squashing or stretching a spring

Table 3.2: Some important forces.

The force that caused the apple to accelerate was the pull of the Earth's gravity. Another name for this force is the **weight** of the apple. The force is shown as an arrow, pulling vertically downwards on the apple (Figure 3.4). It is usual to show the arrow coming from the centre of the apple – its **centre of gravity**. The centre of gravity of an object is defined as the point where its entire weight appears to act.

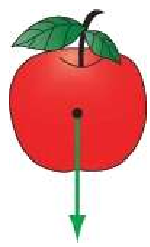


Figure 3.4: The weight of an object is a force caused by the Earth's gravity. It acts vertically down on the object.

Large and small

A large rock has a greater weight than a small rock, but if you push both rocks over a cliff at the same time, they will fall at the same rate. In other words, they have the **same** acceleration, regardless of their mass. This is a surprising result. Common sense may suggest that a heavier object will fall faster than a lighter one. It is said that Galileo dropped a large cannon ball and a small cannon ball from the top of the Leaning Tower of Pisa in Italy, and showed that they landed at the same time. The story illustrates that results are not always what you think they will be – if everyone thought that the two cannon balls would accelerate at the same rate, there would not have been any experiment or story.

In fact, we are used to lighter objects falling more slowly than heavy ones. A feather drifts down to the floor, while a stone falls quickly. But this is because of air resistance. The force of air resistance has a large effect on the falling feather, and almost no effect on the falling stone. When astronauts visited the Moon (where there is virtually no atmosphere and so no air resistance), they were able to show that a feather and a stone fell side-by-side to the ground.

As we saw in Chapter 2, an object falling freely close to the Earth's surface has an acceleration of roughly $9.81\ \mathrm{m\ s^{-2}}$, the acceleration of free fall g.

We can find the force causing this acceleration using $F = ma$. This force is the object's weight. Hence, the weight W of an object is given by:

$$\text{weight} = \text{mass} \times \text{acceleration of free fall}$$

or

$$W = mg$$

KEY EQUATION

$$\begin{aligned} \text{weight} &= \text{mass} \times \text{acceleration of free fall} \\ W &= mg \end{aligned}$$

Question

4 Estimate the mass and weight of each of the following at the surface of the Earth:

a a kilogram of potatoes

b an average student

c a mouse

d a 40-tonne truck.

(For estimates, use $g = 10\ \text{m s}^{-2}$; 1 tonne = 1000 kg.)

On the Moon

The Moon is smaller and has less mass than the Earth, and so its gravity is weaker. If you were to drop a stone on the Moon, it would have a smaller acceleration. Your hand is about 1 m above ground level; a stone takes about 0.45 s to fall through this distance on the Earth, but about 1.1 s on the surface of the Moon. The acceleration of free fall on the Moon is about one-sixth of that on the Earth:

$$g_{\text{Moon}} = 1.6\ \text{m s}^{-2}$$

It follows that objects weigh less on the Moon than on the Earth. They are not completely weightless, because the Moon's gravity is not zero.

Mass and weight

We have now considered two related quantities, mass and weight. It is important to distinguish carefully between these (Table 3.3).

Quantity	Symbol	Unit	In terms of base units	Comment
mass	m	kg	kg	this does not vary from place to place
weight	mg	N	kg m s^{-2}	this is a force – it depends on the strength of gravity

Table 3.3: Distinguishing between mass and weight.

Figure 3.5 shows a vehicle used to travel on the moon, named a moon-buggy. If the moon-buggy breaks down, it will be no easier to get it moving on the Moon than on the Earth. This is because its mass does not change, because it is made from the same atoms and molecules wherever it is. From $F = ma$, it follows that if m does not change, you will need the same force F to start it moving.

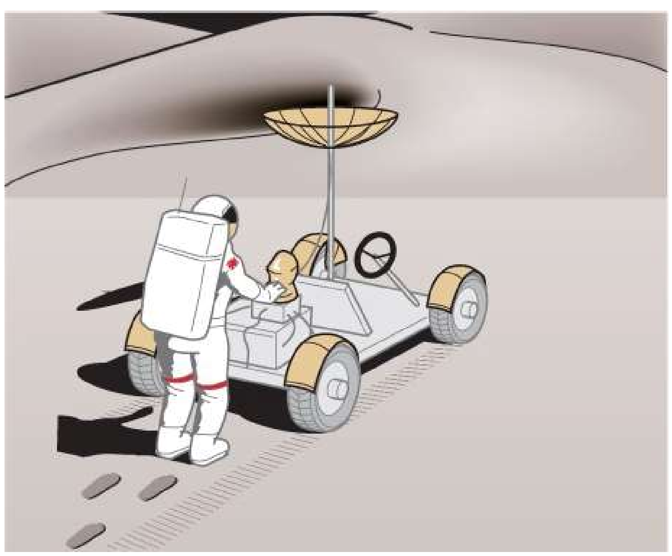


Figure 3.5: The mass of a moon-buggy is the same on the Moon as on the Earth, but its weight is smaller.

However, your moon-buggy will be easier to lift on the Moon, because its weight will be less. From $W = mg$, since g is less on the Moon, it has a smaller weight than when on the Earth.

