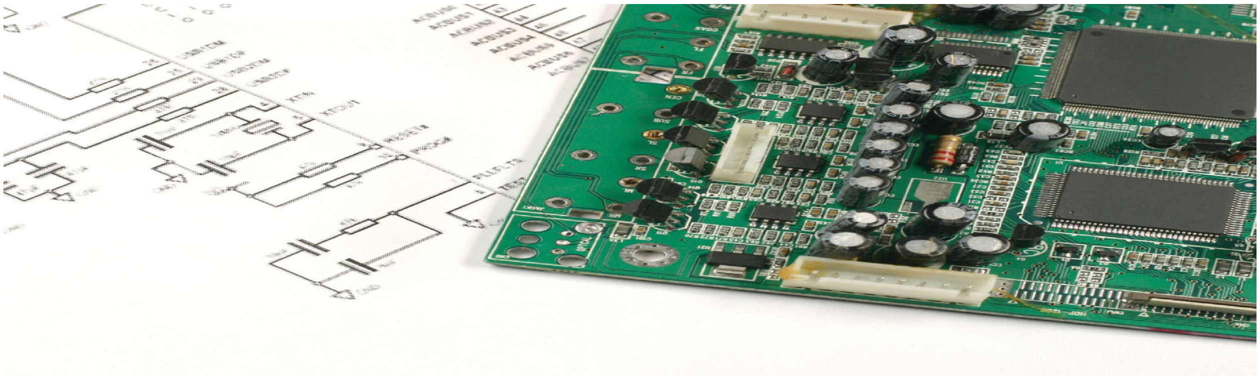

Chapter 23

Capacitance

LEARNING INTENTIONS

In this chapter you will learn how to:

- define capacitance and state its unit, the farad
- solve problems involving charge, voltage and capacitance
- deduce the electric potential energy stored in a capacitor from a potential–charge graph
- deduce and use formulae for the energy stored by a capacitor
- derive and use formulae for capacitances in series and parallel
- recognise and use graphs showing variation of potential difference, current and charge as a capacitor discharges.
- recall and use the time constant for a capacitor–resistor circuit
- use the equation for the discharge of a capacitor through a resistor.

BEFORE YOU START

- In order to avoid an electric shock, electrical engineers regularly connect various points to Earth, even though the equipment is disconnected from the mains supply.
- What does this suggest to you is happening? How can you get a shock when the equipment is not connected to the mains? Discuss with a partner and be prepared to share your thoughts with the rest of the class.

CAPACITORS

Most electronic devices, such as radios, computers and MP3 players, make use of components called capacitors. These are usually quite small, but Figure 23.1 shows a giant capacitor, specially constructed to store electrical energy at the Fermilab particle accelerator in the United States.

Fermilab is a particle physics and accelerator laboratory. Particle accelerators, as the name suggests, accelerate particles, such as protons, up to incredibly high energies. The 'tevatron' at Fermilab can accelerate protons up to energies of approximately 2 TeV (10^{12} eV). High-energy, but short-lasting voltage pulses (100 000 V lasting 10^{-5} s) are required to accelerate the particles. Such pulses would disrupt the public electricity supply. To ensure the public power supply is evenly loaded and is not disrupted by peak pulses, large capacitors (temporary energy storage devices) are continuously charged and discharged 50 times per second.

Wind turbines and solar cells only generate energy in suitable weather conditions. Could huge capacitors be used to store electrical energy generated when the weather conditions are suitable for use at times when it is not? How else could the energy be stored?

Figure 23.1: One of the world's largest capacitors, built to store energy at the Fermilab particle accelerator.

23.1 Capacitors in use

Capacitors are used to store energy in electrical and electronic circuits. This means that they have many valuable applications. For example, capacitors are used in computers; they store energy in normal use and then they gradually release this energy if there is a power failure, so that the computer will operate long enough to save valuable data. Figure 23.2 shows a variety of shapes and sizes of capacitors.

Every capacitor has two leads, each connected to a metal plate. To store energy, these two plates must be given equal and opposite electric charges. Between the plates is an insulating material called the dielectric. Figure 23.3 shows a simplified version of the construction of a capacitor; in practice, many have a spiral form.

Figure 23.2: A variety of capacitors.

To move charge onto the plates of a capacitor, it must be connected to a voltage supply. The negative terminal of the supply pushes electrons onto one plate, making it negatively charged. Electrons are repelled from the other plate, making it positively charged. Figure 23.4 shows that there is a flow of electrons all the way round the circuit.

The two ammeters will give identical readings. The current stops when the potential difference (p.d.) across the capacitor is equal to the electromotive force (e.m.f.) of the supply. We then say that the capacitor is 'fully charged'.

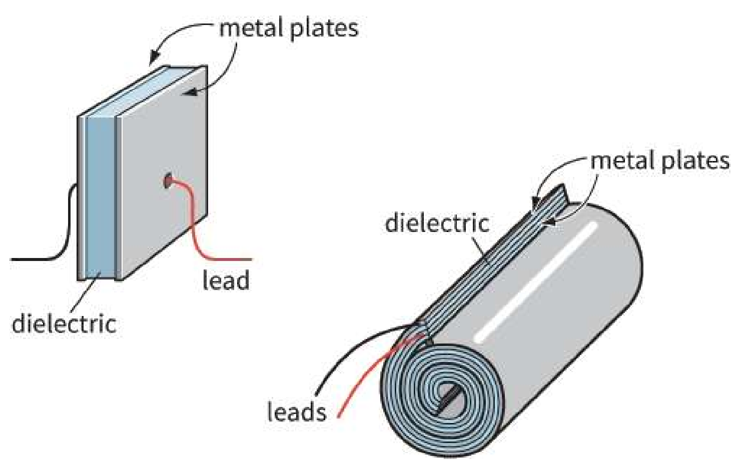

Figure 23.3: The construction of two types of capacitor.

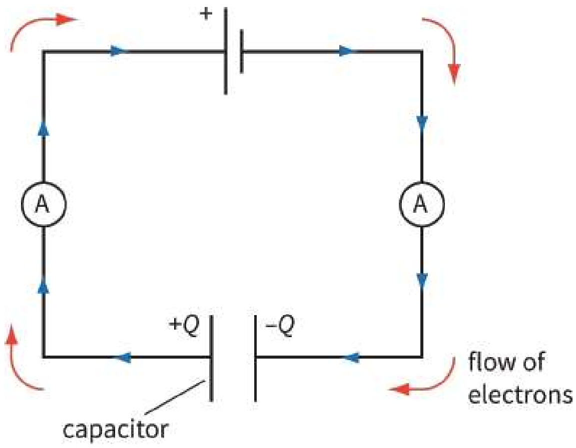

Figure 23.4: The flow of charge when a capacitor is charged up.

Note: The convention is that current is the flow of positive charge. Here, it is free electrons that flow. Electrons are negatively charged; conventional current flows in the opposite direction to the electrons (Figure 23.5).

