SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

I can	See topic...	Needs more work	Almost there	Ready to move on
define and use displacement, speed and velocity	1.1, 1.2, 1.3			
draw and interpret displacement–time graphs	1.4			
describe laboratory methods for determining speed	1.1			
understand the differences between scalar and vector quantities and give examples of each	1.2			
use vector addition to add and subtract vectors that are in the same plane.	1.6, 1.7			

Chapter 2

Accelerated motion

LEARNING INTENTIONS

In this chapter you will learn how to:

- define acceleration
- draw and interpret graphs of speed, velocity and acceleration
- calculate displacement from the area under a velocity–time graph
- calculate velocity and acceleration using gradients of a displacement–time graph and a velocity–time graph
- derive and use the equations of uniformly accelerated motion
- describe an experiment to measure the acceleration of free fall, *g*
- use perpendicular components to represent a vector
- explain projectile motion in terms of uniform velocity and uniform acceleration.

BEFORE YOU START

- Write down definitions of speed and velocity.
- Write a list of all the vectors that you know. Why are some quantities classed as vectors?

QUICK OFF THE MARK

The cheetah (Figure 2.1) has a maximum speed of more than 30 m s^{-1} (108 km/h). A cheetah can reach 20 m s^{-1} from a standing start in just three or four strides, taking only two seconds.

A car cannot increase its speed as rapidly but on a long straight road it can easily travel faster than a cheetah.

How do you think such measurements can be made? What apparatus is needed?

Figure 2.1: The cheetah is the world’s fastest land animal. Its acceleration is impressive, too.

2.1 The meaning of acceleration

In everyday language, the term **accelerating** means 'speeding up'. Anything whose speed is increasing is accelerating. Anything whose speed is decreasing is decelerating.

To be more precise in our definition of acceleration, we should think of it as changing velocity. Any object whose speed is changing or which is changing its direction has **acceleration**. Because acceleration is linked to velocity in this way, it follows that it is a vector quantity.

Some examples of objects accelerating are shown in Figure 2.2.

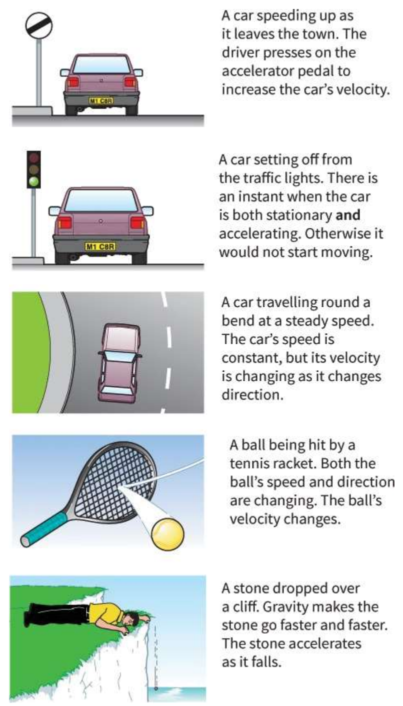


Figure 2.2: Examples of objects accelerating.

2.2 Calculating acceleration

The acceleration of something indicates the rate at which its velocity is changing. Language can get awkward here. Looking at the sprinter in Figure 2.3, we might say, 'The sprinter accelerates faster than the car.' However, 'faster' really means 'greater speed'. It is better to say, 'The sprinter has a greater acceleration than the car.'

Acceleration is defined as follows:

$$\begin{aligned}\text{acceleration} &= \text{rate of change of velocity}\\ \text{average acceleration} &= \frac{\text{change in velocity}}{\text{time taken}}\end{aligned}$$

So to calculate acceleration a, we need to know two quantities – the change in velocity Δv and the time taken Δt:

$$a = \frac{\Delta v}{\Delta t}$$

Sometimes this equation is written differently. We write u for the initial velocity and v for the final velocity (because u comes before v in the alphabet). The moving object accelerates from u to v in a time t (this is the same as the time represented by Δt in the equation). Then the acceleration is given by the equation:

$$a = \frac{v-u}{t}$$

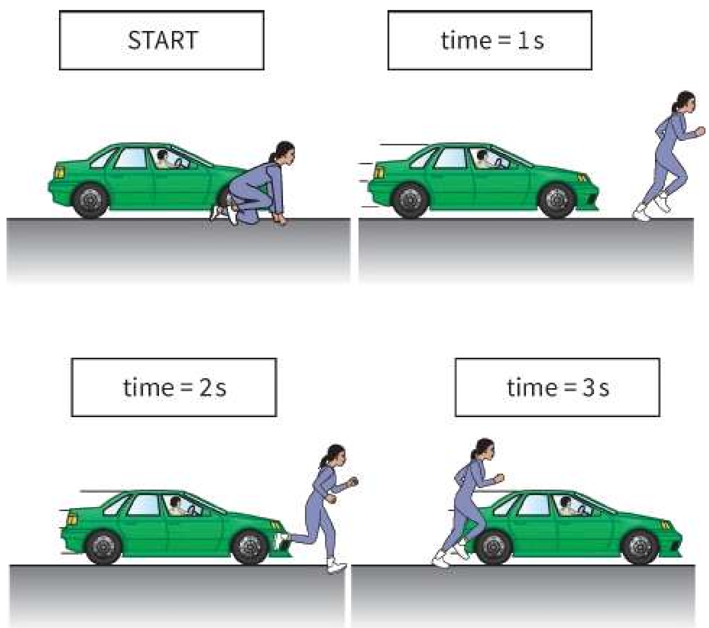


Figure 2.3: The sprinter has a greater acceleration than the car, but her top speed is less.

You must learn the definition of acceleration. It can be put in words or symbols. If you use symbols you must state what those symbols mean.

2.3 Units of acceleration

The unit of acceleration is m s^{-2} (metres per second squared). The sprinter might have an acceleration of 5 m s^{-2}; her velocity increases by 5 m s^{-1} every second. You could express acceleration in other units. For example, an advertisement might claim that a car accelerates from 0 to 60 miles per hour (mph) in 10 s. Its acceleration would then be 6 mph s^{-1} (6 miles per hour per second). However, mixing together hours and seconds is not a good idea, and so acceleration is almost always given in the standard SI unit of m s^{-2}.

WORKED EXAMPLES

1 Leaving a bus stop, a bus reaches a velocity of 8.0 m s^{-1} after 10 s. Calculate the acceleration of the bus.

Step 1 Note that the bus's initial velocity is 0 m s^{-1}.

Therefore:

$$\begin{aligned} \text{change in velocity } \Delta v &= (8.0 - 0)\,\mathrm{m\,s^{-1}} \\ \text{time taken } \Delta t &= 10\,\mathrm{s} \end{aligned}$$

Step 2 Substitute these values in the equation for acceleration:

$$\begin{aligned} \text{acceleration} &= \frac{\Delta v}{\Delta t} \\ &= \frac{8.0}{10} \\ &= 0.80\mathrm{m\,s^{-2}} \end{aligned}$$

2 A sprinter starting from rest has an acceleration of 5.0 m s^{-2} during the first 2.0 s of a race. Calculate her velocity after 2.0 s.

Step 1 Rearranging the equation $a = \frac{v-u}{t}$ gives:

$v = u + at$

Step 2 Substituting the values and calculating gives:

$v = 0 + (5.0 \times 2.0) = 10$ m s^{-1}

3 A train slows down from 60 m s^{-1} to 20 m s^{-1} in 50 s. Calculate the magnitude of the deceleration of the train.

Step 1 Write what you know:

$u = 60$ m s^{-1} $v = 20$ m s^{-1} $t = 50$ s

Step 2 Take care! Here the train's final velocity is less than its initial velocity. To ensure that we arrive at the correct answer, we will use the alternative form of the equation to calculate a.

$$\begin{aligned} a &= \frac{v-u}{t} \\ &= \frac{20-60}{50} = \frac{-40}{50} \\ &= -0.80\ \mathrm{m\,s^{-2}} \end{aligned}$$

The minus sign (negative acceleration) indicates that the train is slowing down. It is decelerating. The magnitude of the deceleration is 0.80 m s^{-2}.

Questions

1 A car accelerates from a standing start and reaches a velocity of 18 m s^{-1} after 6.0 s. Calculate its acceleration.

2 A car driver brakes gently. Her car slows down from 23 m s^{-1} to 11 m s^{-1} in 20 s. Calculate the magnitude (size) of her deceleration. (Note that, because she is slowing down, her acceleration is negative.)

3 A stone is dropped from the top of a cliff. Its acceleration is 9.81 m s^{-2}. How fast is it moving:

a after 1.0 s?

b after 3.0 s?

2.4 Deducing acceleration

The gradient of a velocity–time graph tells us whether the object's velocity has been changing at a high rate or a low rate, or not at all (Figure 2.4). We can deduce the value of the acceleration from the gradient of the graph:

acceleration = gradient of velocity–time graph

KEY IDEA

acceleration = gradient of velocity–time graph

The graph (Figure 2.5) shows how the velocity of a cyclist changed during the start of a sprint race. We can find his acceleration during the first section of the graph (where the line is straight) using the triangle as shown.

The change in velocity Δv is given by the vertical side of the triangle. The time taken Δt is given by the horizontal side.

$$\begin{aligned} \text{acceleration} &= \frac{\text{change in velocity}}{\text{time taken}} \\ &= \frac{25-0}{5} \\ &= 4.0\ \text{m s}^{-2} \end{aligned}$$

A more complex example where the velocity–time graph is curved is shown in Figure 2.18.

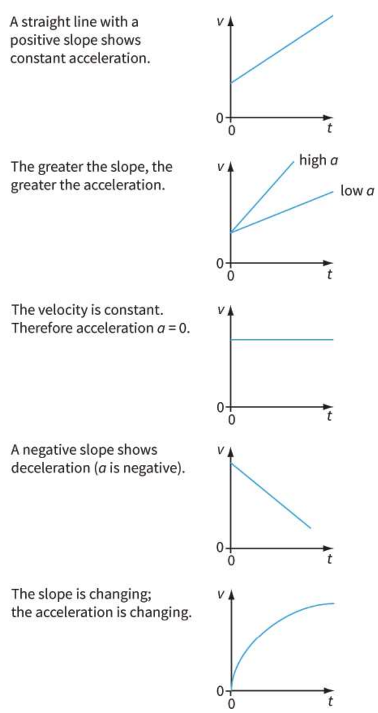


Figure 2.4: The gradient of a velocity–time graph is equal to acceleration.

