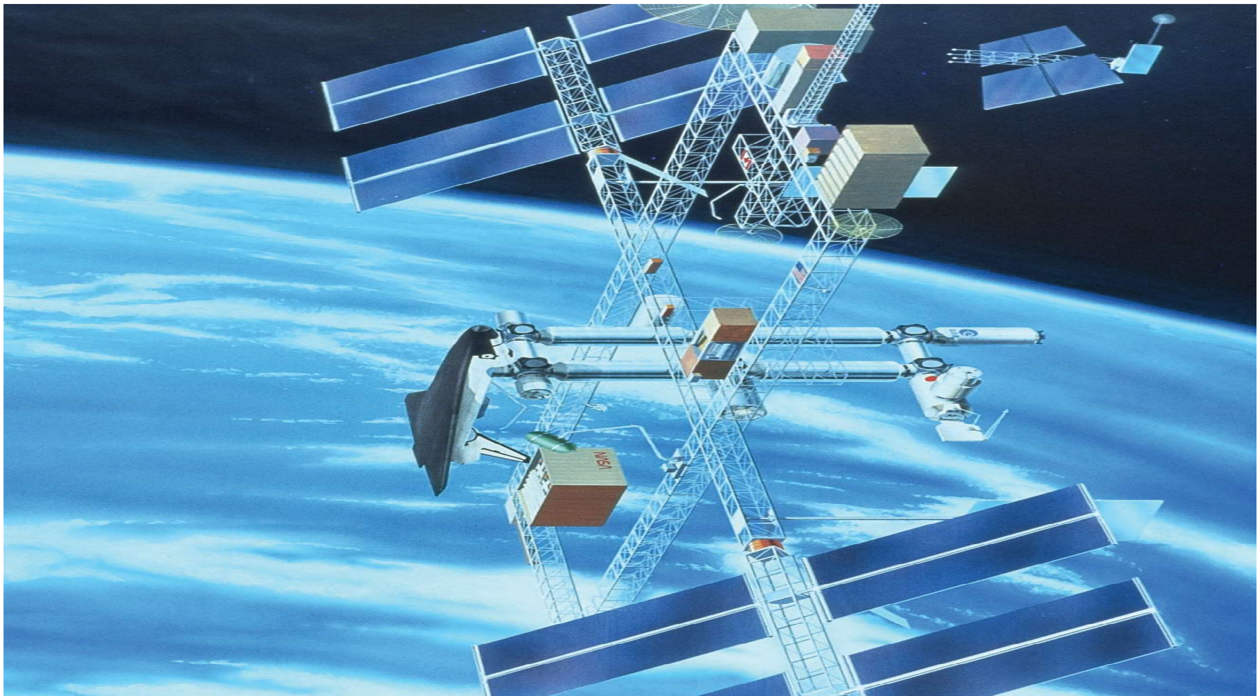

Chapter 14

Stationary waves

LEARNING INTENTIONS

In this chapter you will learn how to:

- explain the formation of stationary waves using graphical methods
- understand experiments to demonstrate stationary waves using microwaves, stretched strings and air columns
- identify nodes and antinodes
- determine the wavelength of sound using stationary waves.

BEFORE YOU START

- Write down the wave equation and use it to estimate the wavelength of ripples on the surface of a pond.
- Write down a few notes about the principle of superposition of waves. This will help you to understand how stationary (standing) waves are formed.

THE BRIDGE THAT BROKE

Figure 14.1a shows the Normandy Bridge under construction in France. When designing bridges, engineers must take into account the possibility of the wind causing a build-up of stationary waves, which may lead the bridge to oscillate violently. Famously, this happened in October 1940 to the Tacoma Narrows Bridge in Washington State, USA. High winds caused the bridge to vibrate with increasing amplitude until it fell apart (Figure 14.1b).

Did you know that the Tacoma Narrows Bridge fell apart because its natural frequency of oscillation matched the thumping frequency of the swirling wind? Do a web search for a videoclip of this momentous event.

Figure 14.1: **a** A suspension bridge under construction. **b** One that failed – the Tacoma Narrows Bridge.

14.1 From moving to stationary

The waves we have considered so far in Chapters 12 and 13 have been **progressive waves**; they start from a source and travel outwards, transferring energy from one place to another. A second important class of waves is **stationary waves** (**standing waves**). These can be observed as follows. Use a long spring or a plastic toy spring. A long rope or piece of rubber tubing will also do. Lay it on the floor and fix one end firmly. Move the other end from side to side so that transverse waves travel along the length of the spring and reflect off the fixed end (Figure 14.2). If you adjust the frequency of the shaking, you should be able to achieve a stable pattern like one of those shown in Figure 14.3. Alter the frequency in order to achieve one of the other patterns.