3.4 Mass and inertia

It took a long time for scientists to develop correct ideas about forces and motion. We will start by thinking about some wrong ideas, and then consider why Galileo, Newton and others decided new ideas were needed.

Observations and ideas

Here are some observations to think about.

- The large tree trunk shown in Figure 3.6 is being pulled from a forest. The elephant provides the
- force needed to pull it along. If the elephant stops pulling, the tree trunk will stop moving.
- A horse is pulling a cart. If the horse stops pulling, the cart stops.
- You are riding a bicycle. If you stop pedaling, the bicycle will come to a halt.
- You are driving along the road. You must keep your foot on the accelerator pedal, otherwise the car will not keep moving.
- You kick a football. The ball rolls along the ground and gradually stops.

In each of these cases, there is a force that makes something move – the pull of the elephant or the horse, your push on the bicycle pedals, the force of the car engine, the push of your foot. Without the force, the moving object comes to a halt. So what conclusion might we draw?

A moving object needs a force to keep it moving.

This might seem a sensible conclusion to draw, but it is wrong. We have not thought about all the forces involved. The missing force is friction.

In each example, friction (or air resistance) makes the object slow down and stop when there is no force pushing or pulling it forwards. For example, if you stop pedaling your cycle, air resistance will slow you down. There is also friction at the axles of the wheels, and this too will slow you down. If you could lubricate your axles and cycle in a vacuum, you could travel along at a steady speed forever, without pedaling!

Figure 3.6: An elephant provides the force needed to pull this tree from the forest.

In the 17th century, astronomers began to use telescopes to observe the night sky. They saw that objects such as the planets could move freely through space. They simply kept on moving, without anything providing a force to push them. Galileo came to the conclusion that this was the natural motion of objects.

- An object at rest will stay at rest, unless a force causes it to start moving.
- A moving object will continue to move at a steady speed in a straight line, unless a force acts on it.

So objects move with a constant velocity, unless a force acts on them. (Being stationary is simply a particular case of this, where the velocity is zero.) Nowadays, it is much easier to appreciate this law of motion, because we have more experience of objects moving with little or no friction such as roller-skates with low-friction bearings, ice skates and spacecraft in empty space. In Galileo's day, people's everyday experience was of dragging things along the ground, or pulling things on carts with high-friction axles. Before Galileo, the orthodox scientific idea was that a force must act all the time to keep an object moving

– this had been handed down from the time of the ancient Greek philosopher Aristotle. So it was a great achievement when scientists were able to develop a picture of a world without friction.

The idea of inertia

The tendency of a moving object to carry on moving is sometimes known as **inertia**.

- An object with a large mass is difficult to stop moving – think about catching a football, compared with a less massive tennis ball moving at the same speed.
- Similarly, a stationary object with a large mass is difficult to start moving – think about pushing a car to get it started.
- It is difficult to make a massive object change direction – think about the way a fully laden supermarket trolley tries to keep moving in a straight line.

All of these examples suggest another way to think of an object's mass; it is a measure of its inertia – how difficult it is to change the object's motion. **Uniform motion** is the natural state of motion of an object.

Here, uniform motion means 'moving with constant velocity' or 'moving at a steady speed in a straight line'.

Newton's first law of motion

The findings on inertia and uniform motion can be summarised as **Newton's first law of motion**:

In fact, this is already contained in the simple equation we have been using to calculate acceleration, $F = ma$. If no resultant force acts on an object ($F = 0$), it will not accelerate ($a = 0$). The object will either remain stationary or it will continue to travel at a constant velocity. If we rewrite the equation as $a = \frac{F}{m}$, we can see that the greater the mass m, the smaller the acceleration a produced by a force F.

Questions

5 Use the idea of inertia to explain why some large cars have power-assisted brakes.

6 A car crashes head-on into a brick wall. Use the idea of inertia to explain why the driver is more likely to come out through the windscreen if he or she is not wearing a seat belt.

Top speed

The vehicle shown in Figure 3.7 is capable of speeds as high as 760 mph, greater than the speed of sound. Its streamlined shape is designed to cut down air resistance and its jet engines provide a strong forwards force to accelerate it up to top speed.

All vehicles have a top speed. But why can't they go any faster? Why can't a car driver keep pressing on the accelerator pedal, and simply go faster and faster?

To answer this, we have to think about the two forces already mentioned: air resistance and the forwards thrust (force) of the engine. The vehicle will accelerate so long as the thrust is greater than the air resistance. When the two forces are equal, the resultant force on the vehicle is zero and the vehicle moves at a steady velocity.

Balanced and unbalanced forces

If an object has two or more forces acting on it, we have to consider whether or not they are 'balanced' (Figure 3.8). Forces on an object are balanced when the resultant force on the object is zero. The object will either remain at rest or have a constant velocity.

We can calculate the **resultant force** by adding up two (or more) forces that act in the same straight line. We must take account of the direction of each force. In the examples in Figure 3.8, forces to the right are positive and forces to the left are negative.

When a car travels slowly, it encounters little air resistance. However, the faster it goes, the more air it has to push out of the way each second and so the greater the air resistance. Eventually, the backwards force of air resistance equals the forwards force provided between the tyres and the road, and the forces on the car are balanced. It can go no faster–it has reached its top speed.

