Chapter 17

Gravitational fields

LEARNING INTENTIONS

In this chapter you will learn how to:

- describe a gravitational field as a field of force and define gravitational field strength g
- represent a gravitational field using field lines
- understand the meaning of centre of mass and use the concept in problems involving uniform spheres
- recall and use Newton's law of gravitation
- solve problems involving the gravitational field strength of a uniform field and the field of a point mass
- understand how the gravitational potential energy, $E = -\frac{Gm_1m_2}{r}$, of two point masses is a consequence of gravitational potential
- define and solve problems involving gravitational potential
- analyse circular orbits in an inverse square law field, including geostationary orbits.

BEFORE YOU START

We have all experienced gravity and the effects it has on us in everyday life.

Imagine what would happen if gravity was 'switched off'. Discuss the effects that it would have, not only on everyday life, but on a much bigger scale. For example, what would happen to the objects on your desk? Can you predict what would happen to the orbit of the Earth?

GRAVITATIONAL FORCES AND FIELDS

Gravity is amazing! It is the first interaction that we experience, and it is an interaction that we take for granted. But without it, there would be no Earth, no Sun, no galaxies, no us!

You have probably seen pictures like Figure 17.1 before – astronauts can float in mid-air in the International Space Station (ISS). This is not because there is no gravity at this altitude, but because the ISS is effectively in freefall as it orbits around the Earth. The only astronauts who have experienced zero gravity are those who experienced it on their way to and from the Moon. Even then, there would be a small gravitational effect due to the pull of the Earth, the Sun and the Moon itself. This is sometimes referred to as microgravity.

All life on Earth evolved in the Earth's gravitational field. The bodies of all animals evolved so that they could cope with the strains and stresses of the forces inherent in living in a gravitational field. Astronauts spending long periods of time in microgravity, for example in the proposed trips to Mars, would find their bodies losing calcium and their muscles, which are used to supporting their weight, getting weaker.

One of the most amazing things about gravity is its role in the development of the Universe. Nebulae are great clouds of dust and (mostly) hydrogen gas in outer space. Irregularities in the density of the cloud lead to more material being attracted to the higher density areas, so this area becomes more and more concentrated as more material is attracted. The process gathers pace. The vast quantity of gravitational potential energy of the spread-out hydrogen becomes kinetic energy of hydrogen atoms that, once great enough, allows nuclear fusion to occur … and a star is born.

Why is this so amazing? Gravity is a very weak interaction; the electromagnetic interaction is far, far stronger. The electric repulsion between two protons in the nucleus of an atom is about 10^{36} times bigger (that is, 10 followed by 35 zeroes!) than the gravitational attraction.

Figure 17.1: An astronaut on board a space shuttle in Earth-orbit.

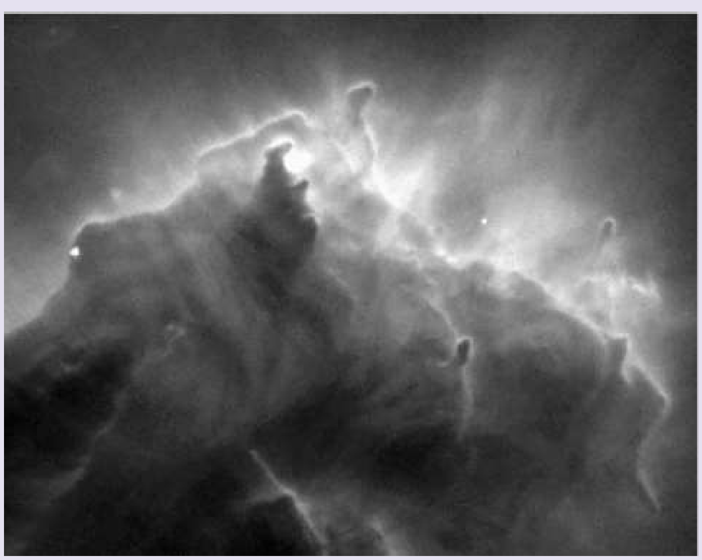

Figure 17.2: Gaseous star nurseries.

Why does the gravitational interaction rule? The gravitational force is always attractive, whereas there are two types of charge (positive and negative) so the electromagnetic interaction can either be attractive or repulsive. In general, the negative and positive charges tend to cancel out, making any large scale object nearly electrically neutral.

17.1 Representing a gravitational field

We can represent the Earth's **gravitational field** by drawing field lines, as shown in Figure 17.3.

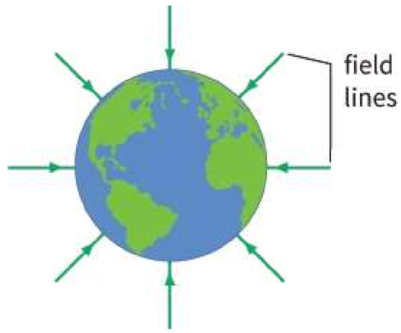

Figure 17.3: The Earth's gravitational field is represented by field lines.

The field lines show two things:

- The arrows on the field lines show us the direction of the gravitational force on a mass placed in the field.
- The spacing of the field lines indicates the strength of the gravitational field–the further apart they are, the weaker the field.

The drawing of the Earth's gravitational field shows that all objects are attracted towards the centre of the Earth. This is true even if they are below the surface of the Earth. The gravitational force gets weaker as you get further away from the Earth's surface – this is shown by the greater separation between the field lines. The Earth is almost a uniform spherical mass, although it does bulge a bit at the equator. The gravitational field of the Earth is as if its entire mass was concentrated at its centre; this is known as its **centre of mass**. As far as any object beyond the Earth's surface is concerned, the Earth behaves as a point mass.

Figure 17.4 shows the Earth's gravitational field closer to its surface. The gravitational field in and around a building on the Earth's surface shows that the gravitational force is directed downwards everywhere and (because the field lines are very nearly parallel and evenly spaced) the strength of the gravitational field is virtually the same at all points in and around the building. This means that your weight is virtually the same everywhere in this gravitational field. Your weight does not become much less when you go upstairs.

