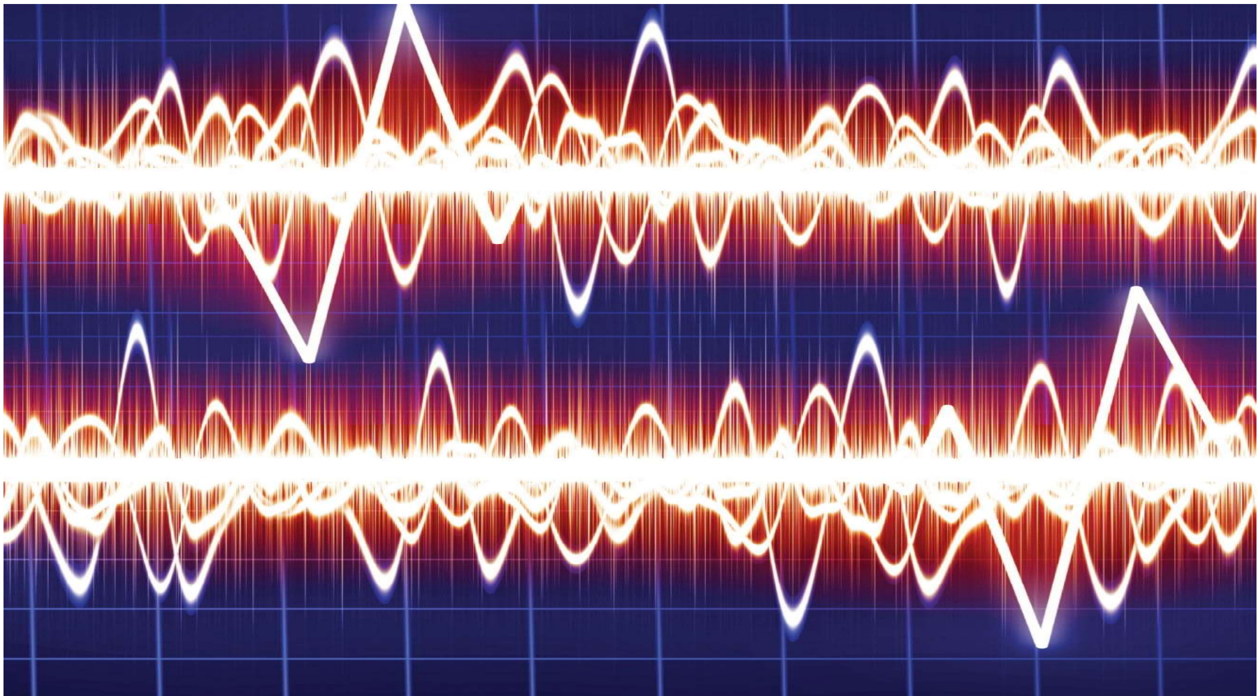

Chapter 18

Oscillations

LEARNING INTENTIONS

In this chapter you will learn how to:

- give examples of free and forced oscillations
- use appropriate terminology to describe oscillations
- use the equation $a = -\omega^2 x$ to define simple harmonic motion (s.h.m.)
- recall and use equations for displacement and velocity in s.h.m.
- draw and use graphical representations of s.h.m.
- describe energy changes during s.h.m.
- recall and use $E = \frac{1}{2}m\omega^2 x_0$, where E is the total energy of a system undergoing simple harmonic motion
- describe the effects of damping on oscillations and draw graphs showing these effects
- understand that resonance involves a maximum amplitude of oscillation
- understand that resonance occurs when an oscillating system is forced to oscillate at its natural frequency.

BEFORE YOU START

- Look at objects that vibrate or move in repetitive motion, such as the pendulum of a clock, a mass on the end of a spring, a toy yo-yo on a string, a branch of a tree in a breeze or an insect's wings. Write down three examples and be ready to share these with the class.
- What does their motion have in common? What differences are there between them? Discuss with a partner.

OSCILLATIONS AND ENGINEERING

When designing new products, designers need to consider unwanted oscillations in the product, whether that be a new vacuum cleaner, a new bridge, a new electric toothbrush or a new aircraft.

Figure 18.1 shows the Millennium Bridge over the river Thames in London. The bridge was revolutionary: a suspension bridge without the large supporting towers that are generally integral to the design of suspension bridges. It was built to celebrate the new millennium and it opened on 10 June 2000; the first new crossing built over the Thames for over 100 years.

Unfortunately, it had to be closed two days later when engineers detected that, when there were a lot of people walking across the bridge, it started to sway and twist. The effect was made worse because people automatically adjusted their walking so that each pace coincided with the movements of the bridge.

Figure 18.1: The Millenium Bridge across the Thames, London.

It took nearly two years before the engineers were able to fix the problem and for the bridge to be reopened – at a cost of nearly five million pounds!

Why do you think a designer, designing a new electric toothbrush, should be aware of the effects of oscillations?

It is not only large oscillations that are dangerous; very small, repeated vibrations will cause cracks to form in metals. You will be aware that if you bend a thin sheet of metal back and forth a few times it becomes easier to bend – a few more times and it will break. This is an extreme example of metal fatigue. Much smaller vibrations repeated often enough will cause microscopic cracks to form at points of high stress. These microscopic cracks will widen and, eventually, the structure will fail.

The first passenger jet airliner, the de Haviland Comet, suffered from this problem and after two of the aircraft disintegrated in mid-air, all Comets were grounded. After meticulous investigation, engineers concluded that the most likely cause of the accidents was metal fatigue at the high stress points near the corners of the ‘almost’, square windows. You may have noticed that the windows in more modern airliners are more oval in shape that avoids the high stress points found at the corners of a square shape.

18.1 Free and forced oscillations

Oscillations and vibrations are everywhere. A bird in flight flaps its wings up and down. An aircraft's wings also vibrate up and down, but this is not how it flies. The wings are long and thin, and they vibrate slightly because they are not perfectly rigid. Many other structures vibrate – bridges when traffic flows across, buildings in high winds.

A more specific term than vibration is **oscillation**. An object **oscillates** when it moves back and forth repeatedly, on either side of some equilibrium position. If we stop the object from oscillating, it returns to the equilibrium position.

We make use of oscillations in many different ways – for pleasure (a child on a swing), for music (the vibrations of a guitar string), for timing (the movement of a pendulum or the vibrations of a quartz crystal). Whenever we make a sound, the molecules of the air oscillate, passing the sound energy along. The atoms of a solid vibrate more and more as the temperature rises.

These examples of oscillations and vibrations may seem very different from one another. In this chapter, we will look at the characteristics that are shared by many oscillations.

Free or forced?

Free

The easiest oscillations to understand are free oscillations. If you pluck a guitar string, it continues to vibrate for some time after you have released it. The guitar string vibrates at a particular frequency (the number of vibrations per unit time). This is called its **natural frequency** of vibration, and it gives rise to the particular note that you hear. Change the length of the string, and you change the natural frequency. Every oscillator has a natural frequency of vibration, the frequency with which it vibrates freely after an initial disturbance.

Forced

Many objects can be forced to vibrate. If you sit on a bus, you may notice that the vibrations from the engine are transmitted to your body, causing you to vibrate with the same frequency. These are not free vibrations of your body; they are forced vibrations. Their frequency is not the natural frequency of vibration of your body, but the forcing frequency of the bus.

In the same way, you can force a metre ruler to oscillate by waving it up and down; however, its natural frequency of vibration will be much greater than this, as you will discover if you hold one end down on the bench and then quickly push down and let go of the other end (Figure 18.2).

Figure 18.2: A ruler vibrating freely at its natural frequency.

Question

1 State which of the following are free oscillations, and which are forced:

- **a** the wing beat of a mosquito
- **b** the movement of the pendulum in a upright clock
- **c** the vibrations of a cymbal after it has been struck
- **d** the shaking of a building during an earthquake.

18.2 Observing oscillations

Many oscillations are too rapid or too small for us to observe. Our eyes cannot respond rapidly enough if the frequency of oscillation is more than about 5 Hz (five oscillations per second); anything faster than this appears as a blur. In order to see the general characteristics of oscillating systems, we need to find suitable systems that oscillate slowly. Practical Activity 18.1 describes three suitable situations to investigate.

PRACTICAL ACTIVITY 18.1

Observing slow oscillations

A mass–spring system

A trolley, loaded with extra masses, is tethered by identical springs in between two clamps (Figure 18.3). Move the trolley to one side and it will oscillate back and forth along the bench. Listen to the sound of the trolley moving. Where is it moving fastest? What happens to its speed as it reaches the ends of its oscillation? What is happening to the springs as the trolley oscillates?

Figure 18.3: A trolley tethered between springs will oscillate freely from side to side.

A long pendulum

A string, at least 2 m long, hangs from the ceiling with a large mass fixed at the end (Figure 18.4). Pull the mass some distance to one side, and let go. The pendulum will swing back and forth at its natural frequency of oscillation. Try to note the characteristics of its motion. In what ways is it similar to the motion of the oscillating trolley? In what ways is it different?

A loudspeaker cone

A signal generator, set to a low frequency (say, 1 Hz), drives a loudspeaker so that it vibrates (Figure 18.5). You need to be able to see the cone of the loudspeaker.

How does this motion compare with that of the pendulum and the mass–spring system? Try using a higher frequency (say, 100 Hz). Use an electronic stroboscope flashing at a similar frequency to show up the movement of the cone. (It may help to paint a white spot on the centre of the cone.) Do you observe the same pattern of movement?

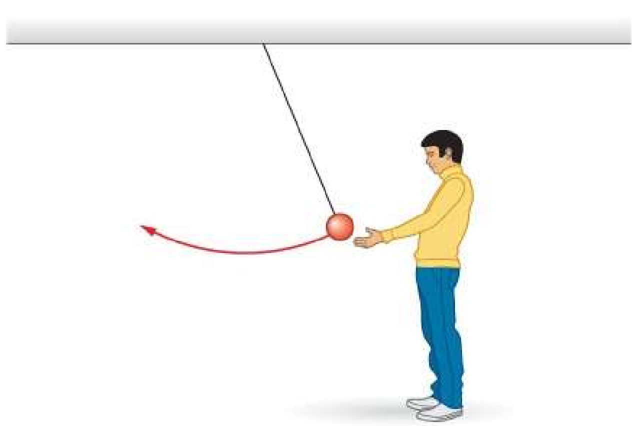

Figure 18.4: A long pendulum oscillates back and forth.

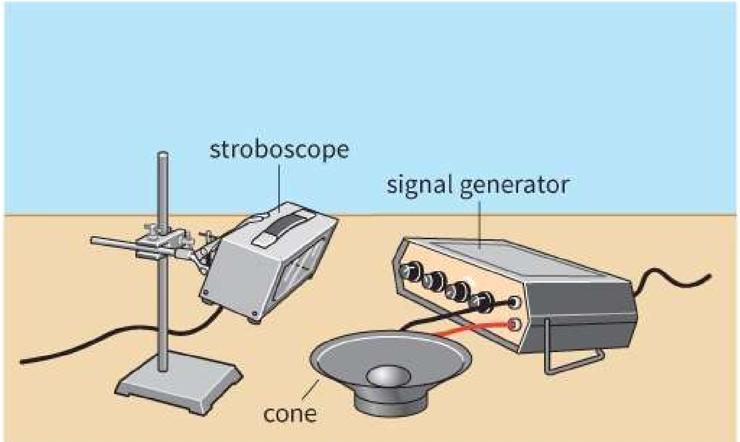

Figure 18.5: A loudspeaker cone forced to vibrate up and down.

