Chapter 27

Alternating currents

LEARNING INTENTIONS

In this chapter you will learn how to:

- understand and use the terms period, frequency and peak value as applied to an alternating current or voltage
- use equations of the form $x = x_0 \sin \omega t$ representing a sinusoidally alternating current or voltage
- recall and use the fact that the mean power in a resistive load is half the maximum power for a sinusoidal alternating current
- distinguish between root-mean-square (r.m.s.) and peak values and recall and use $I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}$ and $V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}$ for a sinusoidal alternating current
- distinguish graphically between half-wave and full-wave rectification
- explain the use of a single diode for the half-wave rectification of an alternating current
- explain the use of four diodes (bridge rectifier) for the full-wave rectification of an alternating current
- analyse the effect of a single capacitor in smoothing, including the effect of the value of capacitance and the load resistance.

BEFORE YOU START

- In pairs, try to recall and explain the relationship for power dissipation in terms of current, potential difference and resistance from Chapter 8.
- The physics of alternating currents has similarities with simple harmonic motion (see Chapter 18). Discuss what you remember about period, frequency and angular frequency.
- Write down what you know about the behaviour of diodes in circuits. What's the most important property of a diode?

- Discuss the discharge of a capacitor through a resistor. Can you remember the factors that affect the time constant of a circuit?

DESCRIBING ALTERNATING CURRENT

In many countries, mains electricity is a supply of alternating current (a.c.). The first mains electricity supplies were developed towards the end of the 19th century; at that time, a great number of different voltages and frequencies were used in different places. In some places, the supply was direct current (d.c.). Nowadays, this has been standardised across much of the world, with standard voltages of 110 V or 230 V (or similar), and frequencies of 50 Hz or 60 Hz.

Mains electricity is transported along many kilometres of high-voltage power lines (cables). Transformers are used for stepping-up and stepping-down alternating voltages between the power stations and the consumers (Figure 27.1). From your prior knowledge of transformers and transmission of electrical energy, can you remember why it is necessary for power lines to use high voltage?

Figure 27.1: This engineer is working on a transformer used for increasing (stepping-up) the size of the alternating voltage to help with the transportation of electrical energy.

27.1 Sinusoidal current

An alternating current can be represented by a graph such as that shown in Figure 27.2. This shows that the current varies regularly. During half of the cycle, the current is positive, and in the other half it is negative. This means that the direction of the current reverses every half cycle. Whenever you use a mains appliance, the charges (free electrons) within the wire and appliance flow backwards and forwards. At any instant in time, the current has a particular magnitude and direction given by the graph.

The graph has the same shape as the graphs used to represent simple harmonic motion (s.h.m.) (see Chapter 18), and it can be interpreted in the same way. In a wire with a.c., the free electrons within the wire move back and forth with s.h.m. The variation of the current with time is a sine curve, so it is described as **sinusoidal**. (In principle, any current whose direction changes between positive and negative can be described as **alternating**, but we will only be concerned with those that have a regular, sinusoidal pattern.)

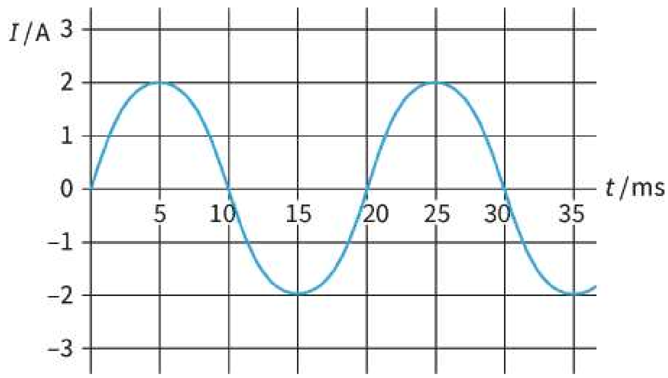


Figure 27.2: A graph to represent a sinusoidal alternating current.

An equation for a.c.

As well as drawing a graph, we can write an equation to represent alternating current. This equation gives us the value of the current I at any time t:

$$I = I_0 \sin \omega t$$

where I is the current at time t, I_0 is the **peak value** of the alternating current and ω is the angular frequency of the supply, measured in rad s^{-1} (radians per second). The peak value is the maximum magnitude of the current. It's very much like the 'amplitude' of the alternating current, except the unit is that of current.

This is related to the frequency f in the same way as for s.h.m.:

$$\omega = 2\pi f$$

and the frequency and period are related by:

$$f = \frac{1}{T}$$

KEY EQUATION

Alternating current:

$$I = I_0 \sin \omega t$$

Remember that your calculator must be in the radian mode when using this equation.

Questions

1 The following questions relate to the graph in Figure 27.2.

a State the value of the current I and its direction when time t = 5 ms.

b Determine the time the current next has the same value, but negative.

c State the time T for one complete cycle (the period of the a.c).

d Determine the frequency of this alternating current.

2 The following questions relate to the graph in Figure 27.2.

a Determine the values of I_0 and ω.

b Write an equation to represent this alternating current.

3 An alternating current, measured in amperes (A), is represented by the equation: $I = 5.0 \sin (120\pi t)$

a Determine the values of I_0, ω, f and T.

b Sketch a graph to represent the current.

27.2 Alternating voltages

Alternating current is produced in power stations by large generators like those shown in Figure 27.3.

Figure 27.3: Generators in the generating hall of a large power station.

As you have already seen in Chapter 26, a generator consists of a coil rotating in a magnetic field. An e.m.f. is induced in the coil according to Faraday's and Lenz's laws of electromagnetic induction.

This e.m.f. V varies sinusoidally, and so we can write an equation to represent it that has the same form as the equation for alternating current:

$$V = V_0 \sin \omega t$$

where V_0 is the **peak value** of the voltage. We can also represent this graphically, as shown in Figure 27.4.