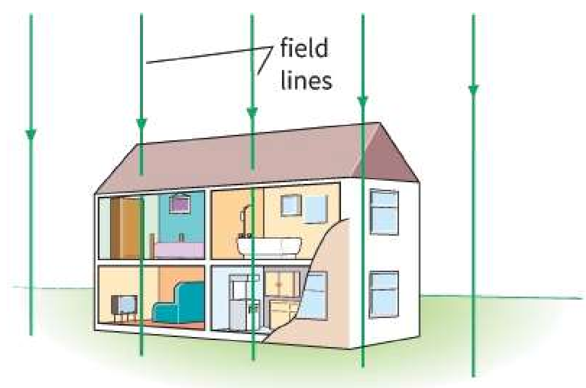

Figure 17.4: The Earth's gravitational field is uniform on the scale of a building.

We describe the Earth's gravitational field as **radial**, since the field lines diverge (spread out) radially from the centre of the Earth. However, on the scale of a building, the gravitational field is **uniform**, since the field lines are equally spaced. Jupiter is a more massive planet than the Earth and so we would represent its gravitational field by showing more closely spaced field lines.

Newton's law of gravitation

Newton used his ideas about mass and gravity to suggest a law of gravitation for two point masses (Figure 17.5).

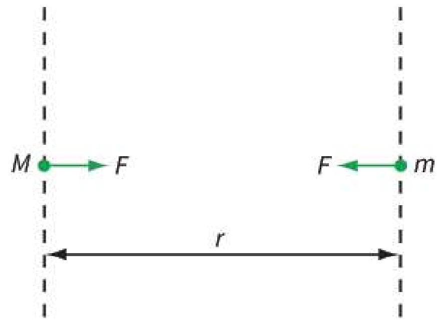

Figure 18.5: Two point masses separated by distance *r*.

Newton considered two point masses M and m separated by a distance r. Each point mass attracts the other with a force F. (According to Newton's third law of motion, the point masses interact with each other and therefore exert equal but opposite forces on each other.)

Newton's law of gravitation states that any two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of their separation.

Note that the law refers to 'point masses' – you can alternatively use the term 'particles'. Things are more complicated if we think about solid bodies that occupy a volume of space. Each particle of one body attracts every particle of the other body and we would have to add all these forces together to work out the force each body has on the other. Newton was able to show that two uniform spheres attract one another with a force that is the same as if their masses were concentrated at their centres (provided their centre-to-centre distance is greater than the sum of their radii).

According to Newton's law of gravitation, we have:

$$\text{force} \propto \text{product of the masses, or } F \propto Mm$$

$$\text{force} \propto \frac{1}{\text{distance}^2} \text{ or } F \propto \frac{1}{r^2}$$

Therefore:

$$F \propto \frac{Mm}{r^2}$$

To make this into an equation, we introduce the gravitational constant G:

$$F = \frac{GMm}{r^2}$$

(The force is attractive, so F is in the opposite direction to r.)

The gravitational constant G is sometimes referred to as the universal gravitational constant because it is believed to have the same value, 6.67×10^{-11} N m^2 kg^{-2}, throughout the Universe. This is important for our understanding of the history and likely long-term future of the Universe.

The equation can also be applied to spherical objects (such as the Earth and the Moon) provided we remember to measure the separation r between the centres of the objects. You may also come across the equation in the form:

$$F = \frac{Gm_1m_2}{r^2}$$

where m_1 and m_2 are the masses of the two bodies.

KEY EQUATION

Newton's law of gravitation:

$$F = \frac{Gm_1m_2}{r^2}$$

Let us examine this equation to see why it seems reasonable. First, each of the two masses is important.

Your weight (the gravitational force on you) depends on your mass and on the mass of the planet you happen to be standing on.

Second, the further away you are from the planet, the weaker its pull. Twice as far away gives one-quarter of the force. This can be seen from the diagram of the field lines in Figure 17.6. If the distance is doubled, the lines are spread out over four times the surface area, so their concentration is reduced to one-quarter. This is called an inverse square law. Inverse square laws are common in physics, light or γ-rays spreading out uniformly from a point source also follow an inverse square law.

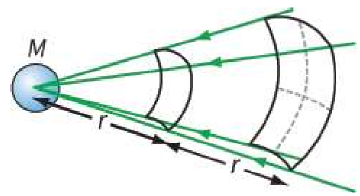

Figure 17.6: Field lines are spread out over a greater surface area at greater distances, so the strength of the field is weaker.

We measure distances from the centre of mass of one body to the centre of mass of the other (Figure 17.7). We treat each body as if its mass were concentrated at one point. The two bodies attract each other with equal and opposite forces, as required by Newton's third law of motion. The Earth pulls on you with a force (your weight) directed towards the centre of the Earth; you attract the Earth with an equal force, directed away from its centre and towards you. Your pull on an object as massive as the Earth has little effect on it. The Sun's pull on the Earth, however, has a very significant effect.

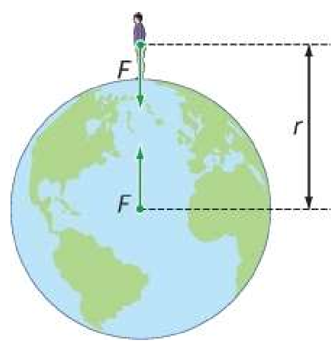

Figure 17.7: A person and the Earth exert equal and opposite attractive forces on each other.

Questions

1 Calculate the gravitational force of attraction between:

- **a** two objects separated by a distance of 1.0 cm and each having a mass of 100 g
- **b** two asteroids separated by a distance of 4.0 10^9 m and each having a mass of 5.0×10^{10} kg
- **c** a satellite of mass 1.4×10^4 kg orbiting the Earth at a distance of 6800 km from the Earth's centre. (The mass of the Earth is 6.0×10^{24} kg.)

2 Estimate the gravitational force of attraction between two people sitting side by side on a park bench. How does this force compare with the gravitational force exerted on each of them by the Earth (in other words, their weight)?

17.2 Gravitational field strength *g*

We can describe how strong or weak a gravitational field is by stating its **gravitational field strength**. We are used to this idea for objects on or near the Earth's surface. The gravitational field strength is the familiar quantity g. Its value is approximately 9.8 m s^{-2}. The weight of a body of mass m is mg.

To make the meaning of g clearer, we should write it as 9.8 N kg^{-1}. That is, each 1 kg of mass experiences a gravitational force of 9.8 N.

The gravitational field strength g at any point in a gravitational field is defined as follows:

The gravitational field strength at a point is the gravitational force exerted per unit mass on a small object placed at that point.

This can be written as an equation:

$$g = \frac{F}{m}$$

where F is the gravitational force on the object and m is the mass of the object. Gravitational field strength has units of N kg^{-1}. This is equivalent to m s^{-2}.