Question

2 If you could draw a velocity–time graph for any of the oscillators described in Practical Activity 18.1, what would it look like? Would it be a curve like the one shown in Figure 18.6a, or triangular (saw-toothed) like the one shown in Figure 18.6b?

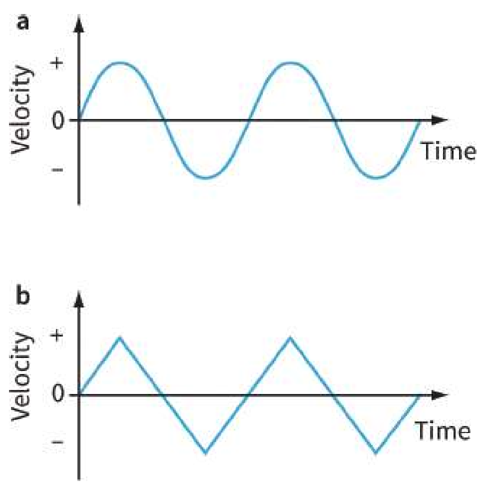

Figure 18.6: Two possible velocity–time graphs for vibrating objects.

18.3 Describing oscillations

All of the examples discussed so far show the same pattern of movement. The trolley accelerates as it moves towards the centre of the oscillation. It is moving fastest at the centre. It decelerates as it moves towards the end of the oscillation. At the extreme position, it stops momentarily, reverses its direction and accelerates back towards the centre again.

Amplitude, period and frequency

Many oscillating systems can be represented by a displacement–time graph like that shown in Figure 18.7. The displacement x varies in a smooth way on either side of the midpoint. The shape of this graph is a sine curve, and the motion is described as **sinusoidal**.

Notice that the displacement changes between positive and negative values, as the object moves through the equilibrium position. The maximum displacement from the equilibrium position is called the **amplitude** x_0 of the oscillation.

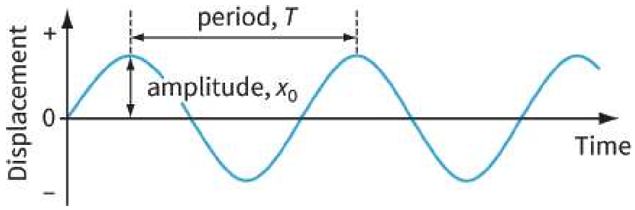

Figure 18.7: A displacement–time graph to show the meaning of amplitude and period.

The displacement–time graph can also be used to show the **period** and frequency of the oscillation. The period T is the time for one complete oscillation. Note that the oscillating object must go from one side to the other and back again (or the equivalent). The frequency f is the number of oscillations per unit time, and so f is the reciprocal of T:

$$\text{frequency} = \frac{1}{\text{period}} \equiv f = \frac{1}{T}$$

The equation can also be written as:

$$\text{period} = \frac{1}{\text{frequency}} \equiv T = \frac{1}{f}$$

Question

3 From the displacement–time graph shown in Figure 18.8, determine the amplitude, period and frequency of the oscillations represented.

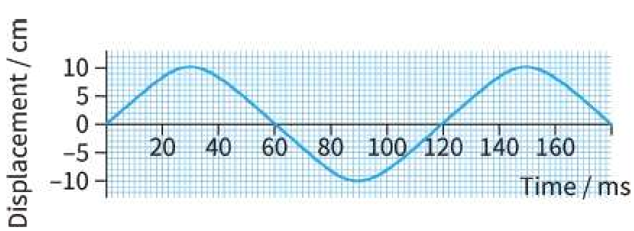

Figure 18.8: A displacement–time graph for an oscillator.

Phase

The term **phase** describes the point that an oscillating mass has reached within the complete cycle of an oscillation. It is often important to describe the **phase difference** between two oscillations. The graph of Figure 18.9a shows two oscillations that are identical except for their phase difference. They are out of step with one another. In this example, they have a phase difference of one-quarter of an oscillation. Phase difference can be measured as a fraction of an oscillation, in degrees or in radians (see Worked example 1).

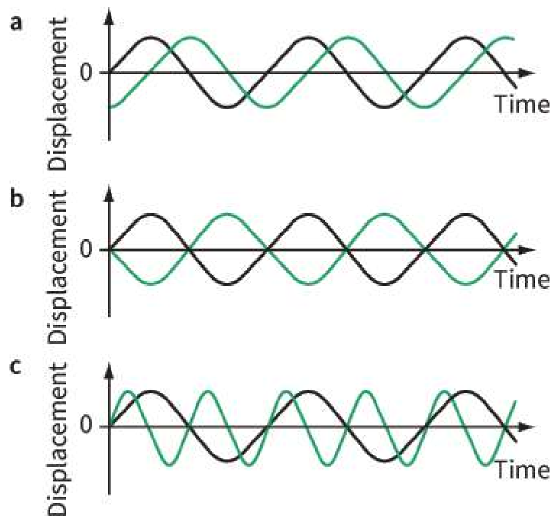

Figure 18.9: Illustrating the idea of phase difference.

WORKED EXAMPLE

1 Figure 18.10 shows displacement–time graphs for two identical oscillators. Calculate the phase difference between the two oscillations. Give your answer in degrees and in radians.

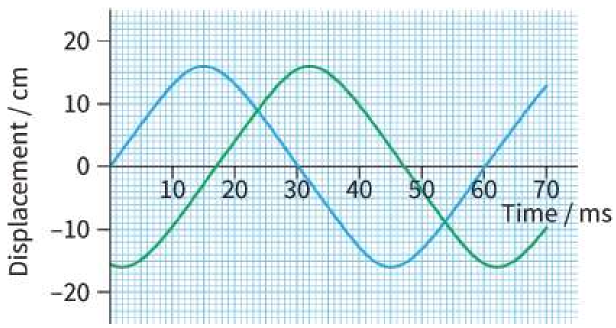

Figure 18.10: The displacement–time graphs of two oscillators with the same period.

Step 1 Measure the time interval t between two corresponding points on the graphs.

t = 17 ms

Step 2 Determine the period T for one complete oscillation.

T = 60 ms

Hint: *Remember that a complete oscillation is when the object goes from one side to the other and back again.*

Step 3 Now you can calculate the phase difference as a fraction of an oscillation.

phase difference = fraction of an oscillation

Therefore:

$$\begin{aligned}\text{phase difference} &= \frac{t}{T}\\ &= \frac{17}{60}\\ &= 0.283 \text{ oscillations}\end{aligned}$$

Step 4 Convert to degrees and radians. There are 360° and 2π rad in one oscillation.

$$\begin{aligned}\text{phase difference} &= 0.283 \times 360^\circ\\ &= 102^\circ \approx 100^\circ\\ \text{phase difference} &= 0.283 \times 2\pi \text{ rad}\\ &= 1.78 \text{ rad} \approx 1.8 \text{ rad}\end{aligned}$$

Question

4 **a** Figure 18.9b shows two oscillations that are out of phase. By what fraction of an oscillation are they out of phase?

b Why would it not make sense to ask the same question about Figure 18.9c?

18.4 Simple harmonic motion

There are many situations where we can observe the special kind of oscillations called **simple harmonic motion (s.h.m.)**. Some are more obvious than others. For example, the vibrating strings of a musical instrument show s.h.m. When plucked or bowed, the strings move back and forth about the equilibrium position of their oscillation. The motion of the tethered trolley in Figure 18.3 and that of the pendulum in Figure 18.4 are also s.h.m. (Simple harmonic motion is defined in terms of the acceleration and displacement of an oscillator – see topic 18.5 Representing s.h.m. graphically.)

Here are some other, less obvious, situations where simple harmonic motion can be found:

- When a pure (single tone) sound wave travels through air, the molecules of the air vibrate with s.h.m.
- When an alternating current flows in a wire, the electrons in the wire vibrate with s.h.m.
- There is a small alternating electric current in a radio or television aerial when it is tuned to a signal in the form of electrons moving with s.h.m.
- The atoms that make up a molecule vibrate with s.h.m. (see, for example, the hydrogen molecule in Figure 18.11a).

Oscillations can be very complex, with many different frequencies of oscillation occurring at the same time. Examples include the vibrations of machinery, the motion of waves on the sea and the vibration of a solid crystal formed when atoms, ions or molecules bond together (Figure 18.11b). It is possible to break down a complex oscillation into a sum of simple oscillations, and so we will focus our attention in this chapter on s.h.m. with only one frequency. We will also concentrate on large-scale mechanical oscillations, but you should bear in mind that this analysis can be extended to the situations already mentioned, and many more besides.

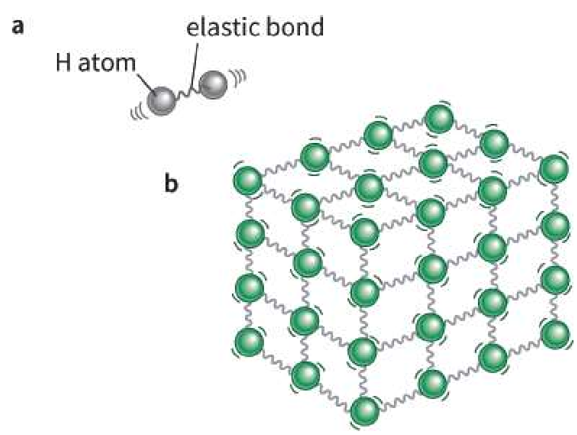

Figure 18.11: We can think of the bonds between atoms as being springy; this leads to vibrations, **a** in a molecule of hydrogen and **b** in a solid crystal.

The requirements for s.h.m.

If a simple pendulum is undisturbed, it is in equilibrium. The string and the mass will hang vertically. To start the pendulum swinging (Figure 18.12), the mass must be pulled to one side of its equilibrium position. The forces on the mass are unbalanced and so it moves back towards its equilibrium position. The mass swings past this point and continues until it comes to rest momentarily at the other side; the process is then repeated in the opposite direction. Note that a complete oscillation in Figure 18.12 is from right to left and back again.

The three requirements for s.h.m. of a mechanical system are:

- a mass that oscillates
- a position where the mass is in equilibrium
- a restoring force that acts to return the mass to its equilibrium position; the restoring force F is directly proportional to the displacement x of the mass from its equilibrium position and is directed towards that point.

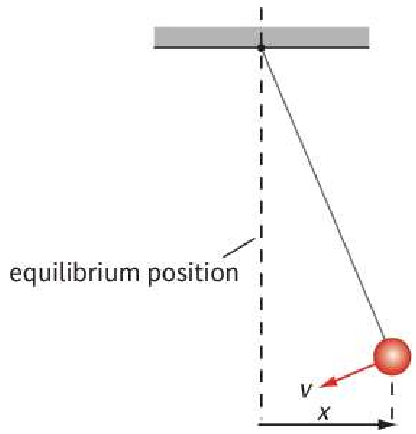

Figure 18.12: This swinging pendulum has positive displacement *x* and negative velocity *v*.

The changes of velocity in s.h.m.

As the pendulum swings back and forth, its velocity is constantly changing. As it swings from right to left (as shown in Figure 18.12) its velocity is negative. It accelerates towards the equilibrium position and then decelerates as it approaches the other end of the oscillation. It has positive velocity as it swings back from left to right. Again, it has maximum speed as it travels through the equilibrium position and decelerates as it swings up to its starting position.