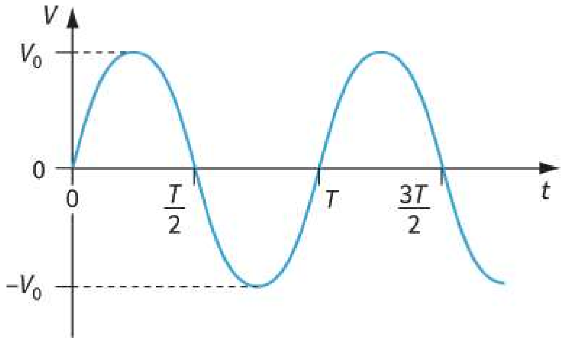


Figure 27.4: An alternating voltage.

Question

4 An alternating voltage V, in volt (V), is represented by the equation:

$V = 300 \sin (100\pi t)$

- **a** Determine the values of V_0, ω and f for this alternating voltage.
- **b** Calculate V when $t = 0.002$ s. (Remember that $100\pi t$ is in radians when you calculate this.)
- **c** Sketch a graph to show **two** complete cycles of this voltage.

Measuring frequency and voltage

An oscilloscope can be used to measure the frequency and voltage of an alternating current. Practical Activity 27.1 explains how to do this. There are two types of oscilloscope. The traditional cathode-ray oscilloscope (CRO) uses an electron beam. The alternative is a digital oscilloscope, which is likely to be much more compact and which can store data and display the traces later.

PRACTICAL ACTIVITY 27.1 MEASUREMENTS USING AN OSCILLOSCOPE

A CRO is an electron beam tube, as shown in Figure 25.4, but with an extra set of parallel plates to produce a horizontal electric field at right angles to the beam (Figure 27.5).

The principles of a cathode-ray oscilloscope (CRO)

The signal into the CRO is a repetitively varying voltage. This is applied to the *y*-input, which deflects the beam up and down using the parallel plates Y_1 and Y_2 shown in Figure 27.5. The time-base produces a p.d. across the other set of parallel plates X_1 and X_2 to move the beam from left to right across the screen.

When the beam hits the screen of the CRO, it produces a small spot of light. If you look at the screen and slow the movement down, you can see the spot move from left to right, while the applied signal moves the spot up and down. When the spot reaches the right side of the screen, it flies back very quickly and waits for the next cycle of the signal to start before moving to the right once again. In this way, the signal is displayed as a stationary trace on the screen. There may be many controls on a CRO, even more than those shown on the CRO illustrated in Figure 27.6.

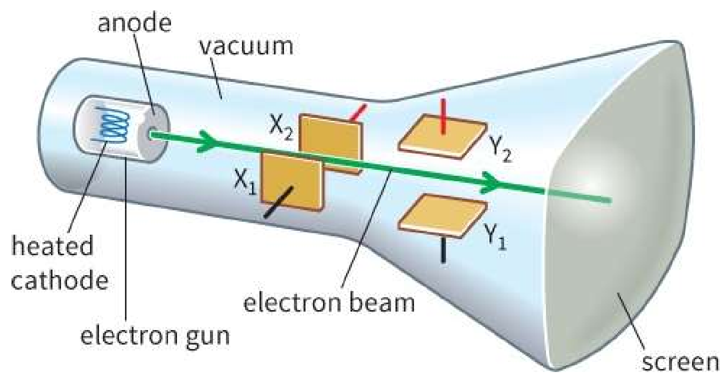


Figure 27.5: The construction of a cathode-ray oscilloscope. Cathode rays (beams of electrons) are produced in the electron gun and then deflected by electric fields before they strike the screen.

The controls

The X-shift and the Y-shift controls move the whole trace in the *x*-direction and the *y*-direction, respectively. The two controls that you must know about are the time-base and the Y-gain, or Y-sensitivity.

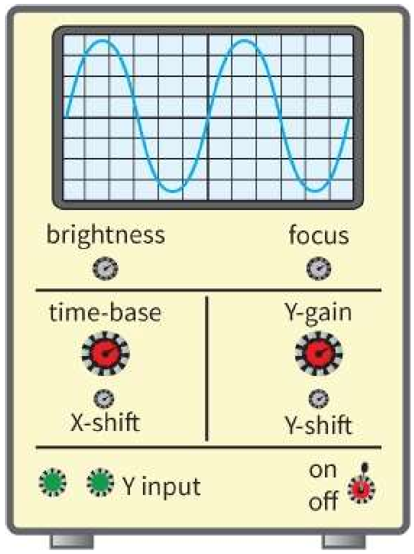


Figure 27.6: The controls of a typical CRO.

You can see in Figure 27.6 that the time-base control has units marked alongside. Let us suppose that this reads 5 ms/cm, although it might be 5 ms/division. This shows that 1 cm (or 1 division) on the *x*-axis represents 5 ms. Varying the time-base control alters the speed with which the spot moves across the screen. If the time-base is changed to 1 ms/cm, then the spot moves faster and each centimetre represents a smaller time.

The Y-gain control has a unit marked in volts/cm, or sometimes volts/division. If the actual marking is 5 V/cm, then each centimetre on the *y*-axis represents 5 V in the applied signal.

It is important to remember that on the CRO screen, the *x*-axis represents **time** and the *y*-axis represents **voltage**.

Determining frequency and amplitude (peak value of voltage)

If you look at the CRO trace shown in Figure 27.7, you can see that the amplitude of the waveform, or the peak value of the voltage, is equivalent to 2 cm and the period of the trace is equivalent to 4 cm.

If the Y-gain or Y-sensitivity setting is 2 V/cm, then the peak voltage is 2 × 2 = 4 V. If the time-base setting is 5 ms/cm, then the period is 4 × 5 = 20 ms.