We can use the definition to determine the gravitational field strength for a point (or spherical) mass. The force between two point masses is given by:

$$F = \frac{GMm}{r^2}$$

So, the gravitational field strength g due to the mass M at a distance of r from its centre is:

$$\begin{aligned} g &= \frac{F}{m} \\ &= \frac{GMm}{r^2 m} \\ &\Rightarrow \frac{GM\cancel{m}}{r^2\cancel{m}} \\ &= \frac{GM}{r^2} \end{aligned}$$

KEY EQUATION

The gravitation field g due to a point mass is:

$$g = \frac{GM}{r^2}$$

where G is the universal gravitational constant, M is the mass and r is the distance from the mass.

You must learn how to derive this equation using Newton's law of gravitation and your understanding of a gravitational field.

Since force is a vector quantity, it follows that gravitational field strength is also a vector. We need to give its direction as well as its magnitude in order to specify it completely. The field strength g is not a constant; it decreases as the distance r increases. The field strength obeys an inverse square law with distance. The field strength will decrease by a factor of four when the distance from the centre is doubled. Close to the Earth's surface, the magnitude of g is about 9.81 N kg^{-1}. Even if you climbed Mount Everest, which is 8.85 km high, the field strength will only decrease by 0.3%.

So the gravitational field strength g at a point depends on the mass M of the body causing the field, and the distance r from its centre (see Worked example 1).

Gravitational field strength g also has units m s^{-2}; it is an acceleration. Another name for g is 'acceleration of free fall'. Any object that falls freely in a gravitational field has this acceleration, approximately 9.8 m s^{-2} near the Earth's surface. In Chapter 2, you learned about different ways to determine an experimental value for g, the local gravitational field strength.

WORKED EXAMPLE

1 The Earth has radius 6400 km. The gravitational field strength on the Earth's surface is 9.81 N kg^{-1}. Use this information to determine the mass of the Earth and its mean density.

Step 1 Write down the quantities given:

$r = 6.4 \times 10^6$ m $g = 9.81$ N kg^{-1}

Step 2 Use the equation $g = \frac{GM}{r^2}$ to determine the mass of the Earth M.

$$\begin{aligned} g &= \frac{GM}{r^2} \\ 9.8 &= \frac{6.67 \times 10^{11} M}{\left(6.4 \times 10^6\right)^2} \\ M &= 9.8 \times \frac{\left(6.4 \times 10^6\right)^2}{6.67 \times 10^{-11}} \\ &= 6.0 \times 10^{24} \text{kg} \end{aligned}$$

Step 3 Use the equation density $= \frac{\text{mass}}{\text{volume}}$ to determine the density ρ of the Earth.

The Earth is a spherical mass. Its volume can be calculated using $\frac{4}{3}\pi r^3$:

$$\begin{aligned} \rho &= \frac{M}{V} \\ &= \frac{6.0 \times 10^{24}}{\frac{4}{3} \times \pi \times \left(6.4 \times 10^6\right)^3} \\ &= 5500 \text{ kg m}^{-3} \end{aligned}$$

We can now see in more detail why the Earth's gravitational field may be considered to be uniform near a planet's surface. Consider a planet of radius R, then, at its surface, the gravitational field strength, $g = \frac{GM}{R^2}$, where G is the universal gravitational constant.

If we move up from the surface, a distance ΔR, the new gravitational field strength, $g_{new} = \frac{GM}{(R+\Delta R)^2}$.

The percentage change in R is $\frac{\Delta R}{R} \times 100\%$ and, using a similar logic to that used when dealing with uncertainties, the percentage change in the gravitational field strength is $2 \times \frac{\Delta R}{R} \times 100\%$.

WORKED EXAMPLE

2 Calculate the change in the gravitational field strength between the Earth's equator and at a height of 10 km above the equator.

(Earth's radius at the equator is 6357 km, mass of the Earth = 5.974×10^{24} kg

$G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2})

Gravitational field strength at the equator =

$$g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 5.974 \times 10^{24}}{\left(6.357 \times 10^6\right)^2} = 9.860 \text{ Nkg}^{-1}$$

percentage change in the field in moving 10 km from the surface

$$= \frac{\Delta R}{R} \times 100\% = \frac{10}{5974} \times 100\% = 0.167\%$$

The change in the field strength = 0.167% of g = 0.167% of 9.860 = 0.01650 Nkg^{-1}

Generally the field strength is quoted to two or three significant figures (9.8 or 9.81 Nkg^{-1}). 10 km is roughly the height at which airliners cruise and even at this height there is no significant change in the field strength, indeed local variation due to different densities of the Earth's crust and the fact that the Earth is not a perfect sphere have a much larger effect.

Looking at this from a different viewpoint, if we refer back to Figure 17.6 and the spreading of the field lines, as we go further from the centre of mass we see that a change in height increases the area subtended by those lines. The area subtended increases by the square of the distance and thus moving a distance ΔR from the surface of the Earth would give a percentage increase in the area subtended by the field lines of $2 \times \frac{\Delta R}{R} \times 100\%$. Following similar logic to the worked example, we can see that a 10 km vertical movement from the Earth's surface leads to a percentage increase in area of 0.16%, which shows that over this distance the lines in Figure 17.4 (a rise of less than 10 metres) are very nearly parallel and the field is very nearly uniform.

Questions

You will need the data in Table 17.1 to answer these questions.

Body	Mass / kg	Radius / km	Distance from Earth / km

Earth	6.0×10^{24}	6 400	–
Moon	7.4×10^{22}	1 740	3.8×10^{5}
Sun	2.0×10^{30}	700 000	1.5×10^{8}

Table 17.1: Data for Questions 3 to 8.

3 Mount Everest is approximately 9.0 km high. Estimate how much less a mountaineer of mass 100 kg (including backpack) would weigh at its summit, compared to her weight at sea level. Would this difference be measurable with bathroom scales?

4 **a** Calculate the gravitational field strength:

i close to the surface of the Moon

ii close to the surface of the Sun.

b Suggest how your answers help to explain why the Moon has only a thin atmosphere, while the Sun has a dense atmosphere.

5 **a** Calculate the Earth's gravitational field strength at the position of the Moon.

b Calculate the force the Earth exerts on the Moon. Hence, determine the Moon's acceleration towards the Earth.

6 Jupiter's mass is 320 times that of the Earth and its radius is 11.2 times the Earth's. The Earth's surface gravitational field strength is 9.81 N kg^{-1}. Calculate the gravitational field strength close to the surface of Jupiter.