This pattern of acceleration–deceleration–changing direction–acceleration again is characteristic of simple harmonic motion. There are no sudden changes of velocity. In the next topic, we will see how we can observe these changes and how we can represent them graphically.

Questions

5 Identify the features of the motion of the trolley in Figure 18.3 that satisfy the three requirements for s.h.m.

6 Explain why the motion of someone jumping up and down on a trampoline is not simple harmonic motion. (Their feet lose contact with the trampoline during each bounce.)

18.5 Representing s.h.m. graphically

If you set up a trolley tethered between springs (Figure 18.13) you can hear the characteristic rhythm of s.h.m. as the trolley oscillates back and forth. By adjusting the mass carried by the trolley, you can achieve oscillations with a period of about two seconds.

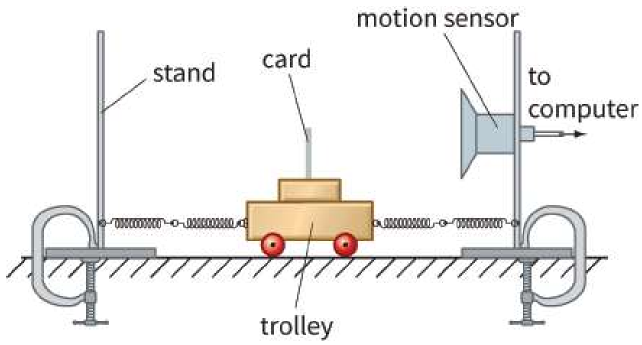

Figure 18.13: A motion sensor can be used to investigate s.h.m. of a spring–trolley system.

The motion sensor allows you to record how the displacement of the trolley varies with time. Ultrasonic pulses from the sensor are reflected by the card on the trolley and the reflected pulses are detected. This 'sonar' technique allows the sensor to determine the displacement of the trolley. A typical screen display is shown in Figure 18.14.

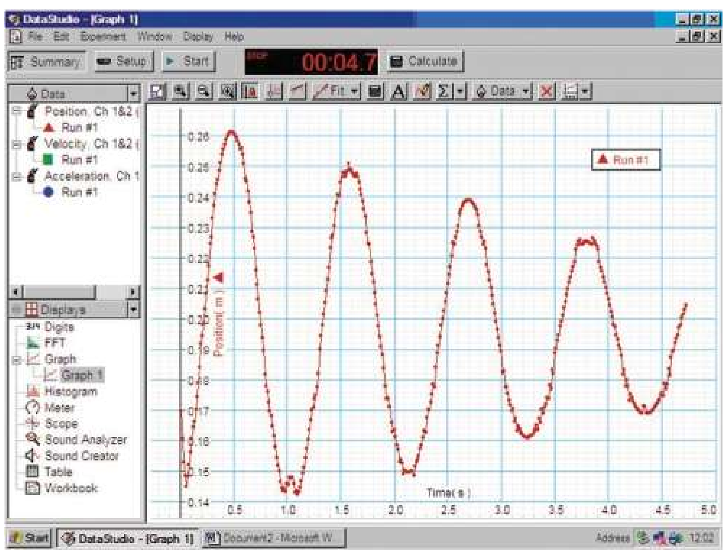

Figure 18.14: A typical displacement–time graph generated by a motion sensor.

The computer can then determine the velocity of the trolley by calculating the rate of change of displacement. Similarly, it can calculate the rate of change of velocity to determine the acceleration.

Idealised graphs of displacement, velocity and acceleration against time are shown in Figure 18.15. We will examine these graphs in sequence to see what they tell us about s.h.m. and how the three graphs are related to one another.

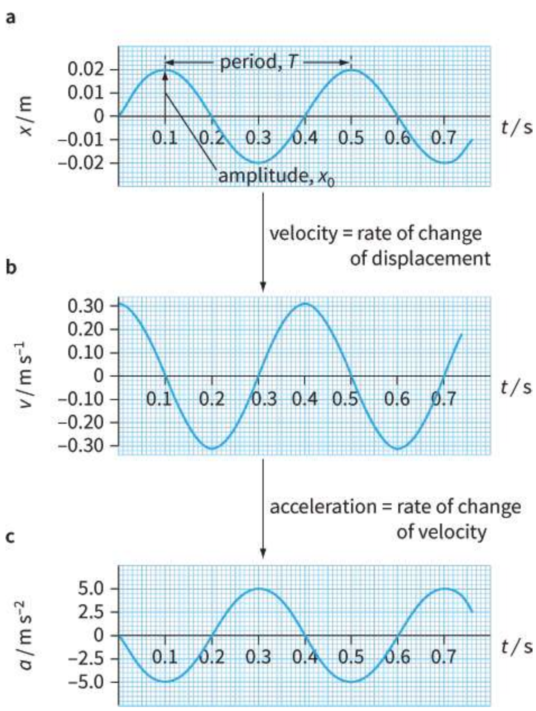

Figure 18.15: Graphs of displacement x, velocity v and acceleration a against time t for s.h.m.

Displacement–time (x–t) graph

The displacement of the oscillating mass varies according to the smooth curve shown in Figure 18.15a. Mathematically, this is a sine curve; its variation is described as sinusoidal. Note that this graph allows us to determine the amplitude x_0 and the period T of the oscillations. In this graph, the displacement x of the oscillation is shown as zero at the start, when t is zero. We have chosen to consider the motion to start when the mass is at the midpoint of its oscillation (equilibrium position) and is moving to the right. We could have chosen any other point in the cycle as the starting point, but it is conventional to start as shown here.

Velocity–time (v–t) graph

The velocity v of the oscillator at any time can be determined from the gradient of the displacement–time graph:

$$v = \frac{\Delta x}{\Delta t}$$

Again, we have a smooth curve (Figure 18.15b), which shows how the velocity v depends on time t. The shape of the curve is the same as for the displacement–time graph, but it starts at a different point in the cycle.

When time $t = 0$, the mass is at the equilibrium position and this is where it is moving fastest. Hence, the velocity has its maximum value at this point. Its value is positive because at time $t = 0$ it is moving towards the right.

Acceleration–time (a–t) graph

Finally, the acceleration a of the oscillator at any time can be determined from the gradient of the velocity–time graph:

$$a = \frac{\Delta v}{\Delta t}$$

This gives a third curve of the same general form (Figure 18.15c), which shows how the acceleration a depends on time t. At the start of the oscillation, the mass is at its equilibrium position. There is no resultant force acting on it so its acceleration is zero. As it moves to the right, the restoring force acts towards the left, giving it a negative acceleration. The acceleration has its greatest value when the mass is displaced farthest from the equilibrium position. Notice that the acceleration graph is 'upside-down' compared with the displacement graph. This shows that:

$$\text{acceleration} \propto -\text{displacement}$$

or

$$a \propto -x$$

In other words, whenever the mass has a positive displacement (to the right), its acceleration is to the left, and vice versa.

18.6 Frequency and angular frequency

The frequency f of s.h.m. is equal to the number of oscillations per unit time. As we saw earlier, f is related to the period T by:

$$f = \frac{1}{T}$$

We can think of a complete oscillation of an oscillator or a cycle of s.h.m. as being represented by 2π radians. (This is similar to a complete cycle of circular motion, where an object moves round through 2π radians.) The phase of the oscillation changes by 2π rad during one oscillation. Hence, if there are f oscillations in unit time, there must be $2\pi f$ radians in unit time. This quantity is the **angular frequency** of the s.h.m. and it is represented by the Greek letter ω (omega).

The angular frequency ω is related to frequency f by the equation:

$$\omega = 2\pi f$$

KEY EQUATION

Relationship of angular frequency ω to frequency f:

$$\omega = 2\pi f$$

Since $f = \frac{1}{T}$, the angular frequency ω is related to the period T of the oscillator by the equation:

$$\omega = \frac{2\pi}{T} \text{ or } T = \frac{2\pi}{\omega}$$

Questions

7 Use the graphs shown in Figure 18.15 to determine the values of the following quantities:

- **a** amplitude
- **b** time period
- **c** maximum velocity
- **d** maximum acceleration.

8 State at what point in an oscillation the oscillator has zero velocity but acceleration towards the right.

9 Look at the x–t graph of Figure 18.15a. When t = 0.1 s, what is the gradient of the graph? State the velocity at this instant.

10 Figure 18.16 shows the displacement–time (x–t) graph for an oscillating mass. Use the graph to determine the following quantities:

- **a** the velocity in cm s^{-1} when t = 0 s
- **b** the maximum velocity in cm s^{-1}
- **c** the acceleration in cm s^{-2} when t = 1.0 s.

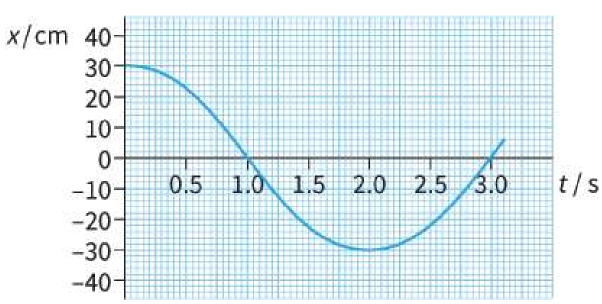

Figure 18.16: A displacement–time graph for an oscillating mass. For Question 10.

In Figure 18.17, a single cycle of s.h.m. is shown, but with the x-axis marked with the phase of the motion in radians.

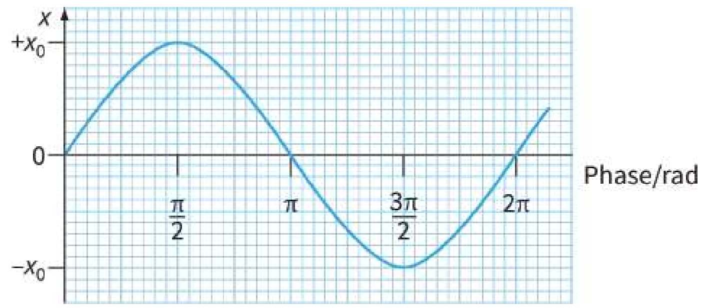

Figure 18.17: The phase of an oscillation varies from 0 to 2π during one cycle.

Questions

11 An object moving with s.h.m. goes through two complete cycles in 1.0 s. Calculate:

- **a** the period T
- **b** the frequency f
- **c** the angular frequency ω.

12 Figure 18.18 shows the displacement–time graph for an oscillating mass. Use the graph to determine the following:

- **a** amplitude
- **b** period
- **c** frequency
- **d** angular frequency
- **e** displacement at A
- **f** velocity at B
- **g** velocity at C.

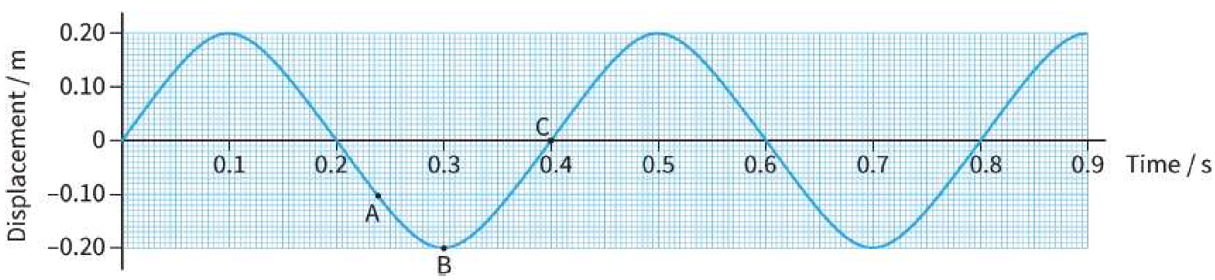

Figure 18.18: A displacement–time graph. For Question 12.