In the example:

$$\begin{aligned} \text{frequency} &= \frac{1}{\text{period}} \\ &= \frac{1}{0.02} \\ &= 50 \text{ Hz} \end{aligned}$$

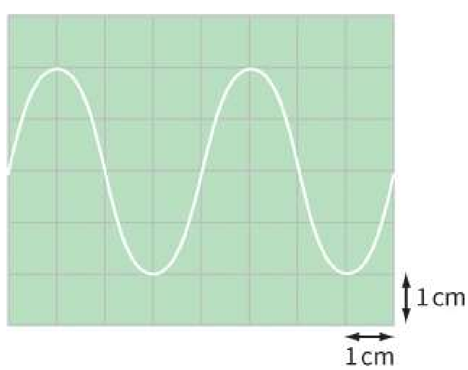


Figure 27.7: A typical trace on the screen of a CRO.

Questions

5 The Y-sensitivity and time-base settings are 5 V/cm and 10 ms/cm. The trace seen on the CRO screen is the one shown in Figure 27.7.

Determine the amplitude, period and frequency of the signal applied to the Y-input of the CRO.

6 Sketch the CRO trace for a sinusoidal voltage of frequency 100 Hz and amplitude 10 V, when the time-base is 10 ms/cm and the Y-sensitivity is 10 V/cm.

27.3 Power and alternating current

We use mains electricity to supply us with energy. If the current and voltage are varying all the time, does this mean that the power is varying all the time too? The answer to this is yes. You may have noticed that some fluorescent lamps flicker continuously, especially if you observe them out of the corner of your eye or when you move your head quickly from one side to the other. A tungsten filament lamp would flicker too, but the frequency of the mains has been chosen so that the filament does not have time to cool down noticeably between peaks in the supply.

Root-mean-square (r.m.s.) values

There is a mathematical relationship between the peak value V_0 of the alternating voltage and a direct voltage that delivers the same average electrical power. The direct voltage is about 70% of V_0. (You might have expected it to be about half, but it is more than this, because of the shape of the sine graph.) This steady direct voltage is known as the **root-mean-square (r.m.s.) value** of the alternating voltage. In the same way, we can think of the root-mean-square value of an alternating current, $I_{\text{r.m.s}}$.

The r.m.s. value of an alternating current is that steady current that delivers the same average power as the a.c. to a resistive load.

The lamps in Practical Activity 27.2 are the 'resistive loads'. A full analysis, which we will come to shortly, shows that $I_{\text{r.m.s.}}$ is related to I_0 by:

$$\begin{aligned} I_{\text{r.m.s.}} &= \frac{I_0}{\sqrt{2}} \\ &\approx 0.707 \times I_0 \end{aligned}$$

This is where the factor of 70% comes from. Note that this factor only applies to sinusoidal alternating currents.

We also have r.m.s. voltage $V_{\text{r.m.s.}}$ across the resistive load. $V_{\text{r.m.s}}$ is related to the peak voltage V_0 by:

$$V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}$$

KEY EQUATIONS

Root-mean-square value:

$$\begin{aligned} I_{\text{r.m.s.}} &= \frac{I_0}{\sqrt{2}} \\ &\approx 0.707 \times I_0 \end{aligned}$$

where I_0 is the peak (maximum) current.

$$\begin{aligned} V_{\text{r.m.s.}} &= \frac{V_0}{\sqrt{2}} \\ &\approx 0.707 \times V_0 \end{aligned}$$

where V_0 is the peak (maximum) voltage.

PRACTICAL ACTIVITY 27.2

Comparing alternating current (a.c.) and direct current (d.c.)

Because the power supplied by an alternating current is varying all the time, we need to have some way of describing the **average power** that is being supplied. To do this, we compare an alternating current with a direct current, and try to find the direct current that supplies the same average power as the alternating current.

Figure 27.8 shows how this can be done in practice. Two filament lamps (our resistive loads) are placed side by side; one is connected to an a.c. supply (on the right) and the other to a d.c. supply (the batteries on the left). The a.c. supply is adjusted so that the two lamps are equally bright, indicating that the two supplies are providing energy at the same average rate. The output voltages are then compared on the double-beam oscilloscope.

A typical trace is shown in Figure 27.9. This shows that the a.c. trace sometimes rises above the steady d.c. trace, and sometimes falls below it. This makes sense: sometimes the a.c. is delivering more power than the d.c., and sometimes less, but the average power is the same for both.

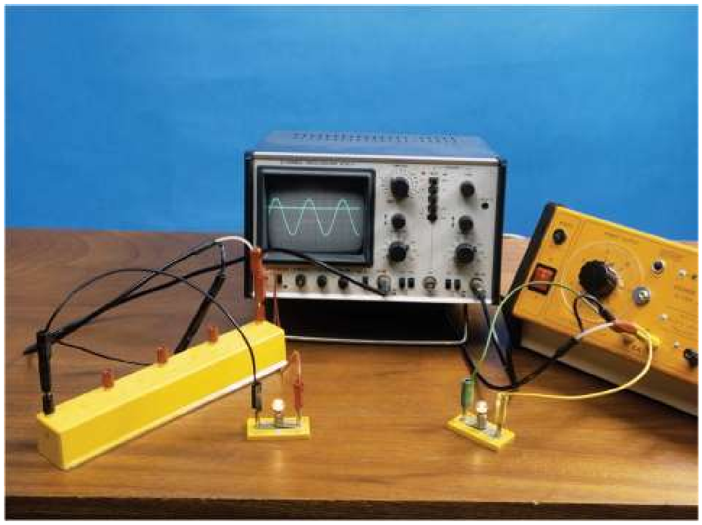


Figure 27.8: Comparing direct and alternating currents that supply the same power. The lamps are equally bright.