7 The Moon and the Sun both contribute to the tides on the Earth's oceans. Which has a bigger pull on each kilogram of seawater, the Sun or the Moon?

8 Astrologers believe that the planets exert an influence on us, particularly at the moment of birth. (They don't necessarily believe that this is an effect of gravity!)

a Calculate the gravitational force on a 4.0 kg baby caused by Mars when the planet is at its closest to the Earth at a distance of 100 000 000 km. Mars has a mass 6.4×10^{23} kg.

b Calculate the gravitational force on the same baby due to its 50 kg mother at a distance of 0.40 m.

17.3 Energy in a gravitational field

As well as the force on a mass in a gravitational field, we can think about its energy. If you lift an object from the ground, you increase its gravitational potential energy (g.p.e.). The higher you lift it, the more work you do on it and so the greater its g.p.e. The object's change in g.p.e. can be calculated as $mg\Delta h$, where Δh is the change in its height (as we saw in Chapter 5).

This approach is satisfactory when we are considering objects close to the Earth's surface. However, we need a more general approach to calculating gravitational energy, for two reasons:

- If we use g.p.e. = $mg\Delta h$, we are assuming that an object's g.p.e. is zero on the Earth's surface. This is fine for many practical purposes but not, for example, if we are considering objects moving through space, far from Earth. For these, there is nothing special about the Earth's surface.
- If we lift an object to a great height, g decreases and we would need to take this into account when calculating g.p.e.

For these reasons, we need to set up a different way of thinking about gravitational potential energy. We start by picturing a mass at infinity, that is, at an infinite distance from all other masses. We say that here the mass has zero potential energy. This is a more convenient way of defining the zero of g.p.e. than using the surface of the Earth.

Now we picture moving the mass to the point where we want to know its g.p.e. As with lifting an object from the ground, we determine the work done to move the mass to the point. The work done on it is equal to the energy transferred to it; that is, its g.p.e., and that is how we can determine the g.p.e. of a particular mass.

17.4 Gravitational potential

In practice, it is more useful to talk about the gravitational potential at a point. This tells us the g.p.e. per unit mass at the point (just as field strength g tells us the force per unit mass at a point in a field). The symbol used for potential is ϕ (Greek letter phi), and unit mass means one kilogram. **Gravitational potential** at a point is defined as the work done per unit mass bringing a unit mass from infinity to the point.

For a point mass M, we can write an equation for ϕ at a distance r from M:

$$\phi = -\frac{GM}{r}$$

where G is the gravitational constant as before. Notice the minus sign; gravitational potential is always negative. This is because, as a mass is brought towards another mass, its g.p.e. decreases. Since g.p.e. is zero at infinity, it follows that, anywhere else, g.p.e. and potential are less than zero; that is, they are negative.

KEY EQUATION

Gravitational potential:

$$\phi = -\frac{GM}{r}$$

Imagine a spacecraft coming from a distant star to visit the Solar System. The variation of the gravitational potential along its path is shown in Figure 17.8. We will concentrate on three parts of its journey:

1. As the spacecraft approaches the Earth, it is attracted towards it. The closer it gets to Earth, the lower its g.p.e. becomes and so the lower its potential.
2. As the spacecraft moves away from the Earth, it has to work against the pull of the Earth's gravity. Its g.p.e. increases and so we can say that the potential increases. The Earth's gravitational field creates a giant 'potential well' in space. We live at the bottom of that well.
3. As the spacecraft approaches the Sun, it is attracted into a much deeper well. The Sun's mass is much greater than the Earth's and so its pull is much stronger and the potential at its surface is more negative than on the Earth's surface.

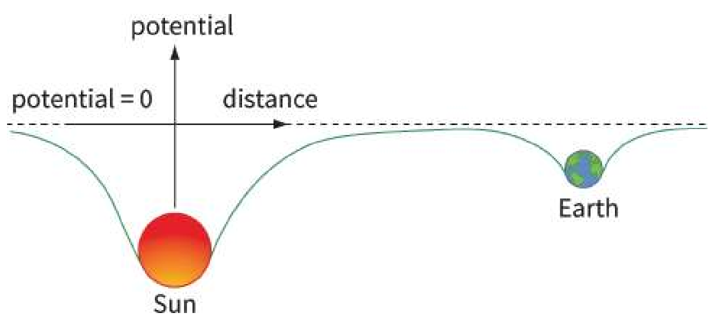

Figure 17.8: The gravitational potential is zero at infinity (far from any mass), and decreases as a mass is approached.

WORKED EXAMPLE

3 A planet has a diameter of 6800 km and a mass of 4.9×10^{23} kg. A rock of mass 200 kg, initially at rest and a long distance from the planet, accelerates towards the planet and hits the surface of the planet.

Calculate the change in potential energy of the rock and its speed when it hits the surface.

Step 1 Write down the quantities given.

$r = 3.4 \times 10^6$ m $\quad M = 4.9 \times 10^{23}$ kg

Step 2 The equation $\phi = -\frac{GM}{r}$ gives the potential at the surface of the planet, that is, the gravitational potential energy per unit mass at that point. So the gravitational potential energy of the rock of mass m at that point is given by:

$$\text{g.p.e.} = -\frac{GMm}{r}$$

The g.p.e. of the rock when it is far away is zero, so the value we calculate using this equation gives the decrease in the rock's g.p.e. during its fall to hit the planet.

$$\begin{aligned} \text{change in g.p.e} &= \frac{6.67\times10^{-11}\times4.9\times10^{23}\times200}{3.4\times10^{6}} \\ &= 1.92\times10^{9}\,\text{J} \\ &\approx 1.9\times10^{9}\,\text{J} \end{aligned}$$

Step 3 In the absence of an atmosphere, all of the g.p.e. becomes kinetic energy of the rock, and so:

$$\begin{aligned} \tfrac{1}{2}mv^2 &= 1.92\times10^{9}\ \text{J} \\ v &= \sqrt{\frac{1.92\times10^{9}\times2}{200}} \\ &= 440\ \text{m s}^{-1} \end{aligned}$$

Note that the rock's final speed when it hits the planet does not depend on the mass of the rock. This is because, if you equate the two equations for k.e. and the change in g.p.e., the mass *m* of the rock cancels.

Questions

You will need the data for the mass and radius of the Earth and the Moon from Table 17.1 to answer this question.