Free fall

Skydivers (Figure 3.9) are rather like cars–at first, they accelerate freely. At the start of the fall, the only force acting on the diver is his or her weight. The acceleration of the diver at the start must therefore be g. Then increasing air resistance opposes their fall and their acceleration decreases. Eventually, they reach a maximum velocity, known as the **terminal velocity**.

At the terminal velocity, the air resistance is equal to the weight. The terminal velocity is approximately 120 miles per hour (about 50 m s^{-1}), but it depends on the skydiver's weight and orientation. Head-first is fastest.

Figure 3.7: The Thrust SSC rocket car broke the world land-speed record in 1997. It achieved a top speed of 763 mph (just over 340 m s^{-1}) over a distance of 1 mile (1.6 km).

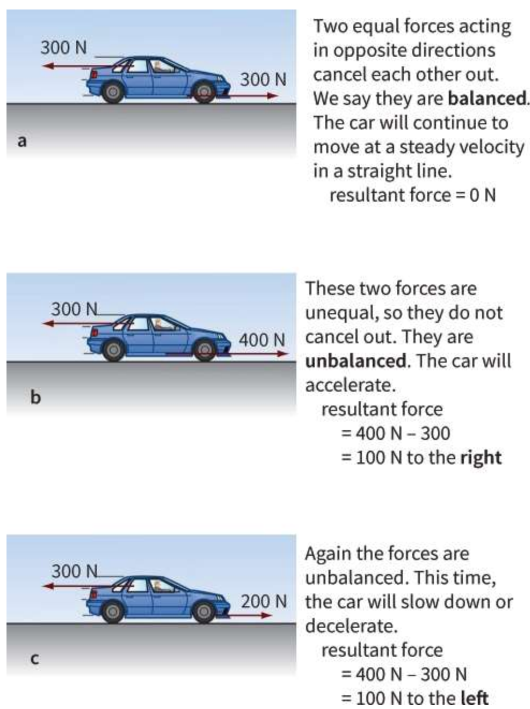


Figure 3.8: Balanced and unbalanced forces.

Figure 3.9: A skydiver falling freely.

The idea of a parachute is to greatly increase the air resistance. Then terminal velocity is reduced, and the parachutist can land safely. Figure 3.10 shows how a parachutist's velocity might change during descent.

Terminal velocity depends on the weight and surface area of the object. For insects, air resistance is much greater relative to their weight than for a human being and so their terminal velocity is quite low. Insects can be swept up several kilometres into the atmosphere by rising air streams. Later, they fall back to Earth uninjured. It is said that mice can survive a fall from a high building for the same reason.

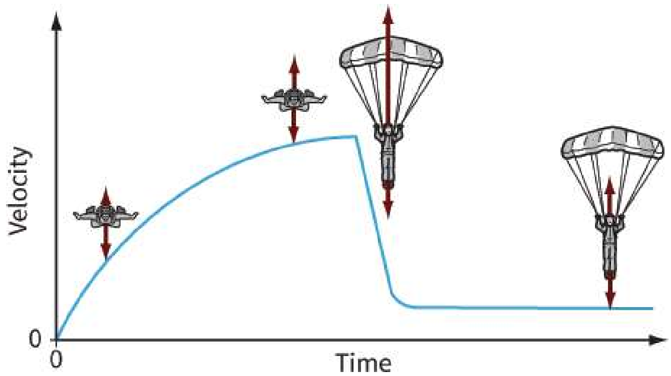


Figure 3.10: The velocity of a parachutist varies during a descent. The force arrows show weight (downwards) and air resistance (upwards).

3.5 Moving through fluids

Air resistance is just one example of the **resistive force** (or viscous force) that objects experience when they move through a fluid, a liquid or a gas. If you have ever run down the beach and into the sea, or tried to wade quickly through the water of a swimming pool, you will have experienced the force of **drag**. The deeper the water gets, the more it resists your movement and the harder you have to work to make progress through it. In deep water, it is easier to swim than to wade.

You can observe the effect of drag on a falling object if you drop a key or a coin into the deep end of a swimming pool. For the first few centimetres, it speeds up, but for the remainder of its fall, it has a steady speed. (If it fell through the same distance in air, it would accelerate all the way.) The drag of water means that the falling object reaches its terminal velocity very soon after it is released. Compare this with a skydiver, who has to fall hundreds of metres before reaching terminal velocity.

Moving through air

We rarely experience drag in air. This is because air is much less dense than water; its density is roughly that of water. At typical walking speed, we do not notice the effects of drag. However, if you want to move faster, the effects can be important. Racing cyclists, like the one shown in Figure 3.11, wear tight-fitting clothing and streamlined helmets.

Figure 3.11: A racing cyclist adopts a posture that helps to reduce drag. Clothing, helmet and even the cycle itself are designed to allow them to go as fast as possible.

Other athletes may take advantage of the drag of air. The runner in Figure 3.12 is undergoing resistance training. The parachute provides a backwards force against which his muscles must work. This should help to develop his muscles.

Figure 3.12: A runner making use of air resistance to build up his muscles.

