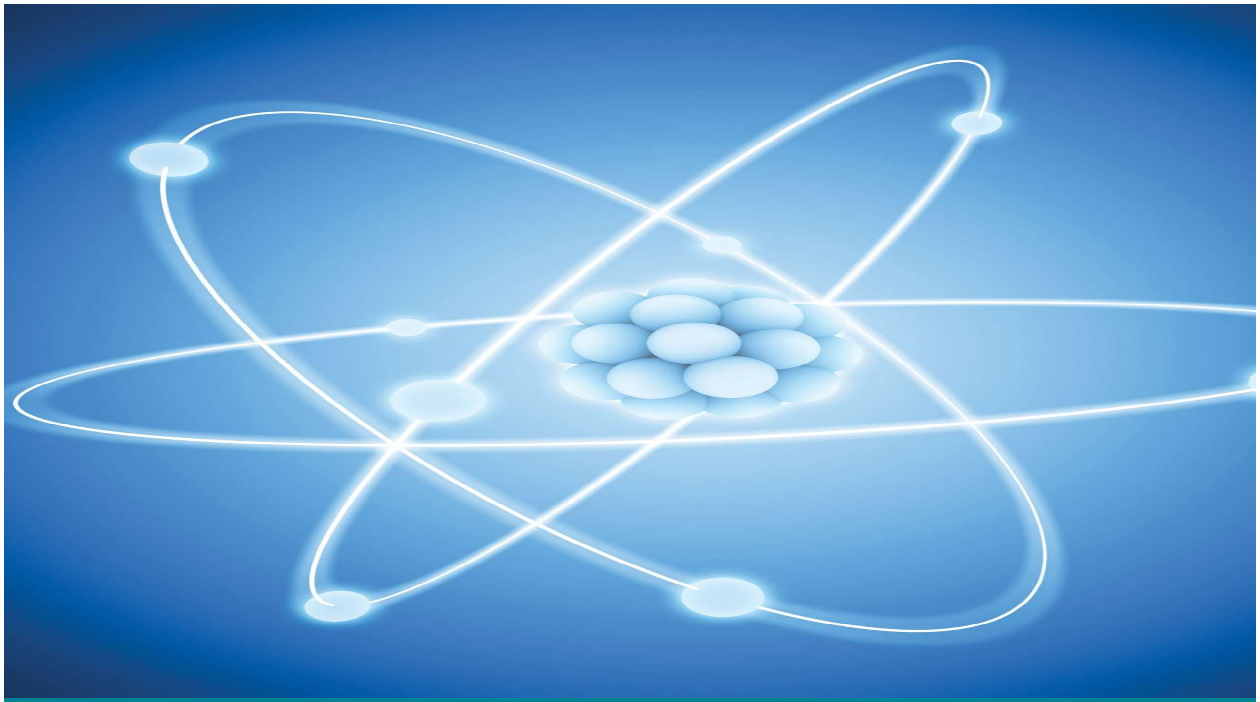

Chapter 29

Nuclear physics

LEARNING INTENTIONS

In this chapter you will learn how to:

- understand the equivalence between energy and mass as represented by $E = mc^2$ and recall and use this equation
- represent simple nuclear reactions by nuclear equations
- define and use the terms mass defect and binding energy
- sketch the variation of binding energy per nucleon with nucleon number
- explain what is meant by nuclear fusion and nuclear fission
- explain the relevance of binding energy per nucleon to nuclear reactions, including nuclear fusion and nuclear fission
- calculate the energy released in nuclear reactions using $E = \Delta mc^2$
- understand that fluctuations in count rate provide evidence for the random nature of radioactive decay
- understand that radioactive decay is both spontaneous and random
- define activity and decay constant, and recall and use $A = \lambda N$
- define half-life
- use $\lambda = \frac{0.693}{t_{\frac{1}{2}}}$
- understand the exponential nature of radioactive decay, and sketch and use the relationship $x = x_0 e^{-\lambda t}$, where x could represent activity, number of undecayed nuclei or received count rate.

BEFORE YOU START

- Background knowledge of radioactivity from Chapter 15 would be useful in the study of this chapter. In pairs, write a summary of what you know.

- Try to remember, then write down, the particles that make up the nucleus and the forces the particles experience.
- Discuss why it is sensible to express the mass of particles in atomic mass units (u).

ENERGY AND THE NUCLEUS

The existence of every living organism on the surface of the Earth, including humans, depends on the light and heat from the Sun. Without the Sun, our planet would be a lifeless rock in space.

The Sun warms our oceans, stirs our atmosphere, creates our climate and, most importantly of all, gives energy to the plants that provide food and oxygen for life on Earth.

How does the Sun produce its energy? The Sun is an active hot ball of gas; it converts mass into energy. The Sun generates about 10^{26} W of radiant power by converting more than a billion kilograms of matter into energy every second. You do not need to worry about the Sun dying out soon – it has lots of mass! The mass of the Sun is about 10^{30} kg. Can you estimate the lifetime of the Sun?

In this chapter, we will examine how nuclear reactions produce energy. We will also look at the stability of nuclei, and how we can model the decay of unstable nuclei with mathematical equations.

Figure 29.1: Our understanding of nuclear physics is important to all life on Earth.

29.1 Balanced equations

When an unstable nucleus undergoes radioactive decay, the nucleus before the decay is often referred to as the **parent** nucleus and the new nucleus formed after the decay process is known as the **daughter** nucleus.

Radioactive decay processes can be represented by balanced equations. As with all equations representing nuclear processes, both nucleon number A and proton number Z are conserved.

- In α decay, the nucleon number decreases by 4 and the proton number decreases by 2.
- In β^- decay, the nucleon number is unchanged and the proton number increases by 1.
- In β+ decay, the nucleon number is unchanged and the proton number decreases by 1.
- In gamma decay, there is no change in either nucleon number or proton number.

The emission of α- and β-particles can be shown on a graph of nucleon number A plotted against proton number Z, as shown in Figure 29.2. The graph will appear different if neutron number is plotted against proton number.

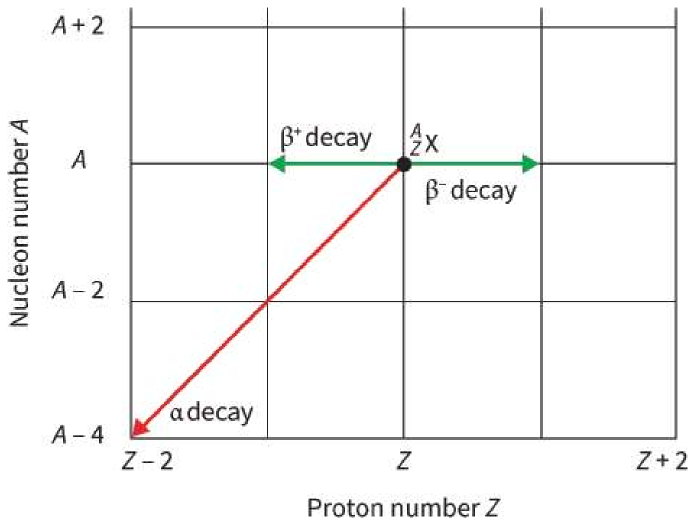

Figure 29.2: Emission of α- and β-particles.

WORKED EXAMPLES

1 Radon is a radioactive gas. The isotope of radon-222 decays by α emission to become a nucleus of polonium (Po). Here is the equation for the decay of a single isotope of radon-222:

$$^{222}_{86}\text{Rn} \rightarrow {}^{218}_{84}\text{Po} + {}^{4}_{2}\text{He}$$

Show that A and Z are conserved.

Compare the nucleon and proton numbers on both sides of the equation for the decay:

nucleon number A: 222 = 218 + 4

proton number Z: 86 = 84 + 2

Hint: *Remember that in α decay, A decreases by four and Z decreases by two. Don't confuse nucleon number A with neutron number N.*

In this case, radon-222 is the parent nucleus and polonium-218 is the daughter nucleus.

2 A carbon-14 nucleus (parent) decays by β⁻ emission to become an isotope of nitrogen (daughter). Here is the equation that represents this decay:

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e}$$

Show that both nucleon number and proton number are conserved.

Compare the nucleon and proton numbers on both sides of the equation for the decay:

nucleon number A: 14 = 14 + 0

proton number Z: 6 = 7 − 1

Hint: *Remember that in β^- decay, A remains the same and Z increases by 1.*

Questions

1 Study the decay equations given in Worked examples 1 and 2, and write balanced equations for the following:

- **a** A nucleus of radon-220 $\left({}^{220}_{86}\mathrm{Rn}\right)$ decays by α emission to form an isotope of polonium, Po.
- **b** A nucleus of a sodium isotope $\left({}^{25}_{11}\mathrm{Na}\right)$ decays by β^{-} emission to form an isotope of magnesium, Mg.