Gravitational constant $G = 6.67 \times 10^{-11}$ N m^2 kg^{-2}.

9 **a** Determine the gravitational potential at the surface of the Earth.

b Determine the gravitational potential at the surface of the Moon.

c Which has the shallower 'potential well', the Earth or the Moon? Draw a diagram similar to Figure 17.8 to compare the 'potential wells' of the Earth and the Moon.

d Use your diagram to explain why a large rocket is needed to lift a spacecraft from the surface of the Earth but a much smaller rocket can be used to launch from the Moon's surface.

Gravitational potential difference

Very often, we consider problems where it is useful to know how much energy is needed to lift a satellite from the surface of a planet or moon to a height where the satellite can be put into orbit. The equation for the change in potential, $\phi = \frac{GM}{r}$, can be used twice, once to find the potential at the surface and once to find the potential at the orbital height. However, it is much easier to combine the two operations and use the equation:

$$\Delta\phi = GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Question

10 During the manned Moon landings in the 1960s, the command module orbited the Moon in an elliptic orbit with a maximum height of 310 km above the surface of the Moon, whilst the lunar module descended and landed on the Moon's surface.

a Explain why the potential energy of the command module varied during its orbit.

b Calculate the maximum gravitational potential difference between the lunar surface and the position of the command module.

Fields: terminology

The words used to describe gravitational (and other) fields can be confusing. Remember:

- **Field strength** tells us about the **force** on unit mass at a point;
- **Potential** tells us about **potential energy** of unit mass at a point.

You will meet the idea of electric field strength in Chapter 21, where it is the force on unit charge. Similarly, when we talk about the potential difference between two points in electricity, we are talking about the difference in electrical potential energy per unit charge. In that chapter, you will meet repulsive fields as well as attractive fields and this should develop your understanding as to why the choice of infinity for the zero of potential is the only sensible choice.

17.5 Orbiting under gravity

For an object orbiting a planet, such as an artificial satellite orbiting the Earth, gravity provides the centripetal force that keeps it in orbit (Figure 17.9). This is a simple situation as there is only one force acting on the satellite–the gravitational attraction of the Earth. The satellite follows a circular path because the gravitational force is at right angles to its velocity.

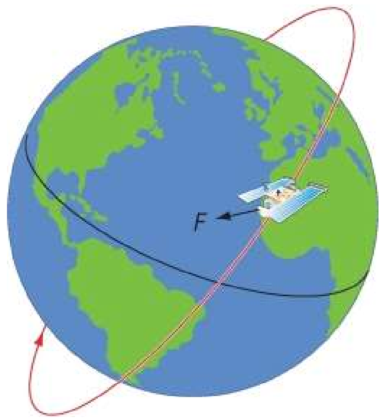

Figure 17.9: The gravitational attraction of the Earth provides the centripetal force on an orbiting satellite.

From Chapter 16, you know that the centripetal force F on a body is given by:

$$F = \frac{mv^2}{r}$$

Consider a satellite of mass m orbiting the Earth at a distance r from the Earth's centre at a constant speed v. Since it is the gravitational force between the Earth and the satellite that provides this centripetal force, we can write:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

where M is the mass of the Earth. (There is no need for a minus sign here as the gravitational force and the centripetal force are both directed towards the centre of the circle.)

Rearranging gives:

$$v^2 = \frac{GM}{r}$$

This equation allows us to calculate, for example, the speed at which a satellite must travel to stay in a circular orbit. Notice that the mass of the satellite m has cancelled out. The implication of this is that all satellites, whatever their masses, will travel at the same speed in a particular orbit. You would find this very reassuring if you were an astronaut on a space walk outside your spacecraft (Figure 17.10). You would travel at the same speed as your craft, despite the fact that your mass is a lot less than its mass. The equation can be applied to the planets of our solar system – M becomes the mass of the Sun.

Figure 17.10: During this space walk, both the astronaut and the spacecraft travel through space at over 8 km s^{-1}.

Now look at Worked example 3.

WORKED EXAMPLE

4 The Moon orbits the Earth at an average distance of 384 000 km from the centre of the Earth. Calculate its orbital speed. (The mass of the Earth is 6.0×10^{24} kg.)

Step 1 Write down the known quantities.

$r = 3.84 \times 10^8$ m $M = 6.0 \times 10^{24}$ kg $v = ?$

Step 2 Use the equation $v^2 = \frac{GM}{r}$ to determine the orbital speed v.

$$
\begin{aligned}
v^2 &= \frac{GM}{r} \\
v^2 &= \frac{6.67\times10^{-11}\times6.0\times10^{24}}{3.84\times10^{8}} \\
v^2 &= 1.04 \times 10^6 \\
v &= \sqrt{1.04 \times 10^6} \\
&= 1020 \text{ m s}^{-1} \\
&\approx 1.0 \times 10^3 \text{m s}^{-1}
\end{aligned}
$$

So, the Moon travels around its orbit at a speed of roughly 1 km s^{-1}.

Question

11 Calculate the orbital speed of an artificial satellite travelling 200 km above the Earth's surface. (The radius of Earth is 6.4×10^6 m and its mass is 6.0×10^{24} kg.)

17.6 The orbital period

It is often more useful to consider the time taken for a complete orbit, the **orbital period** T.

Since the distance around an orbit is equal to the circumference $2\pi r$, it follows that:

$$v = \frac{2\pi r}{T}$$

We can substitute this in the equation for v^2.

This gives:

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

and rearranging this equation gives:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$\text{or } \sqrt{\frac{4\pi r^3}{GM}}$$

This equation shows that the orbital period T is related to the radius r of the orbit. The square of the period is directly proportional to the cube of the radius ($T^2 \propto r^3$). This is an important result. It was first discovered by Johannes Kepler, who analysed the available data for the planets of the Solar System. It was an empirical law (one based solely on experiment) since he had no theory to explain why there should be this relationship between T and r. It was not until Isaac Newton formulated his law of gravitation that it was possible to explain this fact.

17.7 Orbiting the Earth

The Earth has one natural satellite – the Moon – and many thousands of artificial satellites – some spacecraft and a lot of debris. Each of these satellites uses the Earth's gravitational field to provide the centripetal force that keeps it in orbit. In order for a satellite to maintain a particular orbit, it must travel at the correct speed. This is given by the equation in topic 17.5 Orbiting under gravity:

$$V^2 = \frac{GM}{r}$$

It follows from this equation that, the closer the satellite is to the Earth, the faster it must travel. If it travels too slowly, it will fall down towards the Earth's surface. If it travels too quickly, it will move out into a higher orbit.