WORKED EXAMPLES

3 A car of mass 500 kg is travelling along a flat road. The forward force provided between the car tyres and the road is 300 N and the air resistance is 200 N. Calculate the acceleration of the car.

Step 1 Start by drawing a diagram of the car, showing the forces mentioned in the question (Figure 3.13). Calculate the resultant force on the car; the force to the right is taken as positive:

resultant force = 300 − 200 = 100 N

Step 2 Now use $F = ma$ to calculate the car's acceleration:

$$a = \frac{F}{m} = \frac{100}{500} = 0.20\ \text{m s}^{-2}$$

So the car's acceleration is $0.20\ \text{m s}^{-2}$.

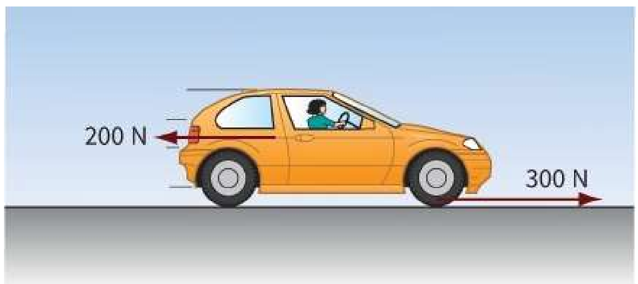


Figure 3.13: The forces on an accelerating car.

4 The maximum forward force a car can provide is 500 N. The air resistance F that the car experiences depends on its speed according to $F = 0.2v^2$, where v is the speed in m s^{-1}. Determine the top speed of the car.

Step 1 From the equation $F = 0.2v^2$, you can see that the air resistance increases as the car goes faster. Top speed is reached when the forward force equals the air resistance. So, at top speed:

$500 = 0.2v^2$

Step 2 Rearranging gives:

$$v^2 = \frac{500}{0.2} = 2500$$

$$v = \sqrt{2500} = 50\ \text{m s}^{-1}$$

So the car's top speed is $50\ \text{m s}^{-1}$ (this is about $180\ \text{km h}^{-1}$).

Questions

7 If you drop a large stone and a small stone from the top of a tall building, which one will reach the ground first? Explain your answer.

8 In a race, downhill skiers want to travel as quickly as possible. They are always looking for ways to increase their top speed. Explain how they might do this. Think about:

a their skis

b their clothing

c their muscles

d the slope.

9 Skydivers jump from a plane at intervals of a few seconds. If two divers wish to join up as they fall, the second must catch up with the first.

a If one diver is more massive than the other, who should jump first? Use the idea of forces and terminal velocity to explain your answer.

b If both divers are equally massive, suggest what the second might do to catch up with the first.

Contact forces and upthrust

We will now think about the forces that act when two objects are in contact with each other. When two objects touch each other, each exerts a force on the other. These are called **contact forces**. For example, when you stand on the floor (Figure 3.14), your feet push downwards on the floor and the floor pushes back upwards on your feet. This is a vital force – the upward push of the floor prevents you from falling downwards under the pull of your weight.

Where do these contact forces come from? When you stand on the floor, the floor becomes slightly compressed. Its atoms are pushed slightly closer together, and the interatomic forces push back against the compressing force. At the same time, the atoms in your feet are also pushed together so that they push back in the opposite direction. (It is hard to see the compression of the floor when you stand on it, but if you stand on a soft material such as foam rubber or a mattress you will be able to see the compression clearly.)

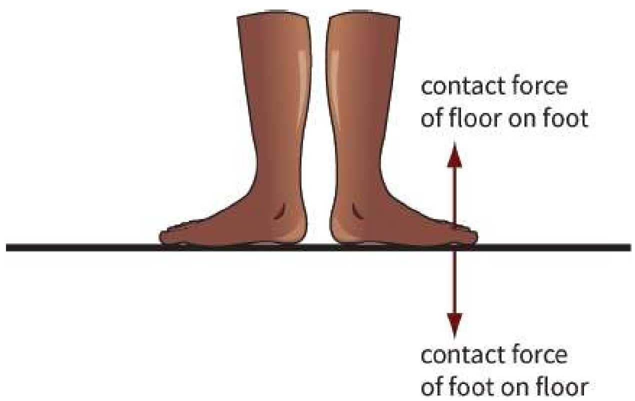


Figure 3.14: Equal and opposite contact forces act when you stand on the floor.

You can see from Figure 3.14 that the two contact forces act in opposite directions. They are also equal in magnitude. As we will see shortly, this is a consequence of Newton's third law of motion.

When an object is immersed in a fluid (a liquid or a gas), it experiences an upward force called **upthrust**. It is the upthrust of water that keeps a boat floating (Figure 3.15), and the upthrust of air that lifts a hot air balloon upwards.

The upthrust of water on a boat can be thought of as the contact force of the water on the boat. It is caused by the pressure of the water pushing upwards on the boat. Pressure arises from the motion of the water molecules colliding with the boat and the net effect of all these collisions is an upwards force.

An object in air, such as a ball, has a very small upthrust acting on it, because the density of the air around it is low. Molecules hit the top surface of the ball pushing down, but only a few more molecules push upwards on the bottom of the ball, so the resultant force upwards, or the upthrust, is low. If the ball is falling, air resistance is greater than this small upthrust but both these forces are acting upwards on the ball.