2 Copy and complete this equation for the β^{-} decay of a nucleus of argon:

${}^{41}_{18}\mathrm{Ar} \rightarrow \mathrm{K} + ?$

29.2 Mass and energy

In Chapter 15, we saw that energy is released when the nucleus of an unstable atom decays. How can we calculate the amount of energy released by radioactive decay? To find the answer to this, we need to think first about the masses of the particles involved.

We will start by considering a stable nucleus, C. This consists of six protons and six neutrons. Fortunately for us (because we have a lot of this form of carbon in our bodies), this is a very stable nuclide. This means that the nucleons are bound tightly together by the strong nuclear force. It takes a lot of energy to pull them apart.

Figure 29.3 shows the results of an imaginary experiment in which we have done just that. On the left-hand side of the balance is a single ${}^{12}_{6}\mathrm{C}$ nucleus. On the right-hand side are six protons and six neutrons, the result of dismantling the nucleus. The surprising thing is that the balance is tipped to the right. The separate nucleons have **greater** mass than the nucleus itself. This means that the law of conservation of mass appears to have been broken. Have we violated what was thought to be a fundamental law of nature, something that was held to be true for hundreds of years?

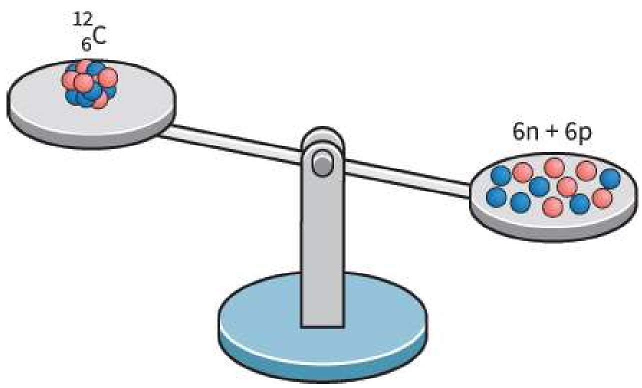

Figure 29.3: The mass of a nucleus is less than the total mass of its component protons and neutrons.

Notice that, in dismantling the ${}^{12}_{6}\mathrm{C}$ nucleus, we have had to do work against the strong nuclear force. The nucleons attract one another with the strong nuclear force when we try to pull them apart. So, we have put energy into the nucleus to pull it apart, and this energy increases the potential energy of the individual nucleons. We can think of the nucleons within the nucleus as sitting in a deep potential well that results from the strong nuclear forces that hold the nucleus together. When we separate nucleons, we lift them out of this potential well, giving them more nuclear potential energy. This potential well is similar to that formed by the electric field around the nucleus; it is this well in which the atomic electrons sit, but it is much, much deeper. This explains why it is much easier to remove an electron from an atom than to remove a nucleon (proton or neutron) from the nucleus.

The problem of **changing** mass remains. To solve this problem, Einstein made the revolutionary hypothesis about energy and mass – to him, they were equivalent. This is not an easy idea. When bodies are in a higher energy state they have more mass than in a lower energy state. A bucket of water at the top of a hill will have more mass than when it is at the bottom because energy has been transferred to it in carrying it up the hill. A tennis ball travelling at 50 m s^{-1} will have more mass than the same tennis ball when stationary. In everyday life, the amount of extra mass is so small that it is not noticeable. However, the large changes in energy that occur in nuclear physics and high-energy physics make the changes in mass significant. Indeed, the increase in mass of particles, such as electrons, as they are accelerated to speeds near to the speed of light is a well-established experimental fact.

Another way to express this is to treat mass and energy as aspects of the same thing. Rather than having separate laws of conservation of mass and conservation of energy, we can combine these two. The total amount of mass and energy in a system is constant. There may be conversions from one to the other, but the total amount of 'mass–energy' remains constant.

Einstein's mass–energy equation

Albert Einstein produced his famous mass–energy equation, which links energy E and mass m:

$$E = mc^2$$

where c is the speed of light in a vacuum (free-space). The value of c is approximately 3.00×10^8 m s^{-1}, but its precise value has been fixed as c = 299 792 458 m s^{-1}.

Generally, we will be concerned with the changes in mass owing to changes in energy, when the equation becomes:

$$\Delta E = \Delta mc^2$$

where ΔE is the change in energy corresponding to a change, Δm in mass and c is the speed of light in a vacuum.

KEY EQUATION

$$\Delta E = \Delta mc^2$$

You may find this equation written in different forms:

$$E = c^2 \Delta m$$
$$E = mc^2$$

According to Einstein's equation:

- the mass of a system **increases** when energy is **supplied** to it
- the mass of a system **decreases** when energy is **released** from it.

Now, if we know the total mass of particles before a nuclear reaction and their total mass after the reaction, we can work out how much energy is released. Table 29.1 gives the mass in kilograms of each of the particles shown in Figure 29.3. Notice that this is described as the **rest mass** of the particle; that is, its mass when it is stationary. The mass of a particle will be greater when it is moving because of its increase in energy. Nuclear masses are measured to a high degree of precision using mass spectrometers, often to seven or eight significant figures.

Particle	**Rest mass / 10^{-27} kg**
$^{1}_{1}\text{p}$	1.672 623
$^{1}_{0}\text{n}$	1.674 929
$^{12}_{6}\text{C}$ nucleus	19.926 483

Table 29.1: Rest masses of some particles. It is worth noting that the mass of the neutron is slightly larger than that of the proton (roughly 0.1% greater).

We can use the mass values to calculate the mass that is **released** as energy when nucleons combine to form a nucleus. So, for our particles in Figure 29.3, we have:

$$\begin{aligned}
\text{mass of system before} &= \text{mass of all the seperate nucleons} \\
&= (6 \times 1.672\ 623 + 6 \times 1.674\ 929) \times 10^{-27}\text{kg} \\
&= 20.085\ 312 \times 10^{-27}\text{kg} \\
\text{mass of system after} &= \text{mass of the carbon-12 nucleus} \\
&= 19.926\ 483 \times 10^{-27}\text{kg} \\
\text{decreases in the mass of the system} &= \Delta m = (20.085\ 312 - 19.926\ 483) \times 10^{-27}\text{kg} \\
&= 0.158\ 829 \times 10^{-27}\text{kg}
\end{aligned}$$

When six protons and six neutrons combine to form the nucleus of carbon-12, there is a very small loss of mass Δm, known as the **mass defect**.

The mass defect of a nucleus is equal to the difference between the total mass of the individual separate nucleons and the mass of the nucleus.

The **loss** in mass implies that energy is **released** in this process. The energy released ΔE is given by Einstein's mass–energy equation. Therefore:

$$\Delta E = \Delta mc^2 = 0.158\ 829 \times 10^{-27} \times \left(3.00 \times 10^8\right)^2$$
$$\Delta E \approx 1.43 \times 10^{-11}\text{J}$$

This may seem like a very small amount of energy, but it is a lot on the atomic scale. For comparison, the amount of energy released in a chemical reaction involving a single carbon atom would typically be of the order of 10^{-18} J, more than a million times smaller.

Now look at Worked example 3.

WORKED EXAMPLES

3 Use the following data to determine the minimum energy required to split a nucleus of oxygen-16 $\left({}^{16}_{8}O\right)$ into its separate nucleons. Give your answer in joules (J).

mass of proton = $1.672\ 623 \times 10^{-27}$ kg

mass of neutron = $1.674\ 929 \times 10^{-27}$ kg

mass of ${}^{16}_{8}O$ nucleus = $26.551\ 559 \times 10^{-27}$ kg

speed of light $c = 3.00 \times 10^{8}$ m s^{-1}

Step 1 Find the difference Δm in kg between the mass of the oxygen nucleus and the mass of the individual nucleons. The ${}^{16}_{8}O$ nucleus has 8 protons and 8 neutrons.

$$\Delta m = \textit{final mass} - \textit{initial mass}$$

$$\Delta m = ((8 \times 1.672\ 623 + 8 \times 1.674\ 929) - 26.551\ 559) \times 10^{-27} kg)$$

$$\Delta m \approx 2.20 \times 10^{-28} kg$$

There is an **increase** in the mass of this system, therefore, external energy must be **supplied** for the splitting of the oxygen-16 nucleus into its totally free nucleons.