13 An atom in a crystal vibrates with s.h.m. with a frequency of 10^{14} Hz. The amplitude of its motion is 2.0×10^{-12} m.

- **a** Sketch a graph to show how the displacement of the atom varies during one cycle.
- **b** Use your graph to estimate the maximum velocity of the atom.

18.7 Equations of s.h.m.

The graph of Figure 18.15a, shown earlier, represents how the displacement of an oscillator varies during s.h.m. We have already mentioned that this is a sine curve. We can present the same information in the form of an equation. The relationship between the displacement x and the time t is as follows:

$$x = x_0 \sin \omega t$$

where x_0 is the amplitude of the motion and ω is its frequency. Sometimes, the same motion is represented using a cosine function, rather than a sine function:

$$x = x_0 \cos \omega t$$

KEY EQUATIONS

Equations of simple harmonic motion:

$$x = x_0 \sin \omega t$$
$$x = x_0 \cos \omega t$$

The difference between these two equations is illustrated in Figure 18.19. The sine version starts at x = 0; that is, the oscillating mass is at its equilibrium position when t = 0.

The cosine version starts at $x = x_0$, so that the mass is at its maximum displacement when t = 0.

Note that, in calculations using these equations, the quantity ωt is in radians. Make sure that your calculator is in radian mode for any calculation (see Worked example 2). The presence of the π in the equation should remind you of this.

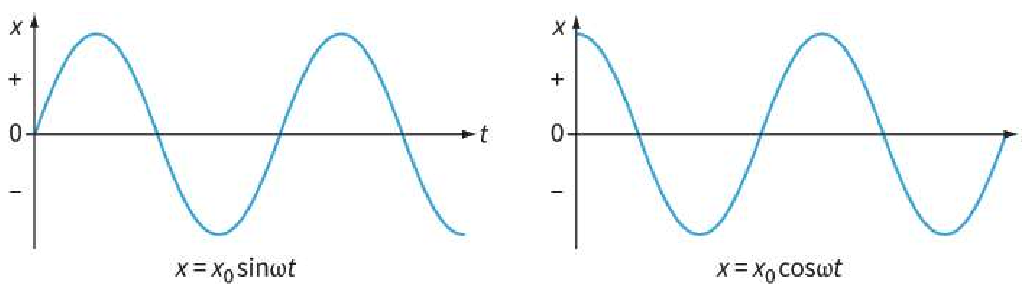

Figure 18.19: These two graphs represent the same simple harmonic motion. The difference in starting positions is related to the sine and cosine forms of the equation for x as a function of t.

Questions

14 The vibration of a component in a machine is represented by the equation:

$x = 3.0 \times 10^{-4} \sin (240\pi t)$

where the displacement x is in metres.

Determine the:

a amplitude

b frequency

c period

of the vibration.

15 A trolley is at rest, tethered between two springs. It is pulled 0.15 m to one side and, when time t = 0, it is released so that it oscillates back and forth with s.h.m. The period of its motion is 2.0 s.

a Write an equation for its displacement x at any time t (assume that the motion is not damped by frictional forces).

b Sketch a displacement–time graph to show two cycles of the motion, giving values where appropriate.

Acceleration and displacement

In s.h.m., an object's acceleration depends on how far it is displaced from its equilibrium position and on the magnitude of the restoring force. The greater the displacement x, the greater the acceleration a. In fact, a is proportional to x. We can write the following equation to represent this:

$$a = -\omega^2 x$$

where a = the acceleration of an object vibrating in s.h.m., ω is the angular frequency of the object, x = displacement

KEY EQUATION

$$a = -\omega^2 x$$

Acceleration of an object vibrating in simple harmonic motion.

This equation shows that a is proportional to x; the constant of proportionality is ω^2. The minus sign shows that, when the object is displaced to the **right**, the direction of its acceleration is to the **left**.

The acceleration is always directed towards the equilibrium position, in the opposite direction to the displacement.

It should not be surprising that angular frequency ω appears in this equation. Imagine a mass hanging on a spring, so that it can vibrate up and down. If the spring is stiff, the force on the mass will be greater; it will be accelerated more for a given displacement and its frequency of oscillation will be higher.

The equation $a = -\omega^2 x$ helps us to define simple harmonic motion. The acceleration a is directly proportional to displacement x; and the minus sign shows that it is in the opposite direction.

An object vibrates in simple harmonic motion if its acceleration is directly proportional to its displacement from its equilibrium position and is in the opposite direction to the displacement.

If a and x were in the same direction (no minus sign), the body's acceleration would increase as it moved away from the fixed point and it would move away faster and faster, never to return.

Figure 18.20 shows the acceleration–displacement (a – x) graph for an oscillator executing s.h.m. Note the following:

- The graph is a straight line through the origin ($a \propto x$).
- It has a negative slope (the minus sign in the equation $a = -\omega^2 x$). This means that the acceleration is always directed towards the equilibrium position.
- The magnitude of the gradient of the graph is ω^2.

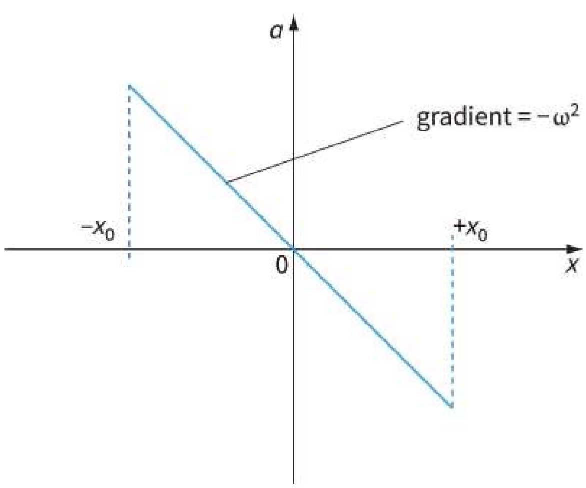

Figure 18.20: Graph of acceleration a against displacement x for an oscillator executing s.h.m.

- The gradient is independent of the amplitude of the motion. This means that the frequency f or the period T of the oscillator is independent of the amplitude and so a simple harmonic oscillator keeps steady time.

If you have studied calculus, you may be able to differentiate the equation for x twice with respect to time to obtain an equation for acceleration and thereby show that the defining equation $a = -\omega^2 x$ is satisfied.

KEY IDEA

We say that the equation $a = -\omega^2 x$ defines simple harmonic motion–it tells us what is required if a body is to perform s.h.m. The equation $x = x_0 \sin \omega t$ is then described as a **solution** to the equation, since it tells us how the displacement of the body varies with time.

WORKED EXAMPLE

2 A pendulum oscillates with frequency 1.5 Hz and amplitude 0.10 m. If it is passing through its equilibrium position when $t = 0$, write an equation to represent its displacement x in terms of amplitude x_0, angular frequency ω and time t. Determine its displacement when $t = 0.50$ s.

Step 1 Select the correct equation. In this case, the displacement is zero when $t = 0$, so we use the sine form:

$$x = x_0 \sin \omega t$$

Step 2 From the frequency f, calculate the angular frequency ω:

$$\begin{aligned} \omega &= 2\pi f \\ &= 2 \times \pi \times 1.5 \\ &= 3.0\pi \end{aligned}$$

Step 3 Substitute values in the equation: $x_0 = 0.10$ m, so:

$x = 0.10 \sin (3.0\pi t)$

Hint: *Remember to put your calculator into radian mode.*

Step 4 To find x when $t = 0.50$ s, substitute for t and calculate the answer:

$$\begin{aligned} x &= 0.10 \sin (2\pi \times 1.5 \times 0.50) \\ &= 0.10 \sin (4.713) \\ &= -0.10 \text{ m} \end{aligned}$$

This means that the pendulum is at the extreme end of its oscillation; the minus sign means that it is at the negative or left-hand end, assuming you have chosen to consider displacements to the right as positive.

(If your calculation went like this:

$$\begin{aligned} x &= 0.10 \sin (2\pi \times 1.5 \times 0.50) \\ &= 0.10 \sin (4.713) \\ &= -8.2 \times 10^{-3} \text{m} \end{aligned}$$

then your calculator was incorrectly set to work in degrees, not radians.)

Equations for velocity

The velocity v of an oscillator varies as it moves back and forth. It has its greatest speed when it passes through the equilibrium position in the middle of the oscillation. If we take time $t = 0$ when the oscillator passes through the middle of the oscillation with its greatest speed v_0, then we can represent the changing velocity as an equation:

$$x = x_0 \cos \omega t$$

We use the cosine function to represent the velocity since it has its maximum value when $t = 0$.

The equation $v = v_0 \cos \omega t$ tells us how v depends on t. We can write another equation to show how the velocity depends on the oscillator's displacement x:

$$v = \pm \omega \sqrt{{x_0}^2 - x^2}$$

This equation can be used to deduce the speed of an oscillator at any point in an oscillation, including its maximum speed.

Maximum speed of an oscillator

If an oscillator is executing simple harmonic motion, it has maximum speed when it passes through its equilibrium position. This is when its displacement x is zero. The maximum speed v_0 of the oscillator depends on the frequency f of the motion and on the amplitude x_0. Substituting $x = 0$ in the equation:

$$v = \omega\sqrt{{x_0}^2 - x^2}$$

$x = 0$ when the speed is at a maximum:

$$v_0 = \omega x_0$$

According to this equation, for a given oscillation:

$$v_0 \propto x_0$$

KEY EQUATION

$$v = \omega\sqrt{{x_0}^2 - x^2}$$

Speed of an oscillator.

A simple harmonic oscillator has a period that is independent of the amplitude. A greater amplitude means that the oscillator has to travel a greater distance in the same time–hence it has a greater speed.

The equation also shows that the maximum speed is proportional to the frequency. Increasing the frequency means a shorter period. A given distance is covered in a shorter time–hence it has a greater speed.

Have another look at Figure 18.15. The period of the motion is 0.40 s and the amplitude of the motion is 0.02 m. The frequency f can be calculated as follows:

$$\begin{aligned} f &= \frac{1}{t} \\ &= \frac{1}{0.40} \\ &= 2.5\text{Hz} \end{aligned}$$

We can now use the equation $v_0 = (2\pi f)x_0$ to determine the maximum speed v_0:

$$v_0 = (2\pi f)x_0 = (2\pi \times 2.5) \times 2.0 \times 10^{-2}$$

$$v_0 \approx 0.31 \text{ m s}^{-1}$$

This is how the values on Figure 18.15b were calculated.

Questions

16 A mass secured at the end of a spring moves with s.h.m. The frequency of its motion is 1.4 Hz.

- **a** Write an equation of the form $a = -\omega^2 x$ to show how the acceleration of the mass depends on its displacement.
- **b** Calculate the acceleration of the mass when it is displaced 0.050 m from its equilibrium position.

17 A short pendulum oscillates with s.h.m. such that its acceleration a (in m s^{-2}) is related to its displacement x (in m) by the equation $a = -300x$. Determine the frequency of the oscillations.

18 The pendulum of a grandfather clock swings from one side to the other in 1.00 s. The amplitude of the oscillation is 12 cm.

