

## > Chapter 25

# Motion of charged particles

### LEARNING INTENTIONS

In this chapter you will learn how to:

- determine the direction of the force on a charge moving in a magnetic field
- recall and use  $F = BQv \sin\theta$
- describe the motion of a charged particle moving in a uniform magnetic field perpendicular to the direction of motion of the particle
- explain how electric and magnetic fields can be used in velocity selection
- understand the origin of the Hall voltage and derive and use the expression  $V_H = \frac{BI}{n tq}$
- understand the use of a Hall probe to measure magnetic flux density.

### BEFORE YOU START

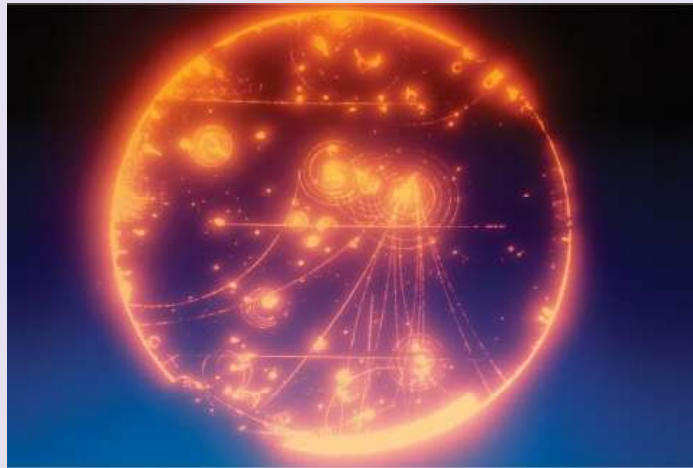
- A current-carrying conductor in a uniform magnetic field experiences a magnetic force  $F$ . Write down the factors that affect this force  $F$  and how you can determine the direction of the force.
- You can get a uniform electric field between two oppositely charged parallel plates. Can you recall and write down the definition for electric field strength  $E$ ?

### MOVING PARTICLES

The world of atomic physics is populated by a great variety of particles – electrons, protons, neutrons, positrons, and many more. Many of these particles are electrically charged, and so their motion is influenced by electric and magnetic fields. Indeed, we use this fact to help us to distinguish one particle from another. Figure 25.1 shows the tracks of particles in a detector called a bubble chamber. A photon (no track) has entered from the top and collided with a proton; the resulting spray of nine particles shows up as the gently curving tracks moving downwards. The tracks curve because the particles are charged and are moving in a magnetic field. The tightly wound spiral tracks are produced by electrons

that, because their mass is small, are more dramatically affected by the field.

In this chapter, we will look at how charged particles behave in electric and magnetic fields and how this knowledge can be used to control beams of charged particles. At the end of the chapter, we will look at how this knowledge was used to discover the electron and to measure its charge and mass.

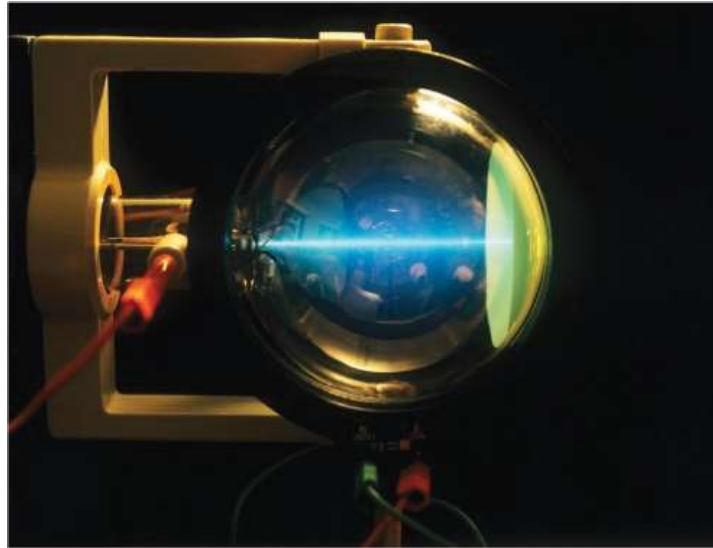


**Figure 25.1:** A bubble chamber image of the tracks of sub-atomic particles. The tracks curve because the charged particles are affected by the presence of a magnetic field.

## 25.1 Observing the force

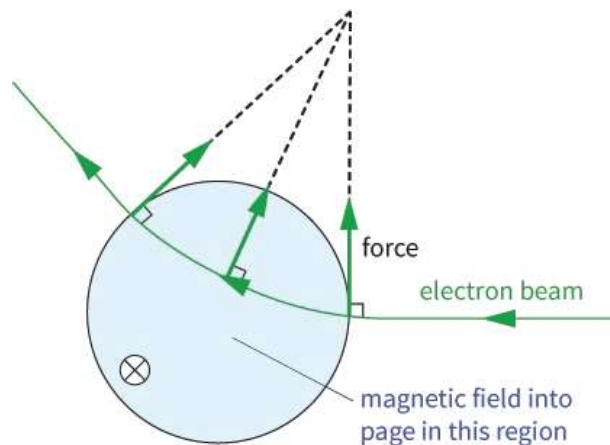
You can use your knowledge of how charged particles and electric currents are affected by fields to interpret diagrams of moving particles. You must always remember that, by convention, the direction of conventional electric current is the direction of flow of positive charge. When electrons are moving, the conventional current is regarded as flowing in the opposite direction.

An electron beam tube (Figure 25.2) can be used to demonstrate the magnetic force on moving charged particles. A beam of electrons is produced by an 'electron gun', and magnets or electromagnets are used to apply a magnetic field.



**Figure 25.2:** An electron beam tube.

You can use such an arrangement to observe the effect of changing the strength and direction of the magnetic field, and the effect of reversing the field.



**Figure 25.3:** A beam of electrons is deflected as it crosses a magnetic field. The magnetic field into the plane of the paper is represented by the cross in the circle.