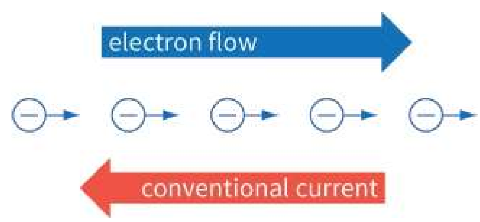

Figure 23.5: A flow of electrons to the right constitutes a conventional current to the left.

Charge on the plates

Think about a capacitor with uncharged plates. Each plate has equal amounts of positive and negative charge. Connecting the capacitor to a supply pulls charge $+Q$ from one plate and transfers it to the other, leaving behind charge $-Q$. The supply does work in separating the charges. Since the two plates now store equal and opposite charges, the total charge on the capacitor is zero. When we talk about the 'charge stored' by a capacitor, we mean the quantity Q, the magnitude of the charge stored on each plate.

To make the capacitor plates store more charge, we would have to use a supply of higher e.m.f. If we connect the leads of the charged capacitor together, electrons flow back around the circuit and the capacitor is discharged.

You can observe a capacitor discharging as follows. Connect the two leads of a capacitor to the terminals of a battery. Disconnect, and then reconnect the leads to a light-emitting diode (LED). It is best to have a protective resistor in series with the LED. The LED will glow briefly as the capacitor discharges.

In any circuit, the charge that flows past a point in a given time is equal to the area under a current–time graph (just as distance is equal to the area under a speed–time graph). So the magnitude of the charge on the plates in a capacitor is given by the area under the current–time graph recorded while the capacitor is being charged up.

The meaning of capacitance

If you look at some capacitors, you will see that they are marked with the value of their **capacitance**. The greater the capacitance, the greater is the charge on the capacitor plates for a given potential difference across it.

The capacitance C of a capacitor is defined by:

$$\text{capacitance} = \frac{\text{charge}}{\text{potential difference}}$$
$$C = \frac{Q}{V}$$

where Q is the magnitude of the charge on each of the capacitor's plates and V is the potential difference across the capacitor.

KEY EQUATION

$$\text{capacitance} = \frac{\text{charge}}{\text{potential difference}}$$

$$C = \frac{Q}{V}$$

The charge on the capacitor may be calculated using the equation:

$$Q = VC$$

This equation shows that the charge depends on two things: the capacitance C and the voltage V (double the voltage means double the charge). Note that it isn't only capacitors that have capacitance. Any object can become charged by connecting it to a voltage. The object's capacitance is then the ratio of the charge to the voltage.

Units of capacitance

The unit of capacitance is the **farad**, F. From the equation that defines capacitance, you can see that this must be the same as the units of charge (coulombs, C) divided by voltage (V):

$$1\ \text{F} = 1\ \text{C V}^{-1}$$

(It is unfortunate that the letter 'C' is used for both capacitance and coulomb. There is room for confusion here!)

In practice, a farad is a large unit. Few capacitors have a capacitance of 1 F. Capacitors usually have their values marked in picofarads (pF), nanofarads (nF) or microfarads (μF):

$1\ \text{pF} = 10^{-12}\ \text{F}$ $1\ \text{nF} = 10^{-9}\ \text{F}$ $1\ \mu\text{F} = 10^{-6}\ \text{F}$

Other markings on capacitors

Many capacitors are marked with their highest safe working voltage. If you exceed this value, charge may leak across between the plates, and the dielectric will cease to be an insulator. Some capacitors (electrolytic ones) must be connected correctly in a circuit. They have an indication to show which end must be connected to the positive of the supply. Failure to connect correctly will damage the capacitor, and can be extremely dangerous.

Questions

1. Calculate the charge on a 220 μF capacitor charged up to 15 V. Give your answer in microcoulombs (μC) and in coulombs (C).
2. A charge of 1.0×10^{-3} C is measured on a capacitor with a potential difference across it of 500 V. Calculate the capacitance in farads (F), microfarads (μF) and picofarads (pF).
3. Calculate the average current required to charge a 50 μF capacitor to a p.d. of 10 V in a time interval of 0.01 s.
4. A student connects an uncharged capacitor of capacitance C in series with a resistor, a cell and a switch. The student closes the switch and records the current I at intervals of 10 s. The results are shown in Table 23.1. The potential difference across the capacitor after 60 s is 8.5 V. Plot a current–time graph, and use it to estimate the value of C.

t / s	0	10	20	30	40	50	60
I / μA	200	142	102	75	51	37	27

Table 23.1 Data for Question 4.

23.2 Energy stored in a capacitor

When you charge a capacitor, you use a power supply to push electrons onto one plate and off the other. The power supply does work on the electrons, so their potential energy increases. You recover this energy when you discharge the capacitor.

If you charge a large capacitor (1000 μF or more) to a potential difference of 6.0 V, disconnect it from the supply and then connect it across a 6.0 V lamp, you can see the lamp glow as energy is released from the capacitor. The lamp will flash briefly. Clearly, such a capacitor does not store much energy when it is charged.

In order to charge a capacitor, work must be done to push electrons onto one plate and off the other (Figure 23.6). At first, there is only a small amount of negative charge on the left-hand plate. Adding more electrons is relatively easy, because there is not much repulsion. As the charge on the plate increases, the repulsion between the electrons on the plate and the new electrons increases, and a greater amount of work must be done to increase the charge on the plate.

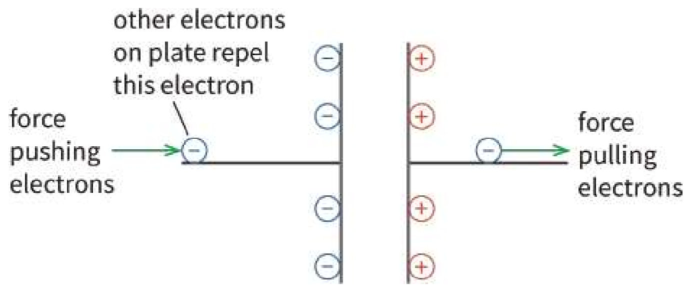

Figure 23.6: When a capacitor is charged, work must be done to push additional electrons against the repulsion of the electrons that are already present.

This can be seen qualitatively in Figure 23.7a. This graph shows how the p.d. *V* increases as the amount of charge *Q* increases. It is a straight line because *Q* and *V* are related by:

$$V = \frac{Q}{C}$$

We can use Figure 23.7a to calculate the work done in charging up the capacitor.

First, consider the work done *W* in moving charge *Q* through a constant p.d. *V*. This is given by:

$$W = QV$$

(You studied this equation in Chapter 9.) From the graph of *Q* against *V* (Figure 23.7b), we can see that the quantity $Q \times V$ is given by the area under the graph.

The area under a graph of p.d. against charge is equal to work done.