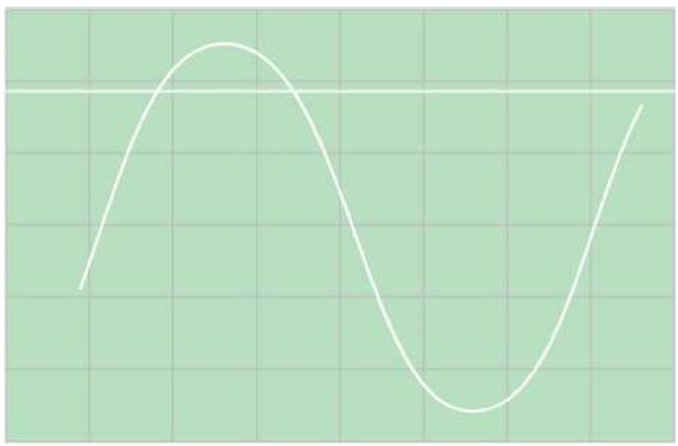


Figure 27.9: The oscilloscope trace from the experiment shown in Figure 27.8.

Questions

7 The alternating current (in ampere, A) in a resistor is represented by the equation: $I = 2.5 \sin(100\pi t)$

Calculate the r.m.s. value for this alternating current.

8 The mains supply to domestic consumers in many European countries has an r.m.s. value of 230 V for the alternating voltage. (Note that it is the r.m.s. value that is generally quoted, not the peak value.)

Calculate the peak value of the alternating voltage.

Calculating power

The importance of r.m.s. values is that they allow us to apply equations from our study of direct current to situations where the current is alternating. So, to calculate the average power dissipated in a resistor, we can use the usual formulae for power:

$$P = I^2 R = IV = \frac{V^2}{R}$$

Remember that it is essential to use the r.m.s. values of I and V, as in Worked example 1. If you use peak values, your answer will be too great by a factor of 2.

Where does this factor of 2 come from? Recall that r.m.s. and peak values are related by:

$$I_0 = \sqrt{2} I_{r.m.s}$$

So, if you calculate I^2R using I_0 instead of $I_{r.m.s.}$, you will introduce a factor of $\left(\sqrt{2}\right)^2$ or 2. The same is true if you calculate power using V_0 instead of $V_{r.m.s.}$. It follows that, for a sinusoidal alternating current, peak power is twice average power.

WORKED EXAMPLE

1 A 20 Ω resistor is connected to an alternating supply. The voltage across the resistor has peak value 25 V.

Calculate the average power dissipated in the resistor.

Step 1 Calculate the r.m.s. value of the voltage.

$$\begin{aligned} V_{r.m.s} &= \frac{V_0}{\sqrt{2}} \\ &= \frac{25}{\sqrt{2}} \\ &= 17.7\ \text{V} \end{aligned}$$

Step 2 Now calculate the average power dissipated. (Remember you must use the r.m.s. value, and not the peak value.)

$$\begin{aligned} P &= \frac{V^2}{R} \\ &= \frac{17.7^2}{20} \\ &= 15.6\ \text{W} \end{aligned}$$

Note that, if we had used V_0 rather than $V_{r.m.s.}$, we would have found:

$$\begin{aligned} P &= \frac{25^2}{20} \\ &= 31.3\ \text{W} \end{aligned}$$

which is double the correct answer.

Questions

9 Calculate the average power dissipated in a resistor of resistance 100 Ω when a sinusoidal alternating current has a peak value of 3.0 A.

10 The sinusoidal voltage across a 1.0 kΩ resistor has a peak value 325 V.

- **a** Calculate the r.m.s. value of the alternating voltage.
- **b** Use $V = IR$ to calculate the r.m.s. current in the resistor.
- **c** Calculate the average power dissipated in the resistor.
- **d** Calculate the **peak** power dissipated in the resistor.

Explaining root-mean-square

We will now briefly consider the origin of the term root-mean-square and show how the factor of $\sqrt{2}$ in the equation $I_0 = \sqrt{2} I_{r.m.s}$ comes about.

The equation $P = I^2R$ shows us that the power P is directly proportional to the square of the current I. Figure 27.10 shows how we can calculate $I^2$for an alternating current. The current I varies sinusoidally, and during half of each cycle it is negative. However, I^2 is always positive (because the square of a negative number is positive). Notice that I^2 varies up and down, and that it has twice the frequency of the current.

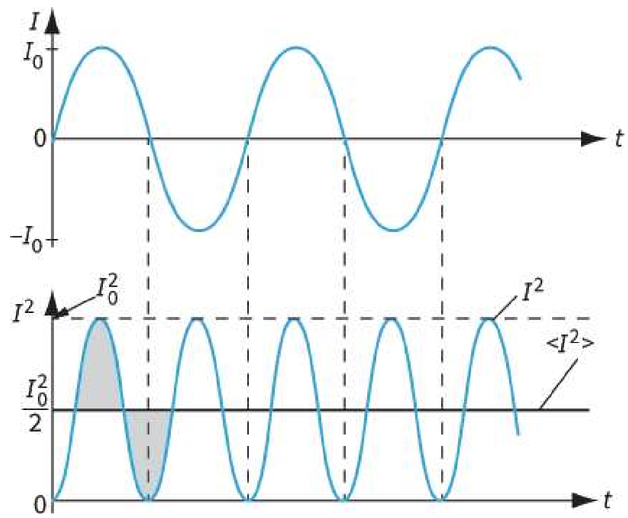


Figure 27.10: An alternating current I is alternately positive and negative, while I^2 is always positive.