- **a** Calculate:
  - **i** the period of its motion
  - **ii** the frequency
  - **iii** the angular frequency.
- **b** Write an equation of the form $a = -\omega^2 x$ to show how the acceleration of the pendulum bob depends on its displacement.
- **c** Calculate the maximum speed of the pendulum bob.
- **d** Calculate the speed of the bob when its displacement is 6 cm.

19 A trolley of mass m is fixed to the end of a spring. The spring can be compressed and extended. The spring has a force constant k. The other end of the spring is attached to a vertical wall. The trolley lies on a smooth horizontal table. The trolley oscillates when it is displaced from its equilibrium position.

a Show that the motion of the oscillating trolley is s.h.m.

b Show that the period T of the trolley is given by the equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

18.8 Energy changes in s.h.m.

During simple harmonic motion, there is a constant exchange of energy between two forms: potential and kinetic. We can see this by considering the mass–spring system shown in Figure 18.21.

When the mass is pulled to one side (to start the oscillations), one spring is compressed and the other is stretched. The springs store elastic potential energy. When the mass is released, it moves back towards the equilibrium position, accelerating as it goes. It has increasing kinetic energy. The potential energy stored in the springs decreases while the kinetic energy of the mass increases by the same amount (as long as there are no heat losses due to frictional forces). Once the mass has passed the equilibrium position, its kinetic energy decreases and the energy is transferred back to the springs. Provided the oscillations are undamped, the total energy in the system remains constant.

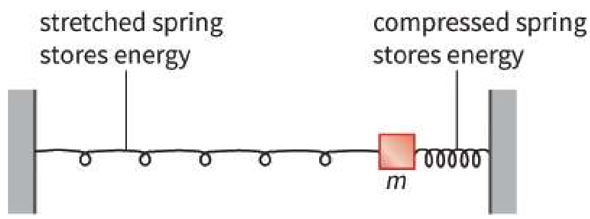

Figure 18.21: The elastic potential energy stored in the springs is converted to kinetic energy when the mass is released.

Energy graphs

We can represent these energy changes in two ways. Figure 18.22 shows how the kinetic energy and elastic potential energy change with time. Potential energy is maximum when displacement is maximum (positive or negative). Kinetic energy is maximum when displacement is zero. The total energy remains constant throughout. Note that both kinetic energy and potential energy go through two complete cycles during one period of the oscillation. This is because kinetic energy is maximum when the mass is passing through the equilibrium position moving to the left and again moving to the right. The potential energy is maximum at both ends of the oscillation.

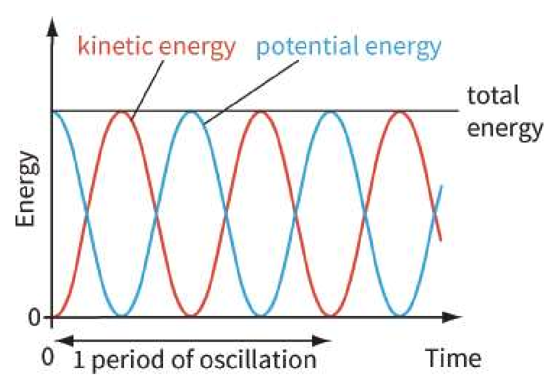

Figure 18.22: The kinetic energy and potential energy of an oscillator vary periodically, but the total energy remains constant if the system is undamped.

A second way to show this is to draw a graph of how potential energy and kinetic energy vary with displacement (Figure 18.23).

The graph shows that:

- kinetic energy is maximum when displacement $x = 0$
- potential energy is maximum when $x = \pm x_0$
- at any point on this graph, the total energy (k.e. + p.e.) has the same value.

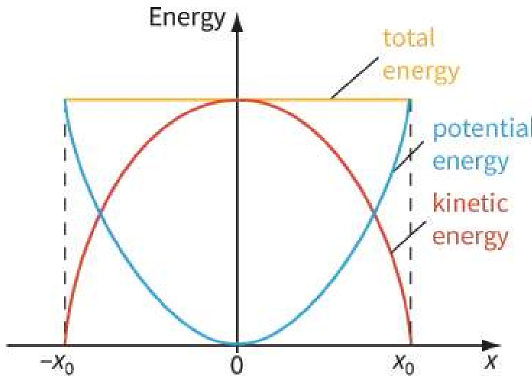

Figure 18.23: The kinetic energy is maximum at zero displacement; the potential energy is maximum at maximum displacement (x_0 and $-x_0$).

It follows that if the maximum speed is v_0 then maximum kinetic energy $= \frac{1}{2}m{v_0}^2$.

At this point in the cycle, all the energy is in the form of kinetic energy, so the total energy of the system is:

$$E_0 = \frac{1}{2}m{v_0}^2$$

Since:

$$v_0 = \omega x_0$$

Then:

$$E_0 = \frac{1}{2}m\omega^2{x_0}^2$$

KEY EQUATION

$$E_0 = \frac{1}{2}m\omega^2{x_0}^2$$

Total energy of a system undergoing simple harmonic motion.

Questions

20 To start a pendulum swinging, you pull it slightly to one side.

- **a** What kind of energy does this transfer to the mass?
- **b** Describe the energy changes that occur when the mass is released.

21 Figure 18.23 shows how the different forms of energy change with displacement during s.h.m. Copy the graph, and show how the graph would differ if the oscillating mass were given only half the initial input of energy.

22 Figure 18.24 shows how the velocity v of a 2.0 kg mass was found to vary with time t during an investigation of the s.h.m. of a pendulum. Use the graph to estimate the following for the mass:

- **a** its maximum velocity
- **b** its maximum kinetic energy
- **c** its maximum potential energy
- **d** its maximum acceleration
- **e** the maximum restoring force that acted on it.

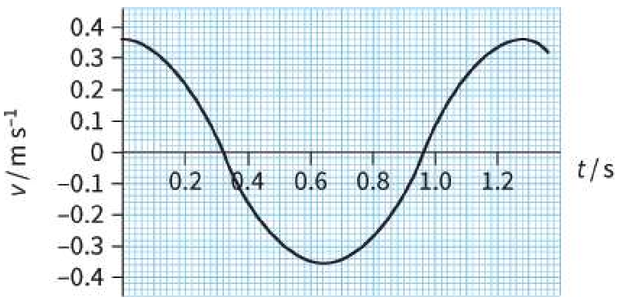

Figure 18.24: A velocity–time graph for a pendulum. For Question 22.

18.9 Damped oscillations

In principle, oscillations can go on for ever. In practice, however, the oscillations we observe around us do not. They die out, either rapidly or gradually. A child on a swing knows that the amplitude of her swinging will decline until eventually she will come to rest, unless she can put some more energy into the swinging to keep it going.

This happens because of friction. On a swing, there is friction where the swing is attached to the frame and there is friction with the air. The amplitude of the child's oscillations decreases as the friction transfers energy away from her to the surroundings.

We describe these oscillations as **damped**. Their amplitude decreases according to a particular pattern. This is shown in Figure 18.25.

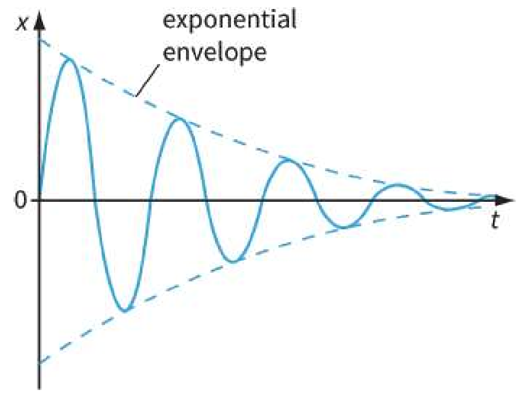

Figure 18.25: Damped oscillations.

The amplitude of damped oscillations does not decrease linearly. It decays exponentially with time. An exponential decay is a particular mathematical pattern that arises as follows. At first, the swing moves rapidly. There is a lot of air resistance to overcome, so the swing loses energy quickly and its amplitude decreases at a high rate. Later, it is moving more slowly. There is less air resistance and so energy is lost more slowly–the amplitude decreases at a lower rate. Hence, we get the characteristic curved shape, which is the 'envelope' of the graph in Figure 18.25.

Notice that the frequency of the oscillations does not change as the amplitude decreases. This is a characteristic of simple harmonic motion. The child may, for example, swing back and forth once every two seconds, and this stays the same whether the amplitude is large or small.

PRACTICAL ACTIVITY 18.2

Investigating damping

You can investigate the exponential decrease in the amplitude of oscillations using a simple laboratory arrangement (Figure 18.26). A hacksaw blade or other springy metal strip is clamped (vertically or horizontally) to the bench. A mass is attached to the free end. This will oscillate freely if you displace it to one side.

A card is attached to the mass so that there is significant air resistance as the mass oscillates. The amplitude of the oscillations decreases and can be measured every five oscillations by judging the position of the blade against a ruler fixed alongside.

A graph of amplitude against time will show the characteristic exponential decrease. You can find the 'half-life' of this exponential decay graph by determining the time it takes to decrease to half its initial amplitude (Figure 18.27).

By changing the size of the card, it is possible to change the degree of damping, and hence alter the half-life of the motion.

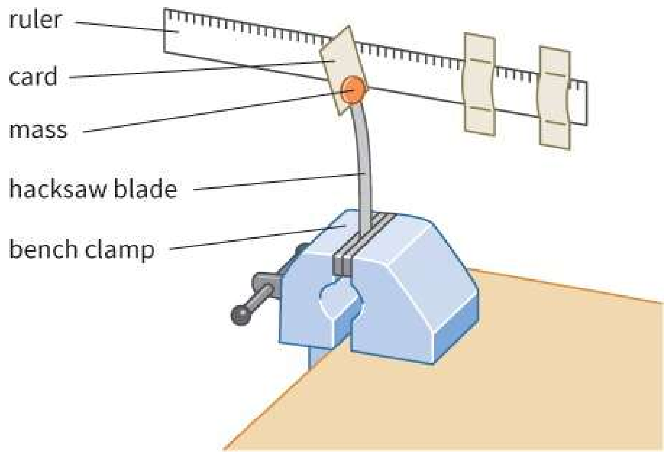

Figure 18.26: Damped oscillations with a hacksaw blade.

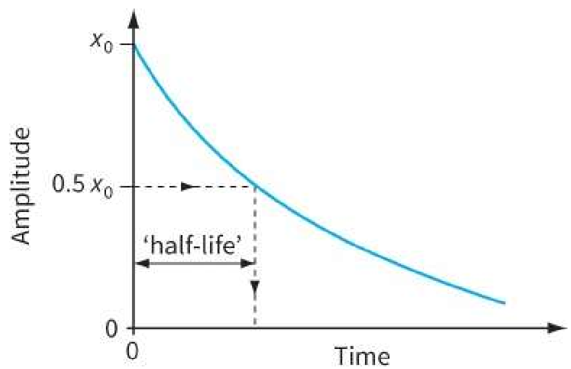

Figure 18.27: A typical graph of amplitude against time for damped oscillations.

Energy and damping

Damping can be very useful if we want to get rid of vibrations. For example, a car has springs (Figure 18.28) that make the ride much more comfortable for us when the car goes over a bump. However, we wouldn't want to spend every car journey vibrating up and down as a reminder of the last bump we went over. So the springs are damped by the shock absorbers, and we return rapidly to a smooth ride after every bump.

Damping is achieved by introducing the force of friction into a mechanical system. In an undamped oscillation, the total energy of the oscillation remains constant. There is a regular interchange between potential and kinetic energy. By introducing friction, damping has the effect of removing energy from the oscillating system, and the amplitude and maximum speed of the oscillation decrease.