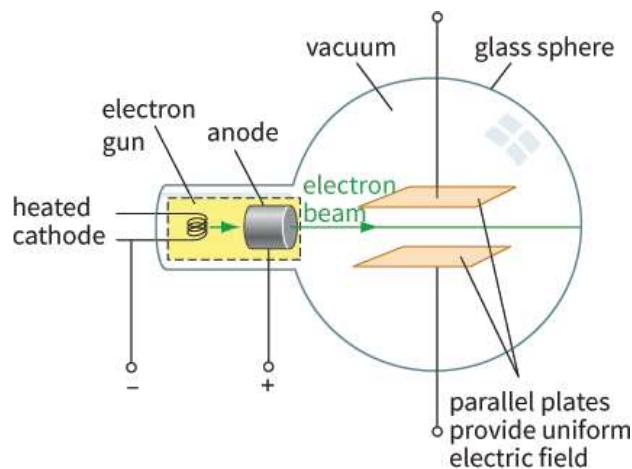
If you are able to observe a beam of electrons like this, you should find that the force on the electrons moving through the magnetic field can be predicted using Fleming's left-hand rule (see [Chapter 24](#)). In Figure 25.3, a beam of electrons is moving from right to left, into a region where a magnetic field is directed into the plane of the paper. Since electrons are negatively charged, they represent a conventional current from left to right. Fleming's left-hand rule predicts that, as the electrons enter the field, the force on them will be upwards and so the beam will be deflected up the page. As the direction of the beam changes, so does the direction of the force. The force due to the magnetic field is always at  $90^\circ$

to the velocity of the electrons. It is this force that gives rise to the motor effect. The electrons in a wire experience a force when they flow across a magnetic field, and they transfer the force to the wire itself. In the past, most oscilloscopes, monitors and television sets made use of beams of electrons. The beams were moved about using magnetic and electric fields, and the result was a rapidly changing image on the screen.

## PRACTICAL ACTIVITY 25.1

### Electron beam tubes

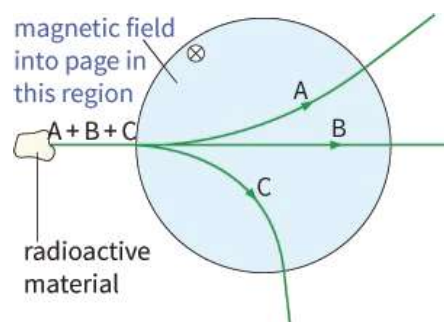
Figure 25.4 shows the construction of a typical tube. The electron gun has a heated cathode. The electrons have sufficient thermal energy to be released from the surface of the heated cathode. These electrons form a cloud around the cathode. The positively charged anode attracts these electrons, and they pass through the anode to form a narrow beam in the space beyond. The direction of the beam can be changed using an electric field between two plates (as in Figure 25.4), or a magnetic field created by electromagnetic coils.



**Figure 25.4:** The construction of an electron beam tube.

## Question

- Figure 25.5 shows how radiation from a radioactive material passes through a region of uniform magnetic field. State and explain whether each type of radiation has positive or negative charge, or is uncharged.



**Figure 25.5:** Three types of radiation passing through a magnetic field.

## Magnetic force on a moving charged particle

Imagine a charged particle moving in a region of uniform magnetic field, with the particle's velocity at right angles to the field. We can make an intelligent guess about the factors that determine the size of the force on the particle (Figure 25.6). It will depend on:

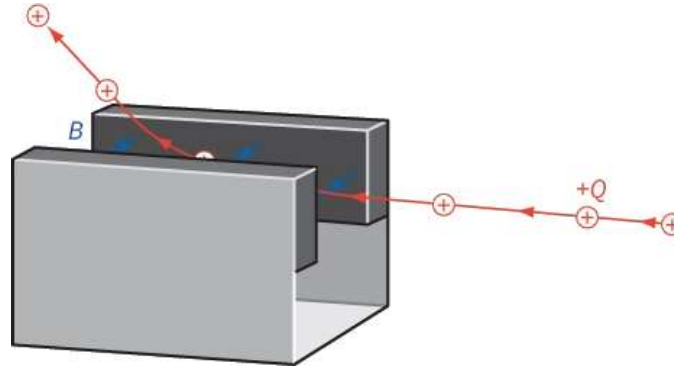
- the magnetic flux density  $B$  (strength of the magnetic field)

- the charge  $Q$  on the particle
- the speed  $v$  of the particle.

The magnetic force  $F$  on a moving particle at right angles to a magnetic field is given by the equation:

$$F = BQv$$

The direction of the force can be determined from Fleming's left-hand rule. The force  $F$  is always at  $90^\circ$  to the velocity of the particle. Consequently, the path described by the particle will be an arc of a circle.



**Figure 25.6:** The path of a charged particle is curved in a magnetic field.

If the charged particle is moving at an angle  $\theta$  to the magnetic field, the component of its velocity at right angles to  $B$  is  $v \sin \theta$ . Hence, the equation becomes:

$$F = BQv \sin \theta$$

where  $B$  is the magnetic flux density,  $Q$  is the charge on the particle,  $v$  is the speed of the particle and  $\theta$  is the angle between the magnetic field and the velocity of the particle.

#### KEY EQUATION

$$F = BQv \sin \theta$$

Magnetic force  $F$  experienced by a charged particle.

We can show that the two equations  $F = BIL$  and  $F = BQv$  are consistent with one another, as follows. Since current  $I$  is the rate of flow of charge, we can write:

$$I = \frac{Q}{t}$$

Substituting in  $F = BIL$  gives:

$$F = \frac{BQL}{t}$$

Now,  $\frac{L}{t}$  is the speed  $v$  of the moving particle, so we can write:

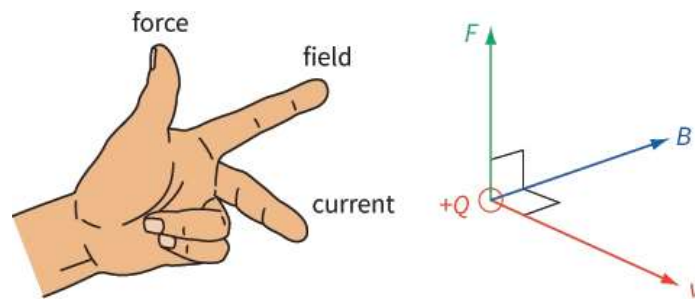
$$F = BQv$$

For an electron, with a charge of  $-e$ , the magnitude of the force is:

$$F = Bev \quad (e = 1.60 \times 10^{-19} \text{ C})$$

The force on a moving charged particle is sometimes called the 'Bev force'; it is this force acting on all the electrons in a wire that gives rise to the 'BIL force'.