Now, if we consider $<I^2>$, the average (mean) value of I^2, we find that its value is half of the square of the peak current (because the graph is symmetrical). That is:

$$< I^2 >= \frac{1}{2}{I_0}^2$$

To find the r.m.s. value of I, we now take the square root of $<I^2>$.

This gives:

$$I_{\text{r.m.s}} = \sqrt{< I^2 >} = \sqrt{\frac{1}{2}{I_0}^2}$$

$$\text{Or}$$

$$I_0 = \sqrt{2} I_{\text{r.m.s}}$$

Summarising this process: to find the r.m.s. value of the current, we find the **r**oot of the **m**ean of the **s**quare of the current – hence **r.m.s**.

27.4 Rectification

Many electrical appliances work with alternating current. Some, like electrical heaters, will work equally well with d.c. or a.c. However, there are many appliances, such as electronic equipment, which require d.c. For these, the alternating mains voltage must be converted to direct voltage by the process of **rectification**.

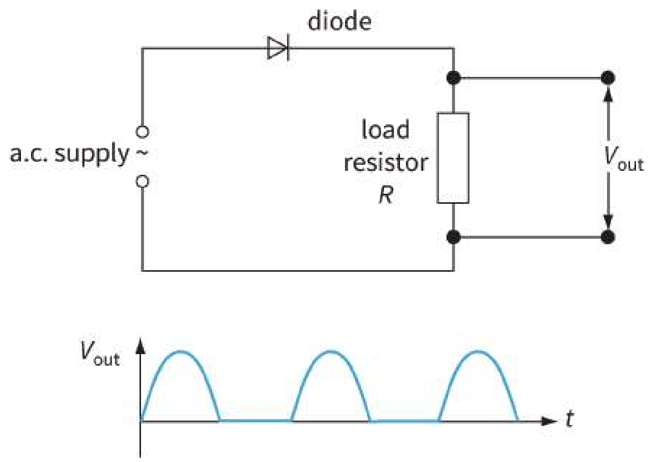


Figure 27.11: Half-wave rectification of a.c. requires a single diode.

A simple way to do this is to use a diode, which is a component that will only allow current in only one direction. (You have already met diodes in Chapter 10.) Figure 27.11 shows a circuit for doing this. An alternating input voltage is applied to a circuit with a diode and a resistor in series. The diode will only conduct during the positive cycles of the input voltage. Hence, there will be a current in the load resistor only during these positive cycles. The output voltage V_{out} across the resistor will fluctuate as shown in the V_{out} against time t graph. This graph is identical to the input alternating voltage, except the negative cycles have been 'chopped-off'.

This type of rectification is known as half-wave rectification. For one-half of the time the voltage is zero, and this means that the power available from a half-wave rectified supply is reduced.

The bridge rectifier

To overcome this problem of reduced power, a bridge rectifier circuit is used. This consists of four diodes connected across the input alternating voltage, as shown in Figure 27.12. The output voltage V_{out} is taken across the load resistor R. The resulting output voltage across the load resistor R is full-wave rectified.

The way in which this works is shown in Figure 27.13.

- During the **positive** cycles of the input voltage, A is positive and B is negative. The diodes 2 and 3 conduct because they are both in forward bias. The diodes 1 and 4 are in reverse bias, and therefore do not conduct. The current in the load resistor R will be downwards. Figure 27.13a shows the direction of the current.
- During the **negative** cycles of the input voltage, B is positive and A is negative. The diodes 4 and 1 conduct because they are now both in forward bias. The diodes 2 and 3 are in reverse bias, and therefore do not conduct. The current in the load resistor R will still be downwards. Figure 27.13b shows the direction of the current.

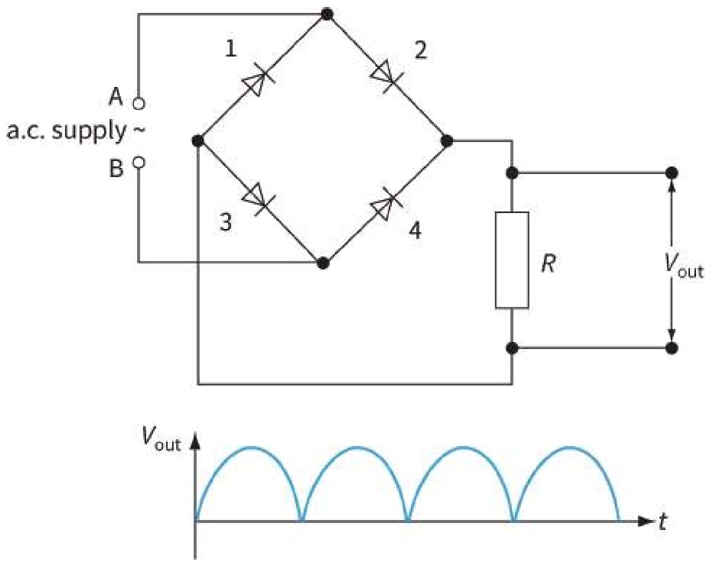


Figure 27.12: Full-wave rectification of a.c. using a diode bridge.

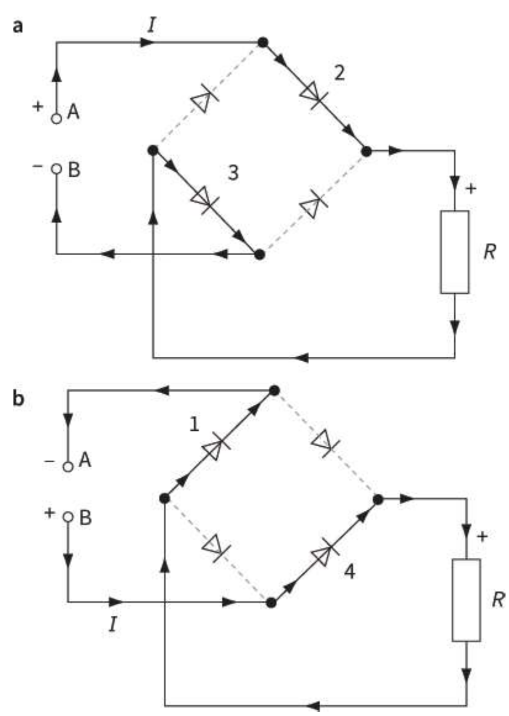


Figure 27.13: Direction of current during full-wave rectification **a** for positive cycles and **b** for negative cycles.