Step 2 Use Einstein's mass–energy equation to determine the energy supplied:

$\Delta E = \Delta mc^2$

$$\text{energy supplied} = 2.20 \times 10^{-28} \times \left(3.00 \times 10^{8}\right)^{2} \approx 1.98 \times 10^{-11}\,\text{J}$$

The value is the minimum energy. If the energy were to be greater than this value, the surplus energy would appear as kinetic energy of the nucleons.

Mass–energy conservation

Einstein pointed out that his equation $\Delta E = \Delta mc^2$ applied to **all** energy changes, not just nuclear processes. So, for example, it applies to chemical changes too. If we burn some carbon, we start off with carbon and oxygen. At the end, we have carbon dioxide and energy. If we measure the mass of the carbon dioxide, we find that it is very slightly less than the mass of the carbon and oxygen at the start of the experiment. The total potential energy of the system will be less than at the start of the experiment, hence the mass is less. In a chemical reaction such as this, the change in mass is very small, less than a microgram if we start with 1 kg of carbon and oxygen. Compare this with the change in mass that occurs during the fission of 1 kg of uranium, described later. The change in mass in a chemical reaction is a much, much smaller proportion of the original mass, which is why we don't notice it.

Questions

3 The Sun releases vast amounts of energy. Its power output is 4.0×10^{26} W. Estimate how much its mass decreases each second because of this energy loss.

4 **a** Calculate the energy released if a ${}^{4}_{2}He$ nucleus is formed from separate stationary protons and neutrons. The masses of the particles are given in Table 29.2.

b Calculate also the energy released per nucleon.

Particle	Mass / 10^{-27} kg
${}^{1}_{1}p$	1.672 623
${}^{1}_{0}n$	1.674 929
${}^{4}_{2}He$	6.644 661

Table 29.2: Masses of some particles.

5 The rest mass of a golf ball is 150 g.

Calculate its increase in mass when it is travelling at 50 m s^{-1}. What is this as a percentage of its rest mass?

Another unit of mass

When calculating energy values using $\Delta E = \Delta mc^2$, it is essential to use values of mass in kg, the SI unit of mass. However, the mass of a nucleus is very small, perhaps 10^{-25} kg, and these numbers are awkward. As an alternative, atomic and nuclear masses are often given in a different unit, the **atomic mass unit** (symbol u). You have already met this alternative unit for mass in Chapter 15.

The conversion factor for atomic mass unit u to kilogram (kg) is:

$$1 \text{ u} = 1.660\,538\,921(73) \times 10^{-27} \text{ kg}$$

To convert the mass of a particle from u to kg, you just multiply by the conversion factor shown–usually 1.6605×10^{-27} is sufficiently accurate.

Table 29.3 shows the masses of proton, neutron and some nuclides in u. It is worth noting that the mass in u is close to the nucleon number A. For example, the mass of uranium-235 nucleus is 235 u.

Nuclide	**Symbol**	**Mass / u**
proton	$^{1}_{1}\text{p}$	1.007 276
neutron	$^{1}_{0}\text{n}$	1.008 665
helium-4	$^{4}_{2}\text{He}$	4.002 602
carbon-12	$^{12}_{6}\text{C}$	12.000 000
potassium-40	$^{40}_{19}\text{K}$	39.963 998
uranium-235	$^{235}_{92}\text{U}$	235.043 930

Table 29.3: Masses of some particles in u. Some have been measured to several more decimal places than are shown here.

Questions

6 **a** The mass of an atom of $^{56}_{26}\text{Fe}$ is 55.934 937 u. Calculate its mass in kg.

b The mass of an atom of $^{16}_{8}\text{O}$ is $2.656\,015 \times 10^{-26}$ kg. Calculate its mass in u.

7 Table 29.3 gives the masses (in u) of several particles.

(Avogadro constant $N_A = 6.02 \times 10^{23}$ mol^{-1}.)

Use the table to determine to three significant figures:

a the mass in kg of a helium-4 nucleus

b the mass in gram (g) of 1.0 mole of uranium-235 nuclei.

29.3 Energy released in radioactive decay

Unstable nuclei may emit α- and β-particles with large amounts of kinetic energy. We can use Einstein's mass–energy equation $\Delta E = \Delta mc^2$ to explain the origin of this energy. Take, for example, the decay of a nucleus of uranium-238. It decays by emitting an α-particle and changes into an isotope of thorium:

$$^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

The uranium nucleus is in a high-energy, relatively unstable state. It emits the α-particle and the remaining thorium nucleus is in a lower, more stable energy state. There is a decrease in the mass of the system. That is, the combined mass of the thorium nucleus and the α-particle is less than the mass of the uranium nucleus. According to Einstein's mass–energy equation, this difference in mass Δm is equivalent to the energy released as kinetic energy of the products. Using the most accurate values available:

$$\begin{aligned}
\text{mass of } {}^{238}_{92}\mathrm{U} \text{ nucleus} &= 3.952\,83 \times 10^{-25}\,\mathrm{kg} \\
\text{total mass of } {}^{234}_{90}\mathrm{Th} \text{ nucleus and } \alpha\text{-particle } \left({}^{4}_{2}\mathrm{He}\right) &= 3.952\,76 \times 10^{-25}\,\mathrm{kg} \\
\text{change in mass } \Delta m &= (3.952\,76 - 3.952\,83) \times 10^{-25}\,\mathrm{kg} \\
&\approx -7.0 \times 10^{-30}\,\mathrm{kg}
\end{aligned}$$

The minus sign shows a decrease in mass, hence, according to the equation $\Delta E = \Delta mc^2$, energy is released in the decay process:

$$\begin{aligned}
\text{energy released} &\approx 7.0 \times 10^{-30} \times \left(3.0 \times 10^{8}\right)^2 \\
&\approx 6.3 \times 10^{-13}\,\mathrm{J}
\end{aligned}$$

This is an enormous amount of energy for a single decay. One mole of uranium-238, which has 6.02×10^{23} nuclei, has the potential to emit total energy equal to about 10^{11} J.

We can calculate the energy released in all decay reactions, including β decay, using the same ideas.

Question

8 A nucleus of beryllium-10 $\left({}^{10}_{4}\mathrm{Be}\right)$ decays into an isotope of boron by β^- emission. The chemical symbol for boron is B.

a Write a nuclear decay equation for the nucleus of beryllium-10.

b Calculate the energy released in this decay and state its form.

(Mass of ${}^{10}_{4}\mathrm{Be}$ nucleus = $1.662\,38 \times 10^{-26}$ kg; mass of boron isotope = $1.662\,19 \times 10^{-26}$ kg; mass of electron = $9.109\,56 \times 10^{-31}$ kg.)

29.4 Binding energy and stability

We can now begin to see why some nuclei are more stable than others. If a nucleus is formed from separate nucleons, energy is released. In order to pull the nucleus apart, energy must be put in; in other words, work must be done against the strong nuclear force that holds the nucleons together. The more energy involved in this, the more stable the nucleus.

The minimum energy needed to completely pull a nucleus apart into its separate nucleons is known as the **binding energy** of the nucleus.

Take care: this is **not** energy stored in the nucleus. On the contrary, it is the energy that must be put in to the nucleus in order to pull it apart. In the example of ${}^{12}_{6}\mathrm{C}$ discussed earlier, we calculated the binding energy from the mass difference between the mass of the ${}^{12}_{6}\mathrm{C}$ nucleus and the masses of the separate protons and neutrons.

In order to compare the stability of different nuclides, we need to consider the **binding energy per nucleon**.

We can determine the binding energy per nucleon for a nuclide as follows:

- Determine the mass defect for the nucleus.
- Use Einstein's mass–energy equation to determine the binding energy of the nucleus by multiplying the mass defect by c^2.
- Divide the binding energy of the nucleus by the number of nucleons to calculate the binding energy per nucleon.

Figure 29.4 shows the variation of binding energy per nucleon with nucleon number A for nuclei. The red dot represents the plot for the iron-56 nuclide, which is from Worked example 4. The greater the value of the binding energy per nucleon, the more tightly bound are the nucleons that make up the nucleus. The most striking observation is that not all nuclides are the same – some nuclides are more tightly bound than others.