Figure 18.28: The springs and shock absorbers in a car suspension system form a damped system.

Question

23 a Sketch graphs to show how each of the following quantities changes during the course of a single complete oscillation of an undamped pendulum: kinetic energy, potential energy, total energy.

b State how your graphs would be different for a lightly damped pendulum.

18.10 Resonance

Resonance is an important physical phenomenon that can appear in a great many different situations. A dramatic example is the Millennium Bridge in London, opened in June 2000 (Figure 18.29). With up to 2000 pedestrians walking on the bridge, it started to sway dangerously. The people also swayed in time with the bridge, and this caused the amplitude of the bridge's oscillations to increase–this is resonance. After three days, the bridge was closed. It took engineers two years to analyse the problem and then add 'dampers' to the bridge to absorb the energy of its oscillations. The bridge was then reopened and there have been no problems since.

You will have observed a much more familiar example of resonance when pushing a small child on a swing. The swing plus child has a natural frequency of oscillation. A small push in each cycle results in the amplitude increasing until the child is swinging high in the air.

Figure 18.29: The 'wobbly' Millennium Bridge in London was closed for nearly two years to correct problems caused by resonance.

PRACTICAL ACTIVITY 18.3

Observing resonance

Resonance can be observed with almost any oscillating system. The system is forced to oscillate at a particular frequency. If the forcing frequency happens to match the natural frequency of oscillation of the system, the amplitude of the resulting oscillations can build up to become very large.

Barton's pendulums

Barton's pendulums is a demonstration of this (Figure 18.30). Several pendulums of different lengths hang from a horizontal string. Each has its own natural frequency of oscillation. The 'driver' pendulum at the end is different; it has a large mass at the end, and its length is equal to that of one of the others. When the driver is set swinging, the others gradually start to move. However, only the pendulum whose length matches that of the driver pendulum builds up a large amplitude so that it is resonating.

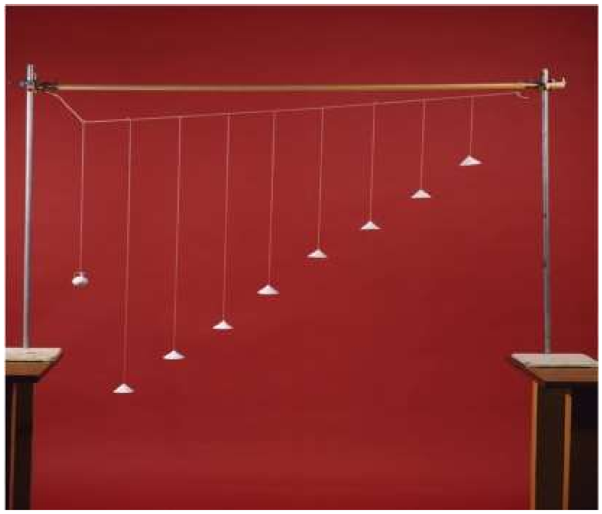

Figure 18.30: Barton's pendulums.

What is going on here? All the pendulums are coupled together by the suspension. As the driver swings, it moves the suspension, which in turn moves the other pendulums. The frequency of the matching pendulum is the same as that of the driver, and so it gains energy and its amplitude gradually builds up. The other pendulums have different natural frequencies, so the driver has little effect.

In a similar way, if you were to push the child on the swing once every three-quarters of an oscillation, you would soon find that the swing was moving backwards as you tried to push it forwards, so that your push would slow it down.

A mass–spring system

You can observe resonance for yourself with a simple mass–spring system. You need a mass on the end of a spring (Figure 18.31), chosen so that the mass oscillates up and down with a natural frequency of about 1 Hz. Now hold the top end of the spring and move your hand up and down rapidly, with an amplitude of a centimetre or two. Very little happens. Now move your hand up and down more slowly, close to 1 Hz.

You should see the mass oscillating with gradually increasing amplitude. Adjust your movements to the exact frequency of the natural vibrations of the mass and you will see the greatest effect.

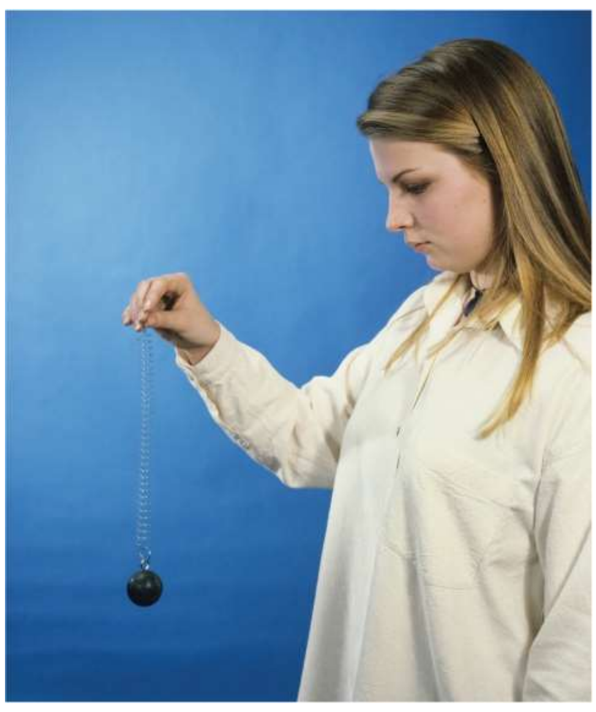

Figure 18.31: Resonance with a mass on a spring.

Defining resonance

For resonance to occur, we must have a system that is capable of oscillating freely. We must also have some way in which the system is forced to oscillate. When the forcing frequency matches the natural frequency of the system, the amplitude of the oscillations grows dramatically.

If the driving frequency does not quite match the natural frequency, the amplitude of the oscillations will increase, but not to the same extent as when resonance is achieved. Figure 18.32 shows how the amplitude of oscillations depends on the driving frequency in the region close to resonance.

In resonance, energy is transferred from the driver to the resonating system more efficiently than when resonance does not occur. For example, in the case of the Millennium Bridge, energy was transferred from the pedestrians to the bridge, causing large-amplitude oscillations.

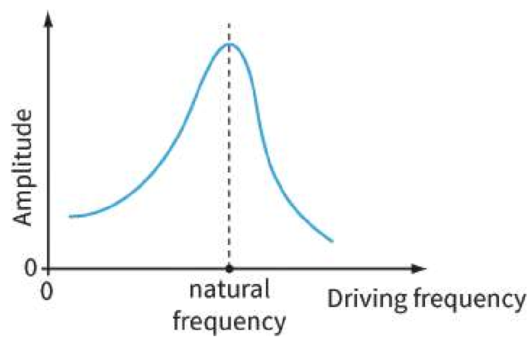

Figure 18.32: Maximum amplitude is achieved when the driving frequency matches the natural frequency of oscillation.

The following statements apply to any system in resonance:

- Its natural frequency is equal to the frequency of the driver.
- Its amplitude is maximum.
- It absorbs the greatest possible energy from the driver.

Resonance and damping

During earthquakes, buildings are forced to oscillate by the vibrations of the Earth. Resonance can occur, resulting in serious damage (Figure 18.33). In regions of the world where earthquakes happen regularly, buildings may be built on foundations that absorb the energy of the shock waves. In this way, the vibrations are 'damped' so that the amplitude of the oscillations cannot reach dangerous levels. This is an expensive business, and so far is restricted to the wealthier parts of the world.

Figure 18.33: Resonance during the Christchurch, New Zealand, earthquake of 22 February 2011 caused the collapse of many buildings. The earthquake, whose epicentre was in Lyttelton, just 10

kilometres south-east of Christchurch's central business district, measured 6.3. Nearly 200 lives were lost.

Damping is useful if we want to reduce the damaging effects of resonance. Figure 18.34 shows how damping alters the resonance response curve of Figure 18.32. Notice that, as the degree of damping is increased, the amplitude of the resonant vibrations decreases. The resonance peak becomes broader. There is also an effect on the frequency at which resonance occurs, which becomes lower as the damping increases.

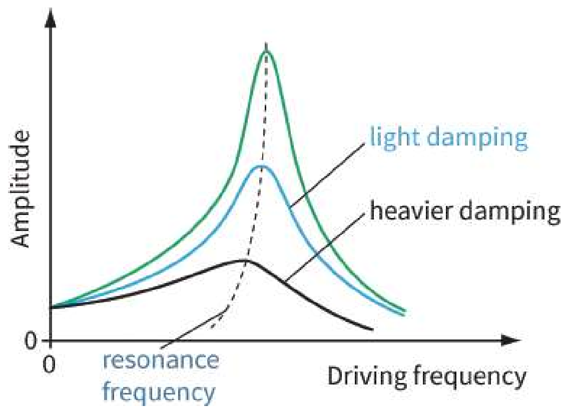

Figure 18.34: Damping reduces the amplitude of resonant vibrations.

An everyday example of damping can be seen on some doors. For example, a restaurant may have a door leading to the kitchen; this door can swing open in either direction. Such a door is designed to close by itself after someone has passed through it. Ideally, the door should swing back quickly without overshooting its closed position. To achieve this, the door hinges (or the closing mechanism) must be correctly damped. If the hinges are damped too lightly, the door will swing back and forth several times as it closes. If the damping is too heavy, it will take too long to close. With **critical damping**, the door will swing closed quickly without oscillating.

Critical damping is the minimum amount of damping required to return an oscillator to its equilibrium position without oscillating. Under-damping results in unwanted oscillations; over-damping results in a slower return to equilibrium (see Figure 18.35). A car's suspension system uses springs to smooth out bumps in the road. It is usually critically damped so that passengers do not experience nasty vibrations every time the car goes over a bump.

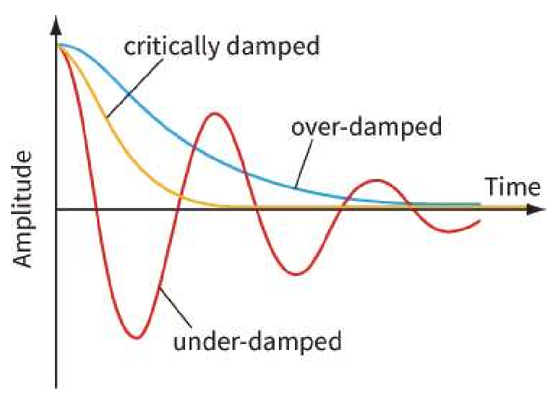

Figure 18.35: Critical damping is just enough to ensure that a damped system returns to equilibrium without oscillating.

Using resonance

As we have seen, resonance can be a problem in mechanical systems. However, it can also be useful. For example, many musical instruments rely on resonance.

Resonance is not confined to mechanical systems. It is made use of in, for example, microwave cooking. The microwaves used have a frequency that matches the natural frequency of vibration of water molecules (the microwave is the 'driver' and the molecule is the 'resonating system'). The water molecules in the food are forced to vibrate and they absorb the energy of the microwave radiation. The water gets hotter and the absorbed energy spreads through the food and cooks or heats it.

Magnetic resonance imaging (MRI) is used in medicine to produce images such as Figure 18.36, showing aspects of a patient's internal organs. Radio waves having a range of frequencies are used, and particular frequencies are absorbed by particular atomic nuclei. The frequency absorbed depends on the type of nucleus and on its surroundings. By analysing the absorption of the radio waves, a computer-generated image can be produced.