Here is an important reminder: the force  $F$  is always at right angles to the particle's velocity  $v$ , and its direction can be found using Fleming's left-hand rule (Figure 25.7).



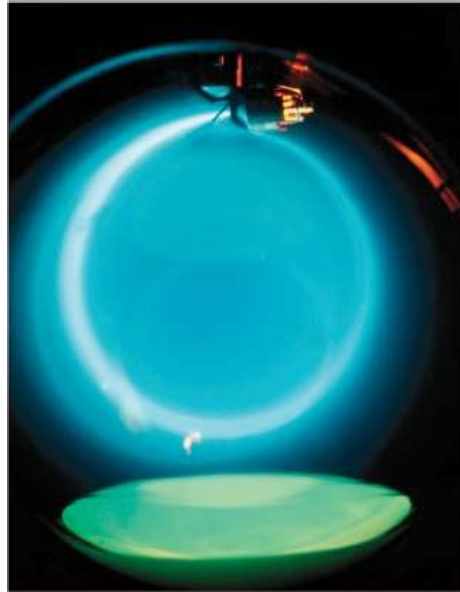
**Figure 25.7:** Fleming's left-hand rule, applied to a moving positive charge.

## Questions

- 2 An electron is moving at  $1.0 \times 10^6 \text{ m s}^{-1}$  in a uniform magnetic field of flux density 0.50 T. Calculate the force on the electron when it is moving:
  - a at right angles to the field
  - b at an angle of  $45^\circ$  to the field.
- 3 Positrons are particles identical to electrons, except that their charge is positive ( $+e$ ). Use a diagram to explain how a magnetic field could be used to separate a mixed beam consisting of both positrons and electrons.

## 25.2 Orbiting charged particles

Consider a charged particle of mass  $m$  and charge  $Q$  moving at right angles to a uniform magnetic field. It will describe a circular path because the magnetic force  $F$  is always perpendicular to its velocity. The magnetic force  $F$ , provides the **centripetal force** on the particle - the direction of the force is always towards the centre of the circle.



**Figure 25.8:** In this fine-beam tube, a beam of electrons is bent around into a circular orbit by a uniform magnetic field. The beam is shown up by the presence of a small amount of gas in the tube. (The electrons travel in an anticlockwise direction.)

Figure 25.8 shows a fine-beam tube. In this tube, a beam of fast-moving electrons is produced by an electron gun. This is similar to the cathode and anode shown in [Figure 25.4](#), but in this case the beam is directed vertically downwards as it emerges from the gun. It enters the spherical tube, which has a uniform horizontal magnetic field. The beam is at right angles to the magnetic field and the  $Bqv$  force pushes it round in a circle.

The fact that the centripetal force is provided by the magnetic force  $BQv$ , gives us a clue as to how we can calculate the radius  $r$  of the orbit of a charged particle in a uniform magnetic field. The centripetal force is given by:

$$\text{centripetal force} = \frac{mv^2}{r}$$

Therefore

$$BQv = \frac{mv^2}{r}$$

Cancelling and rearranging, you get:

$$r = \frac{mv}{BQ}$$

If the charged particles are electrons, then  $Q$  is numerically equal to  $e$ . The equation then becomes:

$$r = \frac{mv}{Be}$$

The momentum  $p$  of the particle is  $mv$ . You can therefore write the equation as:

$$p = Ber$$

The equation  $r = \frac{mv}{Be}$  shows that:

- faster-moving particles move in bigger circles because  $r \propto v$
- particles with greater masses also move in bigger circles because  $r \propto m$



- particles with greater charge move in tighter (smaller) circles because  $r \propto \frac{1}{Q}$
- a stronger field (greater magnetic flux density) makes the particles move in tighter circles because  $r \propto \frac{1}{B}$ .

These ideas have a variety of scientific applications, such as particle accelerators and mass spectrometers. They can also be used to find the charge-to-mass ratio  $\frac{e}{m_e}$  of an electron.

## The charge-to-mass ratio of an electron

Experiments to find the mass of an electron first involve finding the charge-to-mass ratio  $\frac{e}{m_e}$ . This is known as the specific charge on the electron – the word ‘specific’ here means ‘per unit mass’.

Using the equation for an electron travelling in a circle in a magnetic field, we have  $\frac{e}{m_e} = \frac{v}{Br}$ . Clearly, measurements of  $v$ ,  $B$  and  $r$  are needed to determine  $\frac{e}{m_e}$ .

There are difficulties in measuring  $B$  and  $r$ . For example, it is difficult to directly measure  $r$  with a ruler outside the tube in Figure 25.8 because of parallax error. Also,  $v$  must be measured, and you need to know how this is done. One way is to use the potential difference (p.d.)  $V_{ca}$  between the cathode and the anode. This p.d. causes each electron to accelerate as it moves from the cathode to the anode. An individual electron has charge  $-e$ , therefore an amount of work is done on each electron is  $e \times V_{ca}$ . This is equivalent to the kinetic energy of the electron as it leaves the anode – we assume that the electron has zero kinetic energy at the cathode. Therefore:

$$eV_{ca} = \frac{1}{2}m_e v^2$$

where  $m_e$  is the mass of the electron and  $v$  is the final speed of the electron.

Eliminating  $v$  from the equations:

$$eV_{ca} = \frac{1}{2}m_e v^2 \text{ and } r = \frac{m_e v}{Be}$$

gives:  $\frac{e}{m_e} = \frac{2V_{ca}}{r^2 B^2}$

A voltmeter can be used to measure  $V_{ca}$ , and if  $r$  and  $B$  are known, we can calculate the ratio  $\frac{e}{m_e}$ . As you shall see shortly, the charge on the electron  $e$  can be measured more directly, and this allows physicists to calculate the electron mass  $m_e$  from the value of  $\frac{e}{m_e}$ .