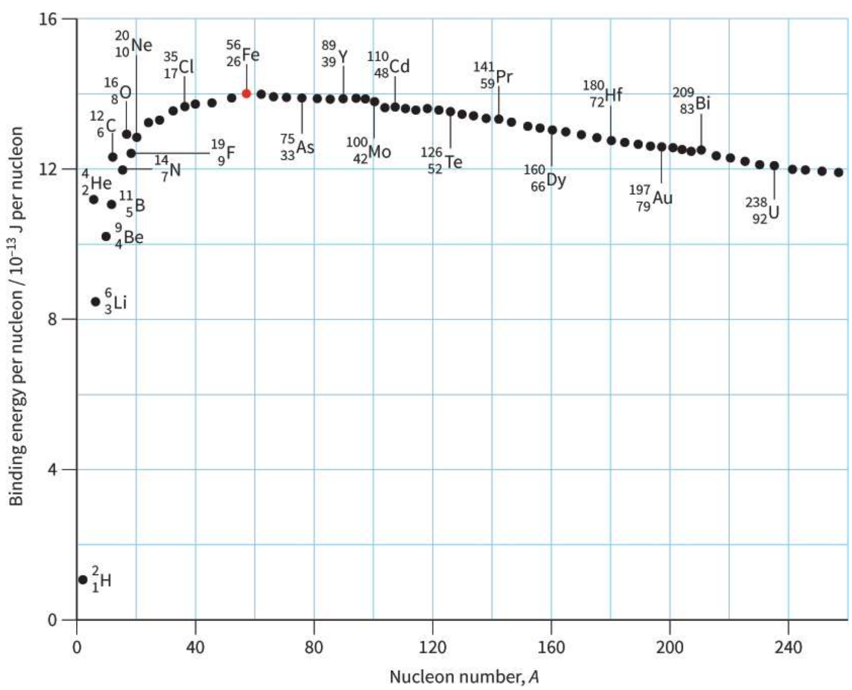

Figure 29.4: This graph shows the binding energy per nucleon for a number of nuclei. The nucleus becomes more stable as binding energy per nucleon increases.

If you further examine this graph, you will see that the general trend is for light nuclei to have low binding energies per nucleon. Note, however, that helium has a much higher binding energy than its place in the Periodic Table might suggest. The high binding energy per nucleon means that it is very stable. Other common stable nuclei include ${}^{12}_{6}\mathrm{C}$ and ${}^{16}_{8}\mathrm{O}$, which can be thought of, respectively, as three and four α-particles bound together (Figure 29.5).

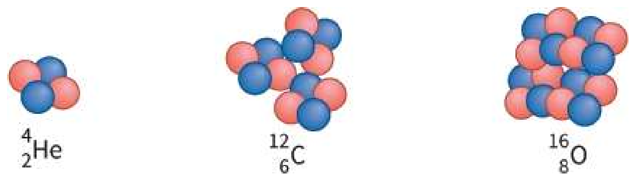

Figure 29.5: More stable nuclei are formed when 'α-particles' are bound together. In $^{12}_{6}\text{C}$ and $^{16}_{8}\text{O}$, the 'α-particles' do not remain separate, as shown here; rather, the protons and neutrons are tightly packed together.

For nuclides with $A > 20$ approximately, there is not much variation in binding energy per nucleon. The greatest value of binding energy per nucleon is found for $^{56}_{26}\text{Fe}$. This isotope of iron requires the most energy per nucleon to dismantle it into separate nucleons; hence iron-56 is the most stable isotope in nature.

WORKED EXAMPLES

4 Use the following data to calculate the binding energy per nucleon for the nuclide $^{56}_{26}\text{Fe}$.

mass of neutron = 1.675×10^{-27} kg

mass of proton = 1.673×10^{-27} kg

mass of $^{56}_{26}\text{Fe}$ nucleus = 9.288×10^{-26} kg

Step 1 Calculate the mass defect.

$$\text{number of neutrons} = 56 - 26 = 30$$

$$\text{mass defect} = \left(30 \times 1.675 \times 10^{-27} + 26 \times 1.673 \times 10^{-27}\right) - 9.288 \times 10^{-26}$$

$$\text{mass defect} = 8.680 \times 10^{-28}\,\text{kg}$$

Step 2 Calculate the binding energy of the nucleus using Einstein's mass-energy Zequation.

$$\text{binding energy} = \Delta mc^2 = 8.680 \times 10^{-28} \times \left(3.00 \times 10^{8}\right)^2$$

$$\text{binding energy} = 7.812 \times 10^{-11}\,\text{J}$$

Step 3 Calculate the binding energy per nucleon.

$$\text{binding energy per nucleon} = \frac{7.812 \times 10^{-11}}{56} \approx 14 \times 10^{-13}\,\text{J}$$

Have another look at Figure 29.4. The value matches with the plot of iron-56.

Questions

9 **a** Explain why hydrogen ^1_1H (proton) cannot appear on the graph shown in Figure 29.4.

b Use Figure 29.4 to estimate the binding energy of the nuclide $^{14}_{7}\text{N}$.

10 The mass of a ^8_4Be nucleus is 1.33×10^{-26} kg. For the nucleus of ^8_4Be, determine:

a the mass defect in kg

b the binding energy of the nucleus in MeV

c the binding energy (in MeV) per nucleon for the nucleus.

Binding energy, fission and fusion

We can use the binding energy graph to help us decide which nuclear processes – fission, fusion, radioactive decay – are likely to occur (Figure 29.6).

Fission

Fission is the process in which a massive nucleus splits to form two smaller fragments (rather than simply emitting α- or β-radiation).

The isotope of uranium-235 can split spontaneously, but such an event is very rare. However, in a process known as **induced** fission, uranium-235 can be made split by absorbing a slow-moving neutron. A typical nuclear reaction is shown:

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{236}_{92}U \rightarrow ^{142}_{56}Ba + ^{92}_{36}Kr + 2^{1}_{0}n$$

The uranium-235 nucleus captures the neutron and becomes a highly unstable nucleus of uranium-236. In a very short period of time, typically a few microseconds, the fission of uranium-236 results in barium-142, krypton-92 and two fast-moving neutrons. Energy is released in the reaction as kinetic energy because the total mass of the system decreases. This is what we would expect from Einstein's mass-energy equation. There is now another alternative way of interpreting this reaction. If we look at Figure 29.6, we see that these two fragments have greater binding energy per nucleon than the original uranium nucleus. Hence, if the uranium nucleus splits in this way, energy will be released. The total binding energy of $^{142}_{56}Ba$ and $^{92}_{36}Kr$, is greater than the binding energy of $^{235}_{92}U$–the difference is the energy released. (Note: the neutron is a lone nucleon, so it has zero binding energy.)

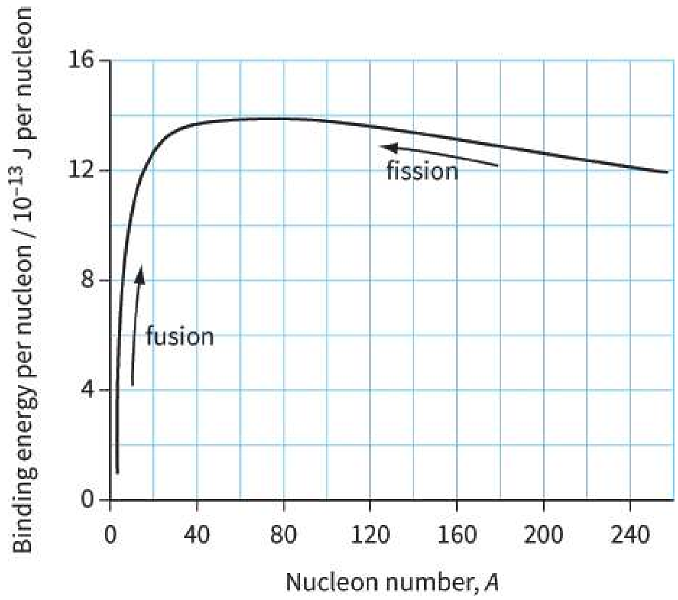

Figure 29.6: Both fusion and fission are processes that tend to increase the binding energy per nucleon of the particles involved.

Fusion

Fusion is the process by which two very light nuclei join together to form a heavier nucleus. This is the process by which energy is released in the Sun, when hydrogen nuclei fuse to form helium nuclei. When two light nuclei join together, the final binding energy of the nucleus formed is greater than the total binding energy of the fusing-nuclei – once again, the difference is the energy released in the fusion reaction. The high binding energy of the $^{4}_{2}He$ nuclide means that it is rare for these nuclei to fuse.

The following fusion reaction is one of the many taking place inside the core of stars, including our Sun:

$$^{2}_{1}H + ^{1}_{1}p \rightarrow ^{3}_{2}He$$

A deuterium nucleus $\left(^{2}_{1}H\right)$ joins together with a proton $\left(^{1}_{1}p\right)$, to make the helium-3 nucleus. The binding energy of deuterium nucleus is 2.2 MeV, and the binding energy of helium-3 nucleus is 7.7 MeV. The energy released in this fusion reaction is 5.5 MeV, which is the difference in the two binding energies. It is worth noting that the binding energy per nucleon of the helium-3 nucleus is greater that of the deuterium nucleus – fusion increases the binding energy per nucleon, as shown on Figure 29.6.