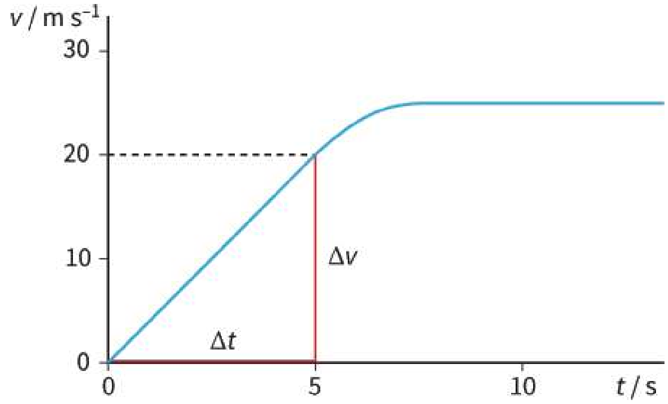


Figure 2.5: Deducing acceleration from a velocity–time graph.

2.5 Deducing displacement

We can also find the displacement of a moving object from its velocity–time graph. This is given by the area under the graph:

displacement = area under velocity–time graph

KEY IDEA

displacement = area **under** velocity–time graph

It is easy to see why this is the case for an object moving at a constant velocity. The displacement is simply velocity × time, which is the area of the shaded rectangle (Figure 2.6a).

For changing velocity, again the area under the graph gives displacement (Figure 2.6b).

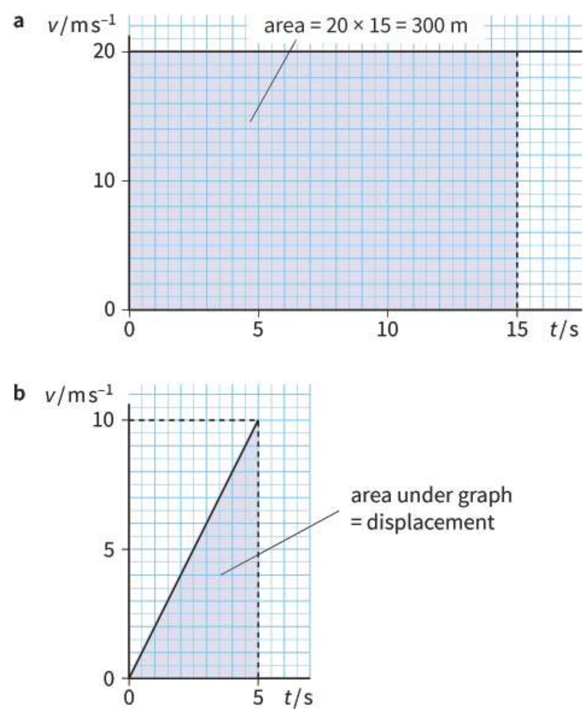


Figure 2.6: The area under the velocity–time graph is equal to the displacement of the object.

So, for this simple case in which the area is a triangle, we have:

$$\begin{aligned} \text{displacement} &= \tfrac{1}{2} \times \text{base} \times \text{height} \\ &= \tfrac{1}{2} \times 5.0 \times 10 \\ &= 25\ \text{m} \end{aligned}$$

It is easy to confuse displacement–time graphs and velocity–time graphs. Check by looking at the quantity marked on the vertical axis.

For more complex graphs, you may have to use other techniques such as counting squares to deduce the area, but this is still equal to the displacement.

(Take care when counting squares: it is easiest when the sides of the squares stand for one unit. Check the axes, as the sides may represent 2 units, 5 units or some other number.)

Questions

4 A lorry driver is travelling at the speed limit on a motorway. Ahead, he sees hazard lights and gradually slows down. He sees that an accident has occurred, and brakes suddenly to a halt. Sketch a velocity–time graph to represent the motion of this lorry.

5 Table 2.1 shows how the velocity of a motorcyclist changed during a speed trial along a straight road.

a Draw a velocity–time graph for this motion.

b From the table, deduce the motorcyclist's acceleration during the first 10 s.

c Check your answer by finding the gradient of the graph during the first 10 s.

d Determine the motorcyclist's acceleration during the last 15 s.

e Use the graph to find the total distance travelled during the speed trial.

Velocity / m s^{-1}	0	15	30	30	20	10	0
Time / s	0	5	10	15	20	25	30

Table 2.1: Data for a motorcyclist.

2.6 Measuring velocity and acceleration

In a car crash, the occupants of the car may undergo a very rapid deceleration. This can cause them serious injury, but can be avoided if an air-bag is inflated within a fraction of a second. Figure 2.7 shows the tiny accelerometer at the heart of the system, which detects large accelerations and decelerations.

The acceleration sensor consists of two rows of interlocking teeth. In the event of a crash, these move relative to one another, and this generates a voltage that triggers the release of the air-bag.

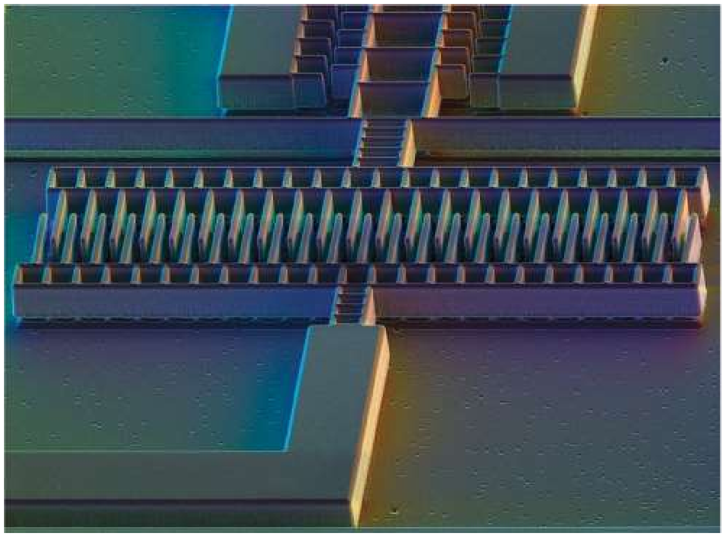


Figure 2.7: A micro-mechanical acceleration sensor is used to detect sudden accelerations and decelerations as a vehicle travels along the road. This electron microscope image shows the device magnified about 1000 times.

At the top of the photograph (Figure 2.7), you can see a second sensor that detects sideways accelerations. This is important in the case of a side impact.

These sensors can also be used to detect when a car swerves or skids, perhaps on an icy road. In this case, they activate the car's stability-control systems.

2.7 Determining velocity and acceleration in the laboratory

In Chapter 1, we looked at ways of finding the velocity of a trolley moving in a straight line. These involved measuring distance and time, and deducing velocity. Practical Activity 2.1 shows how these techniques can be extended to find the acceleration of a trolley.

PRACTICAL ACTIVITY 2.1: LABORATORY MEASUREMENTS OF ACCELERATION

Measurements using light gates

The computer records the time for the first ‘interrupt’ section of the card to pass through the light beam of the light gate (Figure 2.8). Given the length of the interrupt, it can work out the trolley’s initial velocity u. This is repeated for the second interrupt to give final velocity v. The computer also records the time interval $t_3 - t_1$ between these two velocity measurements. Now it can calculate the acceleration a as shown:

$$u = \frac{l_1}{t_2 - t_1}$$

(l_1 = length of first section of the interrupt card)

and

$$v = \frac{l_2}{t_4 - t_3}$$

(l_2 = length of second section of the interrupt card)

Therefore:

$$a = \frac{\text{change in velocity}}{\text{time taken}} = \frac{v - u}{t_3 - t_1}$$

(Note that this calculation gives only an approximate value for a. This is because u and v are **average** speeds over a period of time; for an accurate answer we would need to know the speeds at times t_1 and t_3.)

Sometimes two light gates are used with a card of length l. The computer can still record the times as shown and calculate the acceleration in the same way, with $l_1 = l_2 = l$.

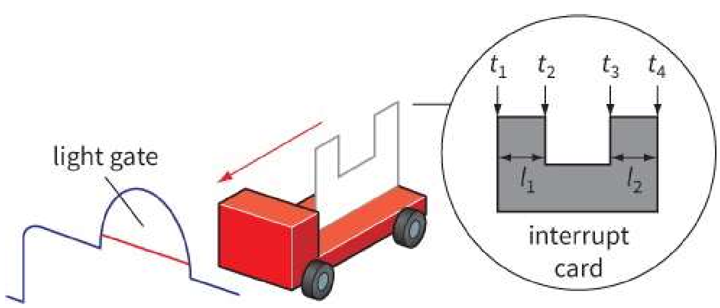


Figure 2.8: Determining acceleration using a single light gate.

Measurements using a ticker-timer

The practical arrangement is the same as for measuring velocity. Now we have to think about how to interpret the tape produced by an accelerating trolley (Figure 2.9).

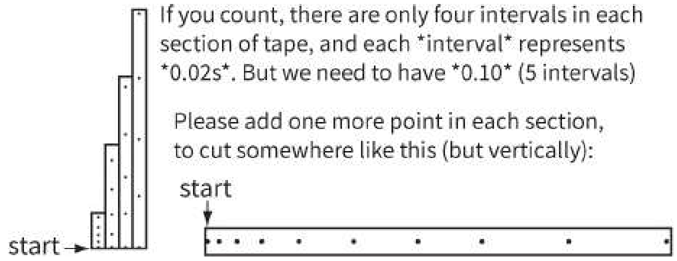


Figure 2.9: Ticker-tape for an accelerating trolley.

The tape is divided into sections, as before, every five dots. Remember that the time interval between adjacent dots is 0.02 s. Each section represents 0.10 s.

By placing the sections of tape side by side, you can picture the velocity–time graph.

The length of each section gives the trolley's displacement in 0.10 s, from which the average velocity during this time can be found. This can be repeated for each section of the tape, and a velocity–time graph drawn. The gradient of this graph is equal to the acceleration. Table 2.2 and Figure 2.10 show some typical results.

The acceleration is calculated to be:

$$\begin{aligned} a &= \frac{\Delta v}{\Delta t} \\ &= \frac{0.93}{0.20} \\ &\approx 4.7\ \mathrm{ms}^{-2} \end{aligned}$$

Section of tape	**Time at start / s**	**Time interval / s**	**Length of section / cm**	**Velocity / m s^{-1}**
1	0.0	0.10	2.3	0.23
2	0.10	0.10	7.0	0.70
3	0.20	0.10	11.6	1.16

Table 2.2: Data for Figure 2.10.

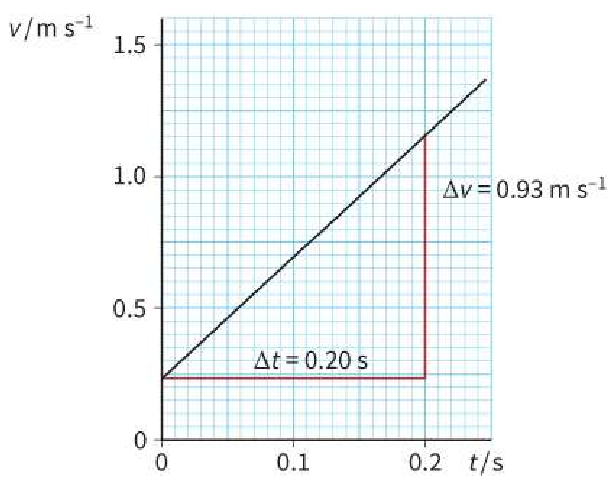


Figure 2.10: Deducing acceleration from measurements of a ticker-tape.