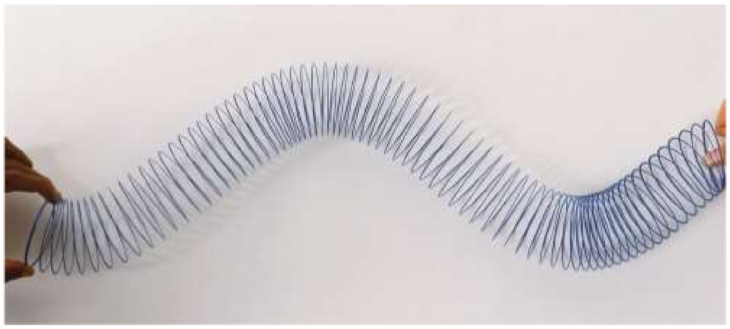

Figure 14.2: A toy spring is used to generate a stationary wave pattern.

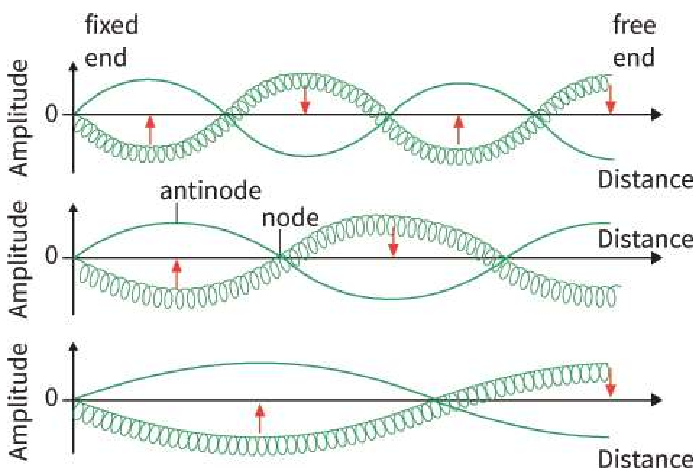

Figure 14.3: Different stationary wave patterns are possible, depending on the frequency of vibration.

You should notice that you have to move the end of the spring with just the right frequency to get one of these interesting patterns. The pattern disappears when the frequency of the shaking of the free end of the spring is slightly increased or decreased.

14.2 Nodes and antinodes

What you have observed is a stationary wave on the long spring. There are points along the spring that remain (almost) motionless while points on either side are oscillating with the greatest amplitude. The points that do not move are called the **nodes** and the points where the spring oscillates with maximum amplitude are called the **antinodes**. At the same time, it is clear that the wave profile is not travelling along the length of the spring. Hence, we call it a stationary wave or a standing wave.

We normally represent a stationary wave by drawing the shape of the spring in its two extreme positions (Figure 14.4). The spring appears as a series of loops, separated by nodes. In this diagram, point A is moving downwards. At the same time, point B in the next loop is moving upwards. The phase difference between points A and B is 180°. Hence, the sections of spring in adjacent loops are always moving in antiphase; they are half a cycle out of phase with one another.

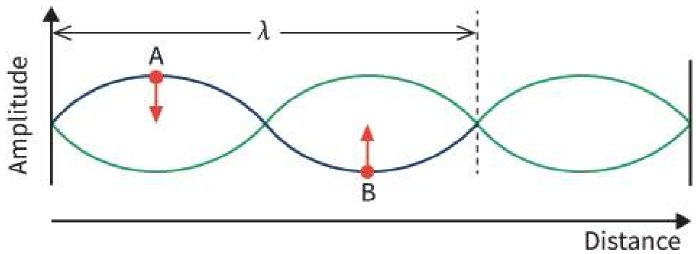

Figure 14.4: The fixed ends of a long spring must be nodes in the stationary wave pattern.

14.3 Formation of stationary waves

Imagine a string stretched between two fixed points, for example, a guitar string. Pulling the middle of the string and then releasing it produces a stationary wave. There is a node at each of the fixed ends and an antinode in the middle. Releasing the string produces two progressive waves travelling in opposite directions. These are reflected at the fixed ends. The reflected waves combine to produce the stationary wave.