Note that in both positive and negative cycles, the current direction in the load resistor *R* is always the same (downwards). This means that the top end of *R* must always be positive.

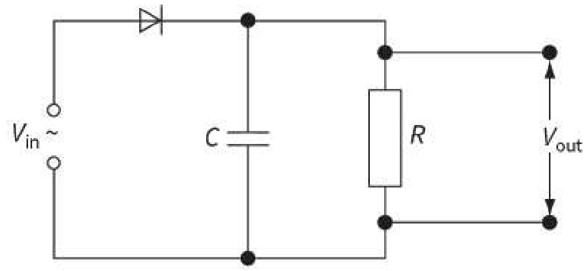
You can construct a bridge rectifier using light-emitting diodes (LEDs) that light up when current flows through them. By connecting this bridge to a slow a.c. supply (for instance, 1 Hz from a signal generator), you can see the sequence in which the diodes conduct during rectification.

Question

11 Explain why, when terminal B in Figure 27.13 is positive (during the negative cycles), only diodes 1 and 4 conduct.

Smoothing

In order to produce steady d.c. from the 'bumpy' d.c. that results from rectification, a smoothing capacitor is necessary in the circuit. This capacitor, of capacitance *C*, is in parallel with the load resistor of resistance *R*. This is shown in Figure 27.14. The idea is that the capacitor charges up and maintains the voltage at a high level. It discharges gradually when the rectified voltage drops, but the voltage soon rises again and the capacitor charges up again. The result is an output voltage with 'ripple'.



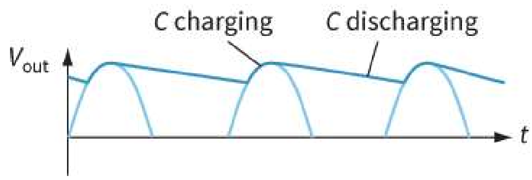


Figure 27.14: A smoothing capacitor is connected across (in parallel with) the load resistor.

The amount of ripple can be controlled by carefully choosing the capacitance *C* of the capacitor and the resistance *R* of the load resistor. A capacitor with a large capacitance value discharges more slowly than a capacitor with a small capacitance value, so will give a smaller ripple. Similarly, if the resistance *R* of the resistor is increased, then this too leads to a slower discharge of the capacitor. You may have already met the physics of discharging capacitors in Chapter 23. So, the size of the ripple can be reduced by increasing the time constant *CR* of the capacitor–resistor circuit. Ideally, though this is definitely not a general rule, *CR* must be much greater than the time interval between the adjacent peaks of the output signal – you want the capacitor to be still discharging between the 'gaps' between the positive cycles. This is illustrated in Worked example 2.

Note that, in Figures 27.11 to 27.14, we have represented the load on the supply by a resistor. This represents any components that are connected to the supply. For example, a rectifier circuit can be used to charge the battery of a mobile phone or provide a direct voltage supply for small radio.

WORKED EXAMPLE

2 Figure 27.15 shows the output voltage from a half-wave rectifier. The load resistor has resistance 1.2 kΩ. A student wishes to smooth the output voltage by placing a capacitor across the load resistor.

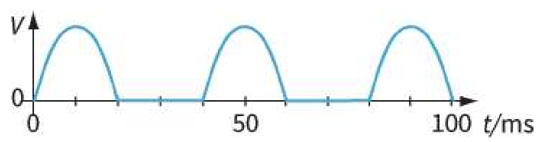


Figure 27.15: Output from a half-wave rectifier.

With the help of a calculation, suggest if a 10 pF capacitor or a 500 μF capacitor would be suitable for this task.

Step 1 Calculate the time constant with the 10 pF capacitor.

time constant = $CR = 10 \times 10^{-9} \times 1.2 \times 10^{3} = 1.2 \times 10^{-5}$ s (= 0.012 ms)

Step 2 Compare the time constant with the time interval between the adjacent peaks of the output signal.

The time constant of 0.012 ms is very small compared with time interval of 40 ms between the adjacent peaks of the output. If this capacitor were to be used, it would discharge far too quickly. There would be no smoothing of the output voltages – the 10 pF capacitor is not suitable.

Step 3 Repeat the steps for the 500 μF capacitor.

time constant = $CR = 500 \times 10^{-6} \times 1.2 \times 10^3 = 0.60$ s (= 600 ms)

Now, the time constant of 600 ms is much larger than 40 ms. This capacitor will not discharge completely between the positive cycles of the half-wave rectified signal. The 500 μF capacitor would be adequate for the smoothing task.

Questions

12 Sketch the following voltage patterns:

a a sinusoidal alternating voltage

b the same voltage as part **a**, but half-wave rectified

c the same voltage as part **b**, but smoothed

d the same voltage as part **a**, but full-wave rectified

e the same voltage as part **d**, but smoothed.

13 A student wires a bridge rectifier incorrectly as shown in Figure 27.16. Explain what you would expect to observe when an oscilloscope is connected across the load resistor *R*.

14 A bridge rectifier circuit is used to rectify an alternating current through a resistor. A smoothing capacitor is connected across the resistor. Figure 27.17 shows how the current varies. Use sketches to show the changes you would expect:

a if the resistance *R* of the resistor is increased

b if the capacitance *C* of the capacitor is decreased.