Measurements using a motion sensor

The computer software that handles the data provided by the motion sensor can calculate the acceleration of a trolley. However, because it deduces velocity from measurements of position, and then calculates acceleration from values of velocity, its precision is relatively poor.

Questions

6 Sketch a section of ticker-tape for a trolley that travels at a steady velocity and then decelerates.

7 Figure 2.11 shows the dimensions of an interrupt card, together with the times recorded as it passed through a light gate. Use these measurements to calculate the acceleration of the card. (Follow the steps outlined in Practical Activity 2.1.)

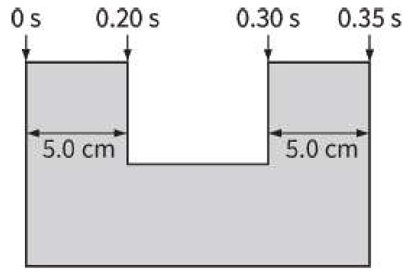


Figure 2.11: For Question 7.

8 Two adjacent five-dot sections of a ticker-tape measure 10 cm and 16 cm, respectively. The interval between dots is 0.02 s. Deduce the acceleration of the trolley that produced the tape.

2.8 The equations of motion

As a space rocket rises from the ground, its velocity steadily increases. It is accelerating (Figure 2.12).

Eventually, it will reach a speed of several kilometres per second. Any astronauts aboard find themselves pushed back into their seats while the rocket is accelerating.

Figure 2.12: A rocket accelerates as it lifts off from the ground.

The engineers who planned the mission must be able to calculate how fast the rocket will be travelling and where it will be at any point in its journey. They have sophisticated computers to do this, using more elaborate versions of the four equations of motion.

There is a set of equations that allows us to calculate the quantities involved when an object is moving with a **constant acceleration**.

The quantities we are concerned with are:

s displacement

u initial velocity

v final velocity

a acceleration

t time taken

The four **equations of motion** are shown above.

Take care using the equations of motion. They can only be used for:

- motion in a straight line
- an object with constant acceleration.

KEY EQUATIONS

The four equations of motion:

equation 1: $v = u + at$

equation 2: $s = \frac{(u+v)}{2} \times t$

equation 3: $s = ut + \frac{1}{2}at^2$

equation 4: $v^2 = u^2 + 2as$

To get a feel for how to use these equations, we will consider some worked examples. In each example, we will follow the same procedure:

Step 1 We write down the quantities that we know, and the quantity we want to find.

Step 2 Then we choose the equation that links these quantities, and substitute in the values.

Step 3 Finally, we calculate the unknown quantity.

We will look at where these equations come from in the next topic, 'Deriving the equations of motion'.

WORKED EXAMPLES

4 The rocket shown in Figure 2.12 lifts off from rest with an acceleration of 20 m s^{-2}. Calculate its velocity after 50 s.

Step 1 What we know:

$u = 0 \text{ m s}^{-1}$

$a = 20 \text{ m s}^{-2}$

$t = 50 \text{ s}$

and what we want to know: v = ?

Step 2 The equation linking u, a, t and v is equation 1:

$v = u + at$

Substituting gives:

$v = 0 + (20 \times 50)$

Step 3 Calculation then gives:

$v = 1000 \text{ m s}^{-1}$

So the rocket will be travelling at 1000 m s^{-1} after 50 s. This makes sense, since its velocity increases by 20 m s^{-1} every second, for 50 s.

You could use the same equation to work out how long the rocket would take to reach a velocity of 2000 m s^{-1}, or the acceleration it must have to reach a speed of 1000 m s^{-1} in 40 s and so on.

5 The car shown in Figure 2.13 is travelling along a straight road at 8.0 m s^{-1}. It accelerates at 1.0 m s^{-2} for a distance of 18 m. How fast is it then travelling?

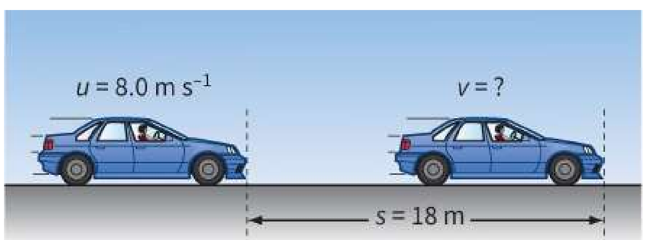


Figure 2.13: For Worked example 5. This car accelerates for a short distance as it travels along the road.

In this case, we will have to use a different equation, because we know the distance during which the car accelerates, not the time.

Step 1 What we know:

$u = 8.0 \text{ m s}^{-1}$

$a = 1.0 \text{ m s}^{-2}$

$s = 18 \text{ m}$

and what we want to know: v = ?

Step 2 The equation we need is equation 4:

$v^2 = u^2 + 2as$

Substituting gives:

$v^2 = 8.0^2 + (2 \times 1.0 \times 18)$

Step 3 Calculation then gives:

$v^2 = 64 + 36 = 100\ \text{m}^2\ \text{s}^{-2}$

$v = 10\ \text{m s}^{-1}$

So the car will be travelling at 10 m s^{-1} when it stops accelerating.

(You may find it easier to carry out these calculations without including the units of quantities when you substitute in the equation. However, including the units can help to ensure that you end up with the correct units for the final answer.)

6 A train (Figure 2.14) travelling at 20 m s^{-1} accelerates at 0.50 m s^{-2} for 30 s. Calculate the distance travelled by the train in this time.

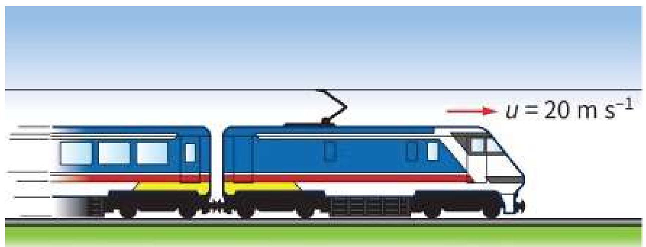


Figure 2.14: For Worked example 6. This train accelerates for 30 s.

Step 1 What we know:

$u = 20\ \text{m s}^{-1}$

$t = 30\ \text{s}$

$a = 0.50\ \text{m s}^{-2}$

and what we want to know: s = ?

Step 2 The equation we need is equation 3:

$s = ut + \frac{1}{2}at^2$

Substituting gives:

$s = (20 \times 30) + \frac{1}{2} \times 0.5 \times (30)^2$

Step 3 Calculation then gives:

$s = 600 + 225 = 825\ \text{m}$

So the train will travel 825 m while it is accelerating.

7 The cyclist in Figure 2.15 is travelling at 15 m s^{-1}. She brakes so that she doesn't collide with the wall. Calculate the magnitude of her deceleration.

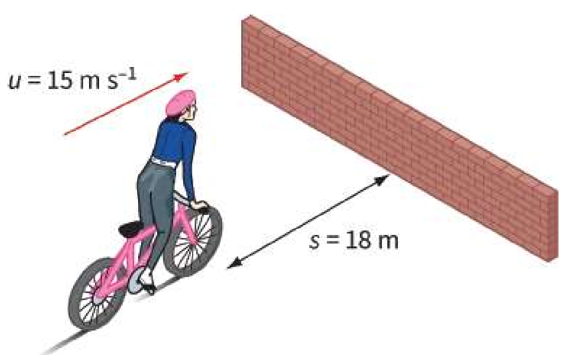


Figure 2.15: For Worked example 7. The cyclist brakes to stop herself colliding with the wall.

This example shows that it is sometimes necessary to rearrange an equation, to make the unknown quantity its subject. It is easiest to do this before substituting in the values.

Step 1 What we know:

$u = 15 \text{ m s}^{-1}$

$v = 0 \text{ m s}^{-1}$

$s = 18 \text{ m}$

and what we want to know: a = ?

Step 2 The equation we need is equation 4:

$v^2 = u^2 + 2as$

Rearranging gives:

$$a = \frac{v^2 - u^2}{2s}$$

$$a = \frac{0^2 - 15^2}{2 \times 18} = \frac{-225}{36}$$

Step 3 Calculation then gives:

$a = -6.25 \text{ m s}^{-2} \approx -6.3 \text{ m s}^{-2}$

So the cyclist will have to brake hard to achieve a deceleration of magnitude 6.3 m s^{-2}. The minus sign shows that her acceleration is negative; in other words, a deceleration.

Questions

9 A car is initially stationary. It has a constant acceleration of 2.0 m s^{-2}.

- **a** Calculate the velocity of the car after 10 s.
- **b** Calculate the distance travelled by the car at the end of 10 s.
- **c** Calculate the time taken by the car to reach a velocity of 24 m s^{-1}.

10 A train accelerates steadily from 4.0 m s^{-1} to 20 m s^{-1} in 100 s.

- **a** Calculate the acceleration of the train.
- **b** From its initial and final velocities, calculate the average velocity of the train.
- **c** Calculate the distance travelled by the train in this time of 100 s.

11 A car is moving at 8.0 m s^{-1}. The driver makes it accelerate at 1.0 m s^{-2} for a distance of 18 m. What is the final velocity of the car?

2.9 Deriving the equations of motion

We have seen how to make use of the equations of motion. But where do these equations come from? They arise from the definitions of velocity and acceleration.

We can find the first two equations from the velocity–time graph shown in Figure 2.16. The graph represents the motion of an object. Its initial velocity is u. After time t, its final velocity is v.

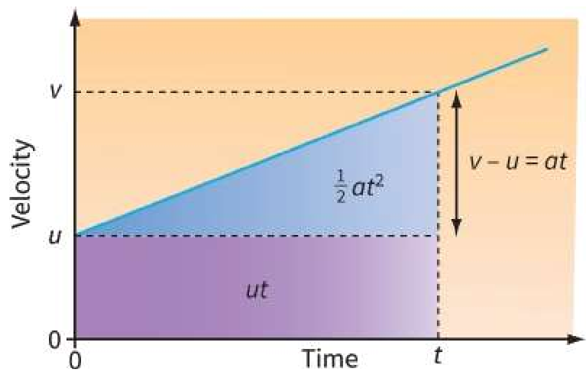


Figure 2.16: This graph shows the variation of velocity of an object with time. The object has constant acceleration.

Equation 1

The graph of Figure 2.16 is a straight line, therefore the object's acceleration a is constant. The gradient (slope) of the line is equal to acceleration.