Figure 14.3 shows how a stationary wave can be set up using a long spring. A stationary wave is formed whenever two progressive waves of the same amplitude and wavelength, travelling in **opposite** directions, superpose. Figure 14.5 uses a displacement–distance graph (s–x) to illustrate the formation of a stationary wave along a long spring (or a stretched length of string):

- At time $t = 0$, the progressive waves travelling to the left and right are in phase. The waves combine **constructively**, giving an amplitude twice that of each wave.
- After a time equal to one-quarter of a period $\left(t = \frac{T}{4}\right)$, each wave has travelled a distance of one quarter of a wavelength to the left or right. Consequently, the two waves are in antiphase (phase difference = 180°). The waves combine **destructively**, giving zero displacement.
- After a time equal to one-half of a period $\left(t = \frac{T}{2}\right)$, the two waves are back in phase again. They once again combine **constructively**.
- After a time equal to three-quarters of a period $\left(t = \frac{3T}{4}\right)$, the waves are in antiphase again. They combine **destructively**, with the resultant wave showing zero displacement.
- After a time equal to one whole period ($t = T$), the waves combine **constructively**. The profile of the spring is as it was at $t = 0$.

This cycle repeats itself, with the long spring showing nodes and antinodes along its length. The separation between adjacent nodes or antinodes tells us about the progressive waves that produce the stationary wave.

A closer inspection of the graphs in Figure 14.5 shows that the separation between adjacent nodes or antinodes is related to the wavelength λ of the progressive wave. The important conclusions are:

- separation between two adjacent nodes (or between two adjacent antinodes) = $\frac{\lambda}{2}$
- separation between adjacent node and antinode = $\frac{\lambda}{4}$

The wavelength λ of any progressive wave can be determined from the separation between neighbouring nodes or antinodes of the resulting stationary wave pattern.

(This separation is $\frac{\lambda}{2}$.) This can then be used to determine either the speed v of the progressive wave or its frequency f by using the wave equation:

$$v = f\lambda$$

KEY EQUATION

$$v = f\lambda$$

The wave equation, where v is the speed of the wave, f is the frequency and λ is the wavelength.

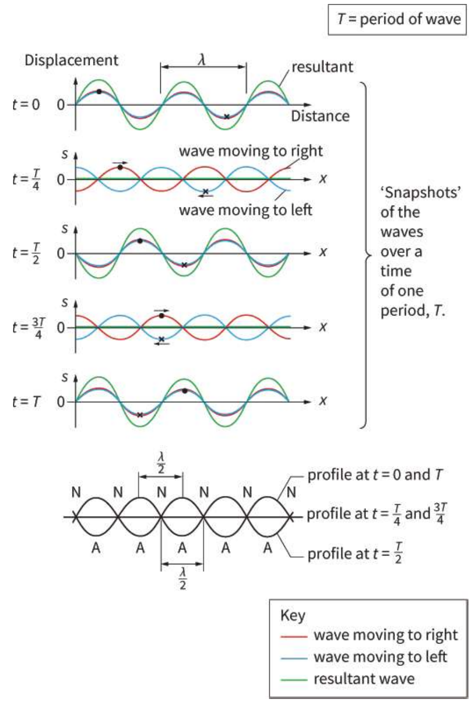

Figure 14.5: The blue-coloured wave is moving to the left and the red-coloured wave to the right. The *principle of superposition* of waves is used to determine the resultant displacement. The profile of the long spring is shown in green.

It is worth noting that a stationary wave does not travel and therefore has no speed. It does not transfer energy between two points like a progressive wave. Table 14.1 shows some of the key features of a progressive wave and its stationary wave.

	Progressive wave	**Stationary wave**
wavelength	λ	λ
frequency	f	f
speed	v	zero

Table 14.1: A summary of progressive and stationary waves.

Question

1 A stationary (standing) wave is set up on a vibrating spring. Adjacent nodes are separated by 25 cm. Calculate:

a the wavelength of the progressive wave

b the distance from a node to an adjacent antinode.

PRACTICAL ACTIVITY 14.1

Observing stationary waves

Here we look at experimental arrangements for observing stationary waves, for mechanical waves on strings, microwaves and sound waves in air columns.

Stretched strings: Melde's experiment

A string is attached at one end to a vibration generator, driven by a signal generator (Figure 14.6). The other end hangs over a pulley and weights maintain the tension in the string. When the signal generator is switched on, the string vibrates with small amplitude. Larger amplitude stationary waves can be produced by adjusting the frequency.