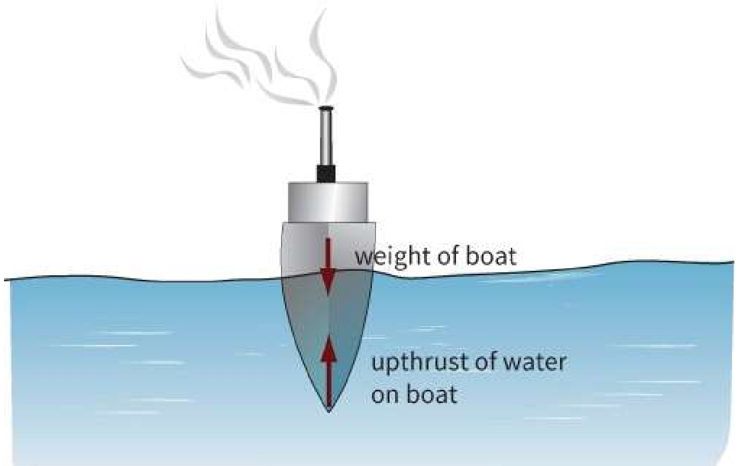


Figure 3.15: Without sufficient upthrust from the water, the boat would sink.

Questions

10 Name these forces:

- **a** the upward push of water on a submerged object
- **b** the force that wears away two surfaces as they move over one another
- **c** the force that pulled the apple off Isaac Newton's tree
- **d** the force that stops you falling through the floor
- **e** the force in a string that is holding up an apple
- **f** the force that makes it difficult to run through shallow water.

11 Draw a diagram to show the forces that act on a car as it travels along a level road at its top speed.

12 Imagine throwing a shuttlecock straight up in the air. Air resistance is more important for shuttlecocks than for a tennis ball. Air resistance always acts in the opposite direction to the velocity of an object.

Draw diagrams to show the two forces, weight and air resistance, acting on the shuttlecock:

- **a** as it moves upwards
- **b** as it falls back downwards.

3.6 Newton's third law of motion

For completeness, we should now consider **Newton's third law of motion**. (There is more about this in Chapter 6.)

When two objects interact, each exerts a force on the other. Newton's third law says that these forces are equal and opposite to each other:

When two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction.

(These two forces are sometimes described as action and reaction, but this is misleading as it sounds as though one force arises as a consequence of the other.
In fact, the two forces appear at the same time and we can't say that one caused the other.)

The two forces that make up a 'Newton's third law pair' have the following characteristics:

- They act on **different** objects.
- They are equal in magnitude.
- They are opposite in direction.
- They are forces **of the same type**.

What does it mean to say that the forces are 'of the same type'? We need to think about the type of interaction which causes the forces to appear.

- Two objects may attract each other because of the gravity of their masses – these are gravitational forces.
- Two objects may attract or repel because of their electrical charges – electrical forces.
- Two objects may touch – contact forces.
- Two objects may be attached by a string and pull on each other – tension forces.
- Two objects may attract or repel because of their magnetic fields – magnetic forces.

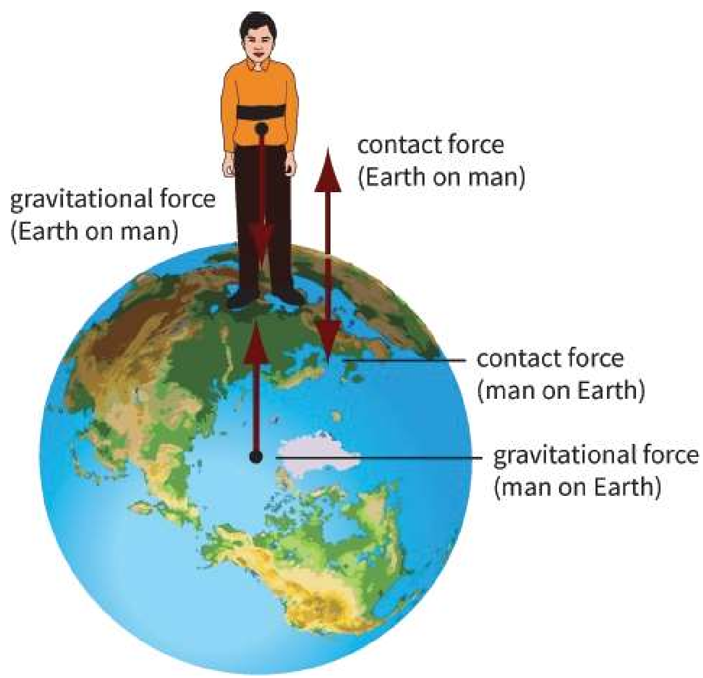


Figure 3.16: For each of the forces that the Earth exerts on you, an equal and opposite force acts on the Earth.

Figure 3.16 shows a person standing on the Earth's surface. The two gravitational forces are a Newton's third law pair, as are the two contact forces. Don't be misled into thinking that the person's weight and the contact force of the floor are a Newton's third law pair. Although they are 'equal and opposite', they do not act on different objects and they are not of the same type.