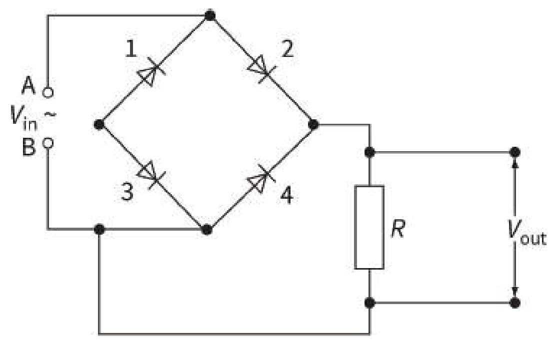


Figure 27.16: A bridge rectifier circuit that is wired incorrectly. For Question 13.

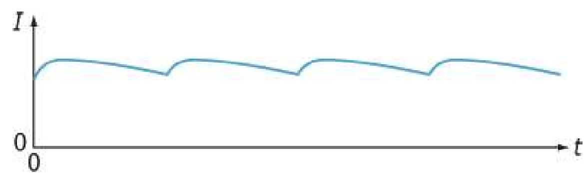


Figure 27.17: A smoothed, rectified current. For Question 14.

REFLECTION

Without looking at your textbook, summarise all the key equations from this chapter.

Make a list of mains operated devices in your laboratory. For each device, determine the power, r.m.s. current and r.m.s. voltage.

Give yourself and a classmate one minute to draw a circuit diagram for a full-wave rectifier circuit. Compare your circuit diagrams. Which diagram was more accurate? How would you make this diagram more accurate if you were to draw it in the future?

SUMMARY

A sinusoidal alternating current can be represented by $I = I_0 \sin \omega t$, where I_0 is the peak value of the current.
The root-mean-square (r.m.s.) value of an alternating current is that steady current that delivers the same average power as the a.c. to a resistive load; for a sinusoidal a.c.: $$I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}} \approx 0.707 \times I_0$$
The relationship between root-mean-square (r.m.s.) voltage $V_{\text{r.m.s}}$ and peak voltage V_0 is: $$V_{\text{r.m.s.}} = \frac{V_0}{\sqrt{2}}$$
The power P dissipated in a resistor can be calculated using the equations: $$P = VI,\ P = I^2R \text{ and } P = \frac{V^2}{R}$$ where V and I are the r.m.s. values of the voltage and current, respectively.
A single diode is used for the half-wave rectification of an alternating current. Four diodes (bridge rectifier) are used for the full-wave rectification of an alternating current.
A capacitor placed in parallel with a resistive load will smooth the rectified alternating voltage. The greater the time constant CR of the capacitor-resistor network, the smaller is the size of the ripple.

EXAM-STYLE QUESTIONS

1 The **maximum** power dissipated in a resistor carrying an alternating current is 10 W.

What is the mean power dissipated in the resistor? **[1]**

A 5.0 W

B 7.1 W

C 10 W

D 14 W

2 The alternating current *I* in ampere (A) in a filament lamp is represented by the equation:

$I = 1.5 \sin(40t)$.

Which of the following is correct? **[1]**

A The angular frequency of the alternating current is 40 rad s^{-1}.

B The frequency of alternating current is 40 Hz.

C The maximum current is 3.0 A.

D The peak voltage is 1.5 V.

3 Write down a general expression for the sinusoidal variation with time *t* of:

a an alternating voltage *V* **[1]**

b an alternating current *I* (you may assume that *I* and *V* are in phase) **[1]**

c the power *P* dissipated due to this current and voltage. **[1]**

[Total: 3]

4 The alternating current *I* in ampere (A) in a circuit is represented by the equation:

$I = 2.0 \sin(50\pi t)$.

a State the peak value of the current. **[1]**

b Calculate the frequency of the alternating current. **[2]**

c Sketch a graph to show **two** cycles of the variation of current with time. Mark the axes with suitable values. **[2]**

d Calculate $I_{r.m.s.}$, the r.m.s. value of current, and mark this on your graph in part c. **[1]**

e Determine **two** values of time *t* at which the current $I = I_{r.m.s.}$. **[3]**

[Total: 9]

5 A heater of resistance 6.0 Ω is connected to an alternating current supply. The output voltage from the supply is 20 V r.m.s.

Calculate:

a the average power dissipated in the heater **[2]**

b the maximum power dissipated in the heater **[1]**

c the energy dissipated by the heater in 5.0 minutes. **[2]**

[Total: 5]

6 An oscilloscope is used to display the variation of voltage across a 200 Ω resistor with time. The trace is shown. The time-base of the oscilloscope is set at 5 ms div^{-1} and the Y-gain at 0.5 V div^{-1}.

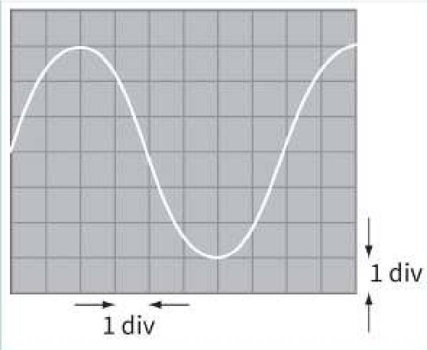


Figure 27.18

Determine:

a the period and hence the frequency of the alternating voltage **[2]**

b the peak voltage and hence the r.m.s. voltage **[2]**

c the r.m.s. current in the resistor **[1]**

d the mean power dissipated in the resistor. **[2]**

[Total: 7]

7 **a** State the relationship between the peak current I_0 and the r.m.s. current I_{rms} for a sinusoidally varying current. **[1]**

b The current in a resistor connected to a steady d.c. supply is 2.0 A. When the same resistor is connected to an a.c. supply, the current in it has a peak value of 2.0 A. The heating effects of the two currents in the resistor are different.

i Explain why the heating effects are different and state which heating effect is the greater. **[2]**

ii Calculate the ratio of the power dissipated in the resistor by the d.c. current to the power dissipated in the resistor by the a.c. current. **[2]**

[Total: 5]

8 A sinusoidal voltage of 6.0 V r.m.s. and frequency 50 Hz is connected to a diode and a resistor R of resistance 400 Ω as shown in the diagram.