If we apply the same idea to the capacitor graph (Figure 23.7a), then the area under the graph is the shaded triangle, with an area of base × height. Hence, the work done in charging a capacitor to a particular p.d. is given by:

$$W = \frac{1}{2}QV$$

Substituting $Q = CV$ into this equation gives two further equations:

$$W = \frac{1}{2}CV^2$$

$$W = \frac{1}{2}\frac{Q^2}{C}$$

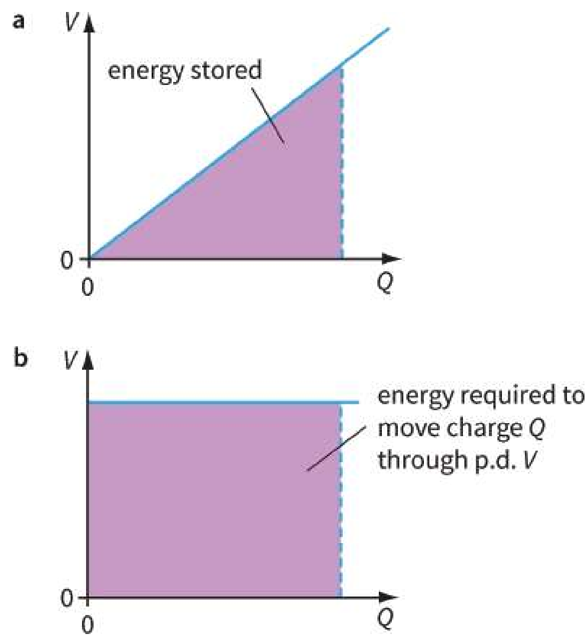

Figure 23.7: The area under a graph of voltage against charge gives a quantity of energy. The area in **a** shows the energy stored in a capacitor; the area in **b** shows the energy required to drive a charge through a resistor.

where W energy stored, Q is the charge on the capacitor, C is the capacitance and V is the potential difference across the capacitor.

These three equations show the work done in charging up the capacitor. This is equal to the energy stored by the capacitor, since this is the amount of energy released when the capacitor is discharged.

We can also see from the second formula $\left(W = \frac{1}{2}CV^2\right)$ that the energy W that a capacitor stores depends on its capacitance C and the potential difference V to which it is charged.

The energy W stored is proportional to the square of the potential difference V ($W \propto V^2$). It follows that doubling the charging voltage means that four times as much energy is stored.

KEY EQUATIONS

Work done by charging a capacitor:

$$W = \frac{1}{2}QV$$
$$W = \frac{1}{2}CV^2$$
$$W = \frac{1}{2}\frac{Q^2}{C}$$

WORKED EXAMPLE

1 A 2000 μF capacitor is charged to a p.d. of 10 V. Calculate the energy stored by the capacitor.

Step 1 Write down the quantities we know:

$$C = 2000\mu F$$
$$V = 10\text{V}$$

Step 2 Write down the equation for energy stored and substitute values:

$$\begin{aligned} W &= \frac{1}{2}CV^2 \\ &= \frac{1}{2} \times 2000 \times 10^{-6} \times 10^2 \\ &= 0.10\ \text{J} \end{aligned}$$

This is a small amount of energy – compare it with the energy stored by a rechargeable battery, typically of the order of 10 000 J. A charged capacitor will not keep an MP3 player running for any length of time.

Questions

5 State the quantity represented by the gradient of the straight line shown in Figure 23.7a.

6 The graph of Figure 23.8 shows how V depends on Q for a particular capacitor.

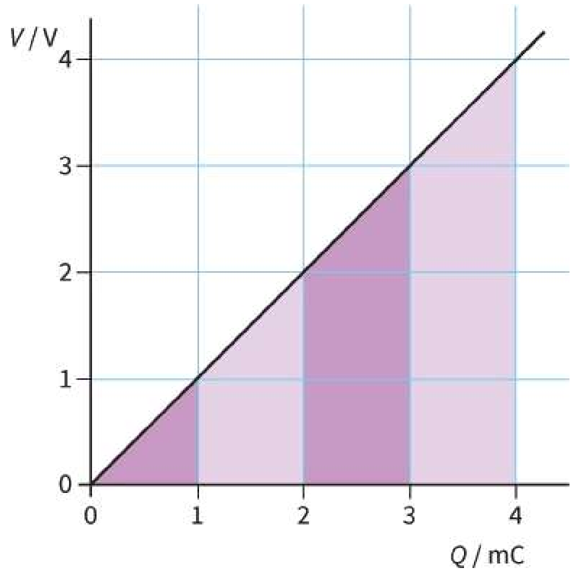

Figure 23.8: The energy stored by a capacitor is equal to the area under the graph of voltage against charge.

The area under the graph has been divided into strips to make it easy to calculate the energy stored. The first strip (which is simply a triangle) shows the energy stored when the capacitor is charged up to 1.0 V. The energy stored is:

$$\begin{aligned} \tfrac{1}{2}QV &= \tfrac{1}{2} \times 1.0\ \text{mC} \times 1.0\ \text{V} \\ &= 0.5\ \text{mJ} \end{aligned}$$

a Calculate the capacitance C of the capacitor.

b Copy Table 23.2 and complete it by calculating the areas of successive strips, to show how W depends on V.

c Plot a graph of W against V. Describe the shape of this graph.

Q / mC	V / V	Area of strip ΔW / mJ	Sum of areas W / mJ
1.0	1.0	0.5	0.5
2.0	2.0	1.5	2.0
3.0			
4.0			

Table 23.2 Data for Question 6.

PRACTICAL ACTIVITY 23.1

Investigating energy stored in a capacitor

If you have a sensitive joulemeter (capable of measuring millijoules, mJ), you can investigate the equation for energy stored. A suitable circuit is shown in Figure 23.9.

The capacitor is charged up when the switch connects it to the power supply. When the switch is altered, the capacitor discharges through the joulemeter. (It is important to wait for the capacitor to discharge completely.) The joulemeter will measure the amount of energy released by the capacitor.

By using capacitors with different values of C, and by changing the charging voltage V, you can investigate how the energy W stored depends on C and V.

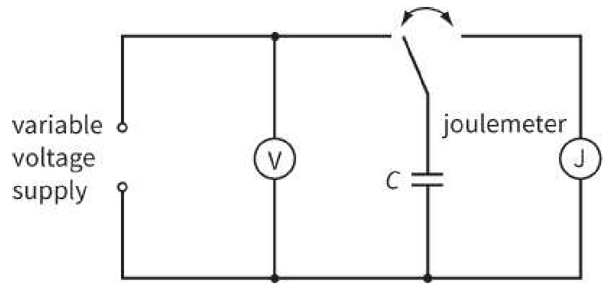

Figure 23.9: With the switch to the left, the capacitor *C* charges up; to the right, it discharges through the joulemeter.

Questions

7 Calculate the energy stored in the following capacitors:

a a 5000 μF capacitor charged to 5.0 V

b a 5000 pF capacitor charged to 5.0 V

c a 200 μF capacitor charged to 230 V.