### WORKED EXAMPLE

- 1** An electron is travelling at right angles to a uniform magnetic field of flux density 1.2 mT. The speed of the electron is  $8.0 \times 10^6 \text{ m s}^{-1}$ .

Calculate the radius of circle described by this electron.

$e = 1.60 \times 10^{-19} \text{ C}$  and  $m_e = 9.11 \times 10^{-31} \text{ kg}$

**Step 1** Calculate the magnetic force on the electron.

$$F = Bev = 1.2 \times 10^{-3} \times 1.60 \times 10^{-19} \times 8.0 \times 10^6$$

$$F = 1.536 \times 10^{-15} \text{ N}$$

**Step 2** Use your knowledge of motion in a circle to determine the radius  $r$ .

$$F = \frac{m_e v^2}{r}$$

Therefore:

$$r = \frac{m_e v^2}{F} = \frac{9.11 \times 10^{-31} \times (8.0 \times 10^6)^2}{1.536 \times 10^{-15}}$$

$$r = 3.8 \times 10^{-2} \text{ m (3.8 cm)}$$

**Note:** You can get the same result by using the equation:

$$r = \frac{m_e v}{Be}$$

## Questions

- 4** Look at the photograph of the electron beam in the fine-beam tube (Figure 25.8).



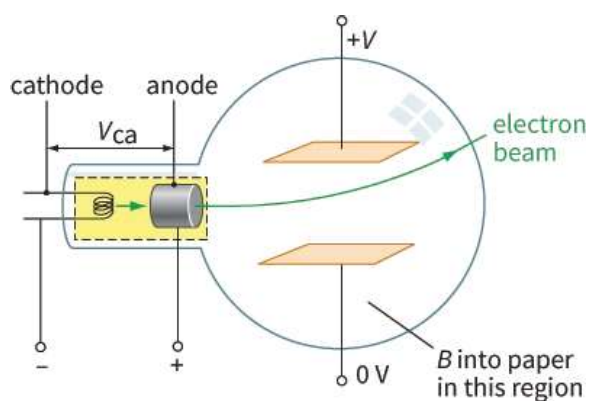
State the direction is the magnetic field (into or out of the plane of the photograph).

- 5 The particles in the circular beam shown in Figure 25.8 all travel round in the same orbit. State what can you deduce about their mass, charge and speed.
- 6 An electron beam in a vacuum tube is directed at right angles to a magnetic field, so that it travels along a circular path.
- Predict the effect on the size and shape of the path that would be produced (separately) by each of the following changes:
- a increasing the magnetic flux density
  - b reversing the direction of the magnetic field
  - c slowing down the electrons
  - d tilting the beam, so that the electrons have a component of velocity along the magnetic field.

## PRACTICAL ACTIVITY 25.2

### The deflection tube

A deflection tube (Figure 25.9) is designed to show a beam of electrons passing through a combination of electric and magnetic fields.



**Figure 25.9:** The path of an electron beam in a deflection tube.

By adjusting the strengths of the electric and magnetic fields, you can balance the two forces on the electrons, and the beam will remain horizontal. The magnetic field is provided by two vertical coils, called Helmholtz coils (Figure 25.10), which give a very uniform field in the space between them.

When the electron beam remains straight, it follows that the electric and magnetic forces on each electron must have the same magnitude and act in opposite directions.



**Figure 25.10:** A pair of Helmholtz coils is used to give a uniform magnetic field.

Therefore: electric force (upwards) = magnetic force (downwards)

$$eE = Bev$$

where  $E$  is the electric field strength between the parallel horizontal plates. The speed  $v$  of the

electrons is simply related to  $E$  and  $B$  because  $e$  in the expression cancels out. Therefore:

$$v = \frac{E}{B}$$

The electric field strength  $E$  is given by:

$$E = \frac{V}{d}$$

where  $V$  is the p.d. between the plates and  $d$  is the distance between the plates. Therefore:

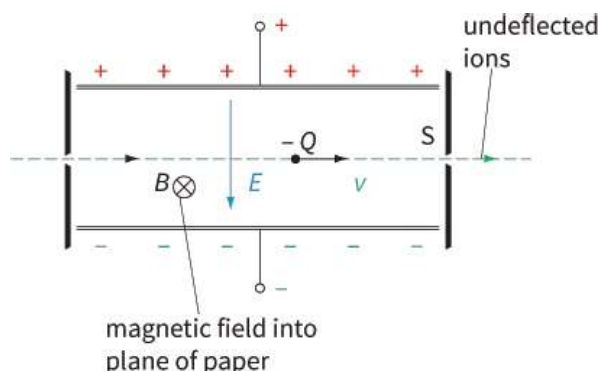
$$v = \frac{V}{Bd}$$

## 25.3 Electric and magnetic fields

Now we will consider in detail what happens when an electron beam passes through an electric field and a magnetic field at the same time.

### Velocity selection

In a device called a **velocity selector**, charged particles of a specific velocity are selected using both electric and magnetic fields. This is used in devices such as mass spectrometers where it is essential to produce a beam of charged particles all moving with the same velocity. The construction of a velocity selector is shown in Figure 25.11.



**Figure 25.11:** A velocity selector - only particles with the correct velocity will emerge through the slit S.

The apparatus is very similar to the deflection tube in Figure 25.9. Two oppositely charged horizontal plates are situated in an evacuated chamber. These plates provide a uniform electric field of strength  $E$  in the space between the plates.

The region between the plates is also occupied by a uniform magnetic field of flux density  $B$  that is at right angles to the electric field. Negatively charged particles (electrons or ions) enter from the left. They all have the same charge  $-Q$  but are travelling at **different speeds**. The magnitude of the electric force  $EQ$  will be the same on all particles as it does not depend on their speed. However, the magnitude of the magnetic force  $BQv$  will be greater for those particles that are travelling faster. Hence, for particles travelling at the desired speed  $v$ , the electric force and the magnetic force must have the same value, but be in opposite directions. The resultant force on the charged particles in the vertical direction must be zero, and all the charged particles with the speed  $v$  will emerge undeflected from the slit S. Therefore:

$$\begin{aligned}QE &= BQv \\v &= \frac{E}{B}\end{aligned}$$

If a charged particle has a speed greater than  $\frac{E}{B}$ , the downward magnetic force on it will be greater than the upward electric force. Thus, it will be deflected downwards and it will hit below slit S.