The acceleration is defined as:

$$a = \frac{(v-u)}{t}$$

which is the gradient of the line. Rearranging this gives the first equation of motion:

$$v = u + at \qquad \text{(equation 1)}$$

Equation 2

Displacement is given by the area under the velocity–time graph. Figure 2.17 shows that the object's average velocity is half-way between u and v. So the object's average velocity, calculated by averaging its initial and final velocities, is given by:

$$\frac{(u+v)}{2}$$

The object's displacement is the shaded area in Figure 2.17. This is a rectangle, and so we have:

displacement = average velocity × time taken

and hence:

$$s = \frac{(u+v)}{2} \times t \qquad \text{(equation 2)}$$

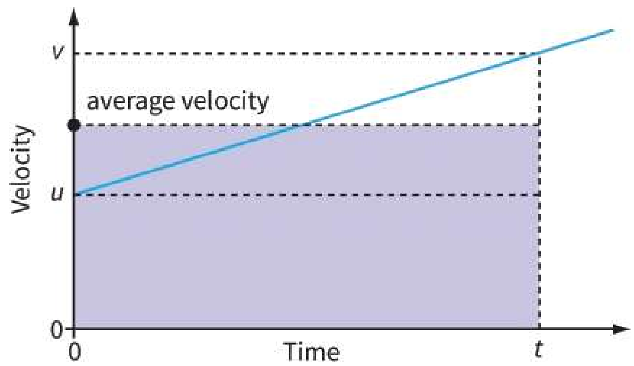


Figure 2.17: The average velocity is half-way between u and v.

Equation 3

From equations 1 and 2, we can derive equation 3:

$$v = u + at \qquad \text{(equation 1)}$$

$$s = \frac{(u+v)}{2} \times t \qquad \text{(equation 2)}$$

Substituting v from equation 1 gives:

$$\begin{aligned} s &= \frac{(u+u+at)}{2} \times t \\ &= \frac{2ut}{2} + \frac{at^2}{2} \end{aligned}$$

So

$$s = ut + \frac{1}{2}at^2 \qquad \text{(equation 3)}$$

Looking at Figure 2.16, you can see that the two terms on the right of the equation correspond to the areas of the rectangle and the triangle that make up the area under the graph. Of course, this is the same area as the rectangle in Figure 2.17.

Equation 4

Equation 4 is also derived from equations 1 and 2:

$$v = u + at \qquad \text{(equation 1)}$$

$$s = \frac{(u+v)}{2} \times t \qquad \text{(equation 2)}$$

Substituting for time t from equation 1 gives:

$$s = \frac{(u+v)}{2} \times \frac{(v-u)}{a}$$

Rearranging this gives:

$$\begin{aligned} 2as &= (u+v)(v-u) \\ &= v^2 - u^2 \end{aligned}$$

or simply:

$$v^2 = u^2 + 2as \qquad \text{(equation 4)}$$

Investigating road traffic accidents

The police frequently have to investigate road traffic accidents. They make use of many aspects of physics, including the equations of motion. The next two questions will help you to apply what you have learned to situations where police investigators have used evidence from skid marks on the road.

Questions

12 Trials on the surface of a new road show that, when a car skids to a halt, its acceleration is $-7.0\ m\ s^{-2}$. Estimate the skid-to-stop distance of a car travelling at a speed limit of $30\ m\ s^{-1}$ (approximately $110\ km\ h^{-1}$ or 70 mph).

13 At the scene of an accident on a country road, police find skid marks stretching for 50 m. Tests on the road surface show that a skidding car decelerates at $6.5\ m\ s^{-2}$. Was the car that skidded exceeding the speed limit of $25\ m\ s^{-1}$ ($90\ km\ h^{-1}$) on this road?

2.10 Uniform and nonuniform acceleration

It is important to note that the equations of motion only apply to an object that is moving with a constant acceleration. If the acceleration a was changing, you wouldn't know what value to put in the equations. Constant acceleration is often referred to as **uniform acceleration**.

The velocity–time graph in Figure 2.18 shows **non-uniform acceleration**. It is not a straight line; its gradient is changing (in this case, decreasing).

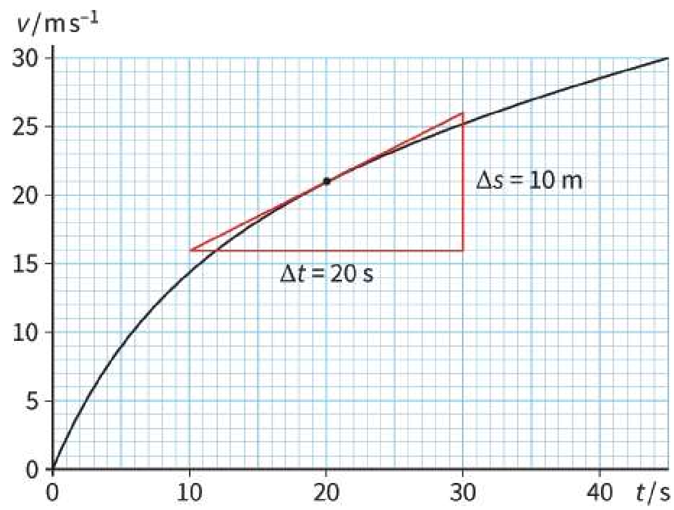


Figure 2.18: This curved velocity–time graph cannot be analysed using the equations of motion.

The acceleration at any instant in time is given by the gradient of the velocity–time graph. The triangle in Figure 2.18 shows how to find the acceleration at t = 20 seconds:

- At the time of interest, mark a point on the graph.
- Draw a **tangent** to the curve at that point.
- Make a large right-angled triangle, and use it to find the gradient.

You can find the change in displacement of the body as it accelerates by determining the area under the velocity–time graph.

To find the displacement of the object in Figure 2.18 between t = 0 and t = 20 s, the most straightforward, but lengthy, method is just to count the number of small squares.

In this case, up to t = 20 s, there are about 250 small squares. This is tedious to count but you can save yourself a lot of time by drawing a line from the origin to the point at 20 s. The area of the triangle is easy to find (200 small squares) and then you only have to count the number of small squares between the line you have drawn and the curve on the graph (about 50 squares)

In this case, each square is 1 m s^{-1} on the y-axis by 1 s on the x-axis, so the area of each square is 1 × 1 = 1 m and the displacement is 250 m. In other cases, note carefully the value of each side of the square you have chosen.

Questions

14 The graph in Figure 2.19 represents the motion of an object moving with varying acceleration. Lay your ruler on the diagram so that it is tangential to the graph at point P.

- **a** What are the values of time and velocity at this point?
- **b** Estimate the object's acceleration at this point.

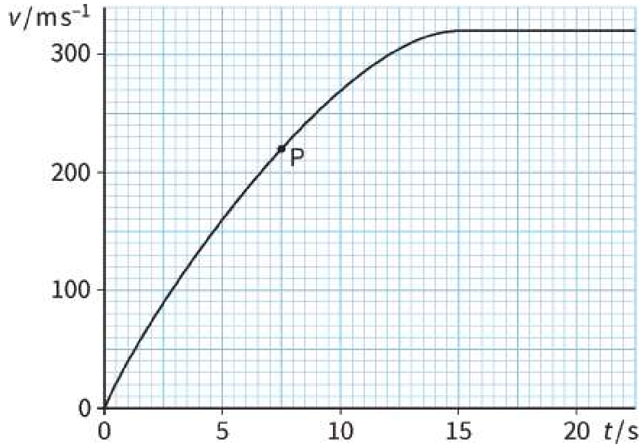


Figure 2.19: For Question 14.

15 The velocity–time graph (Figure 2.20) represents the motion of a car along a straight road for a period of 30 s.

a Describe the motion of the car.

b From the graph, determine the car's initial and final velocities over the time of 30 s.

c Determine the acceleration of the car.

d By calculating the area under the graph, determine the displacement of the car.

e Check your answer to part **d** by calculating the car's displacement using $s = ut\frac{1}{2}at^2$.

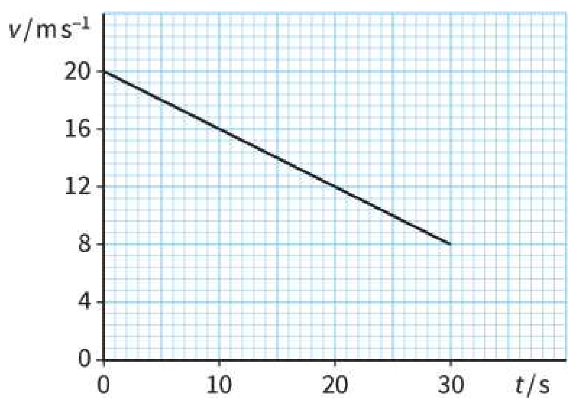


Figure 2.20: For Question 15.

2.11 Acceleration caused by gravity

If you drop a ball or stone, it falls to the ground. Figure 2.21, based on a multiflash photograph, shows the ball at equal intervals of time. You can see that the ball's velocity increases as it falls because the spaces between the images of the ball increase steadily. The ball is accelerating.

A multiflash photograph is useful to demonstrate that the ball accelerates as it falls. Usually, objects fall too quickly for our eyes to be able to observe them speeding up. It is easy to imagine that the ball moves quickly as soon as you let it go, and falls at a steady speed to the ground. Figure 2.21 shows that this is not the case.

If we measure the acceleration of a freely falling object on the surface of the Earth, we find a value of about 9.81 m s^{-2}. This is known as the acceleration of **free fall**, and is given the symbol g:

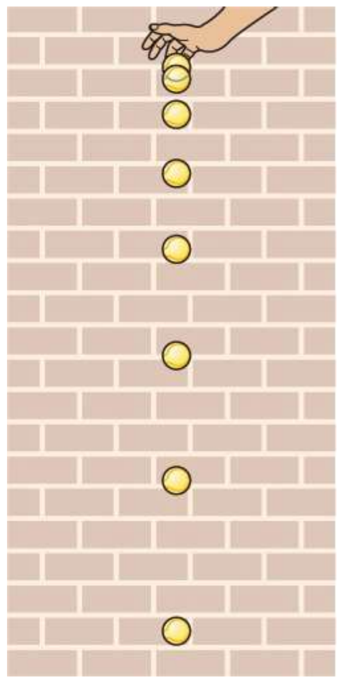


Figure 2.21: This diagram of a falling ball, based on a multiflash photo, clearly shows that the ball's velocity increases as it falls.

$$\text{acceleration of free fall } g = 9.81 \text{ m s}^{-2}$$

The value of g depends on where you are on the Earth's surface, but we usually take g = 9.81 m s^{-2}.