8 Which involves more charge, a 100 μF capacitor charged to 200 V or a 200 μF capacitor charged to 100 V? Which stores more energy?

9 A 10 000 μF capacitor is charged to 12 V, and then connected across a lamp rated at '12 V, 36 W'.

a Calculate the energy stored by the capacitor.

b Estimate the time the lamp stays fully lit. Assume that energy is dissipated in the lamp at a steady rate.

10 In a simple photographic flashgun, a 0.20 F capacitor is charged by a 9.0 V battery. It is then discharged in a flash of duration 0.01 s. Calculate:

a the charge on and energy stored by the capacitor

b the average power dissipated during the flash

c the average current in the flash bulb

d the approximate resistance of the bulb.

23.3 Capacitors in parallel

Capacitors are used in electric circuits to store energy. Situations often arise where two or more capacitors are connected together in a circuit. In this topic, we will look at capacitors connected in parallel. The next topic deals with capacitors in series.

When two capacitors are connected in parallel (Figure 23.10), their combined or total capacitance C_{total} is simply the sum of their individual capacitances C_1 and C_2:

$$C_{total} = C_1 + C_2$$

This is because, when two capacitors are connected together, they are equivalent to a single capacitor with larger plates. The bigger the plates, the more charge that can be stored for a given voltage, and hence the greater the capacitance.

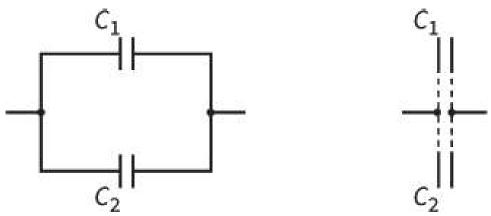

Figure 23.10: Two capacitors connected in parallel are equivalent to a single, larger capacitor.

The total charge Q on two capacitors connected in parallel and charged to a potential difference V is simply given by:

$$Q = C_{total} \times V$$

For three or more capacitors connected in parallel, the equation for their total capacitance becomes:

$$C_{total} = C_1 + C_2 + C_3 + \ldots$$

Capacitors in parallel: deriving the formula

We can derive the equation for capacitors in parallel by thinking about the charge on the two capacitors. As shown in Figure 23.11, C_1 stores charge Q_1 and C_2 stores charge Q_2. Since the p.d. across each capacitor is V, we can write:

$$Q_1 = C_1 V \text{ and } Q_2 = C_2 V$$

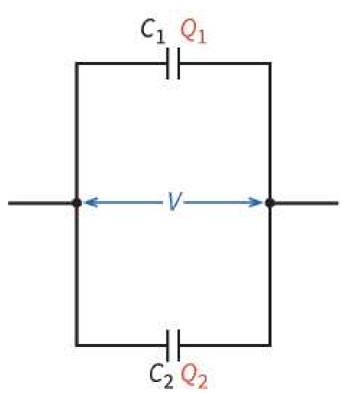

Figure 23.11: Two capacitors connected in parallel have the same p.d. across them, but different amounts of charge.

The total charge is given by the sum of these:

$$Q = Q_1 + Q_2 = C_1 V + C_2 V$$

Since V is a common factor:

$$Q = (C_1 + C_2)V$$

Comparing this with $Q = C_{total}V$ gives the required $C_{total} = C_1 + C_2$. It follows that for three or more capacitors connected in parallel, we have:

$$C_{total} = C_1 + C_2 + C_3 + \ldots$$

Capacitors in parallel: summary

For capacitors in parallel, the following rules apply:

- The p.d. across each capacitor is the same.
- The total charge on the capacitors is equal to the sum of the charges:

$$Q_{total} = Q_1 + Q_2 + Q_3 + \ldots$$

- The total capacitance C_{total} is given by:

$$C_{total} = C_1 + C_2 + C_3 + \ldots$$

KEY EQUATION

$$C_{total} = C_1 + C_2 + C_3 + \ldots$$

The combined capacitance of capacitors in parallel.

You must learn how to derive this equation.

Questions

11 **a** Calculate the total capacitance of two 100 μF capacitors connected in parallel.

b Calculate the total charge they store when charged to a p.d. of 20 V.

12 A capacitor of capacitance 50 μF is required, but the only values available to you are 10 μF, 20 μF and 100 μF (you may use more than one of each value). How would you achieve the required value by connecting capacitors in parallel? Give at least two answers.

23.4 Capacitors in series

In a similar way to the case of capacitors connected in parallel, we can consider two or more capacitors connected in series (Figure 23.12). The total capacitance C_{total} of two capacitors of capacitances C_1 and C_2 is given by:

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Here, it is the reciprocals of the capacitances that must be added to give the reciprocal of the total capacitance. For three or more capacitors connected in series, the equation for their total capacitance is:

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

Figure 23.12: Two capacitors connected in series.

Capacitors in series: deriving the formula

The same principles apply here as for the case of capacitors in parallel. Figure 23.13 shows the situation. C_1 and C_2 are connected in series, and there is a p.d. V across them. This p.d. is divided (it is shared between the two capacitors), so that the p.d. across C_1 is V_1 and the p.d. across C_2 is V_2. It follows that:

$$V = V_1 + V_2$$

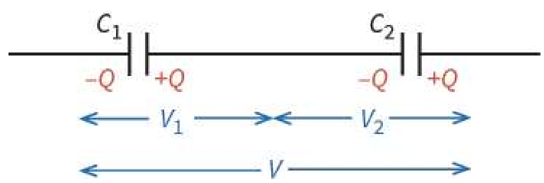

Figure 23.13: Capacitors connected in series store the same charge, but they have different p.d.s across them.

Now we must think about the charge stored by the combination of capacitors. In Figure 23.13, you will see that both capacitors are shown as storing the same charge Q. How does this come about? When the voltage is first applied, charge $-Q$ arrives on the left-hand plate of C_1. This repels charge $-Q$ off the right-hand plate, leaving it with charge $+Q$. Charge $-Q$ now arrives on the left-hand plate of C_2, and this in turn results in charge $+Q$ on the right-hand plate.

KEY EQUATION

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

The combined capacitance of capacitors in series.

You must learn how to derive this equation.

Note that charge is not arbitrarily created or destroyed in this process – the total amount of charge in the system is constant. This is an example of the conservation of charge.

Notice also that there is a central isolated section of the circuit between the two capacitors. Since this is initially uncharged, it must remain so at the end. This requirement is satisfied, because there is charge $-Q$ at one end and $+Q$ at the other. Hence, we conclude that capacitors connected in series store the same charge. This allows us to write equations for V_1 and V_2:

$$V_2 = \frac{Q}{C_1} \text{ and } V_2 = \frac{Q}{C_2}$$

The combination of capacitors stores charge Q when charged to p.d. V, and so we can write:

$$V = \frac{Q}{C_{\text{total}}}$$

Substituting these in $V = V_1 + V_2$ gives:

$$\frac{Q}{C_{\text{total}}} + \frac{Q}{C_1} + \frac{Q}{C_2}$$

Cancelling the common factor of Q gives the required equation:

$$\frac{1}{C_{\text{total}}} + \frac{1}{C_1} + \frac{1}{C_2}$$

Worked example 2 shows how to use this relationship.