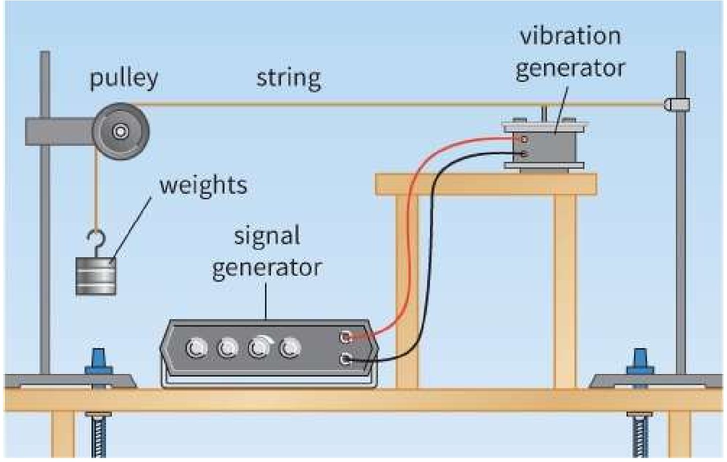

Figure 14.6: Melde's experiment for investigating stationary waves on a string.

The pulley end of the string cannot vibrate; this is a node. Similarly, the end attached to the vibrator can only move a small amount, and this is also a node. As the frequency is increased, it is possible to observe one loop (one antinode), two loops, three loops and more. Figure 14.7 shows a vibrating string where the frequency of the vibrator has been set to produce two loops.

A flashing stroboscope is useful to reveal the motion of the string at these frequencies, which look blurred to the eye. The frequency of vibration is set so that there are two loops along the string; the frequency of the stroboscope is set so that it almost matches that of the vibrations. Now we can see the string moving 'in slow motion', and it is easy to see the opposite movements of the two adjacent loops.

Figure 14.7: When a stationary wave is established, one half of the string moves upwards as the other half moves downwards. In this photograph, the string is moving too fast to observe the effect.

This experiment is known as **Melde's experiment**, and it can be extended to investigate the effect of changing the length of the string, the tension in the string and the thickness of the string.

Microwaves

Start by directing the microwave transmitter at a metal plate, which reflects the microwaves back towards the source (Figure 14.8). Move the probe receiver around in the space between the transmitter and the reflector and you will observe positions of high and low intensity. This is because a stationary wave is set up between the transmitter and the sheet; the positions of high and low intensity are the antinodes and nodes, respectively.

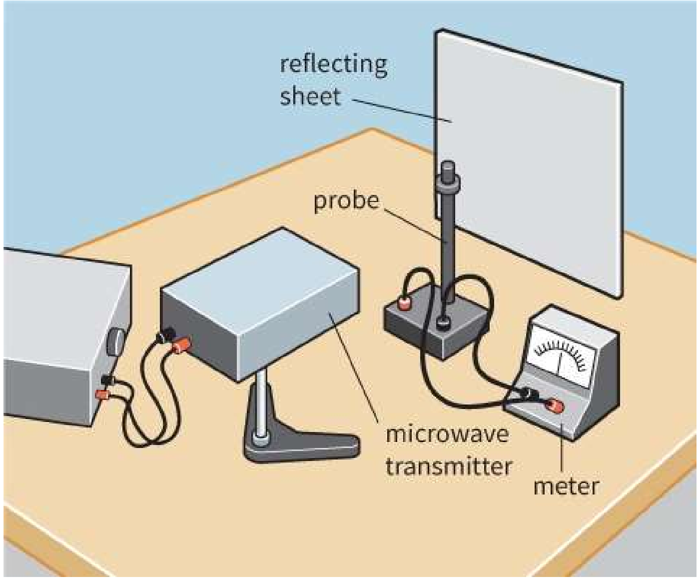

Figure 14.8: A stationary wave is created when microwaves are reflected from the metal sheet.

If the probe is moved along the direct line from the transmitter to the plate, the wavelength of the microwaves can be determined from the distance between the nodes. Knowing that microwaves travel at the speed of light c (3.0×10^8 m s^{-1}), we can then determine their frequency f using the wave equation:

$$c = f\lambda$$

An air column closed at one end

A glass tube (open at both ends) is clamped so that one end dips into a cylinder of water. By adjusting its height in the clamp, you can change the length of the column of air in the tube (Figure 14.9). When you hold a vibrating tuning fork above the open end, the air column may be forced to vibrate and the note of the tuning fork sounds much louder. This is an example of a phenomenon called **resonance**. The experiment described here is known as the resonance tube.