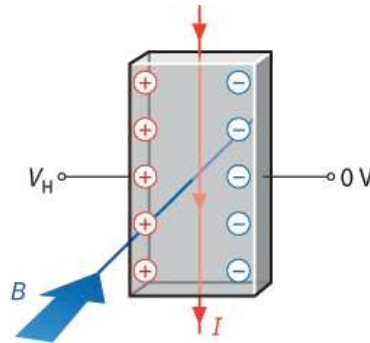
Note that we do not have to concern ourselves with the gravitational force  $mg$  acting on the charged particles as this will be negligible compared with the electric and magnetic forces.

### Question

- 7 This question is about the velocity selector shown in Figure 25.11.
- State the directions of the magnetic and electric forces on a positively charged ion travelling towards the slit S.
  - Calculate the speed of an ion emerging from the slit S when the magnetic flux density is  $0.30 \text{ T}$  and the electric field strength is  $1.5 \times 10^3 \text{ V m}^{-1}$ .
  - Explain why ions travelling with a speed greater than your answer to part b will not emerge from the slit.

## 25.4 The Hall effect

In [Chapter 24](#), you saw how to use a Hall probe to measure magnetic flux density. This probe works on the basis of the **Hall effect**. The Hall effect is the production of a potential difference across an electrical conductor when an external magnetic field is applied in a direction perpendicular to the current.

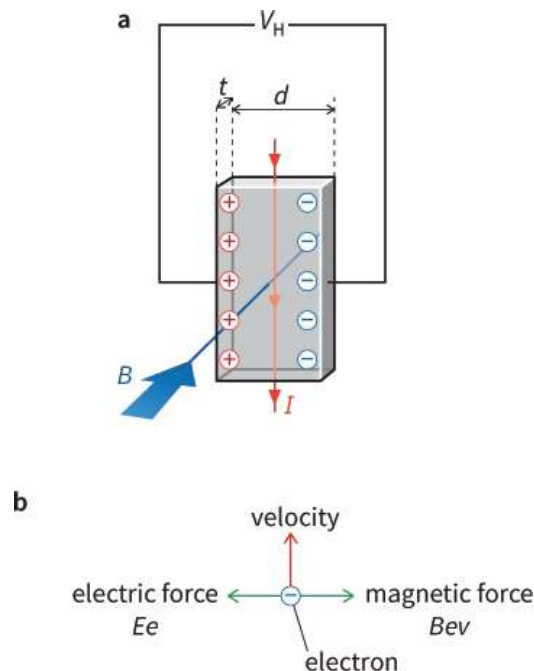


**Figure 25.12:** A Hall voltage is produced across the sides of the slice of conductor (metal).

Consider a slice of conductor with an external magnetic field applied perpendicular to the direction of the current. If the conductor is a **metal**, then the current is due to the flow of electrons. These electrons will experience a magnetic force, which will make them drift towards one side of the conductor, where they will gather. The opposite side of the slice is deficient of electrons. A potential difference, known as the **Hall voltage**, will be developed across the conductor (Figure 25.12). As you will see later, the Hall voltage  $V_H$  for the slice is constant for a given current and is directly proportional to the magnetic flux density  $B$  of the external magnetic field.

### An equation for the Hall voltage

Using what we know about electric current and the forces on electric charges produced by electric and magnetic fields, we can derive an expression for the Hall voltage  $V_H$ . Figure 25.13 shows a current-carrying slice of a metal. The Hall voltage is the voltage that appears between the two opposite sides of the slice.



**Figure 25.13:** **a** The Hall voltage is measured across the slice of metal. **b** The forces on an electron when the electric and magnetic forces are equal and opposite.

As we have seen, this voltage arises because electrons accumulate on one side of the slice. There is a corresponding lack of electrons on the opposite side – this opposite side may be considered to have a positive charge. As a result, there is an electric field set up within the slice between the two sides. The two charged sides may be treated as oppositely charged parallel plates – see [Chapter 21](#). Therefore, the electric field strength  $E$  is related to the Hall voltage  $V_H$  by:

$$E = \frac{V_H}{d}$$

where  $d$  is the width of the slice.

Now, imagine a single electron as it travels with drift velocity  $v$  through the slice. The magnetic field is into the plane of the paper, so this electron will experience a magnetic force  $Bev$  to the right. It will also experience an electric force  $Ee$  to the left.

When the current first starts to flow, there is no Hall voltage and so electrons are pushed to the right by the magnetic field. However, as the charge on the right-hand side builds up, so does the internal electric field and this pushes the electrons in the opposite direction to the magnetic force. Soon, an equilibrium situation is reached, the resultant force on each electron is zero and the electrons are undeflected. Now we can equate the two forces:

$$eE = Bev$$

Substituting for  $E$  we have:

$$\frac{eV_H}{d} = Bev$$

Now recall from [Chapter 8](#) that the current  $I$  is related to the mean drift velocity  $v$  of the electrons by  $I = nAve$ , where  $A$  is the cross-sectional area of the conductor and  $n$  is the number density of charge carriers (in this case, electrons). So, we can substitute for  $v$  to get:

$$\frac{eV_H}{d} = \frac{BeI}{nAe}$$

Making  $V_H$  the subject of the equation (and cancelling  $e$ ) gives:

$$V_H = \frac{BI d}{nAe}$$

The cross-sectional area  $A$  of each side-face of the slice is:

$$A = d \times t$$

where  $t$  is the thickness of the slice.

Substituting and cancelling gives:

$$V_H = \frac{BI}{nte}$$

This equation for the Hall voltage shows that  $V_H$  is directly proportional to the magnetic flux density  $B$  for a given slice and current. That is what makes the Hall effect so useful for measuring  $B$ .

To get a large voltage, it would be desirable to have a material with a smaller value for  $n$  compared with metals. Hall probes use a very thin slice of semiconductor. Semiconductors have a number density many thousands of times smaller than metals, hence the Hall voltage will be thousands of times larger.