Question

13 Describe one 'Newton's third law pair' of forces involved in the following situations. In each case, state the object that each force acts on, the type of force and the direction of the force.

a You step on someone's toe.

b A car hits a brick wall and comes to rest.

c A car slows down by applying the brakes.

d You throw a ball upwards into the air.

3.7 Understanding SI units

Throughout physics, we calculate, measure and use many quantities. All quantities consist of a value and a unit. In physics, we mostly use units from the SI system. These units are all defined with extreme care, and for a good reason. In science and engineering, every measurement must be made on the same basis, so that measurements obtained in different laboratories can be compared. This is important for commercial reasons, too. Suppose an engineering firm in Taiwan is asked to produce a small part for the engine of a car that is to be assembled in India. The dimensions are given in millimetres and the part must be made with an accuracy of a tiny fraction of a millimetre. All concerned must know that the part will fit correctly – it would not be acceptable to use a different millimetre scale in Taiwan and India.

KEY IDEA

All physical quantities have a numerical magnitude (a numerical size) and a unit

Base units, derived units

The metre, kilogram and second are three of the seven SI **base units**. These are defined with great precision so that every standards laboratory can reproduce them correctly.

Other units, such as units of speed (m s^{-1}) and acceleration (m s^{-2}) are known as **derived units** because they are combinations of base units. Some derived units, such as the newton and the joule, have special names that are more convenient to use than giving them in terms of base units. The definition of the newton will show you how this works.

Defining the newton

Isaac Newton (1642–1727) played a significant part in developing the scientific idea of force. Building on Galileo's earlier thinking, he explained the relationship between force, mass and acceleration, which we now write as $F = ma$. For this reason, the SI unit of force is named after him.

We can use the equation $F = ma$ to define the **newton** (N).

One newton is the force that will give a 1 kg mass an acceleration of 1 m s^{-2} in the direction of the force.

$$1\ \text{N} = 1\ \text{kg} \times 1\ \text{m s}^{-2} \text{ or } 1\ \text{N} = 1\ \text{kg m s}^{-2}$$

The seven base units

In mechanics (the study of forces and motion), the units we use are based on three base units: the metre, kilogram and second. As we move into studying electricity, we will need to add another base unit, the ampere. Heat requires another base unit, the kelvin (the unit of temperature).

Table 3.4 shows the seven base units of the SI system. Remember that all other units can be derived from these seven. The equations that relate them are the equations that you will learn as you go along (just as $F = ma$ relates the newton to the kilogram, metre and second). The unit of luminous intensity is not part of the AS & A Level courses.

Base unit	**Symbol**	**Base unit**
length	x, l, s and so on	m (metre)
mass	m	kg (kilogram)
time	t	s (second)
electric current	I	A (ampere)
thermodynamic temperature	T	K (kelvin)
amount of substance	n	mol (mole)
luminous intensity	I	cd (candela)

Table 3.4: SI base quantities and units. In this course, you will learn about all of these except the candela.

KEY IDEA

Length, mass, time, current and temperature are base units in mechanics.

Question

14 The pull of the Earth's gravity on an apple (its weight) is about 1 newton. We could devise a new international system of units by defining our unit of force as the weight of an apple. State as many reasons as you can why this would not be a very useful definition.

Other SI units

Using only seven base units means that only this number of quantities have to be defined with great precision. It would be confusing if more units were also defined. For example, if the density of water were defined as exactly 1 g cm^{-3}, then 1000 cm^3 of a sample of water would have a mass of exactly 1 kg. However, it is unlikely that the mass of this volume of water would equal exactly the mass of the standard kilogram.

All other units can be derived from the base units. This is done using the definition of the quantity. For example, speed is defined as $\frac{\text{distance}}{\text{time}}$, and so the base units of speed in the SI system are **m s^{-1}**.

Since the defining equation for force is $F = ma$, the base units for force are **kg m s^{-2}**.

Equations that relate different quantities must have the same base units on each side of the equation. If this does not happen the equation must be wrong.

When each term in an equation has the same base units the equation is said to be **homogeneous**.

KEY IDEA

Base units on each side of a physics equation are the same.

WORKED EXAMPLE

5 It is suggested that the time T for one oscillation of a swinging pendulum is given by the equation $T^2 = 4\pi^2 \left(\frac{l}{g}\right)$ where l is the length of the pendulum and g is the acceleration due to gravity. Show that this equation is homogeneous.

For the equation to be homogeneous, the term on the left-hand side must have the same base units as all the terms on the right-hand side.

Step 1 The base unit of time T is s. The base unit of the left-hand side of the equation is therefore s^2.

Step 2 The base unit of l is m. The base units of g are m s^{-2}. Therefore, the base unit of the right-hand side is $\frac{m}{(ms^{-2})} = s^2$.

(Notice that the constant $4\pi^2$ has no units.)

Since the base units on the left-hand side of the equation are the same as those on the right, the equation is homogeneous.