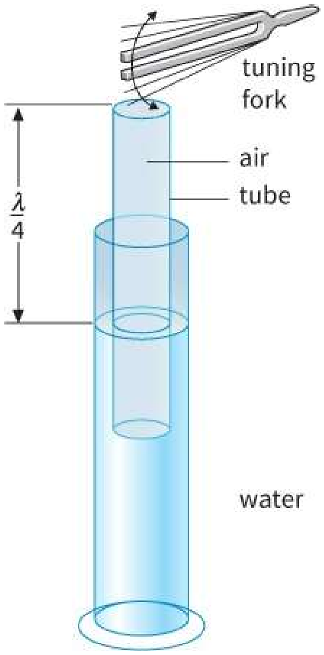

Figure 14.9: A stationary wave is created in the air in the tube when the length of the air column is adjusted to the correct length.

For resonance to occur, the length of the air column must be just right. The air at the bottom of the tube is unable to vibrate, so this point must be a node. The air at the open end of the tube can vibrate most freely, so this is an antinode. Hence, the length of the air column must be one-quarter of a wavelength (Figure 14.10a). (Alternatively, the length of the air column could be set to equal three-quarters of a wavelength – see Figure 14.10b.)

Take care! The representation of stationary sound waves can be misleading. Remember that a sound wave is a longitudinal wave, but the diagram we draw is more like a transverse wave. Figure 14.11a shows how we normally represent a stationary sound wave, while Figure 14.11b shows the direction of vibration of the particles along the wave.

Open-ended air columns

The air in a tube that is open at both ends will vibrate in a similar way to that in a closed column. Take an open-ended tube and blow gently across the top. You should hear a note whose pitch depends on the length of the tube. Now cover the bottom of the tube with the palm of your hand and repeat the process. The pitch of the note now produced will be about an octave lower than the previous note, which means that the frequency is approximately half of the original frequency.

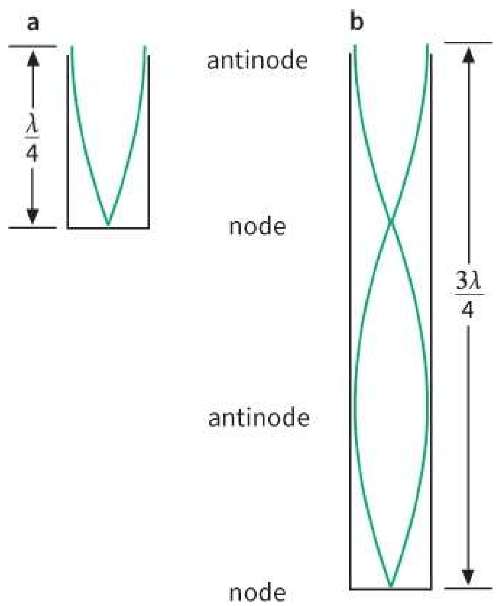

Figure 14.10: Stationary wave patterns for air in a tube with one end closed.

It is rather surprising that a stationary wave can be set up in an open column of air in this way. What is going on? Figure 14.12 compares the situation for open and closed tubes. An open-ended tube has two open ends, so there must be an antinode at each end. There is a node at the midpoint.

For a tube of length l you can see that in the closed tube the stationary wave formed is one-quarter of a wavelength, so the wavelength is $4l$, whereas in the open tube it is half a wavelength, giving a wavelength of $2l$. Closing one end of the tube thus doubles the wavelength of the note and so the frequency halves.

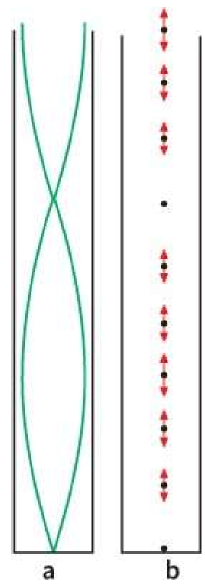

Figure 14.11: **a** The standard representation of a stationary sound wave may suggest that it is a transverse wave. **b** A sound wave is really a longitudinal wave, so that the particles vibrate as shown.

