You can also use the idea of work done to show that the speed of the object moving in a circle remains the same. The work done by a force is equal to the product of the force and the distance moved by the object in the direction of the force. The distance moved by the object in the direction of the centripetal force is zero; hence the work done is zero. If no work is done on the object, its kinetic energy must remain the same and hence its speed is unchanged.

Question

12 An object follows a circular path at a steady speed. Describe how each of the following quantities changes as it follows this path: speed, velocity, kinetic energy, momentum, centripetal force, centripetal acceleration. (Refer to both magnitude and direction, as appropriate.)

Understanding circular motion

Isaac Newton devised an ingenious thought experiment that allows us to think about how an object can remain in a circular orbit around the Earth. Consider a large cannon on some high point on the Earth's surface, capable of firing objects horizontally. Figure 16.10 shows what will happen if we fire them at different speeds.

If the object is fired too slowly, gravity will pull it down towards the ground and it will land at some distance from the cannon. A faster initial speed results in the object landing further from the cannon.

Now, if we try a bit faster than this, the object will travel all the way round the Earth. We have to get just the right speed to do this. As the object is pulled down towards the Earth, the curved surface of the Earth falls away beneath it. The object follows a circular path, constantly falling under gravity but never getting any closer to the surface.

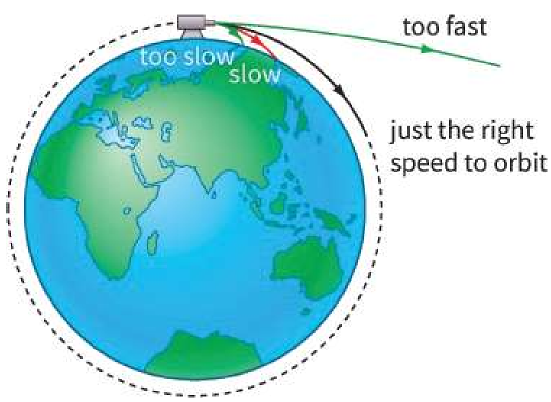

Figure 16.10: Newton's 'thought experiment'.

If the object is fired too fast, it travels off into space, and fails to get into a circular orbit. So we can see that there is just one correct speed to achieve a circular orbit under gravity. (Note that we have ignored the effects of air resistance in this discussion.)

16.6 Calculating acceleration and force

If we spin a bung around in a circle (Figure 16.7), we get a feeling for the factors that determine the centripetal force F required to keep it in its circular orbit. The greater the mass m of the bung and the greater its speed v, the greater is the force F that is required. However, if the radius r of the circle is increased, F is smaller.

Now we will deduce an expression for the **centripetal acceleration** of an object moving around a circle with a constant speed.

Figure 16.11 shows a particle moving round a circle. In time Δt it moves through an angle $\Delta\theta$ from A to B. Its speed remains constant but its velocity changes by Δv, as shown in the vector diagram. Since the narrow angle in this triangle is also $\Delta\theta$, we can say that:

$$\Delta\theta = \frac{\Delta v}{v}$$

Dividing both sides of this equation by Δt and rearranging gives:

$$\frac{\Delta v}{\Delta t} = \frac{v\Delta\theta}{\Delta t}$$

The quantity on the left is $\frac{\Delta v}{\Delta t} = a$, the particle's acceleration.

The quantity on the right is $\frac{\Delta\theta}{\Delta t} = \omega$, the angular velocity.

Substituting for these gives:

$$a = v\omega$$

Using $v = \omega r$, we can eliminate ω from this equation:

$$a = \frac{v^2}{r}$$

where a is the centripetal acceleration, v is the speed and r is the radius of the circle.

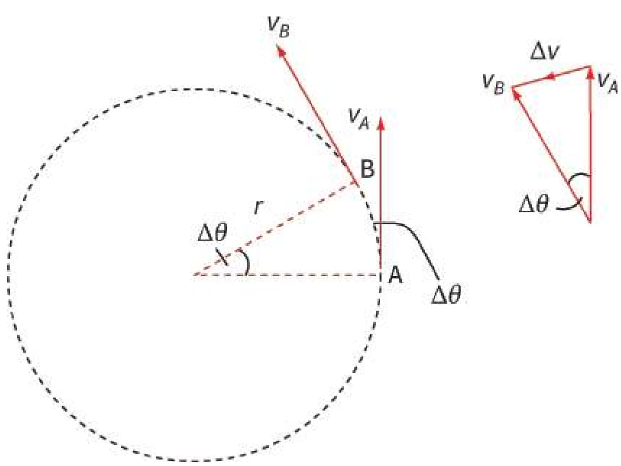

Figure 16.11: Deducing an expression for centripetal acceleration.