WORKED EXAMPLE

2 Calculate the total capacitance of a 300 μF capacitor and a 600 μF capacitor connected in series.

Step 1 The calculation should be done in two steps; this is relatively simple using a calculator with a $\frac{1}{X}$ or x^{-1} key.

Substitute the values into the equation:

$$\frac{1}{C_{\text{total}}} + \frac{1}{C_1} + \frac{1}{C_2}$$

This gives:

$$\frac{1}{C_{\text{total}}} = \frac{1}{300} + \frac{1}{600}$$

$$\frac{1}{C_{\text{total}}} = 0.005 \mu\text{F}^{-1}$$

Step 2 Now take the reciprocal of this value to determine the capacitance in μF:

$$C_{\text{total}} = \frac{1}{0.005} = 200 \text{ F}$$

Notice that the total capacitance of two capacitors in series is less than either of the individual capacitances.

Using the x^{-1} key on your calculator, you can also do this calculation in one step:

$$C_{\text{total}} = (300^{-1} + 600^{-1})^{-1} = 200\ \mu\text{F}$$

Questions

13 Calculate the total capacitance of three capacitors of capacitances 200 μF, 300 μF and 600 μF, connected in series.

14 You have a number of identical capacitors, each of capacitance C. Determine the total capacitance when:

a two capacitors are connected in series

b n capacitors are connected in series

c two capacitors are connected in parallel

d n capacitors are connected in parallel.

23.5 Comparing capacitors and resistors

It is helpful to compare the formulae for capacitors in series and parallel with the corresponding formulae for resistors (Table 23.3).

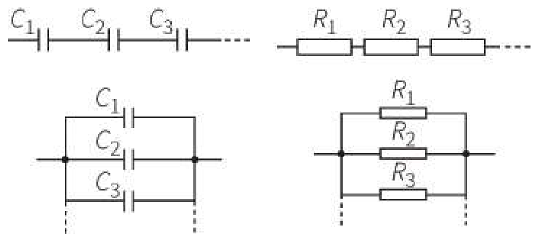

Table 23.3 Capacitors and resistors compared.

Notice that the reciprocal formula applies to capacitors in series but to resistors in parallel. This comes from the definitions of capacitance and resistance. Capacitance indicates how good a capacitor is at storing charge for a given voltage, and resistance indicates how bad a resistor is at letting current through for a given voltage.

23.6 Capacitor networks

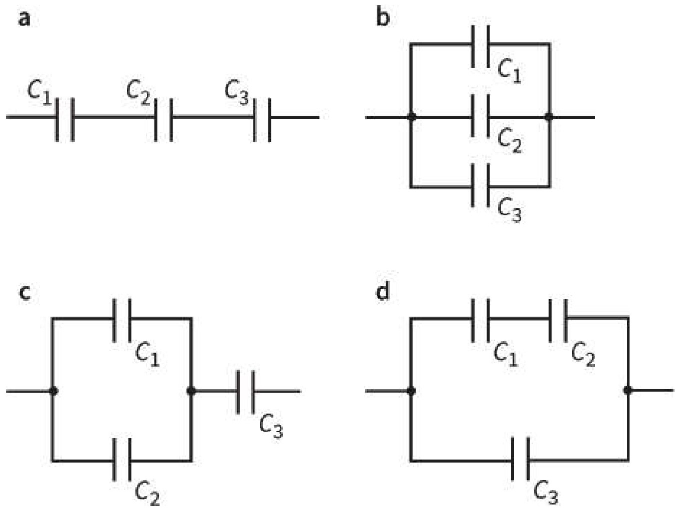

Figure 23.14: Four ways to connect three capacitors.

There are four ways in which three capacitors may be connected together. These are shown in Figure 23.14. The combined capacitance of the first two arrangements (three capacitors in series, three in parallel) can be calculated using the formulae. The other combinations must be dealt with in a different way:

- Figure 23.14a – All in series. Calculate C_{total} as in Table 23.3.
- Figure 23.14b – All in parallel. Calculate C_{total} as in Table 23.3.
- Figure 23.14c – Calculate C_{total} for the two capacitors of capacitances C_1 and C_2, which are connected in parallel, and then take account of the third capacitor of capacitance C_3, which is connected in series.
- Figure 23.14d – Calculate C_{total} for the two capacitors of capacitances C_1 and C_2, which are connected in series, and then take account of the third capacitor of capacitance C_3, which is connected in parallel.

These are the same approaches as would be used for networks of resistors.

Questions

15 For each of the four circuits shown in Figure 23.14, calculate the total capacitance in μF if each capacitor has capacitance 100 μF.

16 Given a number of 100 μF capacitors, how might you connect networks to give the following values of capacitance:

a 400 μF?

b 25 μF?

c 250 μF?

(Note that, in each case, there is more than one correct answer; try to find the answer that requires the minimum number of capacitors.)

17 You have three capacitors of capacitances 100 pF, 200 pF and 600 pF. Determine the maximum and minimum values of capacitance that you can make by connecting them together to form a network. State how they should be connected in each case.

18 Calculate the capacitance in μF of the network of capacitors shown in Figure 23.15.

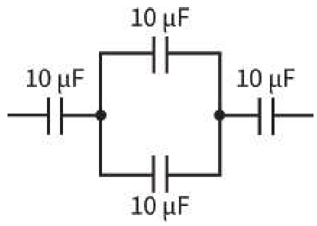

Figure 23.15: A capacitor network. For Question 18.

Sharing charge, sharing energy

If a capacitor is charged and then connected to a second capacitor (Figure 23.16), what happens to the charge and the energy that it stores? Note that, when the capacitors are connected together, they are in parallel, because they have the same p.d. across them. Their combined capacitance C_{total} is equal to the sum of their individual capacitances.

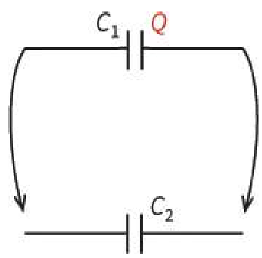

Figure 23.16: Capacitor of capacitance C_1 is charged and then connected across C_2.

Now we can think about the charge stored, Q. This is shared between the two capacitors; the total amount of charge stored must remain the same, since charge is conserved. The charge is shared between the two capacitors in proportion to their capacitances. Now the p.d. can be calculated from $V = \frac{Q}{C}$ and the energy from $W = \frac{1}{2}CV^2$.

If we look at a numerical example, we find an interesting result (Worked example 3).

Figure 23.17 shows an analogy to the situation described in Worked example 3.