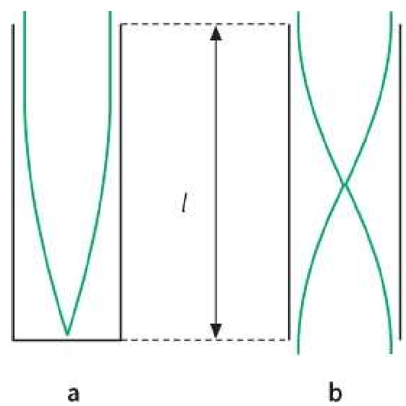

Figure 14.12: Stationary wave patterns for sound waves in **a** a closed tube, and **b** an open tube.

Questions

2 Look at the stationary (standing) wave on the string in Figure 14.7. The length of the vibrating section of the string is 60 cm.

a Determine the wavelength of the progressive wave and the separation of the two neighbouring antinodes.

The frequency of vibration is increased until a stationary wave with three antinodes appears on the string.

b Sketch a stationary wave pattern to illustrate the appearance of the string.

c Calculate the wavelength of the progressive wave on this string.

3 **a** Sketch a stationary wave pattern for the microwave experiment in Practical Activity 14.1. Clearly show whether there is a node or an antinode at the reflecting sheet.

b The separation of two adjacent points of high intensity is found to be 14 mm. Calculate the wavelength and frequency of the microwaves.

4 Explain how two sets of identical but oppositely travelling waves are established in the microwave and air column experiments described in Practical Activity 14.1.

Stationary waves and musical instruments (extension)

The production of different notes by musical instruments often depends on the creation of stationary waves (Figure 14.13). For a stringed instrument, such as a guitar, the two ends of a string are fixed, so nodes must be established at these points. When the string is plucked half-way along its length, it vibrates with an antinode at its midpoint. This is known as the **fundamental mode of vibration** of the string. The fundamental frequency is the **minimum frequency** of a stationary wave for a given system or arrangement.

Figure 14.13: When a guitar string is plucked, the vibrations of the strings continue for some time afterwards. Here, you can clearly see a node close to the end of each string.

Similarly, the air column inside a wind instrument is caused to vibrate by blowing, and the note that is heard depends on a stationary wave being established. By changing the length of the air column, as in a trombone, the note can be changed. Alternatively, holes can be uncovered so that the air can vibrate more freely, giving a different pattern of nodes and antinodes.

In practice, the sounds that are produced are made up of several different stationary waves having different patterns of nodes and antinodes. For example, a guitar string may vibrate with two antinodes along its length. This gives a note having twice the frequency of the fundamental, and is described as a harmonic of the fundamental. The musician's skill is in stimulating the string or air column to produce a desired mixture of frequencies.

The frequency of a harmonic is always a multiple of the fundamental frequency. The diagrams show some of the modes of vibration of a fixed length of string (Figure 14.14) and an air column in a tube of a given length that is closed at one end (Figure 14.15).

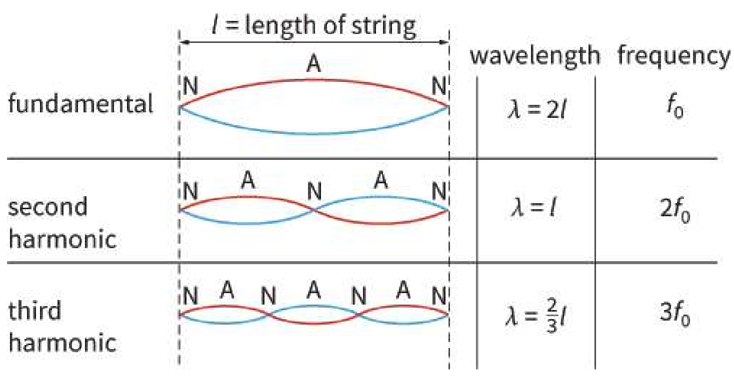

Figure 14.14: Some of the possible stationary waves for a fixed string of length l. The frequency of the harmonics is a multiple of the fundamental frequency f_0.