Question

13 Show that an alternative equation for the centripetal acceleration is $a = \omega^2 r$.

KEY EQUATION

$$a = \frac{v^2}{r}$$
$$a = \mathrm{r}\omega^2$$

Newton's second law of motion

Now that we have an equation for centripetal acceleration, we can use **Newton's second law** of motion to deduce an equation for centripetal force. If we write this law as $F = ma$, we find:

$$\text{centripetal force } F = \frac{mv^2}{r} = mr\omega^2$$

Remembering that an object accelerates in the direction of the resultant force on it, it follows that both F and a are in the same direction, towards the centre of the circle.

Questions

14 Calculate how long it would take a ball to orbit the Earth once, just above the surface, at a speed of 7920 m s^{-1}. (The radius of the Earth is 6400 km.)

15 A stone of mass 0.20 kg is whirled round on the end of a string in a vertical circle of radius 30 cm. The string will break when the tension in it exceeds 8.0 N. Calculate the maximum speed at which the stone can be whirled without the string breaking.

Figure 16.12: The view from the International Space Station, orbiting the Earth over Australia.

16 The International Space Station (Figure 16.12) has a mass of 350 tonnes, and orbits the Earth at an average height of 340 km where the gravitational acceleration is 8.8 m s^{-2}. The radius of the Earth is 6400 km. Calculate:

a the centripetal force on the space station

b the speed at which it orbits

c the time taken for each orbit

d the number of times it orbits the Earth each day.

17 An toy truck of mass 0.40 kg travels round a horizontal circular track of radius 0.50 m. It makes three complete revolutions every 10 seconds.

Calculate:

a its speed

b its centripetal acceleration

c the centripetal force.

18 Mars orbits the Sun once every 687 days at a distance of 2.3×10^{11} m. The mass of Mars is 6.4×10^{23} kg. Calculate:

a the average speed in metres per second

b its centripetal acceleration

c the gravitational force exerted on Mars by the Sun.

Calculating orbital speed

We can use the force equation to calculate the speed that an object must have to orbit the Earth under gravity, as in Newton's thought experiment. The necessary centripetal force $\frac{mv^2}{r}$ is provided by the Earth's gravitational pull mg.

Hence:

$$\begin{aligned} mg &= \tfrac{mv^2}{r} \\ g &= \tfrac{v^2}{r} \end{aligned}$$

where g = 9.81 m s^{-2} is the acceleration of free fall close to the Earth's surface. The radius of its orbit is equal to the Earth's radius, approximately 6400 km. Hence, we have:

$$\begin{aligned} 9.81 &= \tfrac{v^2}{\left(6.4\times10^6\right)} \\ v^2 &= 9.81 \times \left(6.4 \times 10^6\right) \\ v &= \sqrt{9.81\left(6.4 \times 10^6\right)} \\ v &\approx 7.92 \times 10^3 \mathrm{ms}^{-1} \end{aligned}$$

Thus, if you were to throw or hit a ball horizontally at almost 8 km s^{-1}, it would go into orbit around the Earth.

16.7 The origins of centripetal forces

It is useful to look at one or two situations where the physical origin of the centripetal force may not be immediately obvious. In each case, you will notice that the forces acting on the moving object are not balanced – there is a resultant force. An object moving along a circular path is not in equilibrium and the resultant force acting on it is the centripetal force.

1 A car cornering on a level road (Figure 16.13). Here, the road provides two forces. The force N is the normal contact force that balances the weight mg of the car–the car has no acceleration in the vertical direction.

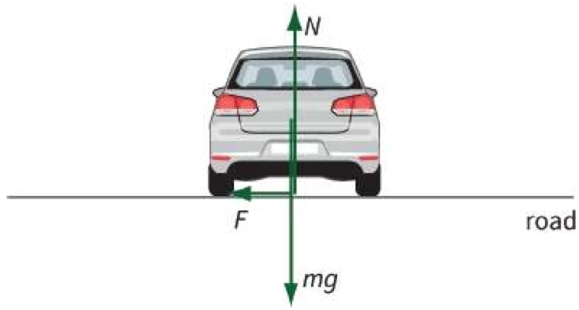

Figure 16.13: This car is moving away from us and turning to the left. Friction provides the centripetal force. N and F are the total normal contact and friction forces (respectively) provided by the contact of all four tyres with the road.