Questions

11 Use the binding energy graph (Figure 29.6) to suggest why fission is unlikely to occur with 'light nuclei' ($A < 20$) and why fusion is unlikely to occur for heavier nuclei ($A > 40$).

12 Use the information given in the fusion section, to determine the binding energy (in MeV) per nucleon of each particle in the following fusion reaction:

$^{2}_{1}H + ^{1}_{1}p \rightarrow ^{3}_{2}He$

Comment on your answers.

29.5 Randomness and radioactive decay

Listen to a counter connected to a Geiger–Müller (GM) tube that is detecting the radiation from a weak source, so that the count rate is about one count per second. Each count represents the detection of a single α-particle or a β-particle or a γ-ray photon. You will notice that the individual counts do not come regularly.

The counter beeps or clicks in a random, irregular manner. If you try to predict when the next clicks will come, you are unlikely to be right.

You can see the same effect if you have a ratemeter, which can measure faster rates (Figure 29.7). The needle fluctuates up and down. Usually, a ratemeter has a control for setting the 'time constant'–the time over which the meter averages out the fluctuations. Usually, this can be set to 1 s or 5 s. The fluctuations are smoothed out more on the 5 s setting.

Figure 29.7: The time constant of this ratemeter can be adjusted to smooth out rapid fluctuations in the count rate.

Figure 29.8 shows a graph of count rate against time, with a smoothing of a few seconds. The count rate decreases with time as the number of radioactive nuclei that are left decreases. The fluctuations either side are caused by the randomness of the decay.

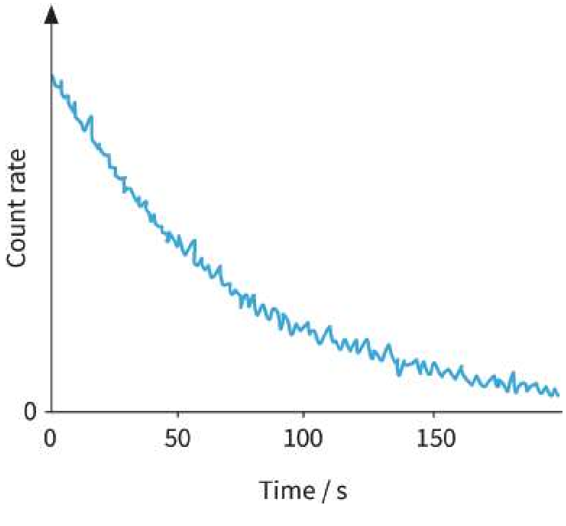

Figure 29.8: Count rate showing randomness of decay.

So, it is apparent that radioactive decay is a random, irregular phenomenon. But is it completely unpredictable? Well, not really. We can measure the average rate of decay. We might measure the number of counts detected in 1000 s, and then calculate the average number per second. We cannot be sure about this average rate either, because the number of counts in 1000 s will fluctuate, too. All of our measurements of radioactive decay are inherently uncertain and imprecise but, by taking averages over a sufficiently long time period, we can reduce or smooth out the random fluctuations to reveal the underlying pattern.

Spontaneous decay

Radioactive decay occurs within the unstable nucleus of an atom. A nucleus emits radiation and becomes the nucleus of an atom of a different element. We cannot predict, for a particular nucleus, when it will happen. If we sit and stare at an individual nucleus, we cannot see any change that will tell us that it is getting ready to decay. And if it doesn't decay in the first hour when we are watching it, we cannot say that it is any more likely to decay in the next hour. What is more, we cannot affect the probability of an individual nucleus decaying, for example, by changing its temperature.

This is slightly odd, because it goes against our everyday experience of the way things around us change. We observe things changing. They gradually age, die, rot away. But this is not how things are on the scale of atoms and nuclei. Many of the atoms of which we are made have existed for billions of years, and will still exist long after we are gone. The nucleus of an atom does not age.

If we look at a very large number of atoms of a radioactive substance, we will see that the number of undecayed nuclei gradually decreases. However, we cannot predict when an **individual** nucleus will decay. Each nucleus 'makes up its own mind' when to decay, independently from its neighbours. This is because neighbouring nuclei do not interact with one another (unlike neighbouring atoms). The nucleus is a tiny fraction of the size of the atom, and the nuclear forces do not extend very far outside the nucleus. So, one nucleus cannot affect a neighbouring nucleus by means of the nuclear force. Being inside a nucleus is a bit like living in a house in the middle of nowhere; you can just see out into the garden, but everything is darkness beyond and the next house is 1000 km away.

The fact that individual nuclei decay independently of their neighbours and of environmental factors, accounts for the random pattern of clicks that we hear from a Geiger counter and the fluctuations of the needle on the ratemeter. Radioactive decay is both **spontaneous** and **random**.

Nuclear decay is spontaneous because:

- the decay of a particular nucleus is not affected by the presence of other nuclei
- the decay of nuclei cannot be affected by chemical reactions or external factors such as temperature and pressure.

Nuclear decay is **random** because:

- it is impossible to predict when a particular nucleus in a sample is going to decay
- each nucleus in a sample has the same chance of decaying per unit time.

29.6 The mathematics of radioactive decay

We have seen that radioactive decay is a random, spontaneous process. Because we cannot say when an individual nucleus will decay, we have to start thinking about very large numbers of nuclei. Even a tiny speck of radioactive material will contain more than 10^{15} nuclei. Then we can talk about the average number of nuclei that we expect to decay in a particular time interval; in other words, we can find out the **average** decay rate. Although we cannot make predictions for individual nuclei, we can say that certain types of nuclei are more likely to decay than others. For example, a nucleus of carbon-12 is stable; carbon-14 decays gradually over thousands of years; carbon-15 nuclei last, on average, a few seconds.

So, because of the spontaneous nature of radioactive decay, we have to make measurements on very large numbers of nuclei and then calculate averages. One quantity we can determine is the probability that an individual nucleus will decay in a particular time interval. For example, suppose we observe one million nuclei of a particular isotope. After one hour, 200 000 have decayed. Then the probability that an individual nucleus will decay in one hour is 0.2 or 20%, since 20% of the nuclei have decayed in this time. (Of course, this is only an approximate value, since we might repeat the experiment and find that only 199 000 decay because of the random nature of the decay. The more times we repeat the experiment, the more reliable our answer will be.)

We can now define the decay constant:

The probability that an individual nucleus will decay per unit time interval is called the **decay constant**, λ.

For the example, we have:

$$\text{decay constant } \lambda = 0.20 \text{ h}^{-1}$$

Note that, because we are measuring the probability of decay per unit time interval, λ has units of h^{-1} (or s^{-1}, day^{-1}, year^{-1}, etc.).

The **activity** A of a radioactive sample is the rate at which nuclei decay or disintegrate.

Activity is measured in decays per second (or h^{-1}, day^{-1}). An activity of one decay per second is one becquerel (1 Bq):

$$1 \text{ Bq} = 1 \text{ s}^{-1}$$

Clearly, the activity of a sample depends on the decay constant λ of the isotope under consideration. The greater the decay constant (the probability that an individual nucleus decays per unit time interval), the greater is the activity of the sample. It also depends on the number of undecayed nuclei N present in the sample.

For a sample of N undecayed nuclei, we have:

$$A = -\lambda N$$

where λ is the decay constant of the isotope and N is the number of undecayed nuclei.

KEY EQUATION

Activity A is given by:

$$A = -\lambda N$$

Activity A is equal to rate of decay of nuclei; therefore $A = \lambda N$.

The minus sign indicates that the number of undecayed nuclei decreases with time. We can omit this minus sign if we just want to determine the magnitude of the activity. So, in calculations, we can just use $A = \lambda N$.

We can also think of the activity as the number of α- or β-particles emitted from the source per unit time. Hence, we can also write the activity A as:

$$A = \frac{\Delta N}{\Delta t}$$

where ΔN is equal to the number of emissions (or decays) in a small time interval of Δt.

WORKED EXAMPLE

5 A radioactive source emits β-particles. The source has an activity of 2.8×10^{7} Bq. Estimate the number of β-particles emitted in a time interval of 2.0 minutes. State one assumption made.

Step 1 Write down the given quantities in SI units.

$A = 2.8 \times 10^7$ Bq $\quad \Delta t = 120$ s

Step 2 Determine the number of β-particles emitted.

$$A = \frac{\Delta N}{\Delta t}$$

$$\begin{aligned}\Delta N &= A \times \Delta t = 2.8 \times 10^7 \times 120 \\ &= 3.36 \times 10^9 \approx 3.4 \times 10^9\end{aligned}$$

We have assumed that the activity remains constant over a period of 2.0 minutes.