Question

12 A satellite orbiting a few hundred kilometres above the Earth's surface will experience a slight frictional drag from the Earth's (very thin) atmosphere. Draw a diagram to show how you would expect the satellite's orbit to change as a result. How can this problem be overcome if it is desired to keep a satellite at a particular height above the Earth?

Observing the Earth

Artificial satellites have a variety of uses. Many are used for making observations of the Earth's surface for commercial, environmental, meteorological or military purposes. Others are used for astronomical observations, benefiting greatly from being above the Earth's atmosphere. Still others are used for navigation, telecommunications and broadcasting.

Figure 17.11 shows two typical orbits. A satellite in a circular orbit close to the Earth's surface, and passing over the poles, completes about 16 orbits in 24 hours. As the Earth turns below it, the satellite 'sees' a different strip of the Earth's surface during each orbit. A satellite in an elliptical orbit has a more distant view of the Earth.

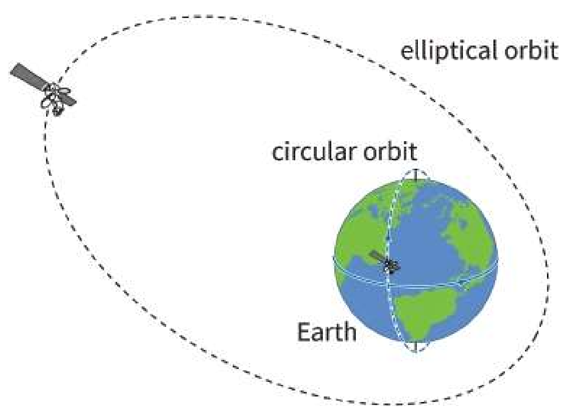

Figure 17.11: Satellites orbiting the Earth.

Geostationary orbits

A special type of orbit is one in which a satellite travels from west to east and is positioned so that, as it orbits, the Earth rotates below it with the same angular speed. The satellite remains above a fixed point on the Earth's equator. This kind of orbit is called a **geostationary orbit**. There are over 300 satellites in such orbits. They are used for telecommunications (transmitting telephone messages around the world) and for satellite television transmissions. A base station on Earth sends the TV signal up to the satellite, where it is amplified and broadcast back to the ground. Satellite receiver dishes are a familiar sight; you will have observed how, in a neighbourhood, they all point towards the same point in the sky. Because the satellite is in a geostationary orbit, the dish can be fixed. Satellites in any other orbits move across the sky so that a tracking system is necessary to communicate with them. Such a system is complex and expensive, and too demanding for the domestic market.

Geostationary satellites have a lifetime of perhaps ten years. They gradually drift out of the correct orbit, so they need a fuel supply for the rocket motors that return them to their geostationary position, and that keep them pointing correctly towards the Earth. Eventually, they run out of fuel and need to be replaced.

We can determine the distance of a satellite in a geostationary orbit using the equation:

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

For a satellite to stay above a fixed point on the equator, it must take exactly 24 hours to complete one orbit (Figure 17.12).

We know:

$$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$$
$$T = 24 \text{ hours} = 86\,400 \text{ s}$$
$$M = 6.0 \times 10^{24}\,\mathrm{kg}$$
$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$\begin{aligned}
86\,400^2 &= \left(\frac{4\pi^2}{6.67\times10^{-11}\times6.0\times10^{24}}\right) r^3 \\
r^3 &= \frac{86\,400^2}{\left(\frac{4\pi^2}{6.67\times10^{-11}\times6.0\times10^{24}}\right)} \\
r^3 &= 7.57 \times 10^{22}\,\mathrm{m^3} \\
r &= \sqrt[3]{7.57 \times 10^{22}} \\
&\approx 4.23 \times 10^{7}\ \mathrm{m}
\end{aligned}$$

So, for a satellite to occupy a geostationary orbit, it must be at a distance of 42 300 km from the centre of the Earth and at a point directly above the equator. Note that the radius of the Earth is 6400 km, so the orbital radius is 6.6 Earth radii from the centre of the Earth (or 5.6 Earth radii from its surface). Figure 17.12 has been drawn to give an impression of the size of the orbit.

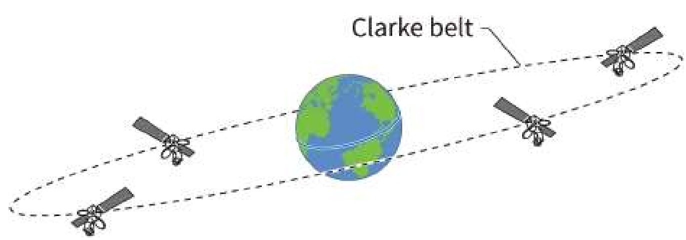

Figure 17.12: Geostationary satellites are parked in the 'Clarke belt', high above the equator. This is a perspective view; the Clarke belt is circular.

Questions

13 For any future mission to Mars, it would be desirable to set up a system of three or four geostationary (or 'martostationary') satellites around Mars to allow communication between the planet and Earth. Calculate the radius of a suitable orbit around Mars.

Mars has mass 6.4×10^{23} kg and a rotational period of 24.6 hours.

14 Although some international telephone signals are sent via satellites in geostationary orbits, most are sent along cables on the Earth's surface. This reduces the time delay between sending and receiving the signal. Estimate this time delay for communication via a satellite, and explain why it is less significant when cables are used.

You will need the following:

- radius of geostationary orbit = 42 300 km
- radius of Earth = 6400 km
- speed of electromagnetic waves in free space $c = 3.0 \times 10^8$ m s^{-1}

REFLECTION

In Question 7, we established that the gravitational force on each kilogram of water on the Earth's surface is greater than that of the Moon; in fact, it is about 175 times greater. However, the Moon affects the tides more than the Sun affects the tides. This seems contradictory.

Find out why and prepare a short paper explaining why.

What were the most interesting discoveries you made while working on this problem?

SUMMARY

The force of gravity is an attractive force between any two objects due to their masses.

The gravitational field strength g at a point is the gravitational force exerted per unit mass on a small object placed at that point:

$$g = \frac{F}{m}$$

The external field of a uniform spherical mass is the same as that of an equal point mass at the centre of the sphere.

Newton's law of gravitation states that:

Any two point masses attract each other with a force that is directly proportional to the product of their masses and inversely proportional to the square of their separation.