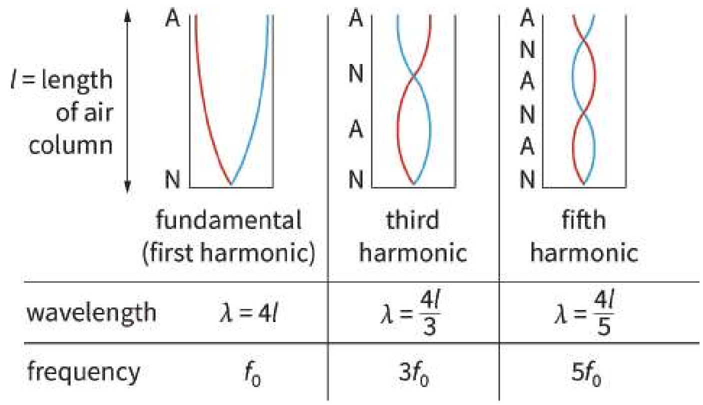

Figure 14.15: Some of the possible stationary waves for an air column, closed at one end. The frequency of each harmonic is an odd multiple of the fundamental frequency f_0.

14.4 Determining the wavelength and speed of sound

Since we know that adjacent nodes (or antinodes) of a stationary wave are separated by half a wavelength, we can use this fact to determine the wavelength λ of a progressive wave. If we also know the frequency f of the waves, we can find their speed v using the wave equation $v = f\lambda$.

One approach uses Kundt's dust tube (Figure 14.16). A loudspeaker sends sound waves along the inside of a tube. The sound is reflected at the closed end. When a stationary wave is established, the dust (fine powder) at the antinodes vibrates violently. It tends to accumulate at the nodes, where the movement of the air is zero. Hence, the positions of the nodes and antinodes can be clearly seen.

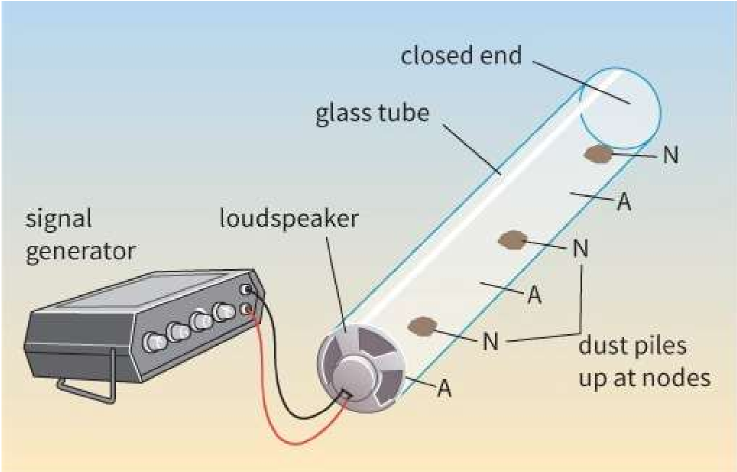

Figure 14.16: Kundt's dust tube can be used to determine the speed of sound.

PRACTICAL ACTIVITY 14.2

Using stationary sound waves to determine λ and v

This method is shown in Figure 14.17; it is the same arrangement as used for microwaves (Practical Activity 14.1). The loudspeaker produces sound waves, and these are reflected from the vertical board. The microphone detects the stationary sound wave in the space between the speaker and the board, and its output is displayed on the oscilloscope. It is simplest to turn off the time-base of the oscilloscope, so that the spot no longer moves across the screen. The spot moves up and down the screen, and the height of the vertical trace gives a measure of the intensity of the sound.

By moving the microphone along the line between the speaker and the board, it is easy to detect nodes and antinodes. For maximum accuracy, we do not measure the separation of adjacent nodes; it is better to measure the distance across several nodes.

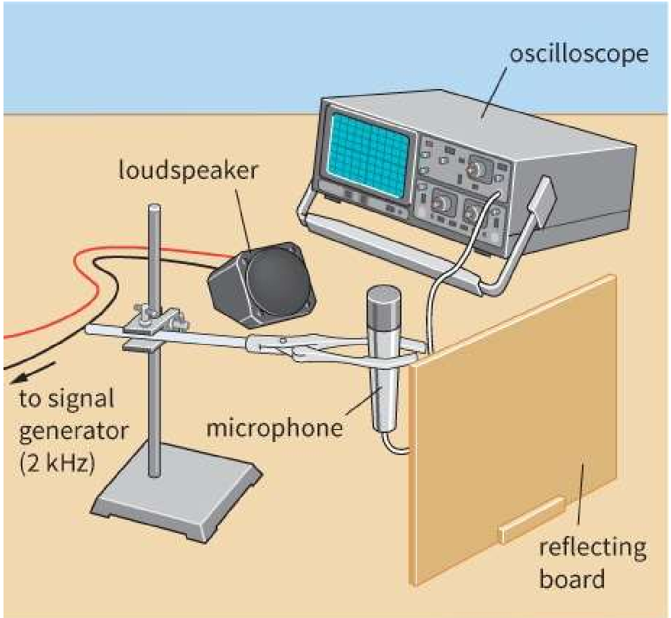

Figure 14.17: A stationary sound wave is established between the loudspeaker and the board.