The second force is the force of friction F between the tyres and the road surface. This is the unbalanced, centripetal force. If the road or tyres do not provide enough friction, the car will not go round the bend along the desired path. The friction between the tyres and the road provides the centripetal force necessary for the car's circular motion.

2 A car cornering on a banked road (Figure 16.14a). Here, the normal contact force N has a horizontal component that can provide the centripetal force. The vertical component of N balances the car's weight. Therefore:

$$\text{vertically} \qquad N\cos\theta = mg$$

$$\text{horizontally} \qquad N\sin\theta = \frac{mv^2}{r}$$

where r is the radius of the circular corner and v is the car's speed.

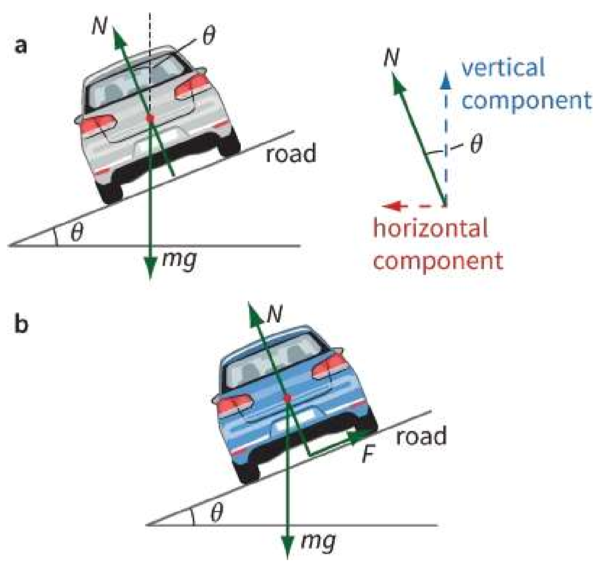

Figure 16.14: a On a banked road, the horizontal component of the normal contact force from the road can provide the centripetal force needed for cornering. **b** For a slow car, friction acts up the slope to stop it from sliding down.

If a car travels around the bend too slowly, it will tend to slide down the slope and friction will act up the slope to keep it on course (Figure 16.14b). If it travels too fast, it will tend to slide up the slope. If friction is insufficient, it will move up the slope and come off the road.

3 An aircraft banking (Figure 16.15a). To change direction, the pilot tips the aircraft's wings. The vertical component of the lift force L on the wings balances the weight. The horizontal component of L provides the centripetal force.

4 A stone being whirled in a horizontal circle on the end of a string – this arrangement is known as a conical pendulum (Figure 16.15b). The vertical component of the tension T is equal to the weight of the stone. The horizontal component of the tension provides the centripetal force for the circular motion.

5 At the fairground (Figure 16.15c). As the cylinder spins, the floor drops away. Friction balances your weight. The normal contact force of the wall provides the centripetal force. You feel as though you are being pushed back against the wall; what you are feeling is the push of the wall on your back.

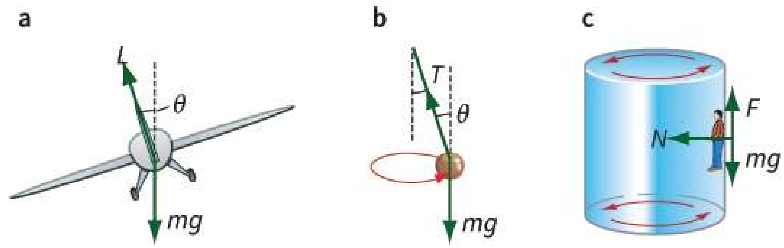

Figure 16.15: Three more ways of providing a centripetal force.

Note that the three situations shown in Figures 16.14a, 16.15a and 16.15b are equivalent. The moving object's weight acts downwards. The second force has a vertical component, which balances the weight, and a horizontal component, which provides the centripetal force.

Questions

19 Explain why it is impossible to whirl a bung around on the end of a string in such a way that the string remains perfectly horizontal.

20 Explain why an aircraft will tend to lose height when banking, unless the pilot increases its speed to provide more lift.