In some semiconductors, the charge carriers are not electrons, but positively charged particles referred to as ‘holes’. We can write a more general equation for the Hall voltage replacing  $e$  with  $q$ , where  $q$  is the charge of an individual charge carrier. This gives  $V_H = \frac{BI}{ntq}$ .

#### KEY EQUATION

Hall voltage:

$$V_H = BI/(ntq)$$

You must learn how to derive this equation.

Positive charges will be deflected in the opposite direction to negative charges, and so we can determine whether the charge carriers are positive or negative by the sign of the Hall voltage.

## Questions

**8** A Hall probe is designed to operate with a steady current of 0.020 A in a semiconductor slice of

thickness 0.05 mm. The number density of charge carriers (electrons) in the semiconductor is  $1.5 \times 10^{23} \text{ m}^{-3}$ .

- a** Calculate the Hall voltage that will result when the probe is placed at right angles to a magnetic field of flux density 0.10 T.

(Elementary charge  $e = 1.60 \times 10^{-19} \text{ C}$ .)

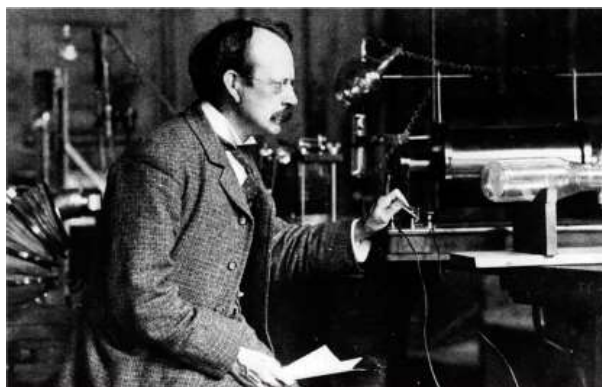
- b** Explain why the current in the Hall probe must be maintained at a constant value.

- 9** Suggest how the Hall effect could be used to determine the number density of charge carriers  $n$  in a semiconducting material.

## 25.5 Discovering the electron

Today, we know a lot about electrons and we use the idea of electrons to explain all sorts of phenomena, including electric current and chemical bonding. However, at the end of the 19th century, physicists were only just beginning to identify the tiny particles that make up matter.

One of the leaders in this field was the English physicist J.J. Thomson (Figure 25.14). In the photograph, he is shown with the deflection tube that he used in his discovery of the electron.



**Figure 25.14:** J.J. Thomson – in 1897, he discovered the electron using the vacuum tube shown here.

His tube was similar in construction to the deflection tube shown in Figure 25.9. At one end was an electron gun that produced a beam of electrons (which he called ‘cathode rays’). Two metal plates allowed him to apply an electric field to deflect the beam, and he could place magnets outside the tube to apply a magnetic force to the beam. Here is a summary of his observations and what he concluded from them:

- The beam in his tube was deflected towards a positive plate and away from a negative plate, so the particles involved must have negative charge. This was confirmed by the deflection of the beam by a magnetic field.
- When the beam was deflected, it remained as a tight, single beam rather than spreading out into a broad beam. This showed that, if the beam consisted of particles, they must all have the same mass, charge and speed. (Lighter particles would have been deflected more than heavier ones; particles with greater charge would be deflected more, and faster particles would be deflected less.)
- By applying both electric and magnetic fields, Thomson was able to balance the electric and magnetic forces so that the beam in the tube remained straight. He could then calculate the charge-to-mass ratio  $\frac{e}{m_e}$  for the particles he had discovered. Although he did not know the value of either  $e$  or  $m_e$  individually, he was able to show that the particles concerned must be much lighter than atoms. They were the particles that we now know as electrons. In fact, for a while, Thomson thought that atoms were made up of thousands of electrons, although his ideas could not explain how so many negatively charged particles could combine to produce a neutral atom.

The magnitude of the charge  $e$  of an electron is very small ( $1.60 \times 10^{-19}$  C) and difficult to measure. The American physicist Robert Millikan devised an ingenious way to do it. He observed electrically charged droplets of oil as they moved in electric and gravitational fields and found that they all had a charge that was a small integer multiple of a particular value, which he took to be the magnitude of the charge on a single electron,  $e$ . Having established a value for  $e$ , he could combine this with Thomson’s value for  $\frac{e}{m_e}$  to calculate the electron mass  $m_e$ .

### Question

**10** The charge-to-mass ratio  $\frac{e}{m_e}$  for the electron is  $1.76 \times 10^{11}$  C kg<sup>-1</sup>.

Calculate the mass of the electron using  $e = 1.60 \times 10^{-19}$  C.

### REFLECTION

Without looking at your textbook, summarise the similarities between a velocity-selector and the Hall effect. Compare your summary with a fellow learner. Did you miss out any key ideas?

Make a list of all the equations leading to  $V_H \propto B$  for a Hall probe.



What things might you want more help with in this topic?

## SUMMARY

The magnetic force on a charged particle moving at right angles to a magnetic field is given by the equation:  $F = BQv$ . For an electron,  $Q = e$ .

For charged particle travelling at an angle  $\theta$  to the magnetic field, the force is given by the equation:

$$F = BQv \sin\theta$$

The direction of the force experienced by a charge moving in a uniform magnetic field can be determined using Fleming's left-hand rule.

A charged particle entering at right angles to a uniform magnetic field describes a circular path because the magnetic force is perpendicular to the velocity.

For an electron describing a circular path in a uniform magnetic field, the centripetal force is provided by  $Bev$ . Therefore:

$$Bev = \frac{mv^2}{r}$$

In a velocity selector, the speed of an undeflected charged particle in a region where electric and magnetic fields are at right angles is given by the equation:

$$v = \frac{E}{B}$$

This speed is independent of the charge of the particle.

In the Hall effect, a potential difference is produced across an electrical conductor when an external magnetic field is applied in a direction perpendicular to the direction of the current.

The Hall voltage is given by:

$$V_H = \frac{BI}{ntq}$$

Note:

$V_H \propto$  magnetic flux density  $B$

$V_H \propto$  current  $I$

$V_H \propto \frac{1}{\text{number density of charge carriers } n}$

$V_H \propto \frac{1}{\text{thickness of slice } t}$

In a Hall probe,  $V_H \propto B$  because the current in the slice is constant.