Capacitors are represented by containers of water. A wide (high capacitance) container is filled to a certain level (p.d.). It is then connected to a container with a smaller capacitance, and the levels equalise. (The p.d. is the same for each.) Notice that the potential energy of the water has decreased, because the height of its centre of gravity above the base level has decreased. Energy is dissipated as heat, as there is friction both within the moving water and between the water and the container.

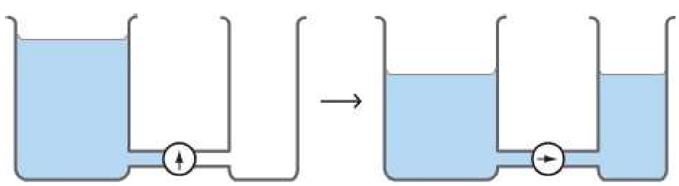

Figure 23.17: An analogy for the sharing of charge between capacitors.

WORKED EXAMPLE

3 Consider two 100 mF capacitors. One is charged to 10 V, disconnected from the power supply, and then connected across the other. Calculate the energy stored by the combination.

Step 1 Calculate the charge and energy stored for the single capacitor.

$$\begin{aligned} \text{initial charge } Q &= VC \\ &= 10 \times 0.10 \\ &= 1.0\text{C} \\ \text{initial stored energy} &= \tfrac{1}{2}CV^2 \\ &= \tfrac{1}{2} \times 0.10 \times 10^2 \\ &= 5.0 \text{ J} \end{aligned}$$

Step 2 Calculate the final p.d. across the capacitors. The capacitors are in parallel and have a total stored charge of 1.0 C.

$C_{total} = C_1 + C_2 = 100 + 100 = 200$ mF

The p.d. V can be determined using $Q = VC$.

$$\begin{aligned} V &= \tfrac{Q}{C} \\ &= \tfrac{1.0}{200} \times 10^{-3} \\ &= 5.0 \text{ V} \end{aligned}$$

This is because the charge is shared equally, with the original capacitor losing half of its charge.

Step 3 Now calculate the total energy stored by the capacitors.

$$\begin{aligned} \text{total energy} &= \tfrac{1}{2}CV^2 \\ &= \tfrac{1}{2} \times 200 \times 10^{-3} \times 5.0^2 \\ &= 2.5 \text{ J} \end{aligned}$$

The charge stored remains the same, but half of the stored energy is lost. The energy goes to heating the connecting wires as the electrons migrate between the capacitors.

Questions

19 Three capacitors, each of capacitance 120 μF, are connected together in series. This network is then connected to a 10 kV supply. Calculate:

- **a** their combined capacitance in μF
- **b** the charge stored
- **c** the total energy stored.

20 A 20 μF capacitor is charged up to 200 V and then disconnected from the supply. It is then connected across a 5.0 μF capacitor. Calculate:

- **a** the combined capacitance of the two capacitors in μF
- **b** the charge they store
- **c** the p.d. across the combination
- **d** the energy dissipated when they are connected together.

Capacitance of isolated bodies

It is not just capacitors that have capacitance – all bodies have capacitance. Yes, even you have capacitance! You may have noticed that, particularly in dry conditions, you may become charged up, perhaps by rubbing against a synthetic fabric. You are at a high voltage and store a significant amount of charge. Discharging yourself by touching an earthed metal object would produce a spark.

If we consider a conducting sphere of radius r insulated from its surroundings and carrying a charge Q it will have a potential at its surface of V, where

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Since $C = \frac{Q}{V}$, it follows that the capacitance of a sphere is $C = 4\pi\varepsilon_0 r$.

Question

21 Estimate the capacitance of the Earth given that it has a radius of 6.4×10^6 m. State any assumptions you make.

23.7 Charge and discharge of capacitors

In Figure 23.18, the capacitor is charged by the battery when the switch is connected to terminal P. When first connected to P, a current is observed in the microammeter. The current starts off quite large and gradually decreases to zero. When connected to terminal Q, the capacitor discharges through the resistor and a current in the opposite direction is observed. As with the previous current, it starts off large and gradually falls to zero.

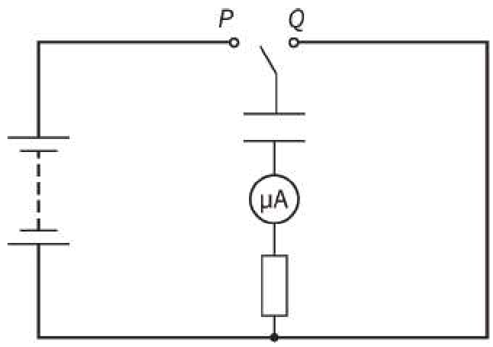

Figure 23.18 A circuit to charge and discharge a capacitor.

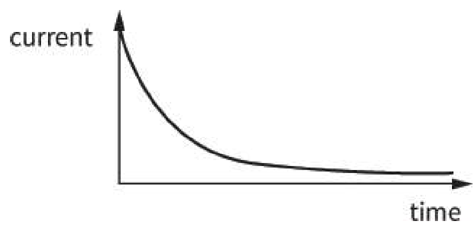

Figure 23.19 A graph showing how the current changes with time when a capacitor discharges through a resistor.

This shape of this graph it is quite common in sciences and it occurs in different situations – you will come across it again in radioactive decay in Chapter 29. In this case, it comes from the fact that, as charge flows off the capacitor, the potential difference reduces and so the current (the charge flowing per unit time) in the circuit also decreases. In radioactive decay, it occurs because as atoms decay, there are fewer atoms left to Charles's law and, therefore, fewer decays per unit time.

This type of decay is called **exponential decay** and is described by the formula:

$$x = x_0 e^{-ky}$$

where x is the dependent variable, y is the independent variable, k and x_0 are constants and e is the exponential function (a naturally occurring number of value 2.7118 28 ...).

Question

22 In the circuit in Figure 23.18, the resistance has a resistance of 2000 Ω, the capacitor has a capacitance of 1000 μF and the battery has an e.m.f. of 12 V.

- **a** Calculate:
  - **i** the potential difference across the capacitor when it is fully charged by the battery
  - **ii** the charge stored by the capacitor when it is fully charged
  - **iii** the current in the resistor when the switch is first connected to terminal Q.
- **b** Explain what happens to the amount of charge stored on the plates in the moments after the switch is first connected to terminal Q.
- **c** Based on your answer to part b, explain what effect this has on:
  - **i** the potential difference across the capacitor
  - **ii** the current in the resistor.

Once you have worked through Question 22, you should understand why the current gradually reduces: it reduces because of the current itself, as it takes charge off the plates.

What is the effect of changing the resistance in the circuit? There will be no change in the initial potential difference across the capacitor, but the initial current through the resistor will be changed. Increased resistance will mean decreased current, so charge flows off the capacitor plates more slowly and, therefore, the capacitor will take longer to discharge. Conversely, decreasing the resistance will cause the capacitor to discharge more quickly.

What is the effect of increasing the capacitance of the capacitor? The initial p.d. across the capacitor is, again, unchanged. So, with an unchanged resistance, the initial current will be unchanged. However, there will be more charge on the capacitor and so it will take longer to discharge.