A radio or television also depends on resonance for its tuning circuitry. The aerial picks up signals of many different frequencies from many transmitters. The tuner can be adjusted to resonate at the frequency of the transmitting station you are interested in, and the circuit produces a large-amplitude signal for this frequency only.

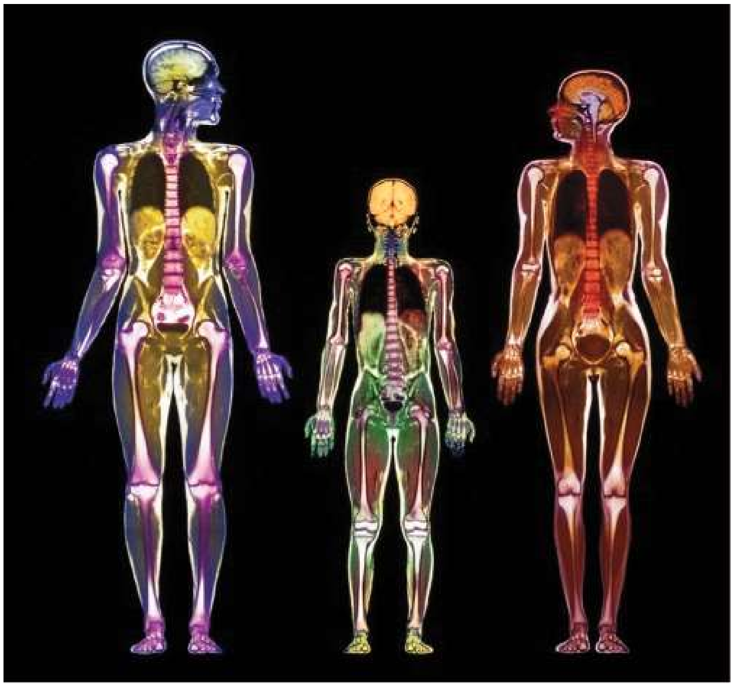

Figure 18.36: This magnetic resonance imaging (MRI) picture shows a man, a woman and a nine-year-old child. The image has been coloured to show up the bones (white), lungs (dark) and other organs.

Big ideas in physics

This study of simple harmonic motion illustrates some important aspects of physics:

- Physicists often take a complex problem (such as how the atoms in a solid vibrate) and reduce it to a simpler, more manageable problem (such as how a mass–spring system vibrates). This is simpler because we know that the spring obeys Hooke's law, so that force is proportional to displacement.
- Physicists generally feel happier if they can write mathematical equations that will give numerical answers to problems. The equation $a = -\omega^2 x$, which describes s.h.m., can be solved to give the sine and cosine equations we have considered earlier.
- Once physicists have solved one problem like this, they look around for other situations where they can use the same ideas all over again. So the mass–spring theory also works well for vibrating atoms and molecules, for objects bobbing up and down in water, and in many other situations.
- Physicists also seek to modify the theory to fit a greater range of situations. For example, what happens if the vibrating mass experiences a frictional force as it oscillates? (This is damping, as discussed earlier.) What happens if the spring doesn't obey Hooke's law? (This is a harder question to answer.)

Your A Level physics course will help you to build up your appreciation of some of these big ideas–fields (magnetic, electric, gravitational), energy and so on.

Question

24 Give an example of a situation where resonance is a problem, and a second example where resonance is useful. In each example, state what the oscillating system is and what forces it to resonate.

REFLECTION

You might have observed some of the terms and the mathematical equations in this chapter share many characteristics with those used in circular motion.

Make a list of the similarities and differences between the terms used in the two examples.

Make a list of equations used in the two examples. How are they related?

Can you use these similarities to help you understand simple harmonic motion further?

What things might you need help with to understand the chapter even better?

SUMMARY

Many systems, mechanical and otherwise, will oscillate freely when disturbed from their equilibrium position.

Some oscillators have motion described as **simple harmonic motion** (s.h.m.). For these systems, graphs of displacement, velocity and acceleration against time are sinusoidal curves–see Figure 18.37.

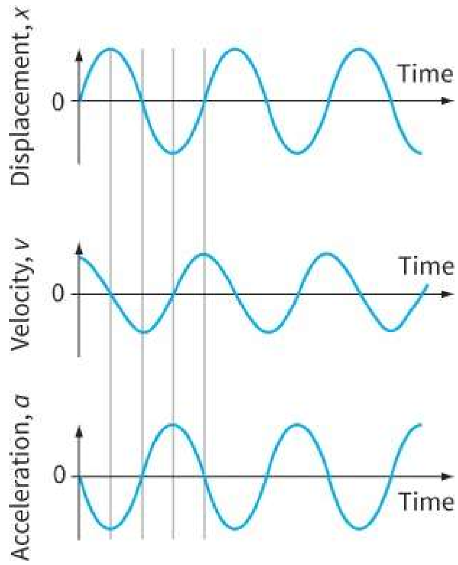

Figure 18.37: Graphs for s.h.m.

During a single cycle of s.h.m., the phase changes by 2π radians. The angular frequency ω of the motion is related to its period T and frequency f:

$$f = \frac{2\pi}{T} \text{ and } \omega = 2\pi f$$

In s.h.m., displacement x and velocity v and acceleration can be represented as functions of time t by equations of the form:

$$x = x_0 \sin \omega t \quad \text{and} \quad v = v_0 \cos \omega t \text{ and } a = -a_0 \sin \omega t$$

A body executes simple harmonic motion if its acceleration is directly proportional to its displacement from its equilibrium position, and is always directed towards the equilibrium position.

Acceleration a in s.h.m. is related to displacement x by the equation:

$$a = -\omega^2 x$$

The maximum speed v_0 in s.h.m. is given by the equation:

$$v_0 = \omega x_0$$

The frequency or period of a simple harmonic oscillator is independent of its amplitude.

In s.h.m., there is a regular interchange between kinetic energy and potential energy.

Resistive forces remove energy from an oscillating system. This is known as damping. Damping causes the amplitude to decay with time.

Critical damping is the minimum amount of damping required to return an oscillator to its equilibrium position without oscillating.

When an oscillating system is forced to vibrate close to its natural frequency, the amplitude of vibration increases rapidly. The amplitude is maximum when the forcing frequency matches the natural frequency of the system; this is resonance.
Resonance can be a problem, but it can also be very useful.

EXAM-STYLE QUESTIONS

1 A mass, hung from a spring, oscillates with simple harmonic motion.

Which statement is correct? **[1]**

A The force on the mass is directly proportional to the angular frequency of the oscillation.

B The force on the mass is greatest when the displacement of the bob is greatest.

C The force on the mass is greatest when the speed of the bob is greatest.

D The force on the mass is inversely proportional to the time period of the oscillation.

2 The bob of a simple pendulum has a mass of 0.40 kg. The pendulum oscillates with a period of 2.0 s and an amplitude of 0.15 m.

At one point in its cycle it has a potential energy of 0.020 J.

What is the kinetic energy of the pendulum bob at this point? **[1]**

A 0.024 J

B 0.044 J

C 0.14 J

D 0.18 J

3 State and justify whether the following oscillators show simple harmonic motion:

a a basketball being bounced repeatedly on the ground. **[2]**

b a guitar string vibrating **[2]**

c a conducting sphere vibrating between two parallel, oppositely charged metal plates **[1]**

d the pendulum of a grandfather clock. **[2]**

[Total: 7]

4 The pendulum of a clock is displaced by a distance of 4.0 cm and it oscillates in s.h.m. with a period of 1.0 s.

a Write down an equation to describe the displacement x of the pendulum bob with time t. **[2]**

b Calculate:

i the maximum velocity of the pendulum bob **[2]**

ii its velocity when its displacement is 2.0 cm. **[1]**

[Total: 5]

5 A 50 g mass is attached to a securely clamped spring. The mass is pulled downwards by 16 mm and released, which causes it to oscillate with s.h.m. of time period of 0.84 s.

a Calculate the frequency of the oscillation. **[1]**

b Calculate the maximum velocity of the mass. **[1]**

c Calculate the maximum kinetic energy of the mass and state at which point in the oscillation it will have this velocity. **[2]**

d Write down the maximum gravitational potential energy of the mass (relative to its equilibrium position). You may assume that the damping is negligible. **[1]**

[Total: 5]

6 In each of the three graphs, **a**, **b** and **c** in Figure 18.38, give the phase difference between the two curves:

i as a fraction of an oscillation **[1]**

ii in degrees **[1]**

iii in radians. **[1]**

[Total: 3]

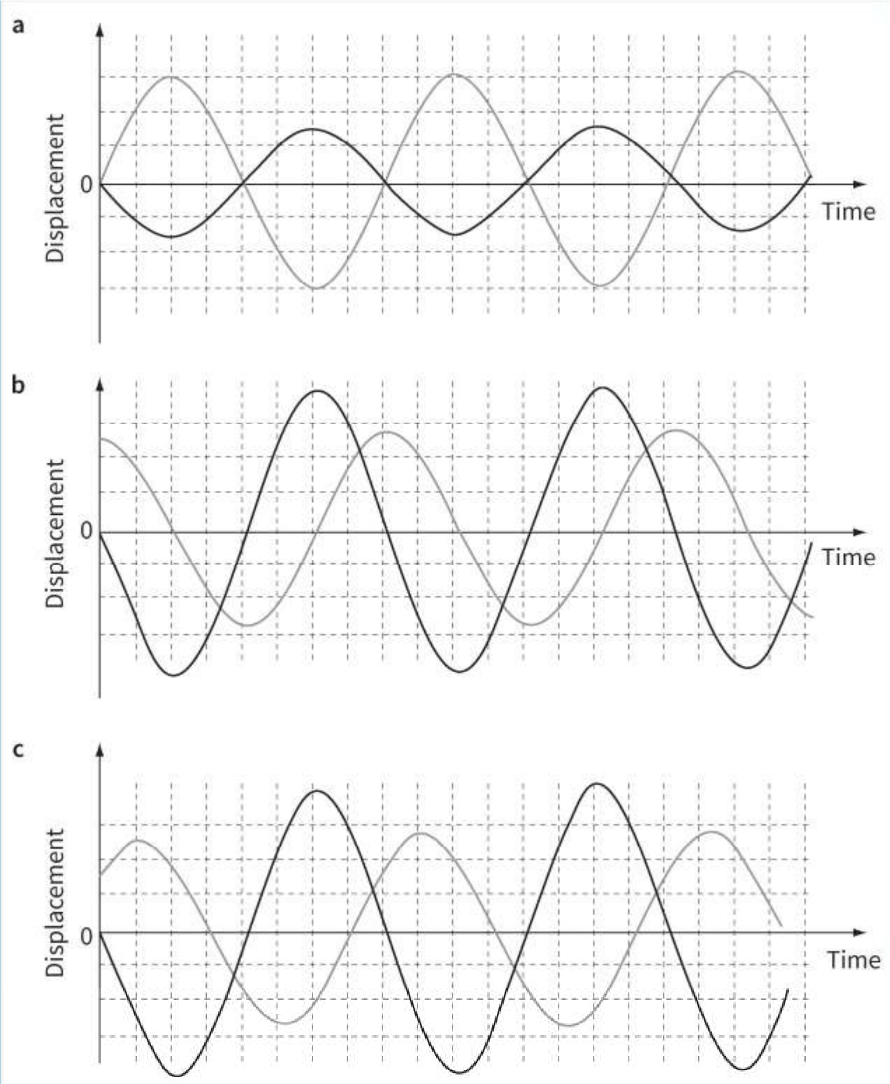

Figure 18.38

7 **a** Determine the frequency and the period of the oscillation described by this graph. **[2]**