The equation for Newton's law of gravitation is:

$$F = \frac{GMm}{r^2}$$

The gravitational field strength at a point is the gravitational force exerted per unit mass on a small object placed at that point:

$$g = \frac{GM}{r^2}$$

On or near the surface of the Earth, the gravitational field is uniform, so the value of g is approximately constant. Its value is equal to the acceleration of free fall.

The gravitational potential at a point is the work done in bringing unit mass from infinity to that point.

The gravitational potential of a point mass is given by:

$$\phi = -\frac{GM}{r}$$

The orbital period of a satellite is the time taken for one orbit.

The orbital period can be found by equating the gravitational force $\frac{GM}{r^2}$ to the centripetal force $\frac{mv^2}{r}$.

The orbital speed of a planet or satellite can be determined using the equation:

$$v^2 = \frac{GM}{r}$$

Geostationary satellites have an orbital period of 24 hours and are used for telecommunications transmissions and for television broadcasting.

EXAM-STYLE QUESTIONS

1 An astronaut is on a planet of mass $0.50M_E$ and radius $0.75r_E$, where M_E is the mass of the Earth and r_E is the radius of the Earth.

What is the gravitational field strength at the surface of the planet? **[1]**

A 6.5 N kg^{-1}

B 8.7 N kg^{-1}

C 11 N kg^{-1}

D 12 N kg^{-1}

2 Consider the dwarf planet Pluto to be an isolated sphere of radius 1.2×10^6 m and mass of 1.27×10^{22} kg.

What is the gravitational potential at the surface of Pluto? **[1]**

A −0.59 J kg^{-1}

B -7.1×10^5 J kg^{-1}

C 0.59 J kg^{-1}

D 7.1×10^5 J kg^{-1}

3 Two small spheres each of mass 20 g hang side by side with their centres 5.00 mm apart. Calculate the gravitational attraction between the two spheres. **[3]**

4 It is suggested that the mass of a mountain could be measured by the deflection from the vertical of a suspended mass. This diagram shows the principle.

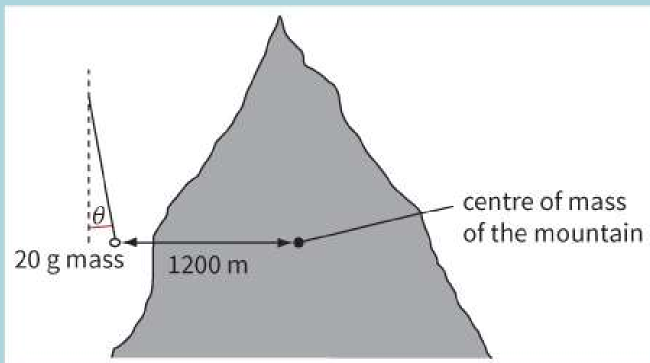

Figure 17.13

a Copy the diagram and draw arrows to represent the forces acting on the mass. Label the arrows. **[3]**

b The whole mass of the mountain, 3.8×10^{12} kg, may be considered to act at its centre of mass. Calculate the horizontal force on the mass due to the mountain. **[2]**

c Compare the force calculated in part **b** with the Earth's gravitational force on the mass. **[2]**

[Total: 7]

5 This diagram shows the Earth's gravitational field.

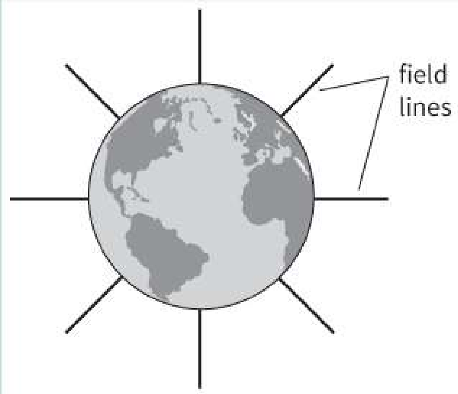

Figure 17.14

a Copy the diagram and add arrows to show the direction of the field. **[1]**

b Explain why the formula for potential energy gained ($mg\Delta h$) can be used to find the increase in potential energy when an aircraft climbs to a height of 10 000 m, but cannot be used to calculate the increase in potential energy when a spacecraft travels from the Earth's surface to a height of 10 000 km. **[2]**

[Total: 3]

6 Mercury, the smallest of the eight recognised planets, has a diameter of 4.88×10^6 m and a mean density of 5.4×10^3 kg m^{-3}.

a Calculate the gravitational field at its surface. **[5]**

b A man has a weight of 900 N on the Earth's surface. What would his weight be on the surface of Mercury? **[2]**

[Total: 7]

7 Calculate the potential energy of a spacecraft of mass 250 kg when it is 20 000 km from the planet Mars. (Mass of Mars = 6.4×10^{23} kg, radius of Mars = 3.4×10^6 m.) **[3]**

8 Ganymede is the largest of Jupiter's moons, with a mass of 1.48×10^{23} kg. It orbits Jupiter with an orbital radius of 1.07×10^6 km and it rotates on its own axis with a period of 7.15 days. It has been suggested that to monitor an unmanned landing craft on the surface of Ganymede a geostationary satellite should be placed in orbit around Ganymede.

a Calculate the orbital radius of the proposed geostationary satellite. **[2]**

b Suggest a difficulty that might be encountered in achieving a geostationary orbit for this moon. **[1]**

[Total: 3]

9 The Earth orbits the Sun with a period of 1 year at an orbital radius of 1.50×10^{11} m. Calculate:

a the orbital speed of the Earth **[3]**

b the centripetal acceleration of the Earth **[2]**

c the Sun's gravitational field strength at the Earth. **[1]**

[Total: 6]

10 The planet Mars has a mass of 6.4×10^{23} kg and a diameter of 6790 km.

a **i** Calculate the acceleration due to gravity at the planet's surface. **[2]**

ii Calculate the gravitational potential at the surface of the planet. **[2]**

b A rocket is to return some samples of Martian material to Earth. Write down how much energy each kilogram of matter must be given to escape completely from Mars' gravitational field. **[1]**

c Use your answer to part **b** to show that the minimum speed that the rocket must reach to escape from the gravitational field is 5000 m s^{-1}. **[2]**

d Suggest why it has been proposed that, for a successful mission to Mars, the craft that takes the astronauts to Mars will be assembled at a space station in Earth orbit and launched from there, rather than from the Earth's surface. [2]

[Total: 9]

11 a Explain what is meant by the **gravitational potential at a point**. **[2]**

b This diagram shows the gravitational potential near a planet of mass *M* and radius *R*.