6 A sample consists of 1000 undecayed nuclei of a nuclide whose decay constant is 0.20 s^{-1}. Determine the initial activity of the sample. Estimate the activity of the sample after 1.0 s.

Step 1 Since activity $A = \lambda N$, we have:

$A = 0.20 \times 1000 = 200\ s^{-1} = 200$ Bq

Step 2 After 1.0 s, we might expect 800 nuclei to remain undecayed.

The activity of the sample would then be:

$A = 0.2 \times 800 = 160\ s^{-1} = 160$ Bq

(In fact, it would be slightly higher than this. Since the rate of decay decreases with time all the time, less than 200 nuclei would decay during the first second.)

Count rate

Although we are often interested in finding the activity of a sample of radioactive material, we cannot usually measure this directly. This is because we cannot easily detect **all** of the radiation emitted. Some will escape past our detectors, and some may be absorbed within the sample itself. A (Geiger–Muller) GM tube placed in front of a radioactive source therefore only detects a fraction of the activity. The further it is from the source, the smaller the count rate. Therefore, our measurements give a received **count rate** R that is significantly lower than the activity A. If we know how efficient our detecting system is, we can deduce A from R. If the level of background radiation is significant, then it must be subtracted to give the **corrected** count rate.

Questions

13 A sample of carbon-15 initially contains 500 000 undecayed nuclei. The decay constant for this isotope of carbon is 0.30 s^{-1}.

Calculate the initial activity of the sample.

14 A small sample of radium gives a received count rate of 20 counts per minute in a detector. It is known that the counter detects only 10% of the decays from the sample. The sample contains 1.5×10^9 undecayed nuclei. Calculate the decay constant of this form of radium.

15 A radioactive sample is known to emit α-, β- and γ-radiations.

Suggest **four** reasons why the count rate measured by a Geiger counter placed next to this sample would be lower than the activity of the sample.

29.7 Decay graphs and equations

The activity of a radioactive substance gradually diminishes as time goes by. The atomic nuclei emit radiation and become different substances. The pattern of radioactive decay is an example of a very important pattern found in many different situations, a pattern called **exponential decay**. Figure 29.9 shows the decay graphs for three different isotopes, each with a different rate of decay.

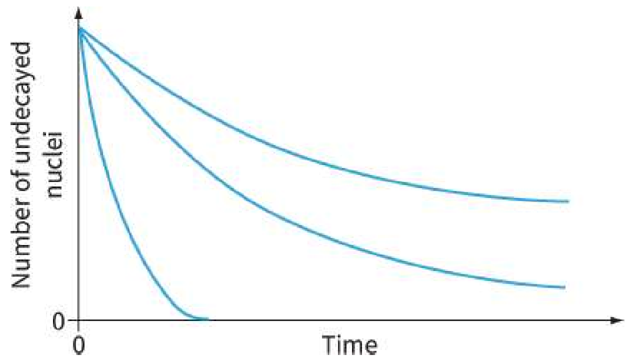

Figure 29.9: Some radioactive materials decay faster than others.

Although the three graphs look different, they all have something in common – their shape. They are curved lines having a special property. If you know what is meant by the **half-life** of an isotope, then you will understand what is special about the shape of these curves.

The half-life $t_{\frac{1}{2}}$ of an isotope is the mean time taken for half of the active nuclei in a sample to decay.

In a time equal to one half-life, the activity of the sample will also halve. This is because activity is directly proportional to the number of undecayed nuclei ($A \propto N$). It takes the same amount of time again for half of the remainder of the nuclei to decay, and a third half-life for half of the new remainder to decay (Figure 29.10).

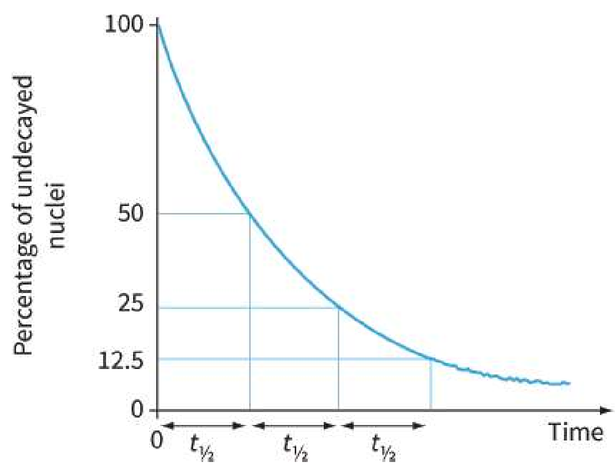

Figure 29.10: All radioactive decay graphs have the same characteristic shape.

In principle, the graph never reaches zero; it just gets closer and closer. In practice, when only a few undecayed nuclei remain, the graph will cease to be a smooth curve (because of the random nature of the decay) and it will eventually reach zero. We use the idea of half-life because we cannot say when a sample will have completely decayed.

Mathematical equations for radioactive decay

We can write an equation to represent the graph shown in Figure 29.10. If we start with N_0 undecayed nuclei, then the number N that remain undecayed after time t is given by:

$$N = N_0 \, e^{-\lambda t}$$

In this equation, λ is the decay constant of an isotope, as before. (You may also see this written as $N = N_0 \exp(-\lambda t)$.) Note that you must take care with units. If λ is in s^{-1}, then the time t must be in s.

The symbol e represents the number e = 2.71828..., a special number in the same way that π is a special number. You will need to be able to use the e^x button on your calculator to solve problems involving e.

PRACTICAL ACTIVITY 31.1

Determining half-life

If you are to determine the half-life of a radioactive substance in the laboratory, you need to choose something that will not decay too quickly or too slowly. In practice, the most suitable isotope is protactinium-234, which decays by emitting β^--radiation. This is available in a bottle containing a solution of a uranium compound (uranyl(VI) nitrate) (Figure 29.11). By shaking the bottle, you can separate the protactinium into the top layer of solvent in the bottle. The counter allows you to measure the decay of the protactinium.

After recording the number of counts in consecutive 10-second intervals over a period of a few minutes, you can then draw a graph, and use it to find the half-life of protactinium-234.

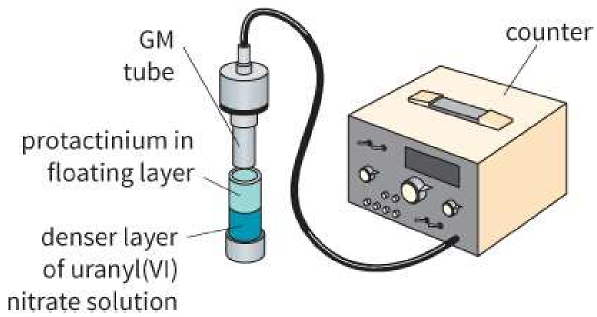

Figure 29.11: Practical arrangement for observing the decay of protactinium-234.

The activity A of a sample is directly proportional to the number of undecayed nuclei N. Hence the activity of the sample decreases exponentially:

$$A = A_0 e^{-\lambda t} \quad (A_0 \text{ is the activity at time } t = 0.)$$

Usually, we measure the corrected count rate R in the laboratory rather than the activity or the number of undecayed nuclei. Since the count rate is a fraction of the activity, it too decreases exponentially with time:

$$R = R_0 e^{-\lambda t} \quad (R_0 \text{ is the corrected count rate at time } t = 0.)$$

KEY EQUATION

$$x = x_0 e^{-\lambda t}$$

where x can represent activity A, number of undecayed nuclei N or received count rate R.

(λ is the decay constant and x is the quantity left at time t.)

Now look at Worked examples 7 and 8.

WORKED EXAMPLES

7 Suppose we start an experiment with 1.0×10^{15} undecayed nuclei of an isotope for which λ is equal to $0.02\ s^{-1}$. Determine the number of undecayed nuclei after 20 s.

Step 1 In this case, we have $N_0 = 1.0 \times 10^{15}$, $\lambda = 0.02\ s^{-1}$ and $t = 20$ s. Substituting in the equation gives:

$N = 1.0 \times 10^{15}\ e^{-0.02 \times 20}$

Step 2 Use the e^x button and calculate N.

$N = 1.0 \times 10^{15}\ e^{-0.02 \times 20} = 6.7 \times 10^{14}$

8 A sample initially contains 1000 undecayed nuclei of an isotope whose decay constant $\lambda = 0.10$ min $^{-1}$. Draw a graph to show how the sample will decay over a period of 10 minutes.