A Hall probe uses a semiconducting material rather than a metal because the smaller number density of charge carriers gives a larger Hall voltage.

## EXAM-STYLE QUESTIONS

- 1 A scientist is doing an experiment on a beam of electrons travelling at right angles to a uniform magnetic field of flux density  $B$ . The graph shows the variation of the magnetic force  $F$  acting on an electron with the speed  $v$  of the electron.

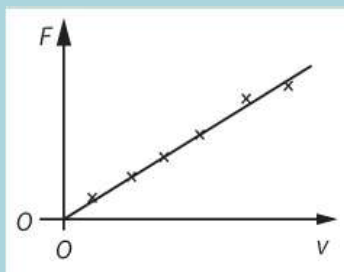


Figure 25.15

The gradient of the graph is  $G$ . The magnitude of the charge on the electron is  $e$ .

What is the correct relationship for the magnetic flux density  $B$ ?

[1]

- A  $B = G$
- B  $B = G \times e$
- C  $B = \frac{G}{e}$
- D  $B = \frac{e}{G}$

- 2 The magnetic force  $BQv$  causes an electron to travel in a circle in a uniform magnetic field.

Explain why this force does not cause an increase in the speed of the electron.

[3]

- 3 An electron beam is produced from an electron gun in which each electron is accelerated through a potential difference (p.d.) of 1.6 kV. When these electrons pass at right angles through a magnetic field of flux density 8.0 mT, the radius of curvature of the electron beam is 0.017 m.

Calculate the ratio  $\frac{e}{m_e}$  (known as the specific charge of the electron).

[4]

- 4 Two particles, an  $\alpha$ -particle and a  $\beta^-$ -particle, are travelling through a uniform magnetic field. They have the same speed and their velocities are at right angles to the field. Determine the ratio of:

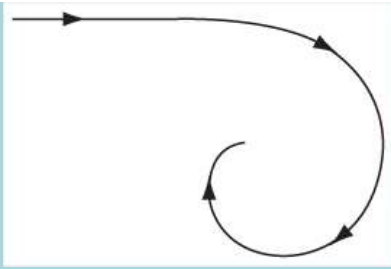
- a the mass of the  $\alpha$ -particle to the mass of the  $\beta^-$ -particle [2]
- b the charge of the  $\alpha$ -particle to the charge of the  $\beta^-$ -particle [2]
- c the force on the  $\alpha$ -particle to the force on the  $\beta^-$ -particle [2]
- d the radius of the  $\alpha$ -particle's orbit to the radius of the  $\beta^-$ -particle's orbit. [2]

[Total: 8]

- 5 A moving charged particle experiences a force in an electric field and also in a magnetic field. State two differences between the forces experienced in the two types of field.

[2]

- 6 This diagram shows the path of an electron as it travels in air. The electron rotates clockwise around a uniform magnetic field into the plane of the paper, but the radius of the orbit decreases in size.

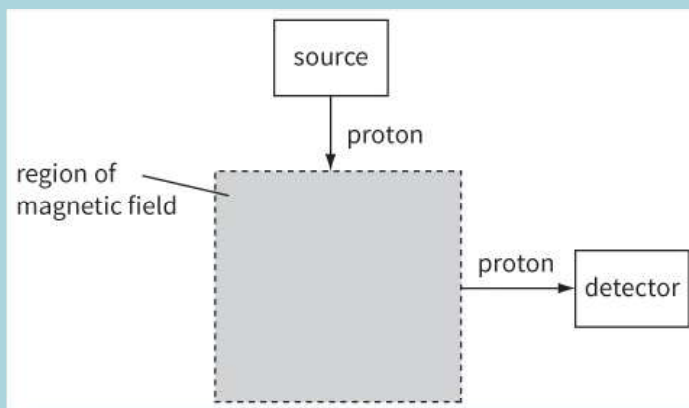


**Figure 25.16**

- a
  - i Explain the origin of the force that causes the electron to spiral in this manner. [2]
  - ii Explain why the radius of the circle gradually decreases. [2]
- b At one point in the path, the speed of the electron is  $1.0 \times 10^7 \text{ m s}^{-1}$  and the magnetic flux density is 0.25 T. Calculate:
  - i the force on an electron at this point due to the magnetic field [2]
  - ii the radius of the arc of the circular path at this point. [2]

**[Total: 8]**

- 7 This diagram shows an arrangement to deflect protons from a source to a detector using a magnetic field. The charge on each proton is  $+e$ . A uniform magnetic field exists only within the area shown. Protons move from the source to the detector in the plane of the paper.

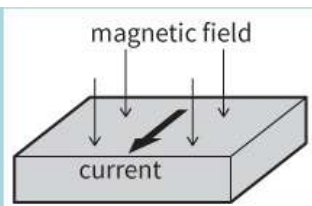


**Figure 25.17**

- a
  - i Copy the diagram and sketch the path of a proton from the source to the detector. Draw an arrow at two points on the path to show the direction of the force on the proton produced by the magnetic field. [3]
  - ii State the direction of the magnetic field within the area shown. [1]
- b The speed of a proton as it enters the magnetic field is  $4.0 \times 10^6 \text{ m s}^{-1}$ . The magnetic flux density is 0.25 T. Calculate:
  - i the magnitude of the force on the proton caused by the magnetic field [1]
  - ii the radius of curvature of the path of the proton in the magnetic field. [2]
- c Two changes to the magnetic field in the area shown are made. These changes allow an electron with the same speed as the proton to be deflected along the same path as the proton. State the two changes made. [2]

**[Total: 9]**

- 8 This diagram shows a thin slice of semiconductor material carrying a current in a magnetic field at right angles to the current.