Questions

15 Determine the base units of:

a pressure $\left(= \frac{\text{force}}{\text{area}}\right)$

b energy (= force × distance)

c density $\left(= \frac{\text{mass}}{\text{volume}}\right)$

16 Use base units to prove that the following equations are homogeneous.

a pressure = density × acceleration due to gravity × depth

b distance travelled $=$ initial speed $\times$ time $+ \frac{1}{2}$acceleration $\times$ time^2 $\left(s = ut + \frac{1}{2}at^2\right)$

Prefixes

Each unit in the SI system can have **multiples** and **sub-multiples** to avoid using very high or low numbers. For example, 1 millimetre (mm) is one thousandth of a metre and 1 micrometre (μm) is one millionth of a metre.

The **prefix** comes before the unit. In the unit mm, the first m is the prefix milli and the second m is the unit metre. You will need to recognise a number of prefixes for the AS & A Level courses, as shown in Table 3.5.

You must take care when using prefixes.

Multiples			Sub-multiples		
Multiple	**Prefix**	**Symbol**	**Multiple**	**Prefix**	**Symbol**
10^3	kilo	k	10^{-1}	deci	d
10^6	mega	M	10^{-2}	centi	c
10^9	giga	G	10^{-3}	milli	m
10^{12}	tera	T	10^{-6}	micro	µ
			10^{-9}	nano	n
			10^{-12}	pico	p

Table 3.5: Multiples and sub-multiples.

Squaring or cubing prefixes

For example:

$$\begin{aligned} 1\text{ cm} &= 10^{-2}\text{m} \\ \text{so } 1\text{ cm}^2 &= \left(10^{-2}\text{m}\right)^2 = 10^{-4}\text{m}^2 \\ \text{and } 1\text{ cm}^3 &= \left(10^{-2}\text{m}\right)^3 = 10^{-6}\text{m}^3 \end{aligned}$$

Writing units

You must leave a small space between each unit when writing a speed such as 3 m s^{-1}, because if you write it as 3 ms^{-1} it would mean 3 millisecond^{-1}.

WORKED EXAMPLE

6 The density of water is 1.0 g cm^{-3}. Calculate this value in kg m^{-3}.

Step 1 Find the conversions for the units:

1 g = 1 × 10^{-3} kg

1 cm^3 = 1 × 10^{-6} m^3

Step 2 Use these in the value for the density of water:

$$\begin{aligned} 1.0\text{ gcm}^{-3} &= \tfrac{1.0\times1\times10^{-3}}{1\times10^{-6}} \\ &= 1.0\times10^{3}\text{kgm}^{-3} \end{aligned}$$

Questions

17 **a** Find the area of one page of this book in cm^2 and then convert your value to m^2.

b If the uncertainty in measuring one side of the page is 0.1 cm find the uncertainty in the area. This can be done by either taking the largest value of each side when you multiply them together and then finding the difference from you value in part **a** or using a combination of the percentage uncertainties (see Chapter P1). Try both methods.

18 Write down, in powers of ten, the values of these quantities:

a 60 pA

b 500 MW

c 20 000 mm.

REFLECTION

Did you find it difficult to understand that Newton's third law of motion relates forces that act on **different** bodies?

SUMMARY

An object will remain at rest or in a state of uniform motion unless it is acted on by an external force. This is Newton's first law of motion.
For a body of constant mass, the acceleration is directly proportional to the resultant force applied to it. Resultant force *F*, mass *m* and acceleration *a* are related by the equation: resultant force = mass × acceleration ($F = ma$) This is a form of Newton's second law of motion.
When two bodies interact, the forces they exert on each other are equal in magnitude and opposite in direction. This is Newton's third law of motion.
The acceleration produced by a force is in the same direction as the force. Where there are two or more forces, we must determine the resultant force.
A newton (N) is the force required to give a mass of 1 kg an acceleration of 1 m s^{-2} in the direction of the force.
The greater the mass of an object, the more it resists changes in its motion. Mass is a measure of the object's inertia.
The weight of an object is a result of the pull of gravity on it: weight = mass × acceleration of free fall ($W = mg$)
Terminal velocity is reached when the fluid resistance is equal to the weight of the object.
Physics equations are homogenous and have the same base units on each side. The main base units are m, kg, s, A and K (the thermodynamic unit for temperature).

EXAM-STYLE QUESTIONS

1 Which list contains only SI base units? **[1]**

A ampere, kelvin, gram

B kilogram, metre, newton

C newton, second, ampere

D second, kelvin, kilogram

2 The speed *v* of a wave travelling a wire is given by the equation

$$v = \left(\frac{Tl}{m}\right)^n$$

where T is the tension in the wire that has mass m and length l.

In order for the equation to be homogenous, what is the value of n? **[1]**

A $\frac{1}{2}$

B 1

C 2

D 4

3 When a golfer hits a ball his club is in contact with the ball for about 0.000 50 s and the ball leaves the club with a speed of 70 m s^{-1}. The mass of the ball is 46 g.

a Determine the mean accelerating force. **[4]**

b What mass, resting on the ball, would exert the same force as in part **a**? **[2]**

[Total: 6]

4 The mass of a spacecraft is 70 kg. As the spacecraft takes off from the Moon, the upwards force on the spacecraft caused by the engines is 500 N. The acceleration of free fall on the Moon is 1.6 N kg^{-1}.