Step 1 We have $N_0 = 1000$ and $\lambda = 0.10\ \text{min}^{-1}$. Hence, we can write the equation for this decay:

$N = 1000\ e^{-0.10 \times t}$

Step 2 Calculate values of the number N of undecayed nuclei at intervals of 1.0 min (60 s); this gives Table 29.4 and the graph shown in Figure 29.12.

t / min	0	1.0	2.0	3.0	4.0	5.0
N	1000	905	819	741	670	607
t / min	6.0	7.0	8.0	9.0	10.0	
N	549	497	449	407	368	

Table 29.4: For Worked example 8.

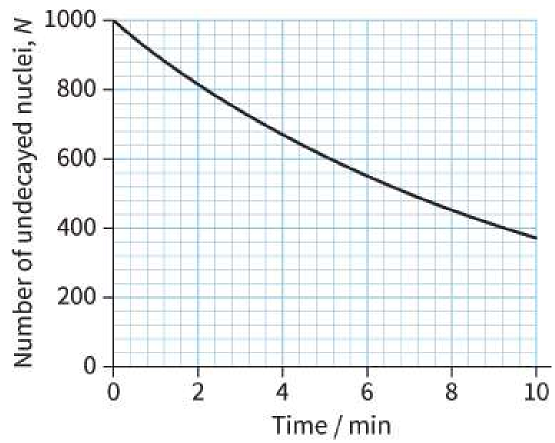

Figure 29.12: Radioactive decay graph.

Questions

16 The isotope nitrogen-13 has a half-life of 10 min. A sample initially contains 8.0×10^{10} undecayed nuclei.

a Write down an equation to show how the number undecayed N depends on time t.

b Calculate how many undecayed nuclei will remain after 10 min, and after 20 min.

c Determine how many nuclei will decay during the first 30 min.

17 A sample of an isotope for which $\lambda = 0.10\ \text{s}^{-1}$ contains 5.0×10^9 undecayed nuclei at the start of an experiment. Determine:

a the number of undecayed nuclei after 50 s

b its activity after 50 s.

18 The value of λ for protactinium-234 is $9.6 \times 10^{-3}\ \text{s}^{-1}$. Table 29.5 shows the number of undecayed nuclei N in a sample.

Copy and complete Table 29.5. Draw a graph of N against t, and use it to find the half-life $t_{\frac{1}{2}}$ of protactinium-234.

t / s	0	20	40	60	80	100	120	140
N	400	330						

Table 29.5: Data for Question 18.

29.8 Decay constant λ and half-life $t_{\frac{1}{2}}$

An isotope that decays rapidly has a short half-life $t_{\frac{1}{2}}$. Its decay constant must be large, since the probability per unit time of an individual nucleus decaying must be high. What is the connection between the decay constant and the half-life?

$$\text{If } e^x = y\text{, then } x = \ln y$$

In a time equal to one half-life $t_{\frac{1}{2}}$, the number of undecayed nuclei is halved. Hence the equation:

$$N = N_0 e^{-\lambda t}$$

becomes:

$$\frac{N}{N_0} = e^{\left(-\lambda t_{\frac{1}{2}}\right)} = \frac{1}{2}$$

Therefore:

$$\begin{aligned} e^{\lambda t_{\frac{1}{2}}} &= 2 \\ \lambda t_{\frac{1}{2}} &= \ln 2 \\ &\approx 0.693 \end{aligned}$$

The half-life of an isotope and the decay constant are inversely proportional to each other. That is:

$$\begin{aligned} \lambda &= \frac{\ln 2}{t_{\frac{1}{2}}} \\ &= \frac{0.693}{t_{\frac{1}{2}}} \end{aligned}$$

Thus, if we know either $t_{\frac{1}{2}}$ or λ, we can calculate the other. For a nuclide with a very long half-life, we might not wish to sit around waiting to measure the half-life; it is easier to determine λ by measuring the activity (and using $A = \lambda N$) and use that to determine $t_{\frac{1}{2}}$.

Note that the units of λ and $t_{\frac{1}{2}}$ must be compatible; for example, λ in s^{-1} and $t_{\frac{1}{2}}$ in s.

KEY EQUATION

Half-life and decay constant are related as follows:

$$\begin{aligned} \lambda &= \frac{\ln 2}{t_{\frac{1}{2}}} \\ &= \frac{0.693}{t_{\frac{1}{2}}} \end{aligned}$$

Questions

19 Figure 29.13 shows the decay of an isotope of caesium, $^{134}_{55}Cs$. Use the graph to determine the half-life of this nuclide in years, and hence find the decay constant in $year^{-1}$.

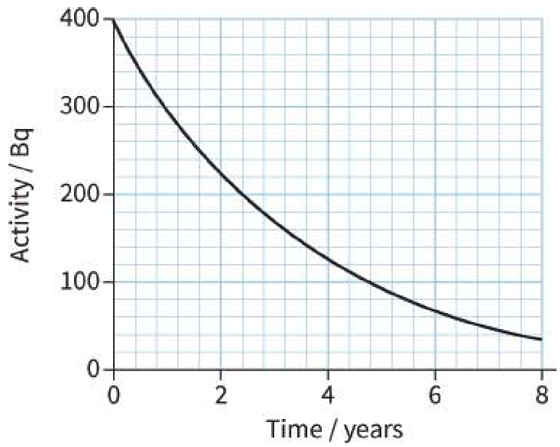

Figure 29.13: Decay graph for an isotope of caesium. For Question 19.

20 The decay constant of a particular isotope is $3.0 \times 10^{-4}\ s^{-1}$. Calculate how long it will take for the activity of a sample of this substance to decrease to one-eighth of its initial value.

21 The isotope ${}^{16}_{7}N$ decays with a half-life of 7.4 s.

a Calculate the decay constant for this nuclide.

b A sample of N initially contains 5000 nuclei. Calculate how many will remain after a time of:

i 14.8 s

ii 20.0 s.

22 A sample contains an isotope of half-life $t_{\frac{1}{2}}$.

a Show that the fraction *f* of nuclei in the sample that remain undecayed after a time *t* is given by the equation:

$f = \left(\frac{1}{2}\right)^n$ where $n = \frac{1}{t_{\frac{1}{2}}}$

b Calculate the fraction *f* after each of the following times:

i $t_{\frac{1}{2}}$

ii $2t_{\frac{1}{2}}$

iii $2.5t_{\frac{1}{2}}$

iv $8.3t_{\frac{1}{2}}$

REFLECTION

Without looking at your textbook, list all equations that contain decay constant λ.

What information can you get from the gradient of a graph of N against t?

Have a competition with a classmate. Use the internet for about 5 mins to find an isotope with the shortest half-life and the longest half-life.

What did this competition reveal about you as a learner?

SUMMARY

Nuclear reactions can be represented by equations of the form:

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e}$$

Einstein's mass–energy equation $\Delta E = \Delta mc^2$ relates mass changes to energy changes.

The mass defect is equal to the difference between the mass of the separate nucleons and that of the nucleus.

The mass of nuclear particles may be measured in atomic mass unit (u), where:

$$1\ \mathrm{u} \approx 1.660 \times 10^{-27}\ \mathrm{kg}$$

The binding energy of a nucleus is the minimum energy required to break up the nucleus into separate nucleons.

The binding energy per nucleon indicates the relative stability of different nuclides.

The variation of binding energy per nucleon shows that energy is released when light nuclei undergo fusion and when heavier nuclei undergo fission, because these processes increase the binding energy per nucleon and, hence, result in more stable nuclides.

Nuclear decay is a spontaneous and random process. This unpredictability means that count rates tend to fluctuate, and we have to measure average quantities.

The half-life $t_{\frac{1}{2}}$ of an isotope is the mean time taken for half of the active nuclei in a sample to decay.

The decay constant λ is the probability that an individual nucleus will decay per unit time interval.

The activity A of a sample is related to the number of undecayed nuclei in the sample N by: $A = \lambda N$

The decay constant and half-life are related by the equation:

$$\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{0.693}{t_{\frac{1}{2}}}$$

We can represent the exponential decrease of a quantity with time t by an equation of the form:

$$x = x_0\, \mathrm{e}^{-\lambda t}$$

where x can be activity A, count rate R or number of undecayed nuclei N.