If we drop an object, its initial velocity u = 0. How far will it fall in time t? Substituting in $s = ut\frac{1}{2}at^2$ gives displacement s:

$$\begin{aligned} s &= \tfrac{1}{2} \times 9.81 \times t^2 \\ &= 4.9 \times t^2 \end{aligned}$$

Hence, by timing a falling object, we can determine g.

Questions

16 If you drop a stone from the edge of a cliff, its initial velocity u = 0, and it falls with acceleration g = 9.81 m s^{-2}. You can calculate the distance s it falls in a given time t using an equation of motion.

- **a** Copy and complete Table 2.3, which shows how s depends on t.
- **b** Draw a graph of s against t.
- **c** Use your graph to find the distance fallen by the stone in 2.5 s.
- **d** Use your graph to find how long it will take the stone to fall to the bottom of a cliff 40 m high. Check your answer using the equations of motion.

Time / s	0	1.0	2.0	3.0	4.0
Displacement / m	0	4.9			

Table 2.3: Time t and displacement s data fo

17 An egg falls off a table. The floor is 0.8 m from the table-top.

a Calculate the time taken to reach the ground.

b Calculate the velocity of impact with the ground.

2.12 Determining *g*

One way to measure the acceleration of free fall g would be to try bungee-jumping (Figure 2.22). You would need to carry a stopwatch, and measure the time between jumping from the platform and the moment when the elastic rope begins to slow your fall. If you knew the length of the unstretched rope, you could calculate g.

There are easier methods for finding g that can be used in the laboratory. These are described in Practical Activity 2.2.

Figure 2.22: A bungee-jumper falls with initial acceleration g.

PRACTICAL ACTIVITY 2.2: LABORATORY MEASUREMENTS OF *g*

Measuring *g* using an electronic timer

In this method, a steel ball-bearing is held by an electromagnet (Figure 2.23). When the current to the magnet is switched off, the ball begins to fall and an electronic timer starts. The ball falls through a trapdoor, and this breaks a circuit to stop the timer. This tells us the time taken for the ball to fall from rest through the distance h between the bottom of the ball and the trapdoor.

Here is how we can use one of the equations of motion to find g:

displacement $s = h$

time taken = t

initial velocity $u = 0$

acceleration $a = g$

Substituting in $s = ut + \frac{1}{2}at^2$ gives:

$h = \frac{1}{2}gt^2$

and for any values of h and t we can calculate a value for g.

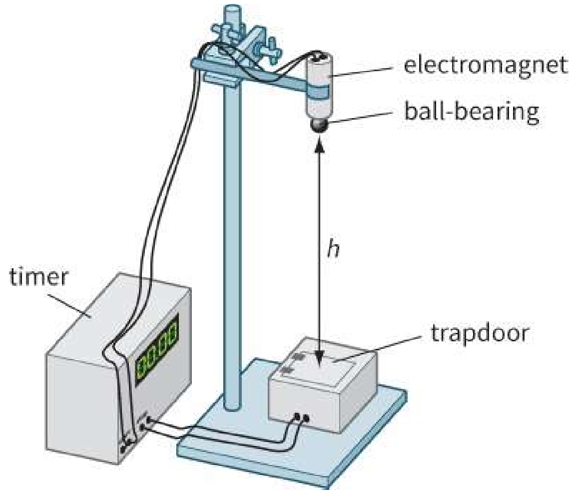


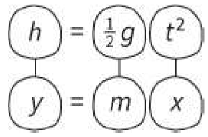
Figure 2.23: The timer records the time for the ball to fall through the distance h.

A more satisfactory procedure is to take measurements of t for several different values of h. The height of the ball bearing above the trapdoor is varied systematically, and the time of fall measured several times to calculate an average for each height. Table 2.4 and Figure 2.24 show some typical results. We can deduce g from the gradient of the graph of h against t^2.

The equation for a straight line through the origin is:

$$y = mx$$

In our experiment we have:



h / m	t / s	t^2 / s^2
0.27	0.25	0.063
0.39	0.30	0.090
0.56	0.36	0.130
0.70	0.41	0.168
0.90	0.46	0.212

Table 2.4: Data for Figure 2.24. These are mean values.

The gradient of the straight line of a graph of h against t^2 is equal to $\frac{g}{2}$.

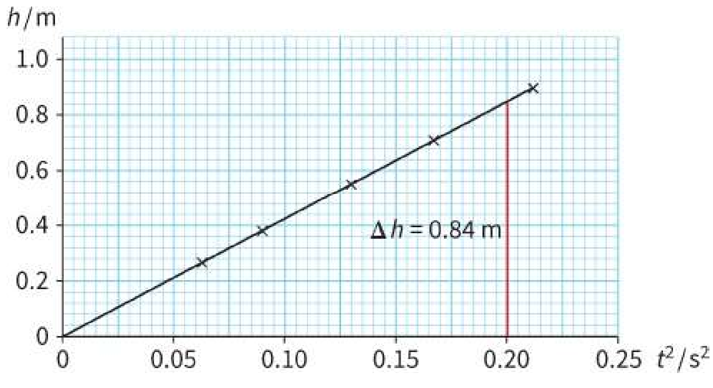


Figure 2.24: The acceleration of free fall can be determined from the gradient.

Therefore:

$$\begin{aligned} \text{gradient} &= \tfrac{g}{2} \\ &= \tfrac{0.84}{0.20} \\ &= 4.2 \end{aligned}$$

$$g = 4.2 \times 2 = 8.4 \text{ m s}^{-2}$$

Sources of uncertainty

The electromagnet may retain some magnetism when it is switched off, and this may tend to slow the ball's fall. Consequently, the time t recorded by the timer may be longer than if the ball were to fall completely freely. From $h = \frac{1}{2}gt^2$, it follows that, if t is too great, the experimental value of g will be too small. This is an example of a systematic error – all the results are systematically distorted so that they are too great (or too small) as a consequence of the experimental design.

Measuring the height h is awkward. You can probably only find the value of h to within ±1 mm at best. So there is a random error in the value of h, and this will result in a slight scatter of the points on the graph, and a degree of uncertainty in the final value of g.

If you just have one value for h and the corresponding value for t you can use the uncertainty in h and t to find the uncertainty in g.

The percentage uncertainty in g = percentage uncertainty in h + 2 × percentage uncertainty in t.

For more about errors and combining uncertainties, see Chapter P1.

Measuring g using a ticker-timer

Figure 2.25 shows a weight falling. As it falls, it pulls a tape through a ticker-timer. The spacing of the dots on the tape increases steadily, showing that the weight is accelerating. You can analyse the tape to find the acceleration, as discussed in Practical Activity 2.1.

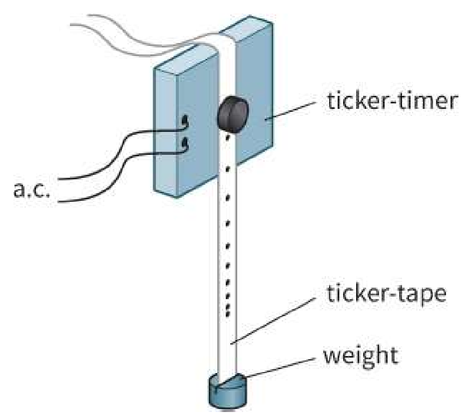


Figure 2.25: A falling weight pulls a tape through a ticker-timer.

This is not a very satisfactory method of measuring g. The main problem arises from friction between the tape and the ticker-timer. This slows the fall of the weight and so its acceleration is less than g. (This is another example of a systematic error.)

The effect of friction is less of a problem for a large weight, which falls more freely. If measurements are made for increasing weights, the value of acceleration gets closer and closer to the true value of g.

Measuring g using a light gate

Figure 2.26 shows how a weight can be attached to a card 'interrupt'. The card is designed to break the light beam twice as the weight falls. The computer can then calculate the velocity of the weight twice as it falls, and hence find its acceleration:

$$\text{initial velocity } u = \tfrac{x}{t_2 - t_1}$$

$$\text{final velocity } v = \tfrac{x}{t_4 - t_3}$$

Therefore:

$$\text{acceleration } a = \tfrac{v - u}{t_3 - t_1}$$

The weight can be dropped from different heights above the light gate. This allows you to find out whether its acceleration is the same at different points in its fall. This is an advantage over Method 1, which can only measure the acceleration from a stationary start.

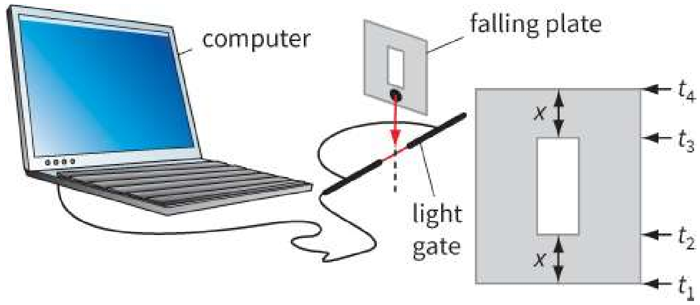


Figure 2.26: The weight accelerates as it falls. The upper section of the card falls more quickly through the light gate.

WORKED EXAMPLE

8 To get a rough value for g, a student dropped a stone from the top of a cliff. A second student timed the stone's fall using a stopwatch. Here are their results:

estimated height of cliff = 30 m

time of fall = 2.6 s

Use the results to estimate a value for g.

Step 1 Calculate the average speed of the stone:

average speed of stone during fall $= \frac{30}{2.6} = 11.5\ \text{m s}^{-1}$

Step 2 Find the values of v and u:

final speed $v = 2 \times 11.5\ \text{m s}^{-1} = 23.0\ \text{m s}^{-1}$

initial speed $u = 0\ \text{m s}^{-1}$

Step 3 Substitute these values into the equation for acceleration:

$$a = \frac{v-u}{t} = \frac{23.0}{2.6} = 8.8\ \text{m s}^{-2}$$

Note that you can reach the same result more directly using $s = ut + \frac{1}{2}at^2$, but you may find it easier to follow what is going on using the method given here. We should briefly consider why the answer is less than the expected value of g = 9.81 m s^{-2}. It might be that the cliff was higher than the student's estimate. The timer may not have been accurate in switching the stopwatch on and off. There will have been air resistance that slowed the stone's fall.

Questions

18 A steel ball falls from rest through a height of 2.10 m. An electronic timer records a time of 0.67 s for the fall.

a Calculate the average acceleration of the ball as it falls.

b Suggest reasons why the answer is not exactly 9.81 m s^{-2}.

c Suppose the height is measured accurately but the time is measured to an uncertainty of ±0.02 s. Calculate the percentage uncertainty in the time and the percentage uncertainty in the average acceleration. You can do this by repeating the calculation for g using a time of 0.65 s. You can find out more about uncertainty in Chapter P1.