21 If you have ever been down a water-slide (a flume) (Figure 16.16) you will know that you tend to slide up the side as you go around a bend. Explain how this provides the centripetal force needed to push you around the bend. Explain why you slide higher if you are going faster.

Figure 16.16: A water-slide is a good place to experience centripetal forces.

REFLECTION

In order to increase the proportion of heavy water (deuterium oxide) in a sample of water, the scientists in the ‘Manhattan Project’ used a centrifuge. Prepare a short talk on the aim of the Manhattan Project and to explain how a centrifuge works.

Think about an astronaut far away from any planets. Discuss with a partner whether the string on a conical pendulum could rotate in a horizontal plane, and what speed it would start to rotate in this manner. What conclusions did you each come to? Did discussing this question with a partner help you to understand the concepts?

SUMMARY

Angles can be measured in radians. An angle of 2π rad is equal to 360°.
An object moving at a steady speed along a circular path has uniform circular motion.
The angular displacement θ is a measure of the angle through which an object moves in a circle.
The angular velocity ω is the rate at which the angular displacement changes: $$\omega = \frac{\Delta\theta}{\Delta t}$$
For an object moving with uniform circular motion, speed and angular velocity are related by $v = \omega r$.
An object moving in a circle is not in equilibrium; it has a resultant force acting on it.
The resultant force acting on an object moving in a circle is directed towards the centre of the circle and is at right angles to the velocity of the object.
An object moving in a circle has a centripetal acceleration a given by: $$a = \frac{v^2}{r} = r\omega^2$$
The magnitude of the force F acting on an object of mass m moving at a speed v in a circle of radius r is given by: $$F = \frac{mv^2}{r} = mr\omega^2$$

EXAM-STYLE QUESTIONS

1 Which statement is correct? **[1]**

A There is a resultant force on an object moving along a circular path at constant speed away from the centre of the circle causing it to be thrown outwards.

B There is a resultant force on an object moving along a circular path at constant speed towards the centre of the circle causing it to be thrown outwards.

C There is a resultant force on an object moving along a circular path at constant speed towards the centre of the circle causing it to move in the circle.

D There is zero resultant force on an object moving along a circular path at constant speed because it is in equilibrium.

2 When ice-dancers spin, as shown in the diagram, the first dancer's hand applies a centripetal force to the second dancer's hand.

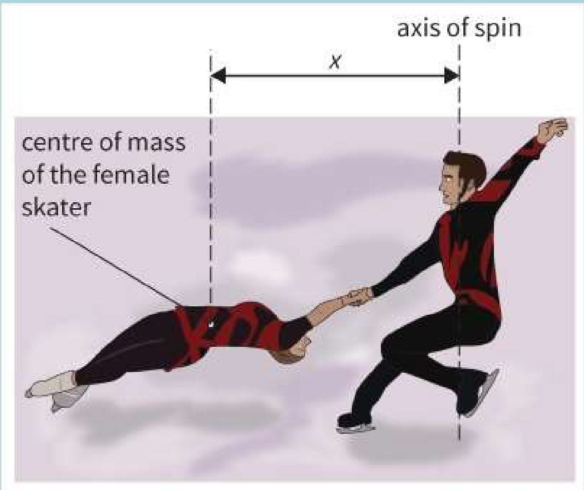

Figure 16.17

In which case is the centripetal force the greatest? **[1]**

	x / m	**Speed of the female skater's centre of mass / m s^{-1}**
A	0.45	9.0
B	0.45	10.0
C	0.50	9.0
D	0.50	10.0

Table 16.1

3 **a** Explain what is meant by a **radian**. **[1]**

b A body moves round a circle at a constant speed and completes one revolution in 15 s. Calculate the angular speed of the body. **[2]**

[Total: 3]

4 This diagram shows part of the track of a roller-coaster ride in which a truck loops the loop. When the truck is at the position shown, there is no reaction force between the wheels of the truck and the track. The diameter of the loop in the track is 8.0 m.