From this, we can see that the time taken for a capacitor to discharge depends on both the capacitance and the resistance in the circuit. The quantity RC is called the **time constant** of the circuit. It is written using the Greek letter tau (τ).

KEY EQUATION

$$\tau = RC$$

Time constant for a capacitor discharging.

Question

23 Show that the unit of the time constant (RC) is the second.

The equation for the exponential decay of charge on a capacitor is:

$$I = I_0 \exp\left(-\frac{t}{RC}\right)$$

where I is the current, I_0 is the initial current, t is time and RC is the time constant.

The current at any time is directly proportional to the potential difference across the capacitor, which in turn is directly proportional to charge across the plate. The equation also describes the change in the potential difference and the charge on the capacitor.

So:

$$V = V_0 \exp\left(-\frac{t}{RC}\right)$$

where V is the p.d, and V_0 is the initial p.d.

And:

$$Q = Q_0 \exp\left(-\frac{t}{RC}\right)$$

where Q is the charge and Q_0 is the initial charge.

KEY EQUATIONS

Exponential decay of charge on a capacitor:

$$I = I_0 \exp\left(-\frac{t}{RC}\right)$$
$$Q = Q_0 \exp\left(-\frac{t}{RC}\right)$$
$$V = V_0 \exp\left(-\frac{t}{RC}\right)$$

WORKED EXAMPLE

4 The potential difference across the plates of a capacitor of capacitance 500 μF is 240 V. The capacitor is connected across the terminals of a 600 Ω resistor.

Find the time taken for the current to fall to 0.10 A.

Step 1 Calculate the initial current:

$$\begin{aligned} I_0 &= \frac{V}{R} \\ &= \frac{240}{600} \\ &= 0.40\text{ A} \end{aligned}$$

Step 2 Calculate the time constant:

$$\begin{aligned} \tau &= RC \\ &= 600 \times 500 \times 10^{-6} \\ &= 0.30 \text{ s} \end{aligned}$$

Step 3 Substitute into the equation:

$$\begin{aligned} I &= I_0 \exp\left(-\frac{t}{RC}\right) \\ 0.10 &= 0.40 \exp\left(-\frac{t}{0.30}\right) \\ \frac{0.10}{0.40} &= \exp\left(-\frac{t}{0.30}\right) \\ 0.25 &= \exp\left(-\frac{t}{0.30}\right) \end{aligned}$$

Step 4 e comes from the antilog of the natural logarithm (ln) such that ln (e^x) = x

Taking ln of both sides:

$$\begin{aligned} \ln 0.25 &= -\frac{t}{0.30} \\ -1.386 &= -\frac{t}{0.30} \\ t &= 1.386 \times 0.30 \\ &= 0.41\ s \end{aligned}$$

Question

24 A 400 μF capacitor is charged using a 20 V battery. It is connected across the ends of a 600 Ω resistor with 20 V potential difference across its plates.

- **a** Calculate the charge stored on the capacitor.
- **b** Calculate the time constant for the discharging circuit.
- **c** Calculate the time it takes the charge on the capacitor to fall to 2.0 mC.
- **d** State the potential difference across the plates when the charge has fallen to 2.0 mC.

REFLECTION

In Worked example 3, we showed that when a charged capacitor is connected to an identical uncharged capacitor, half the energy is dissipated in driving the charge through the circuit and is transformed to thermal energy. If we had superconducting connectors – ones that conduct electricity without any energy losses – what would happen? Discuss with a partner.

What did you find satisfying about discussing this problem?

SUMMARY

Capacitors are constructed from two metal sheets ('plates'), separated by an insulating material. A capacitor stores equal and opposite amounts of charge on its plates.
For a capacitor, the charge stored is directly proportional to the p.d. between the plates: $$Q = VC$$
Capacitance is the charge stored per unit of p.d.
A farad is a coulomb per volt: 1 F = 1 C V^{-1}.
Capacitors store energy. The energy W stored at p.d. V is: $$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$
The formula $W = \frac{1}{2}QV$ is deduced from the area under a graph of potential difference against charge.
For capacitors connected in parallel and in series, the combined capacitances are: parallel: $C_{total} = C_1 + C_2 + C_3 + \ldots$ series: $\frac{1}{C_{total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ These formulae are derived from conservation of charge and addition of p.d.s.
The graphs for the discharge current, charge stored and potential difference across a capacitor are all examples of exponential decay.
The time constant for circuits containing capacitance and resistance is: $\tau = CR$
The graphs of discharge current, charge stored and potential difference across a capacitor are all of the form: $$x = x_0 \exp\left(-\frac{t}{RC}\right)$$

EXAM-STYLE QUESTIONS

1 A capacitor has a potential difference of 6.0 V across its plates and stores 9.0 mJ of energy.

Which row in the table gives the capacitance of the capacitor and the charge on its plates? **[1]**

	Capacitance / μF	Charge / mC
A	500	3.0
B	500	18
C	3000	3.0
D	3000	18

Table 23.3

2 A capacitor in an electronic circuit is designed to slowly discharge through an indicator lamp.

It is decided that the time taken for the capacitor to discharge needs to be increased. Four changes are suggested:

1 Connect a second capacitor in parallel with the original capacitor.

2 Connect a second capacitor in series with the original capacitor.

3 Connect a resistor in parallel with the lamp.

4 Connect a resistor in series with the lamp.

Which suggestions would lead to the discharge time being increased? **[1]**

A 1 and 3 only

B 1 and 4 only

C 2 and 3 only

D 2 and 4 only

3 A 470 μF capacitor is connected across the terminals of a battery of e.m.f. 9 V. Calculate the charge on the plates of the capacitor. **[1]**

4 Calculate the p.d. across the terminals of a 2200 μF capacitor when it has a charge of 0.033 C on its plates. **[1]**

5 Calculate the capacitance of a capacitor if it stores a charge of 2.0 C when there is a potential difference of 5000 V across its plates. **[1]**

6 Calculate the energy stored when a 470 μF capacitor has a potential difference of 12 V across its plates. **[1]**

7 Calculate the energy stored on a capacitor if it stores 1.5 mC of charge when there is a potential difference of 50 V across it. **[1]**

8 A 5000 μF capacitor has a p.d. of 24 V across its plates.

a Calculate the energy stored on the capacitor. **[1]**

b The capacitor is briefly connected across a bulb and half the charge flows off the capacitor. Calculate the energy dissipated in the lamp. **[3]**

[Total: 4]

9 A 4700 μF capacitor has a p.d. of 12 V across its terminals. It is connected to a resistor and the charge leaks away through the resistor in 2.5 s.

a Calculate the energy stored on the capacitor. **[1]**

b Calculate the charge stored on the capacitor. **[1]**

c Estimate the average current through the resistor. **[1]**

d Estimate the resistance of the resistor. **[2]**

e Suggest why the last two quantities can only be estimates. **[1]**

[Total: 6]