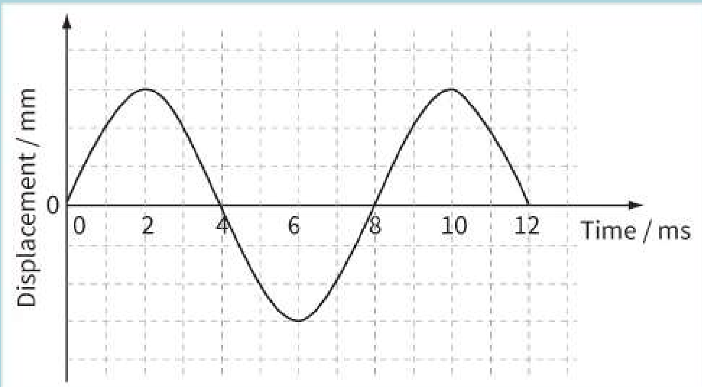

Figure 18.39

b Use a copy of the graph and on the same axes sketch:

i the velocity of the particle **[1]**

ii the acceleration of the particle. **[2]**

[Total: 5]

8 These graphs show the displacement of a body as it vibrates between two points.

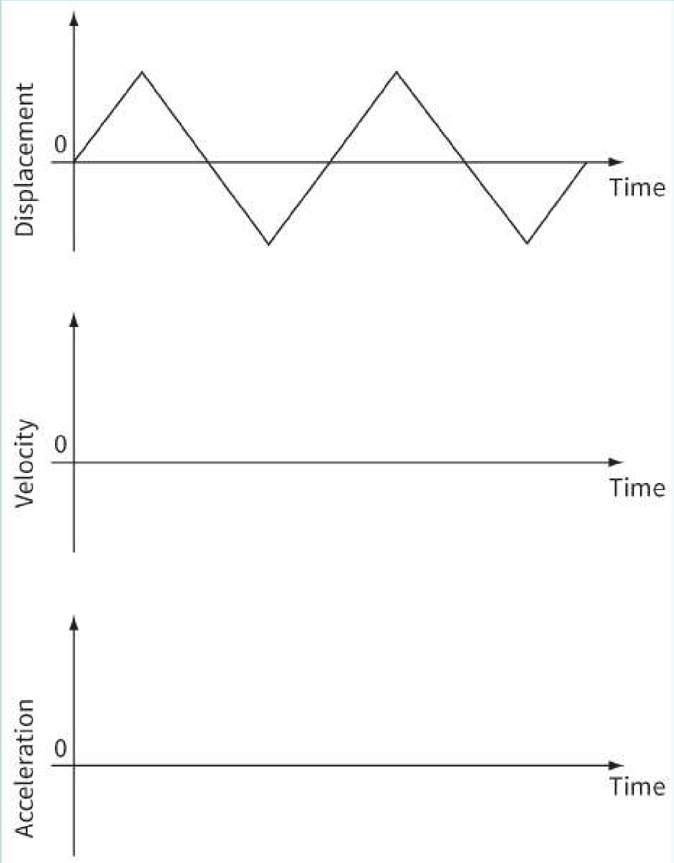

Figure 18.40

a State and explain whether the body is moving with simple harmonic motion. **[1]**

b Make a copy of the three graphs.

i On the second set of axes on your copy show the velocity of the body as it vibrates. **[1]**

ii On the third set of axes on your copy, show the acceleration of the body. **[2]**

[Total: 4]

9 This diagram shows the piston of a small car engine that oscillates in the cylinder with a motion that approximates simple harmonic motion at 4200 revs per minute (1 rev = 1 cycle). The mass of the piston is 0.24 kg.

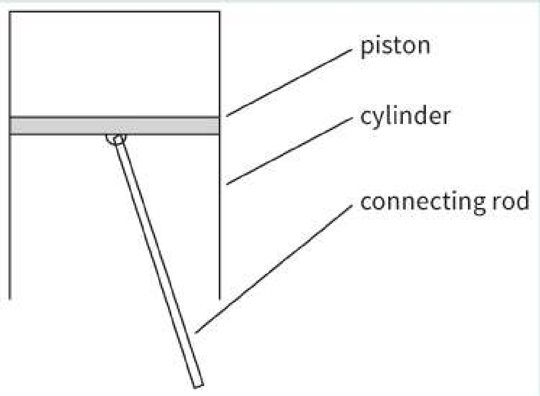

Figure 18.41

a Explain what is meant by simple harmonic motion. **[2]**

b Calculate the frequency of the oscillation. **[1]**

c The amplitude of the oscillation is 12.5 cm. Calculate:

i the maximum speed at which the piston moves **[2]**

ii the maximum acceleration of the piston **[2]**

iii the force required on the piston to produce the maximum acceleration. **[1]**

[Total: 8]

10 This diagram shows a turntable with a rod attached to it a distance 15 cm from the centre. The turntable is illuminated from the side so that a shadow is cast on a screen.

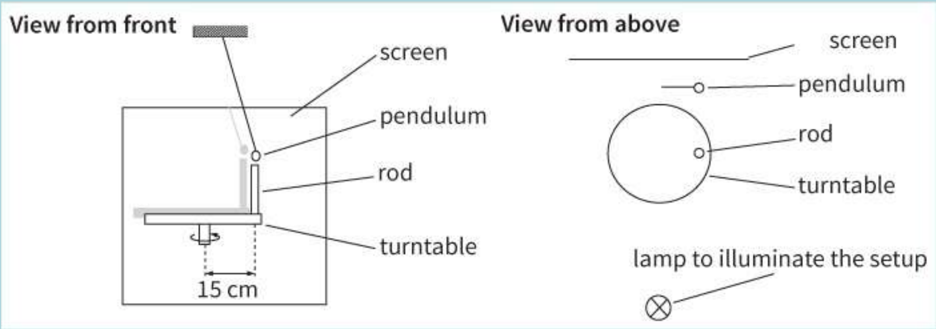

Figure 18.42

A simple pendulum is placed behind the turntable and is set oscillating so that it has an amplitude equal to the distance of the rod from the centre of the turntable.

The speed of rotation of the turntable is adjusted. When it is rotating at 1.5 revolutions per second the shadow of the pendulum and the rod are found to move back and forth across the screen exactly in phase.

a Explain what is meant by the term in phase. **[1]**

b Write down an equation to describe the displacement x of the pendulum from its equilibrium position and the angular frequency of the oscillation of the pendulum. **[1]**

c The turntable rotates through 60° from the position of maximum displacement shown in the diagram.

i Calculate the displacement (from its equilibrium position) of the pendulum at this point. **[3]**

ii Calculate its speed at this point. **[2]**

iii Through what further angle must the turntable rotate before it has this speed again? **[1]**

[Total: 8]

11 When a cricket ball hits a cricket bat at high speed it can cause a standing wave to form on the bat. In one such example, the handle of the bat moved with a frequency of 60 Hz with an amplitude of 2.8 mm.

The vibrational movement of the bat handle can be modelled on simple harmonic motion.

a State the conditions for simple harmonic motion. **[2]**

b Calculate the maximum acceleration of the bat handle. **[2]**

c Given that the part of the bat handle held by the cricketer has a mass of 0.48 kg, calculate the maximum force produced on his hands. **[1]**

d The oscillations are damped and die away after about five complete cycles. Sketch a displacement–time graph to show the oscillations. **[2]**

[Total: 7]

12 Seismometers are used to detect and measure the shock waves that travel through the Earth due to earthquakes.

This diagram shows the structure of a simple seismometer. The shock wave will cause the mass to vibrate, causing a trace to be drawn on the paper scroll.

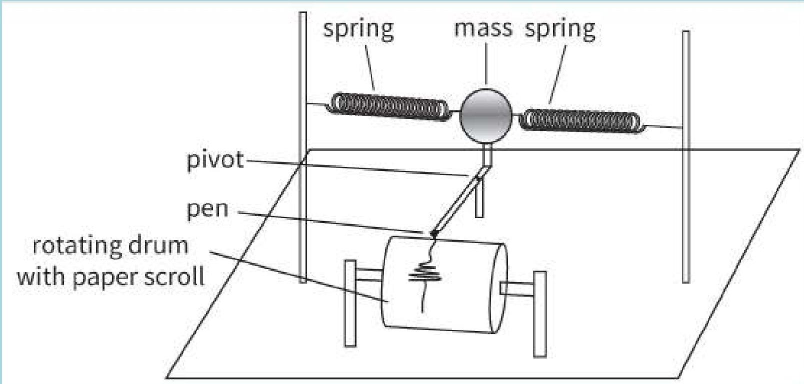

Figure 18.43

a The frequency of a typical shock wave is between 30 and 40 Hz. Explain why the natural frequency of the spring–mass system in the seismometer should be very much less than this range of frequencies. **[3]**

This graph shows the acceleration of the mass against its displacement when the seismometer is recording an earthquake.

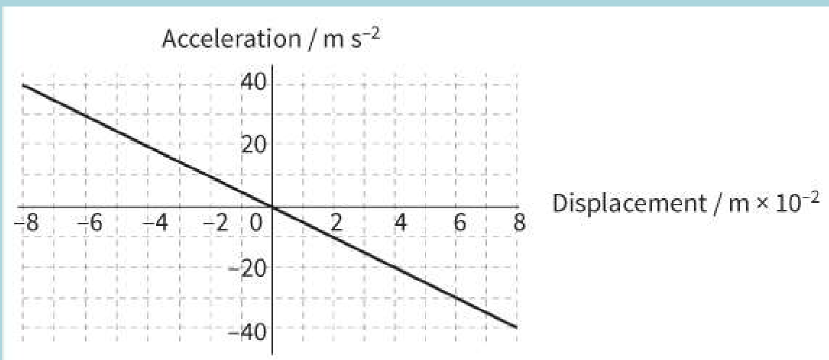

Figure 18.44

b What evidence does the graph give that the motion is simple harmonic? **[2]**

c Use information from the graph to calculate the frequency of the oscillation. **[4]**

[Total: 9]

SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

I can	See topic…	Needs more work	Almost there	Ready to move on
understand the terms displacement, amplitude, period, frequency, angular frequency and phase difference	18.3			
express the period in terms of both frequency and angular frequency	18.3, 18.6			
understand that in simple harmonic motion there is a varying force on the oscillator, which is proportional to the displacement of the oscillator from a point and it is always directed towards that point	18.4			
recall, use and understand the importance of the equation: $a = -\omega^2 x$	18.7			
understand that the solution to the equation $a = -\omega^2 x$ is $x = x_0 \sin \omega t$	18.7			
use the equation: $v = v_0 \cos \omega t$	18.7			
use the equation: $v = \pm\omega\sqrt{(x_0^2 - x^2)}$	18.7			
understand the interchange between potential and kinetic energy in simple harmonic motion	18.8			
understand that the total energy of a simple harmonic oscillator remains constant and is determined by the amplitude of the oscillator, its mass and its frequency	18.8			
recall and use the equation $E = \frac{1}{2} m\omega^2 x_0$ for the total energy of an oscillator	18.8			
understand that a resistive force acting on an oscillator causes damping	18.9			
understand the term critical damping	18.10			
sketch displacement graphs showing the different types of damping	18.5			
understand the concept of resonance	18.10			
understand that resonance occurs when the driving frequency equals the natural frequency of the oscillating system.	18.10			