Determine:

a the weight of the spacecraft on the Moon **[2]**

b the resultant force on the spacecraft **[2]**

c the acceleration of the spacecraft. **[2]**

[Total: 6]

5 A metal ball is dropped into a tall cylinder of oil. The ball initially accelerates but soon reaches a terminal velocity.

a By considering the forces on the metal ball bearing, explain why it first accelerates but then reaches terminal velocity. **[3]**

b State how you would show that the metal ball reaches terminal velocity. Suggest one cause of random errors in your readings. **[4]**

[Total: 7]

6 Determine the speed in m s^{-1} of an object that travels:

a 3.0 μm in 5.0 ms **[2]**

b 6.0 km in 3.0 Ms **[2]**

c 8.0 pm in 4.0 ns. **[2]**

[Total: 6]

7 This diagram shows a man who is just supporting the weight of a box. Two of the forces acting are shown in the diagram. According to Newton’s third law, each of these forces is paired with **another** force.

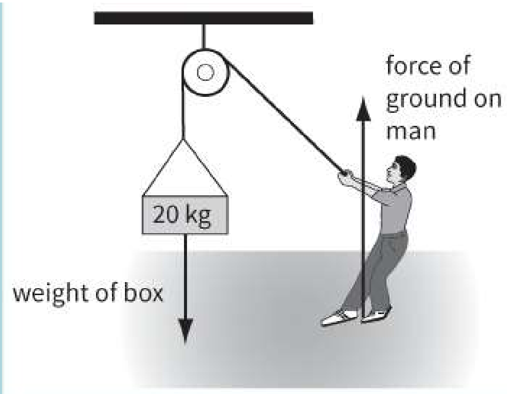


Figure 3.17

For **a** the weight of the box, and **b** the force of the ground on the man, state:

i the body that the other force acts upon **[2]**

ii the direction of the other force **[2]**

iii the type of force involved. **[2]**

[Total: 6]

8 A car starts to move along a straight, level road. For the first 10 s, the driver maintains a constant acceleration of 1.5 m s^{-2}. The mass of the car is 1.1×10^3 kg.

a Calculate the driving force provided by the wheels, when:

i the force opposing motion is negligible **[1]**

ii the total force opposing the motion of the car is 600 N. **[1]**

b Calculate the distance travelled by the car in the first 10 s. **[2]**

[Total: 4]

9 These are the speed–time graphs for two falling balls:

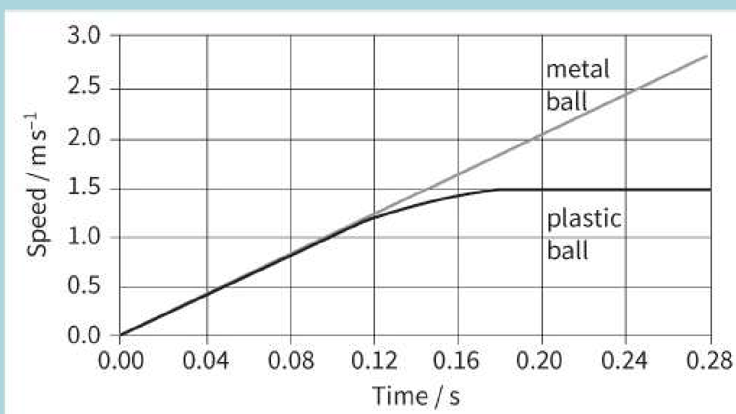


Figure 3.18

a Determine the terminal velocity of the plastic ball. **[1]**

b Both balls are of the same size and shape but the metal ball has a greater mass.

Explain, in terms of Newton's laws of motion and the forces involved, why the plastic ball reaches a constant velocity but the metal ball does not. **[3]**

c Explain why both balls have the same initial acceleration. **[2]**

[Total: 6]

10 A car of mass 1200 kg accelerates from rest to a speed of 8.0 m s^{-1} in a time of 2.0 s.

a Calculate the forward driving force acting on the car while it is accelerating. Assume that, at low speeds, all frictional forces are negligible. **[2]**

b At high speeds the resistive frictional force F produced by air on a body moving with velocity v is given by the equation $F = bv^2$, where b is a constant.

i Derive the base units of force in the SI system. **[1]**

ii Determine the base units of b in the SI system. **[1]**

iii The car continues with the same forward driving force and accelerates until it reaches a top speed of 50 m s^{-1}. At this speed the resistive force is given by the equation $F = bv^2$. Determine the value of b for the car. **[2]**

iv Use your value for b in **iii** and the driving force calculated in part **a** to calculate the acceleration of the car when the speed is 30 m s^{-1}. **[2]**

v **Sketch** a graph showing how the value of F varies with v over the range 0 to 50 m s^{-1}. Use your graph to describe what happens to the acceleration of the car during this time. **[2]**

[Total: 10]

11 a Explain what is meant by the mass of a body and the weight of a body. **[3]**

b State and explain one situation in which the weight of a body changes while its mass remains constant. **[2]**

c State the difference between the base units of mass and weight in the SI system. **[2]**

[Total: 7]

12 a State Newton's second law of motion in terms of acceleration. **[2]**

b When you jump from a wall on to the ground, it is advisable to bend your knees on landing.

i State how bending your knees affects the time it takes to stop when hitting the ground. **[1]**

ii Using Newton's second law of motion, explain why it is sensible to bend your knees. **[2]**

[Total: 5]