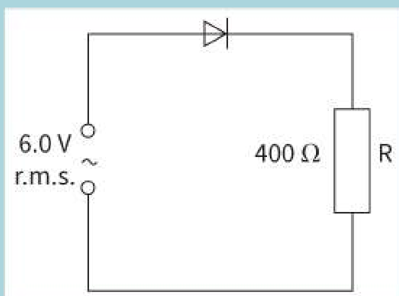


Figure 27.19

a Sketch a graph showing the variation with time of both the supply waveform (use a dotted line) and the voltage across R (use a solid line). Put numerical scales on both the voltage and time axes. **[4]**

b An uncharged capacitor C is connected across R. When the 6.0 V r.m.s. supply is switched on, the capacitor charges fully during the first quarter of a cycle. You may assume that the p.d. across the diode is zero when it conducts. For the next three-quarters of the first cycle, the diode stops conducting and the p.d. across R falls to one-half of the peak value. During this time the mean p.d. across R is 5.7 V.

For the last three-quarters of the first cycle, calculate:

i the time taken **[1]**

ii the mean current in R **[2]**

[2]

iii the charge flowing through R

iv the capacitance of C. **[2]**

c Explain why the diode stops conducting during part of each cycle in part **b**. **[2]**

[Total: 13]

9 The rectified output from a circuit is connected to a resistor R of resistance 1000 Ω. Graph A shows the variation with time *t* of the p.d. *V* across the resistor. Graph B shows the variation of *V* when a capacitor is placed across R to smooth the output.

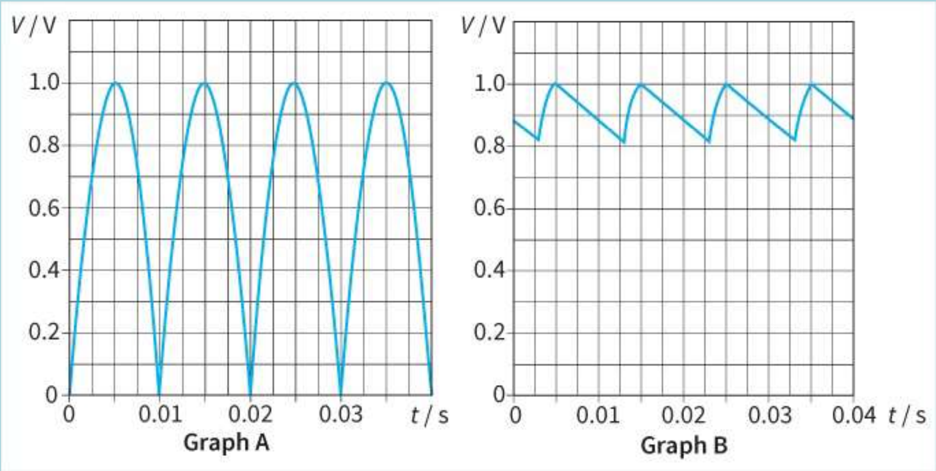


Figure 27.20

Explain how the rectification is achieved. Draw a circuit diagram to show the components involved. **[6]**

b Explain the action of the capacitor in smoothing the output. **[3]**

c Using graph B between $t = 0.005$ and $t = 0.015$ s, determine:

i the time during which the capacitor is charging **[1]**

ii the mean value of the p.d. across R **[1]**

iii the average power dissipated in R. **[2]**

[Total: 13]

10 Electrical energy is supplied by a high-voltage power line that has a total resistance of 4.0 Ω. At the input to the line, the root-mean-square (r.m.s.) voltage has a value of 400 kV and the input power is 500 MW.

a **i** Explain what is meant by **root-mean-square voltage**. **[2]**

ii Calculate the minimum voltage that the insulators that support the line must withstand without breakdown. **[2]**

b **i** Calculate the value of the r.m.s. current in the power line. **[2]**

ii Calculate the power loss on the line. **[2]**

iii Suggest why it is an advantage to transmit the power at a high voltage. **[2]**

[Total: 10]

11 A student has designed a full-wave rectifier circuit.

The output voltage for this circuit is taken across a resistor of resistance 120 Ω. The variation of the output voltage with time is shown.

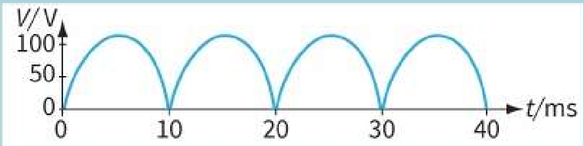


Figure 27.21

A capacitor is now connected across the resistor. The graph shows the new variation of the output voltage with time.

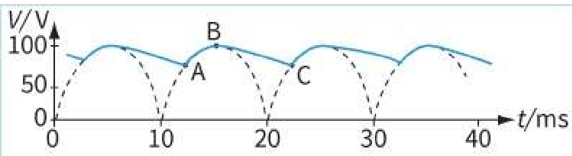


Figure 27.22

a Explain the variation of the output variation between points:

i AB **[1]**

ii BC. **[1]**

b Use the second graph to determine the value of the capacitance *C*. **[3]**

(You may use the equation $V = V_0 e^{-\frac{t}{CR}}$ from Chapter 23.)

[Total: 5]