10 An electronics engineer is designing a circuit in which a capacitor of capacitance of 4700 μF is to be connected across a potential difference of 9.0

V. He has four 4700 μF, 6 V capacitors available. Draw a diagram to show how the four capacitors could be used for this purpose. **[1]**

11 Calculate the different capacitances that can be made from three 100 μF capacitors. For each value, draw the network that is used. **[4]**

12 This diagram shows three capacitors connected in series with a cell of e.m.f. 1.5 V.

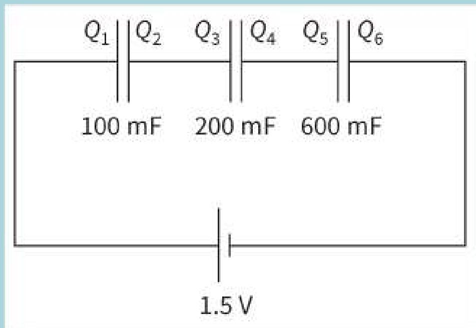

Figure 23.20

a Calculate the charges Q_1 to Q_6 on each of the plates. **[5]**

b Calculate the p.d. across each capacitor. **[3]**

[Total: 8]

13 a State one use of a capacitor in a simple electric circuit. **[1]**

b This is a circuit used to investigate the discharge of a capacitor, and a graph showing the change in current with time when the capacitor is discharged.

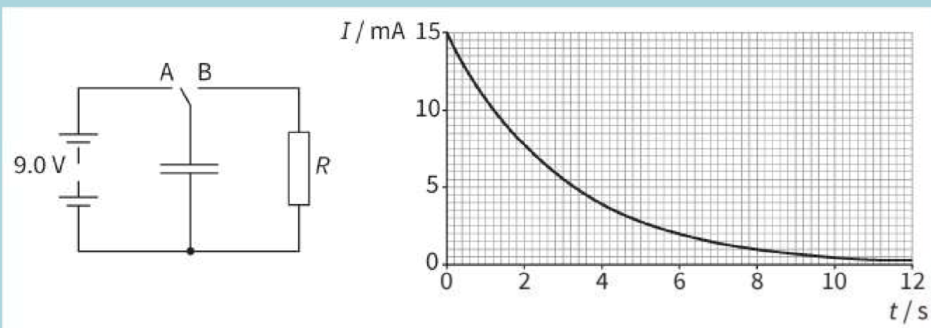

Figure 23.21

i Deduce the resistance R of the resistor. **[2]**

ii Explain why the current decreases as the capacitor discharges. **[2]**

iii The charge on the capacitor is equal to the area under the graph. Estimate the charge on the capacitor when the potential difference across it is 9.0 V. **[2]**

iv Calculate the capacitance of the capacitor. **[2]**

[Total: 9]

14 The spherical dome on a Van de Graaff generator has a diameter of 40 cm and the potential at its surface is 5.4 kV.

a i Calculate the charge on the dome. **[2]**

ii Calculate the capacitance of the dome. **[2]**

An earthed metal plate is moved slowly towards the sphere but does not touch it. The sphere discharges through the air to the plate. This graph shows how the potential at the surface of the sphere changes during the discharge.

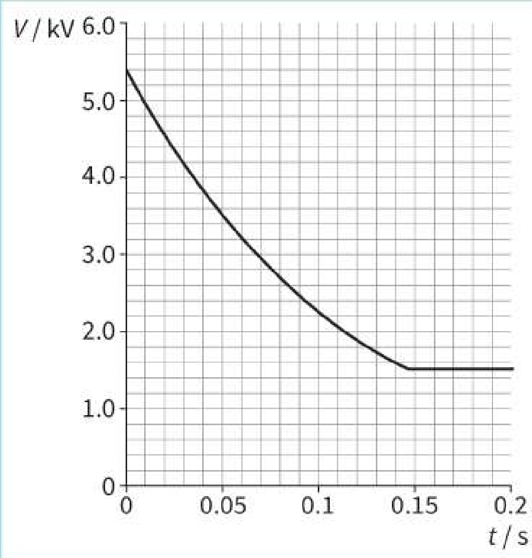

Figure 23.22

b Calculate the energy that is dissipated during the discharge. **[5]**

c Suggest why the discharge ceases while there is still some charge on the dome. **[2]**

[Total: 11]

15 **a** Show that the capacitance *C* of an isolated conducting sphere of radius *r* is given by the formula:

$C = 4\pi\varepsilon_0 r$ **[2]**

This diagram shows two identical conducting brass spheres of radius 10Cm mounted on insulating stands. Sphere A has a charge of $+5.0 \times 10^{-8}$ C and sphere B is uncharged.

Figure 23.23

b **i** Calculate the potential at the surface of sphere A. **[2]**

ii Calculate the energy stored on sphere A. **[2]**

Sphere B is brought up to sphere A and is touched to it so that the charge is shared between the two spheres, before being taken back to its original position.

c **i** Calculate the energy stored on each sphere. **[3]**

ii Suggest why there is a change in the total energy of the system. **[1]**

[Total: 10]

16 **a** Define the term capacitance of a capacitor. **[2]**

b This is a circuit that can be used to measure the capacitance of a capacitor.

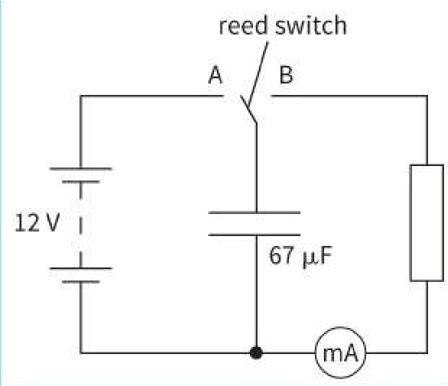

Figure 23.24

The reed switch vibrates back and forth at a frequency of 50 Hz. Each time it makes contact with A, the capacitor is charged by the battery so that there is a p.d. of 12 V across it. Each time it makes contact with B, it is fully discharged through the resistor.

i Calculate the charge that is stored on the capacitor when there is a p.d. of 12 V across it. **[2]**

ii Calculate the average current in the resistor. **[2]**

iii Calculate the average power dissipated in the resistor. **[3]**

c A second capacitor of the same value is connected in series with the first capacitor.

Discuss the effect on both the current recorded and the power dissipated in the resistor. **[4]**

[Total: 13]

17 a Explain what is meant by the time constant of a circuit containing capacitance and resistance. **[2]**

b A circuit contains capacitors of capacitance 500 μF and 2000 μF in series with each other and in series with a resistance of 2.5 kΩ.

i Calculate the effective capacitance of the capacitors in series. **[2]**

ii Calculate the charge on the capacitor plates when there is a potential difference of 50 V across the plates. **[2]**

iii Calculate the time taken for the charge on the plates to fall to 5% of the charge when there was a p.d. of 50 V across the plates. **[2]**

[Total: 8]