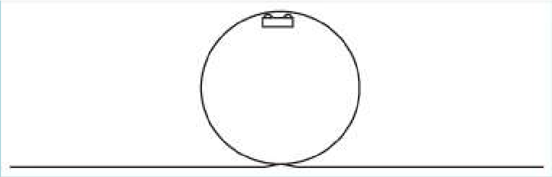

Figure 16.18

a Explain what provides the centripetal force to keep the truck moving in a circle. **[1]**

b Given that the acceleration due to gravity g is 9.8 m s^{-2}, calculate the speed of the truck. **[3]**

[Total: 4]

5 This diagram shows a toy of mass 60 g placed on the edge of a rotating turntable. **[1]**

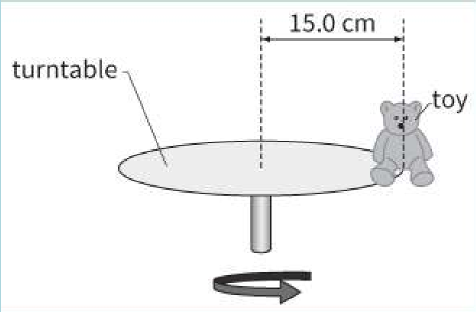

Figure 16.19

a The radius of the turntable is 15.0 cm. The turntable rotates, making 20 revolutions every minute. Calculate the resultant force acting on the toy. **[3]**

b Explain why the toy falls off when the speed of the turntable is increased. **[2]**

[Total: 6]

6 One end of a string is secured to the ceiling and a metal ball of mass 50 g is tied to its other end. The ball is initially at rest in the vertical position. The ball is raised through a vertical height of 70 cm, as shown. The ball is then released. It describes a circular arc as it passes through the vertical position.

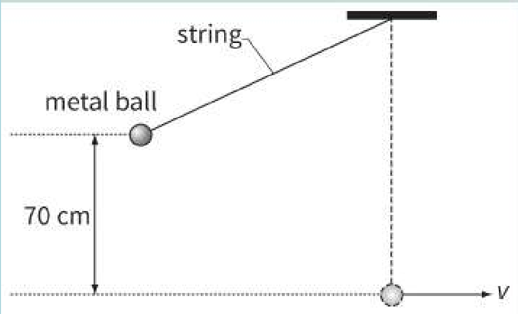

Figure 16.20

The length of the string is 1.50 m.

a Ignoring the effects of air resistance, determine the speed v of the ball as it passes through the vertical position. **[2]**

b Calculate the tension T in the string when the string is vertical. **[3]**

c Explain why your answer to part b is not equal to the weight of the ball. **[2]**

[Total: 7]

7 A car is travelling round a bend when it hits a patch of oil. The car slides off the road onto the grass verge. Explain, using your understanding of circular motion, why the car came off the road. **[2]**

8 This diagram shows an aeroplane banking to make a horizontal turn. The aeroplane is travelling at a speed of 75 m s^{-1} and the radius of the turning circle is 800 m.

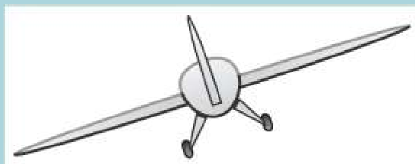

Figure 16.21

a Copy the diagram. On your copy, draw and label the forces acting on the aeroplane. **[2]**

b Calculate the angle that the aeroplane makes with the horizontal. **[4]**

[Total: 6]

9 **a** Explain what is meant by the term **angular speed**. **[2]**

b This diagram shows a rubber bung, of mass 200 g, on the end of a length of string being swung in a horizontal circle of radius 40 cm. The string makes an angle of 56° with the vertical.

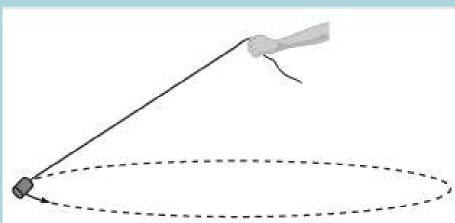

Figure 16.22

Calculate:

i the tension in the string **[2]**

ii the angular speed of the bung **[3]**

iii the time it takes to make one complete revolution. **[1]**

[Total: 8]

10 **a** Explain what is meant by a **centripetal acceleration**. **[2]**

b A teacher swings a bucket of water, of total mass 5.4 kg, round in a vertical circle of diameter 1.8 m.

i Calculate the minimum speed that the bucket must be swung at so that the water remains in the bucket at the top of the circle. **[3]**

ii Assuming that the speed remains constant, what will be the force on the teacher's hand when the bucket is at the bottom of the circle? **[2]**

[Total: 7]

11 In training, military pilots are given various tests. One test puts them in a seat on the end of a large arm that is then spun round at a high speed, as shown.