19 In an experiment to determine the acceleration due to gravity, a ball was timed electronically as it fell from rest through a height h. The times t shown in Table 2.5 were obtained.

a Plot a graph of h against t^2.

b From the graph, determine the acceleration of free fall g.

c Comment on your answer.

Height *h* / m	0.70	1.03	1.25	1.60	1.99
Time *t* / s	0.99	1.13	1.28	1.42	1.60

Table 2.5: Height h and time t data for Question 19.

20 In Chapter 1, we looked at how to use a motion sensor to measure the speed and position of a moving object. Suggest how a motion sensor could be used to determine g.

2.13 Motion in two dimensions: projectiles

A curved trajectory

A multiflash photograph can reveal details of the path, or trajectory, of a projectile. Figure 2.27 shows the trajectories of a projectile – a bouncing ball. Once the ball has left the child's hand and is moving through the air, the only force acting on it is its weight.

The ball has been thrown at an angle to the horizontal. It speeds up as it falls – you can see that the images of the ball become further and further apart. At the same time, it moves steadily to the right. You can see this from the even spacing of the images across the picture.

The ball's path has a mathematical shape known as a parabola. After it bounces, the ball is moving more slowly. It slows down, or decelerates, as it rises – the images get closer and closer together.

We interpret this picture as follows. The vertical motion of the ball is affected by the force of gravity, that is, its weight. When it rises it has a vertical deceleration of magnitude g, which slows it down, and when it falls it has an acceleration of g, which speeds it up. The ball's horizontal motion is unaffected by gravity. In the absence of air resistance, the ball has a constant velocity in the horizontal direction. We can treat the ball's vertical and horizontal motions separately, because they are independent of one another.

Figure 2.27: A bouncing ball is an example of a projectile. This multiflash photograph shows details of its motion that would escape the eye of an observer.

Components of a vector

In order to understand how to treat the velocity in the vertical and horizontal directions separately we start by considering a constant velocity.

If an aeroplane has a constant velocity v at an angle θ as shown in Figure 2.28, then we say that this velocity has two effects or **components**, v_N in a northerly direction and v_E in an easterly direction. These two components of velocity add up to make the actual velocity v.

This process of taking a velocity and determining its effect along another direction is known as **resolving** the velocity along a different direction. In effect, splitting the velocity into two components at right angles is the reverse of adding together two vectors – it is splitting one vector into two vectors along convenient directions.

KEY EQUATIONS

For a velocity v at an angle θ to the x-direction the components are:

x-direction: $v \cos \theta$

y-direction: $v \sin \theta$

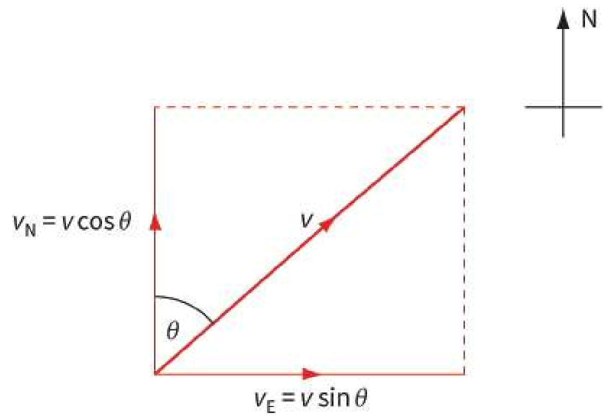


Figure 2.28: Components of a velocity. The component due north is $v_N = v\cos\theta$ and the component due east is $v_E = v\sin\theta$.

To find the component of any vector (for example, displacement, velocity, acceleration) in a particular direction, we can use the following strategy:

Step 1 Find the angle θ between the vector and the direction of interest.

Step 2 Multiply the vector by the cosine of the angle θ.

So the component of an object's velocity v at angle θ to v is equal to $v\cos\theta$ (Figure 2.28).

Question

21 Find the x- and y-components of each of the vectors shown in Figure 2.29. (You will need to use a protractor to measure angles from the diagram.)

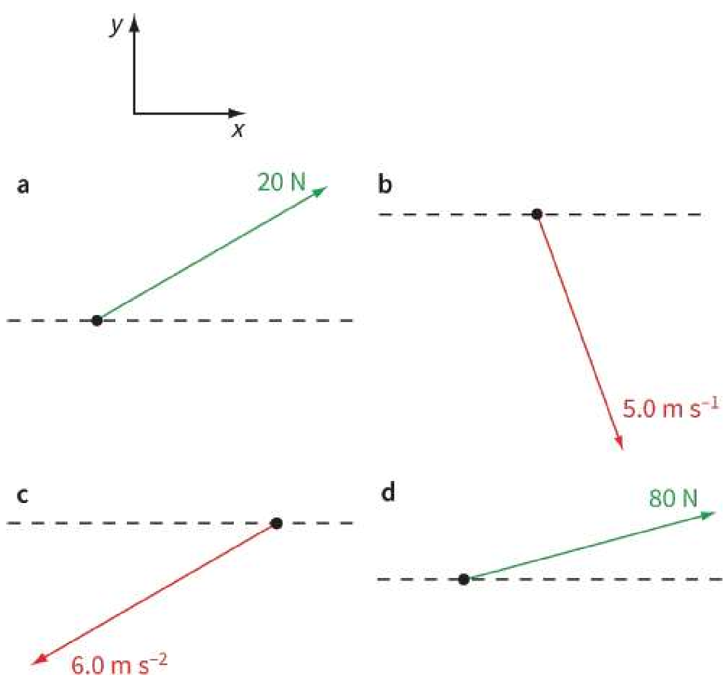


Figure 2.29: The vectors for Question 21.

2.14 Understanding projectiles

We will first consider the simple case of a projectile thrown straight up in the air, so that it moves vertically. Then we will look at projectiles that move horizontally and vertically at the same time.

Up and down

A stone is thrown upwards with an initial velocity of 20 m s^{-1}. Figure 2.30 shows the situation.

It is important to use a consistent sign convention here. We will take upwards as positive, and downwards as negative. So the stone's initial velocity is positive, but its acceleration g is negative. We can solve various problems about the stone's motion by using the equations of motion.

How high?

How high will the stone rise above ground level of the cliff?

As the stone rises upwards, it moves more and more slowly – it decelerates because of the force of gravity.

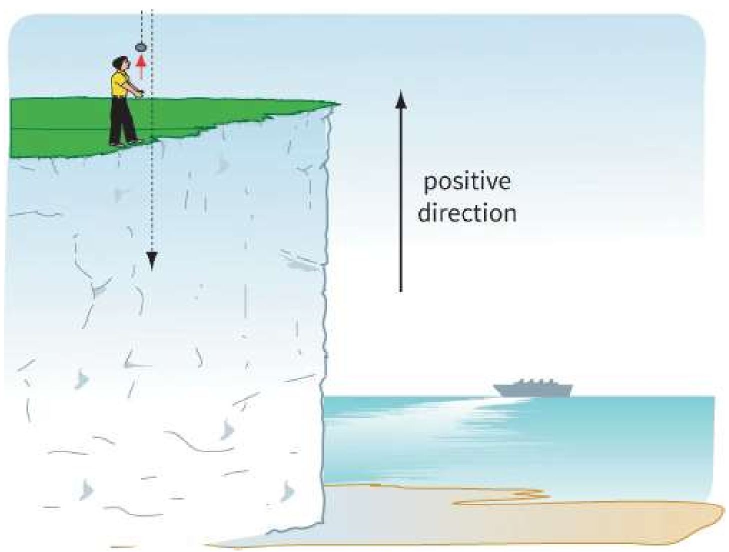


Figure 2.30: Standing at the edge of the cliff, you throw a stone vertically upwards. The height of the cliff is 25 m.

At its highest point, the stone's velocity is zero. So the quantities we know are:

$$\begin{aligned} \text{initial velocity} &= u = 20\ \text{m s}^{-1} \\ \text{final velocity} &= v = 0\ \text{m s}^{-1} \\ \text{acceleration} &= a = -9.81\ \text{m s}^{-2} \\ \text{displacement} &= s = ? \end{aligned}$$

The relevant equation of motion is $v^2 = u^2 + 2as$. Substituting values gives:

$$\begin{aligned} 0^2 &= 20^2 + 2 \times (-9.81) \times s \\ 0 &= 400 - 19.62\ \text{s} \\ s &= \frac{400}{19.62} \\ &= 20.4\ \text{m} \approx 20\ \text{m} \end{aligned}$$

The stone rises 20 m upwards before it starts to fall again.

How long?

How long will it take from leaving your hand for the stone to fall back to the clifftop?

When the stone returns to the point from which it was thrown, its displacement s is zero. So:

$$s = 0$$
$$u = 20 \text{ m s}^{-1}$$
$$a = -9.81 \text{ m s}^{-2}$$
$$t = ?$$

Substituting in $s = ut + \frac{1}{2}at^2$ gives:

$$\begin{aligned} 0 &= 20t \times \tfrac{1}{2}(-9.81) \times t^2 \\ &= 20t - 4.905t^2 \\ &= (20 - 4.905t) \times t \end{aligned}$$

There are two possible solutions to this:

- t = 0 s; in other words, the stone had zero displacement at the instant it was thrown
- t = 4.1 s; in other words, the stone returned to zero displacement after 4.1 s, which is the answer we are interested in.

Falling further

The height of the cliff is 25 m. How long will it take the stone to reach the foot of the cliff?

This is similar to the last example, but now the stone's final displacement is 25 m below its starting point. By our sign convention, this is a negative displacement and s = −25 m.

Questions

22 In the example in 'Falling further', calculate the time it will take for the stone to reach the foot of the cliff.

23 A ball is fired upwards with an initial velocity of 30 m s^{-1}. Table 2.6 shows how the ball's velocity changes. (Take g = 9.81 m s^{-2}.)

a Copy and complete the table.

b Draw a graph to represent the data.

c Use your graph to deduce how long the ball took to reach its highest point.

Velocity / m s^{-1}	30	20.19				
Time / s	0	1.0	2.0	3.0	4.0	5.0

Table 2.6: For Question 23.

Vertical and horizontal at the same time

Here is an example to illustrate what happens when an object travels vertically and horizontally at the same time.

In a toy, a ball-bearing is fired horizontally from a point 0.4 m above the ground. Its initial velocity is 2.5 m s^{-1}. Its positions at equal intervals of time have been calculated and are shown in Table 2.7. These results are also shown in Figure 2.31. Study the table and the graph. You should notice the following:

- The horizontal distance increases steadily. This is because the ball's horizontal motion is unaffected by the force of gravity. It travels at a steady velocity horizontally so we can use $v = \frac{s}{t}$.
- The vertical distances do not show the same pattern. The ball is accelerating downwards so we must use the equations of motion. (These figures have been calculated using g = 9.81 m s^{-2}.)