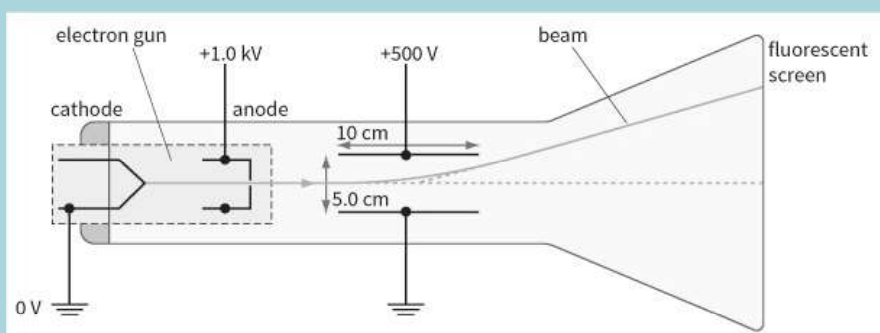


**Figure 25.18**

- a** The current in the slice is due to the movement of free electrons.
- i** Add + and – signs to the diagram to show the charge separation caused by the Hall effect. Explain why the charges separate. [3]
  - ii** Explain how an electron can eventually move in a straight line along the slice. [1]
- b** The Hall voltage is measured using the same slice of semiconductor, the same current and the same magnetic field, but with the laboratory at two temperatures, one significantly higher than the other.
- Describe and explain the changes in the magnitude of the number density, the drift velocity of the charge carriers and the Hall voltage in the two experiments. [5]

[Total: 9]

- 9** This diagram shows an electron tube. Electrons emitted from the cathode accelerate towards the anode and then pass into a uniform electric field created by two oppositely charged horizontal metal plates.



**Figure 25.19**

- a i** Explain why the beam curves upwards between the plates. [2]
- ii** Explain how the pattern formed on the fluorescent screen shows that all the electrons have the same speed as they leave the anode. [2]
- b** Write down an equation relating the speed of the electrons  $v$  to the potential difference  $V_{ac}$  between the anode and the cathode. [1]
- c** The deflection of the beam upwards can be cancelled by applying a suitable uniform magnetic field in the space between the parallel plates.
- i** State the direction of the magnetic field for this to happen. [1]
  - ii** Write down an equation relating the speed of the electrons  $v$ , the electric field strength  $E$  that exists between the plates and the magnetic flux density  $B$  needed to make the electrons pass undeflected between the plates. [2]
  - iii** Determine the value of  $B$  required, using the apparatus shown in the diagram, given that for an electron the ratio  $\frac{e}{m_e} = 1.76 \times 10^{11} \text{ C Kg}^{-1}$ . [4]

[Total: 12]

- 10** Protons and helium nuclei from the Sun pass into the Earth's atmosphere above the poles, where the magnetic flux density is  $6.0 \times 10^{-5} \text{ T}$ . The particles are moving at a speed of  $1.0 \times 10^6 \text{ m s}^{-1}$  at right angles to the magnetic field in this region. The magnetic field can be assumed to be uniform.

- a** Calculate the radius of the path of a proton as it passes above the Earth's pole. [3]

mass of a helium nucleus =  $6.8 \times 10^{-27}$  kg

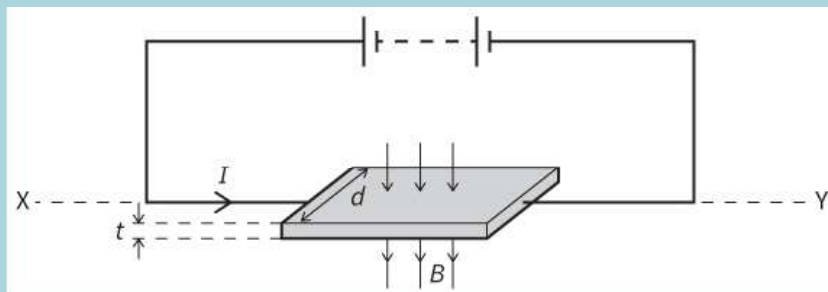
charge on a helium nucleus =  $3.2 \times 10^{-19}$  C

- b** Sketch a diagram to show the deflection caused by the magnetic field to the paths of a proton and of a helium nucleus that both have the same initial velocity as they enter the magnetic field.

[2]

[Total: 5]

- 11** This diagram shows a thin slice of metal of thickness  $t$  and width  $d$ . The metal slice is in a magnetic field of flux density  $B$  and carries a current  $I$ , as shown.



**Figure 25.20**

- a** Copy the diagram and mark:
- i** the side of the slice that becomes negative because of the Hall effect [1]
  - ii** where a voltmeter needs to be placed to measure the Hall voltage. [1]
- b** Derive an expression for the Hall voltage in terms of  $I$ ,  $B$ ,  $t$ , the number density of the charge carriers  $n$  in the metal and the charge  $e$  on an electron. [3]
- c** Given that  $I = 40$  mA,  $d = 9.0$  mm,  $t = 0.030$  mm,  $B = 0.60$  T,  $e = 1.6 \times 10^{-19}$  C and  $n = 8.5 \times 10^{28}$  m<sup>-3</sup>, calculate:
- i** the mean drift velocity  $v$  of the free electrons in the metal [2]
  - ii** the Hall voltage across the metal slice [2]
  - iii** the percentage uncertainty in the mean drift velocity  $v$  of the electrons, assuming the percentage uncertainties in the quantities are as shown. [1]

Quantity	% uncertainty
Current $I$	1.3
Width $d$	2.5
Thickness $t$	3.0
Number density of charge carriers $n$	0.2

**Table 25.1**

- d**
- i** Explain why, in terms of the movement of electrons, the Hall voltage increases when  $I$  increases. [2]
  - ii** A Hall probe used to determine the magnetic flux density of a magnetic field uses a thin slice of a semiconductor rather than metal. Explain why a semiconductor is used. [2]
- e** Explain why, when the slice of metal is rotated about the horizontal axis XY, the Hall voltage varies between a maximum positive value and a maximum negative value. [2]

[Total: 16]

## SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

I can	See topic...	Needs more work	Almost there	Ready to move on
use Fleming's left-hand rule to determine the direction of the force on a charge moving in a magnetic field	25.1			
recall and use: $F = BQv \sin\theta$	25.1			
describe the motion of a charged particle moving in a uniform magnetic field perpendicular to the direction of motion of the particle	25.2			
explain how electric and magnetic fields can be used in velocity selection of charged particles	25.3			
understand the Hall effect and the origin of the Hall voltage	25.4			
derive and use the expression for Hall voltage: $V_H = \frac{BI}{ntq}$	25.4			
use a Hall probe to measure magnetic flux density.	25.4			