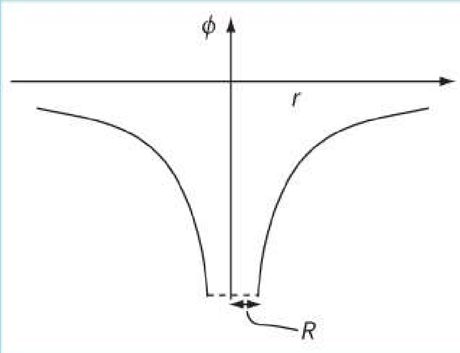

Figure 17.15

On a copy of the diagram, draw similar curves:

i for a planet of the same radius but of mass 2*M*–label this i. **[2]**

ii for a planet of the same mass but of radius 2*R*–label this ii. **[2]**

c Use the graphs to explain from which of these three planets it would require the least energy to escape. **[2]**

d Venus has a diameter of 12 100 km and a mass of 4.87×10^{24} kg.

Calculate the energy needed to lift one kilogram from the surface of Venus to a space station in orbit 900 km from the surface. **[4]**

[Total: 12]

12 a Explain what is meant by the gravitational field strength at a point. **[2]**

This diagram shows the dwarf planet, Pluto, and its moon, Charon. These can be considered to be a double planetary system orbiting each other about their joint centre of mass.

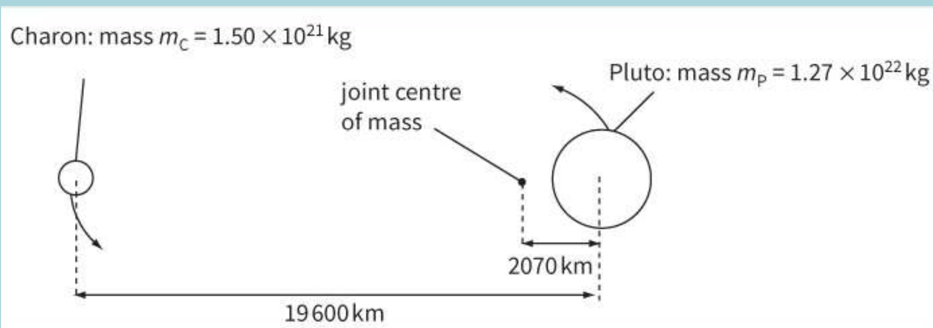

Figure 17.16

b Calculate the gravitational pull on Charon due to Pluto. **[3]**

c Use your result to part **b** to calculate Charon’s orbital period. **[3]**

d Explain why Pluto’s orbital period must be the same as Charon’s. **[1]**

[Total: 9]

13 This diagram shows the variation of the Earth’s gravitational field strength with distance from its centre.

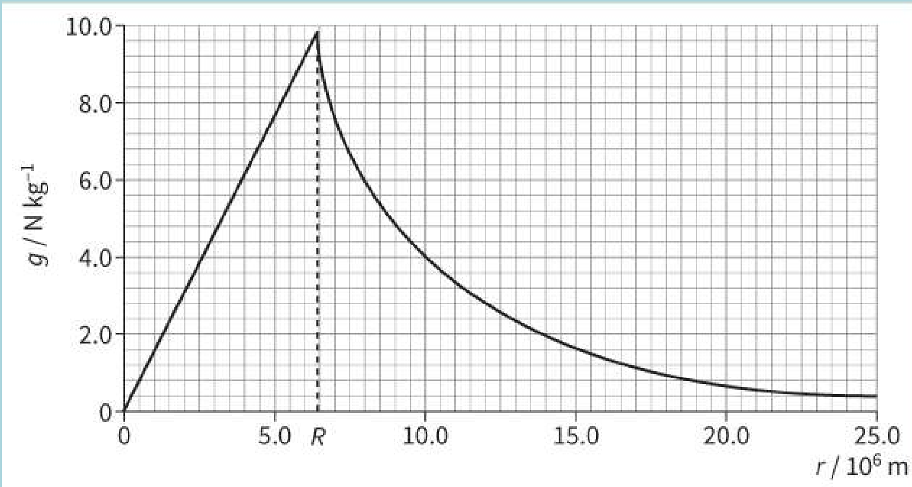

Figure 17.17

a Determine the gravitational field strength at a height equal to 2*R* above the Earth’s surface, where *R* is the radius of the Earth. **[1]**

b A satellite is put into an orbit at this height. State the centripetal acceleration of the satellite. **[1]**

c Calculate the speed at which the satellite must travel to remain in this orbit. **[2]**

d **i** Frictional forces mean that the satellite gradually slows down after it has achieved a circular orbit. Draw a diagram of the initial circular orbital path of the satellite, and show the resulting orbit as frictional forces slow the satellite down. **[1]**

ii Suggest and explain why there is not a continuous bombardment of old satellites colliding with the Earth. **[2]**

[Total: 7]

SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

I can	See topic...	Needs more work	Almost there	Ready to move on
understand the nature of the gravitational field	17.1			
represent and interpret a gravitational field using field lines	17.1			
recall and use Newton's law of gravitation: $F = \frac{Gm_1m_2}{r^2}$	17.1			
understand why g is approximately constant near the Earth's surface	17.2			
derive from Newton's law of gravitation: $g = \frac{GM}{r^2}$	17.2			
recall and use the equation: $g = \frac{GM}{r^2}$	17.2			
understand that the gravitational potential at infinity is zero	17.3			
define gravitational potential at a point, φ, as the work done in bringing unit mass from infinity to that point	17.4			
understand that the gravitational potential decreases, being more negative, as an object moves closer to a second object	17.4			
recall and use the formula that the gravitational potential $\phi = -\frac{GM}{r}$	17.4			
use the formula: $\phi = GM\left(\frac{1}{r^1} - \frac{1}{r^2}\right)$	17.4			
understand that the potential energy of two point masses is equal to: $F = -\frac{Gm_1m_2}{r}$	17.1			
solve problems involving circular orbits of satellites by relating the gravitational force to the centripetal acceleration of the satellite	17.5			
understand that a satellite in a geostationary satellite remains above the same point on the Earth's surface	17.7			
understand that a geostationary satellite has an orbital period of 24 hours and travels from west to east.	17.7			