Time / s	Horizontal distance / m	Vertical distance / m
0.00	0.00	0.000
0.04	0.10	0.008
0.08	0.20	0.031
0.12	0.30	0.071
0.16	0.40	0.126
0.20	0.50	0.196
0.24	0.60	0.283

0.28	0.70	0.385

Table 2.7: Data for the example of a moving ball, as shown in Figure 2.31.

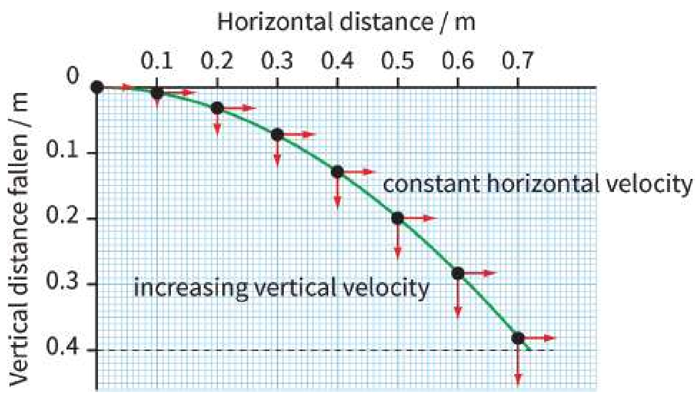


Figure 2.31: This sketch shows the path of the ball projected horizontally. The arrows represent the horizontal and vertical components of its velocity.

You can calculate the distance s fallen using the equation of motion $s = ut + \frac{1}{2}at^2$. (The initial vertical velocity u = 0.)

The horizontal distance is calculated using:

horizontal distance = 2.5 × t

The vertical distance is calculated using:

$$\text{vertical distance} = \frac{1}{2} \times 9.81 \times t^2$$

KEY IDEA

In the absence of air resistance, an object has constant velocity horizontally and constant acceleration vertically.

WORKED EXAMPLES

9 A stone is thrown horizontally with a velocity of 12 m s^{-1} from the top of a vertical cliff.

Calculate how long the stone takes to reach the ground 40 m below and how far the stone lands from the base of the cliff.

Step 1 Consider the ball's vertical motion. It has zero initial speed vertically and travels 40 m with acceleration 9.81 m s^{-2} in the same direction.

$s = ut + \frac{1}{2}at^2$

$40 = 0 + \frac{1}{2} \times 9.81 \times t^2$

So, t = 2.86 s.

Step 2 Consider the ball's horizontal motion. The ball travels with a constant horizontal velocity, 12 m s^{-1}, as long as there is no air resistance.

distance travelled = $u \times t$ = 12 × 2.86 = 34.3 m

Hint: *You may find it easier to summarise the information like this:*

vertically s = 40 u = 0 a = 9.81 t = ? v = ?

horizontally u = 12 v = 12 a = 0 t = ? s = ?

10 A ball is thrown with an initial velocity of 20 m s^{-1} at an angle of 30° to the horizontal (Figure 2.32). Calculate the horizontal distance travelled by the ball (its **range**).

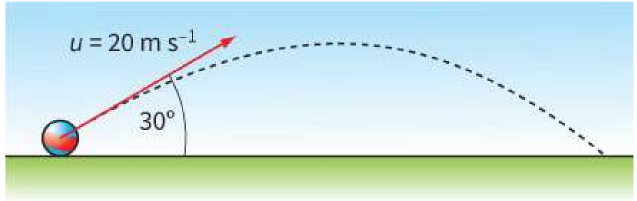


Figure 2.32: For Worked example 10.

Step 1 Split the ball's initial velocity into horizontal and vertical components:

initial velocity = u = 20 m s^{-1}

horizontal component of initial velocity v= u cos θ = 20 × cos 30° = 17.3 m s^{-1}

vertical component of initial velocity = u sin θ = 20 × sin 30° = 10 m s^{-1}

Step 2 Consider the ball's vertical motion. How long will it take to return to the ground? In other words, when will its displacement return to zero?

u = 10 m s^{-1} a = −9.81 m s^{-2} s = 0 t = ?

Using $s = ut + \frac{1}{2}at^2$, we have:

$0 = 10t - 4.905t^2$

This gives t = 0 s or t = 2.04 s.

So, the ball is in the air for 2.04 s.

Step 3 Consider the ball's horizontal motion. How far will it travel horizontally in the 2.04 s before it lands? This is simple to calculate, since it moves with a constant horizontal velocity of 17.3 m s^{-1}.

$$\begin{aligned}\text{horizontal displacement } s &= 17.3 \times 2.04 \\ &= 35.3 \text{ m}\end{aligned}$$

Hence the horizontal distance travelled by the ball (its range) is about 35 m.

Questions

24 A stone is thrown horizontally from the top of a vertical cliff and lands 4.0 s later at a distance 12.0 m from the base of the cliff. Ignore air resistance.

- **a** Calculate the horizontal speed of the stone.
- **b** Calculate the height of the cliff.

25 A stone is thrown with a velocity of 8.0 m s^{-1} into the air at an angle of 40° to the horizontal.

- **a** Calculate the vertical component of the velocity.
- **b** State the value of the vertical component of the velocity when the stone reaches its highest point. Ignore air resistance.
- **c** Use your answers to part **a** and part **b** to calculate the time the stone takes to reach its highest point.
- **d** Calculate the horizontal component of the velocity.
- **e** Use your answers to part **c** and part **d** to find the horizontal distance travelled by the stone as it climbs to its highest point.

26 The range of a projectile is the horizontal distance it travels before it reaches the ground. The greatest range is achieved if the projectile is thrown at 45° to the horizontal.

A ball is thrown with an initial velocity of 40 m s^{-1}. Calculate its greatest possible range when air resistance is considered to be negligible.

REFLECTION

- Could you easily teach somebody a proof of the equations of motion? How would you do this?
- What do you find unexpected about projectile motion?

SUMMARY

Acceleration is equal to the rate of change of velocity. It is a vector, has units m s^{-2} and can be found from the gradient of a velocity–time graph. The area under this graph is the change in displacement.
Acceleration, velocity, displacement and time for a uniform acceleration are related by the equations of motion, which you should know how to derive and use.
The acceleration of free fall is taken as 9.81 m s^{-2} and you should know an experiment to measure this quantity.
Vector quantities can be resolved into components. Components at right angles to one another can be treated independently. For a velocity v at an angle θ to the x-direction the components are: x-direction: $v \cos \theta$ y-direction: $v \sin \theta$
In the absence of air resistance, projectiles involve a constant acceleration downwards and a constant velocity horizontally. These can be treated independently of one another.

EXAM-STYLE QUESTIONS

1 An aircraft, starting from rest accelerates uniformly along a straight runway. It reaches a speed of 200 km h^{-1} and travels a distance of 1.4 km.

What is the acceleration of the aircraft along the runway? **[1]**

A 1.1 m s^{-2}

B 2.2 m s^{-2}

C 3.0 m s^{-2}

D 6.0 m s^{-2}

2 A ball is thrown with a velocity of 10 m s^{-1} at an angle of 30° to the horizontal. Air resistance has a negligible effect on the motion of the ball.

Figure 2.33

What is the velocity of the ball at the highest point in its path? **[1]**

A 0

B 5.0 m s^{-1}

C 8.7 m s^{-1}

D 10 m s^{-1}

3 A trolley travels along a straight track. The variation with time t of the velocity v of the trolley is shown. **[1]**

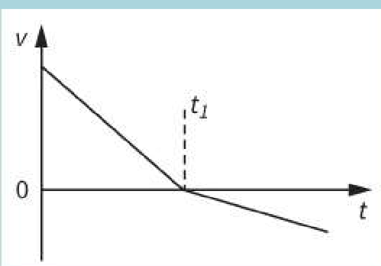
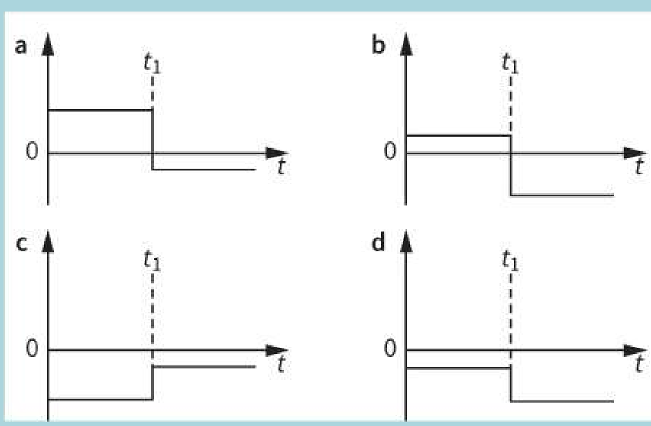


Figure 2.34

Which graph shows the variation with time of the acceleration a of the trolley?



4 A motorway designer can assume that cars approaching a motorway enter a slip road with a velocity of 10 m s^{-1} and reach a velocity of 30 m s^{-1} before joining the motorway. Calculate the minimum length for the slip road, assuming that vehicles have an acceleration of 4.0 m s^{-2}. **[4]**

5 A train is travelling at 50 m s^{-1} when the driver applies the brakes and gives the train a constant deceleration of magnitude 0.50 m s^{-2} for 100 s. Describe what happens to the train. Calculate the distance travelled by the train in 100 s. **[7]**

6 A boy stands on a cliff edge and throws a stone vertically upwards at time t = 0. The stone leaves his hand at 20 m s^{-1}. Take the acceleration of the ball as 9.81 m s^{-2}.

a Show that the equation for the displacement of the ball is:

$s = 20t - 4.9t^2$ **[2]**

b Calculate the height of the stone 2.0 s after release and 6.0 s after release. **[3]**

c Calculate the time taken for the stone return to the level of the boy's hand. You may assume the boy's hand does not move vertically after the ball is released. **[4]**

[Total: 9]

7 This graph shows the variation of velocity with time of two cars, A and B, which are travelling in the same direction over a period of time of 40 s.