EXAM-STYLE QUESTIONS

1 Which expression is correct for determining the energy (in electronvolt eV) produced from a mass change of 1 u? **[1]**

A $1.0 \times (3.00 \times 10^8)^2$

B $1.66 \times 10^{-27} \times (3.00 \times 10^8)^2$

C $1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 \times 1.60 \times 10^{-19}$

D $1.66 \times 10^{-27} \times \frac{(3.00\times10^8)^2}{1.60\times10^{-19}}$

2 A student determines the half-life of an isotope to be 66 ± 5 s.

What is the absolute uncertainty in the decay constant? **[1]**

A $8.0 \times 10^{-4}\ s^{-1}$

B $1.1 \times 10^{-3}\ s^{-1}$

C $5.3 \times 10^{-2}\ s^{-1}$

D $7.6 \times 10^{-2}\ s^{-1}$

3 An antiproton is identical to a proton except that it has negative charge. When a proton and an antiproton collide, they are annihilated and two photons are formed. In annihilation, all the mass of the particles is converted into energy.

a Calculate the energy released in the reaction. **[3]**

b Calculate the energy released if 1 mole of protons and 1 mole of antiprotons were annihilated by this process. **[3]**

(Mass of a proton = mass of an antiproton = 1.67×10^{-27} kg.)

[Total: 6]

4 Calculate the mass that would be annihilated to release 1 J of energy. **[2]**

5 In a nuclear reactor, the mass converted to energy takes place at a rate of 70 $\mu g\ s^{-1}$. Calculate the maximum power output from the reactor assuming that it is 100% efficient. **[3]**

6 The equation shows the radioactive decay of radon-222.

$${}^{222}_{86}Rn \rightarrow {}^{218}_{84}Po + {}^{4}_{2}\alpha + \gamma$$

Calculate the total energy output from this decay and state what forms of energy are produced. **[6]**

(Mass of ${}^{222}_{86}Rn$ = 221.970 u, mass of ${}^{222}_{84}Po$ = 217.963 u, mass of ${}^{4}_{2}\alpha$ = 4.002u, 1 u is the unified atomic mass unit = 1.660×10^{-27} kg.)

(Hint: find the mass defect in u, then convert to kg.)

7 A carbon-12 atom consists of six protons, six neutrons and six electrons. The unified atomic mass unit (u) is defined as $\frac{1}{12}$ the mass of the carbon-12 atom.

Calculate:

a the mass defect in kilograms **[2]**

b the binding energy **[2]**

c the binding energy per nucleon. **[2]**

(Mass of a proton = 1.007 276 u, mass of a neutron = 1.008 665 u, mass of an electron = 0.000 548 u.)

[Total: 6]

8 The fusion reaction that holds most promise for the generation of electricity is the fusion of tritium ${}^{3}_{1}H$ and deuterium ${}^{2}_{1}H$. The following equation shows the process:

$${}^{3}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + {}^{1}_{1}H$$

Calculate:

a the change in mass in the reaction **[3]**

b the energy released in the reaction **[2]**

c the energy released if one mole of deuterium were reacted with one mole of tritium. **[2]**

(Mass of ${}^{3}_{1}H$ = 3.015 500 u, mass of ${}^{2}_{1}H$ = 2.013 553 u, mass of ${}^{4}_{2}H$ = 4.001 50 u; mass of ${}^{1}_{1}H$ = 1.007 276 u.)

[Total: 7]

9 The initial activity a sample of 1 mole of radon-220 is $8.02 \times 10^{21}\ s^{-1}$. Calculate:

a the decay constant for this isotope **[3]**

b the half-life of the isotope. **[2]**

[Total: 5]

10 The graph of count rate against time for a sample containing indium-116 is shown.

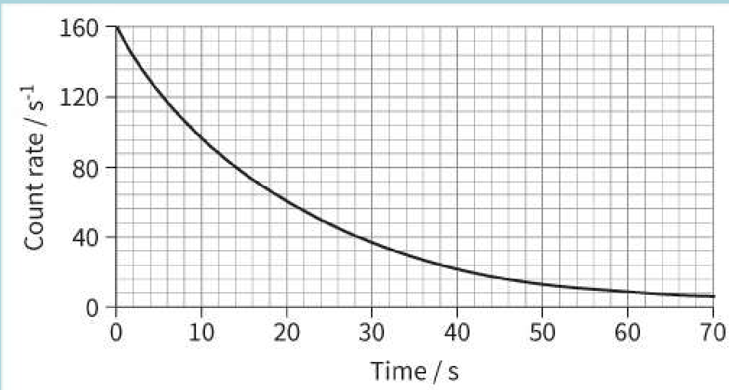

Figure 29.14

a Use the graph to determine the half-life of the isotope. **[2]**

b Calculate the decay constant. **[2]**

[Total: 4]

11 The proportions of different isotopes in rocks can be used to date the rocks. The half-life of uranium-238 is 4.9×10^{9} years. A sample has 99.2% of the proportion of this isotope compared with newly formed rock.

a Calculate the decay constant in y^{-1} for this isotope of uranium. **[2]**

b Calculate the age of the rock in years. **[3]**

[Total: 5]

12 The table shows the received count rate when a sample of the isotope vanadium-52 decays.

Time / min	0	1	2	3	4	5	6	7	8
Count rate / s^{-1}	187	159	134	110	85	70	60	56	40

Table 29.6

a **i** Sketch a graph of the count rate against the time. **[2]**

ii Comment on the scatter of the points. **[1]**

b From the graph, determine the half-life of the isotope. **[1]**

c Describe the changes to the graph that you would expect if you were given a larger sample of the isotope. **[2]**

[Total: 6]

13 This question is about the nucleus of uranium-235 (${}^{235}_{92}U$), which has a mass of 3.89×10^{-25} kg.

a State the number of protons and neutrons in this nucleus. **[1]**

b The radius r of a nucleus is given by the equation:

$r = 1.41 \times 10^{-15} A^{\frac{1}{3}}$

where A is the nucleon number of the nucleus.

Calculate the density of the $^{235}_{92}\mathrm{U}$ nucleus. **[3]**

c Explain why the total mass of the nucleons is different from the mass of the U nucleus. **[2]**

d Without calculations, explain how you can determine the binding energy per nucleon for the uranium-235 nucleus from its mass and the masses of a proton and a neutron. **[4]**

[Total: 10]

14 **a** Explain what is meant by **nuclear fusion** and explain why it only occurs at very high temperatures. **[3]**

b The main reactions that fuel the Sun are the fusion of hydrogen nuclides to form helium nuclides. However, other reactions do occur. In one such reaction, known as the triple alpha process, three helium nuclei collide and fuse to form a carbon-12 nucleus.

i Explain why temperatures higher than those required for the fusion of hydrogen are needed for the triple alpha process. **[1]**

ii Calculate the energy released in the triple alpha process. **[3]**

(Mass of a helium $\left(^{4}_{2}\mathrm{He}\right)$ nucleus = 4.001 506 u, mass of a carbon $\left(^{12}_{6}\mathrm{C}\right)$ nucleus = 12.000 000 u, 1 u = 1.660×10^{-27}kg.)

[Total: 7]

15 The isotope of polonium, $^{218}_{81}\mathrm{Po}$, decays by the emission of an α-particle with a half-life of 183 s.

a In an accident at a reprocessing plant some of this isotope, in the form of dust, is released into the atmosphere.

Explain why a spillage in the form of a dust is far more dangerous to health than a liquid spillage. **[2]**

b It is calculated that 2.4 g of the isotope is released into the atmosphere. The molar mass of polonium is 218 g mol^{-1}.

Calculate the initial activity of the released polonium. **[4]**

c It is felt that it would safe to re-enter the laboratory when the activity falls to background, about 10 Bq.

Calculate how many hours must pass before it is safe to re-enter the laboratory. **[3]**

[Total: 9]

16 A nuclear reactor is fuelled by fission of uranium. The output from the reactor is 200 MW. The following equation describes a typical fission reaction:

$$^{235}_{92}\mathrm{U} + ^{1}_{0}\mathrm{n} \rightarrow ^{236}_{92}\mathrm{U} \rightarrow ^{87}_{35}\mathrm{Br} + ^{146}_{57}\mathrm{La} + 3^{1}_{0}\mathrm{n}$$

a State and explain into what form the majority of the energy released in the reaction is transformed. **[2]**

b **i** Calculate the energy released in the reaction. The kinetic energy of the captured neutron is negligible. **[2]**

ii Assume that the energy released in this fission is typical of all fissions of U-236. Calculate how many fissions occur each second. **[1]**

iii Calculate the mass of uranium-235 that is required to run the reactor for 1 year. **[3]**

(Mass of $^{235}_{92}\mathrm{U}$ = 3.90×10^{-25} kg, mass of $^{87}_{35}\mathrm{Br}$ = 1.44×10^{-25} kg, mass of $^{146}_{57}\mathrm{La}$ = 2.42×10^{-25} kg, mass of neutron = 1.67×10^{-27} kg, 1 year = 3.15×10^{7} s, molar mass of uranium-235 = 235 g mol^{-1}.)

[Total: 8]