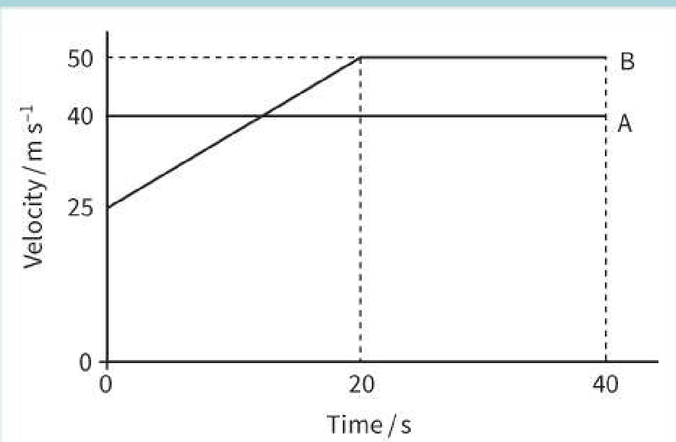


Figure 2.35

Car A, travelling at a constant velocity of 40 m s^{-1}, overtakes car B at time t = 0. In order to catch up with car A, car B immediately accelerates uniformly for 20 s to reach a constant velocity of 50 m s^{-1}. Calculate:

a the distance that A travels during the first 20 s **[2]**

b the acceleration and distance of travel of B during the first 20 s **[5]**

c the additional time taken for B to catch up with A **[2]**

d the distance each car will have then travelled since t = 0. **[2]**

[Total: 11]

8 An athlete competing in the long jump leaves the ground with a velocity of 5.6

m s^{-1} at an angle of 30° to the horizontal.

a Determine the vertical component of the velocity and use this value to find the time between leaving the ground and landing. **[4]**

b Determine the horizontal component of the velocity and use this value to find the horizontal distance travelled. **[4]**

[Total: 8]

9 This diagram shows an arrangement used to measure the acceleration of a metal plate as it falls vertically.

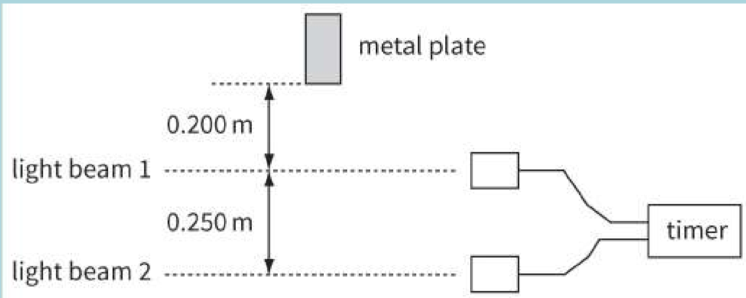


Figure 2.36

The metal plate is released from rest and falls a distance of 0.200 m before breaking light beam 1. It then falls a further 0.250 m before breaking light beam 2.

a Calculate the time taken for the plate to fall 0.200 m from rest. (You may assume that the metal plate falls with an acceleration equal to the acceleration of free fall.) **[2]**

b The timer measures the speed of the metal plate as it falls through each light beam. The speed as it falls through light beam 1 is 1.92 m s^{-1} and the speed as it falls through light beam 2 is 2.91 m s^{-1}.

i Calculate the acceleration of the plate between the two light beams. **[2]**

ii State and explain one reason why the acceleration of the plate is not equal to the acceleration of free fall. **[2]**

[Total: 6]

10 This is a velocity–time graph for a vertically bouncing ball.

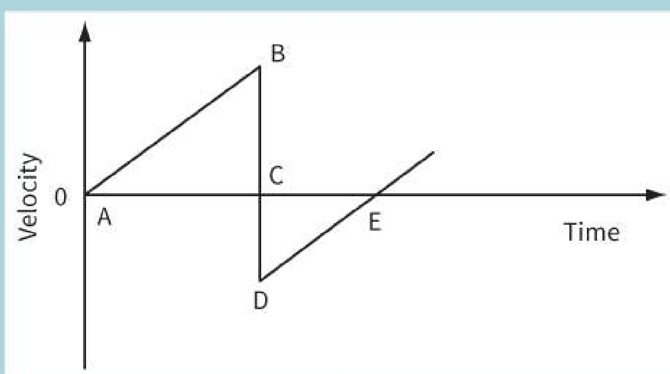


Figure 2.37

The ball is released at A and strikes the ground at B. The ball leaves the ground at D and reaches its maximum height at E. The effects of air resistance can be neglected.

a State:

i why the velocity at D is negative **[1]**

ii why the gradient of the line AB is the same as the gradient of line DE **[1]**

iii what is represented by the area between the line AB and the time axis **[1]**

iv why the area of triangle ABC is greater than the area of triangle CDE. **[1]**

b The ball is dropped from rest from an initial height of 1.2 m. After hitting the ground the ball rebounds to a height of 0.80 m. The ball is in contact with the ground between B and D for a time of 0.16 s.

Using the acceleration of free fall, calculate:

i the speed of the ball immediately before hitting the ground **[2]**

ii the speed of the ball immediately after hitting the ground **[2]**

iii the acceleration of the ball while it is in contact with the ground. State the direction of this acceleration. **[3]**

[Total: 11]

11 A student measures the speed *v* of a trolley as it moves down a slope. The variation of *v* with time *t* is shown in this graph.

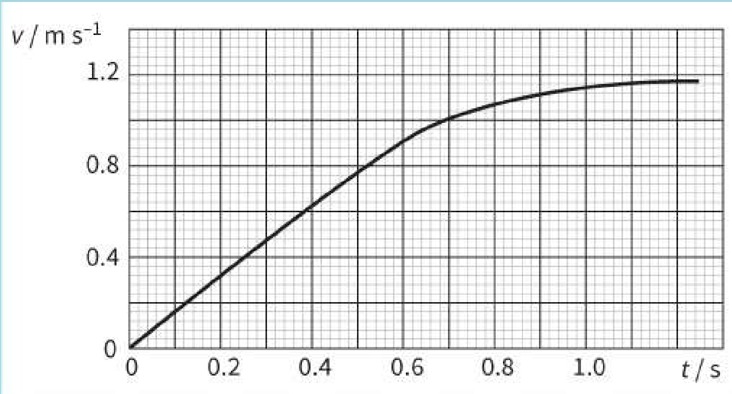


Figure 2.38

a Use the graph to find the acceleration of the trolley when $t = 0.70$ s. **[2]**

b State how the acceleration of the trolley varies between $t = 0$ and $t = 1.0$ s. Explain your answer by reference to the graph. **[3]**

c Determine the distance travelled by the trolley between $t = 0.60$ and $t = 0.80$ s. **[3]**

d The student obtained the readings for *v* using a motion sensor. The readings may have random errors and systematic errors. Explain how these two types of error affect the velocity–time graph. **[2]**

[Total: 10]

12 A car driver is travelling at speed *v* on a straight road. He comes over the top of a hill to find a fallen tree on the road ahead. He immediately brakes hard but travels a distance of 60 m at speed *v* before the brakes are applied. The skid marks left on the road by the wheels of the car are of length 140 m, as shown.

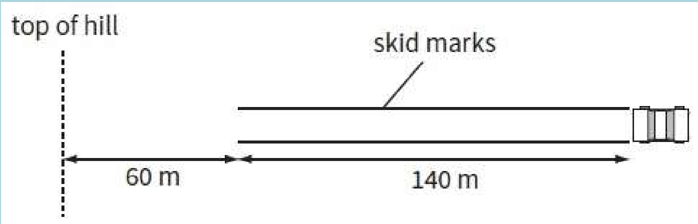


Figure 2.39

The police investigate whether the driver was speeding and establish that the car decelerates at 2.0 m s^{-2} during the skid.

a Determine the initial speed *v* of the car before the brakes are applied. **[2]**

b Determine the time taken between the driver coming over the top of the hill and applying the brakes. Suggest whether this shows whether the driver was alert to the danger. **[2]**

c The speed limit on the road is 100 km/h. Determine whether the driver was breaking the speed limit. **[2]**

[Total: 6]

13 A hot-air balloon rises vertically. At time $t = 0$, a ball is released from the balloon. This graph shows the variation of the ball's velocity v with t. The ball hits the ground at $t = 4.1$ s.

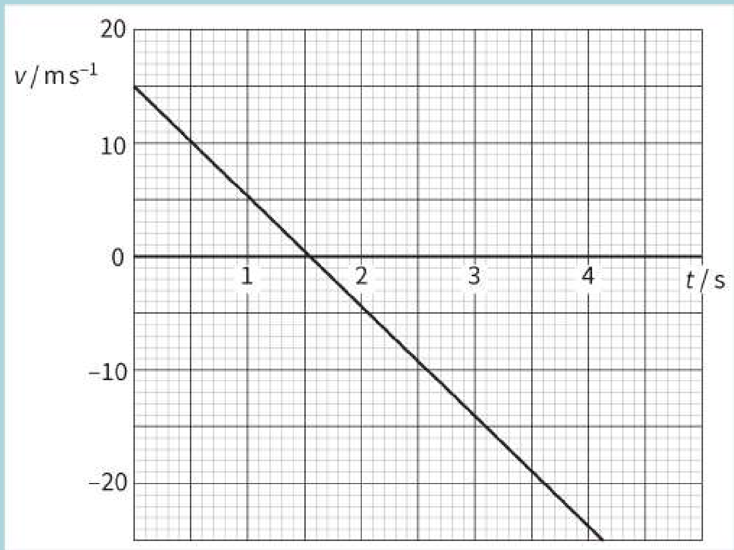


Figure 2.40

a Explain how the graph shows that the acceleration of the ball is constant. **[1]**

b Use the graph to:

i determine the time at which the ball reaches its highest point **[1]**

ii show that the ball rises for a further 12 m between release and its highest point **[2]**

iii determine the distance between the highest point reached by the ball and the ground. **[2]**

c The equation relating v and t is $v = 15 - 9.81t$. State the significance in the equation of:

i the number 15 **[1]**

ii the negative sign. **[1]**

[Total: 8]

14 An aeroplane is travelling horizontally at a speed of 80 m s^{-1} and drops a crate of emergency supplies.

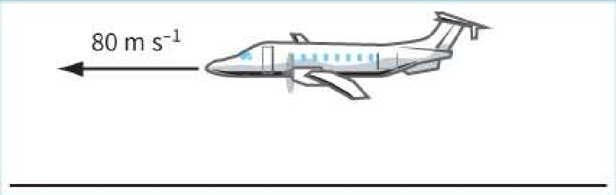


Figure 2.41

To avoid damage, the maximum vertical speed of the crate on landing is 20 m s^{-1}. You may assume air resistance is negligible.

a Calculate the maximum height of the aeroplane when the crate is dropped. **[2]**

b Calculate the time taken for the crate to reach the ground from this height. **[2]**

c The aeroplane is travelling at the maximum permitted height. Calculate the horizontal distance travelled by the crate after it is released from the aeroplane. **[1]**

[Total: 5]

SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

I can	See topic...	Needs more work	Almost there	Ready to move on
define acceleration	2.1			
calculate displacement from the area under a velocity–time graph	2.5			
calculate velocity using the gradient of a displacement–time graph	2.6			
calculate acceleration using the gradient of a velocity–time graph	2.4			
derive and use the equations of uniformly accelerated motion	2.10			
describe an experiment to measure the acceleration of free fall, g	2.11, 2.12			
use perpendicular components to represent a vector	2.13			
explain projectile motion using uniform velocity in one direction and uniform acceleration in a perpendicular direction and do calculations on this motion.	2.14			