Questions

5 **a** For the arrangement shown in Figure 14.17, suggest why it is easier to determine accurately the position of a node rather than an antinode.

b Explain why it is better to measure the distance across several nodes.

6 For sound waves of frequency 2500 Hz, it is found that two nodes are separated by 20 cm, with three antinodes between them.

a Determine the wavelength of these sound waves.

b Use the wave equation $v = f\lambda$ to determine the speed of sound in air.

REFLECTION

Explain to your classmates the difference between progressive sound waves and stationary sound waves.

Sketch four possible stationary wave patterns in a tube closed at just one end. Show these to your fellow learners. What grade would you give yourself for the patterns? Why?

SUMMARY

Stationary waves are formed when two identical progressive waves travelling in opposite directions meet and superpose. This usually happens when one wave is a reflection of the other.
A stationary wave has a characteristic pattern of nodes and antinodes.
A node is a point where the amplitude is always zero.
An antinode is a point of maximum amplitude.
Adjacent nodes (or adjacent antinodes) are separated by a distance equal to half a wavelength of the progressive wave.
We can use the wave equation $v = f\lambda$ to determine the speed v or the frequency f of a progressive wave. The wavelength λ is found using the nodes or antinodes of the stationary wave pattern.

EXAM-STYLE QUESTIONS

1 Which statement is *not* correct about stationary waves? **[1]**

A A stationary wave always has transverse oscillations.

B A stationary wave must have at least one node.

C The separation between two adjacent nodes is $\frac{\lambda}{2}$, where λ is the wavelength of the progressive wave.

D The superposition of two progressive waves travelling in opposite directions will produce a stationary wave.

2 A string is fixed between points X and Y.

A stationary wave pattern is formed on the stretched string.

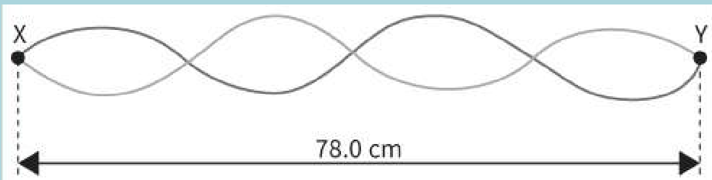

Figure 14.18

The distance between X and Y is 78.0 cm. The string vibrates at a frequency of 120 Hz.

What is the speed of the progressive wave on the string? **[1]**

A $11.7\ \text{m s}^{-1}$

B $23.4\ \text{m s}^{-1}$

C $46.8\ \text{m s}^{-1}$

D $93.6\ \text{m s}^{-1}$

3 This diagram shows a stationary wave on a string.

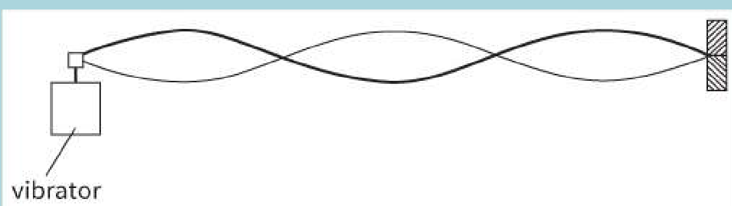

Figure 14.19

a On a copy of the diagram, label one **node** (N) and one **antinode** (A). **[1]**

b Mark on your diagram the wavelength of the progressive wave and label it λ. **[1]**

c The frequency of the vibrator is doubled. Describe the changes in the stationary wave pattern. **[1]**

[Total: 3]

4 A tuning fork that produces a note of 256 Hz is placed above a tube that is nearly filled with water. The water level is lowered until resonance is first heard.

a Explain what is meant by the term **resonance**. **[1]**

b The length of the column of air above the water when resonance is first heard is 31.2 cm.

Calculate the speed of the sound wave. **[2]**

[Total: 3]

5 **a** State **two** similarities and **two** differences between progressive waves and stationary waves. **[4]**

b This diagram shows an experiment to measure the speed of a sound in a string. The frequency of the vibrator is adjusted until the stationary wave shown is formed.

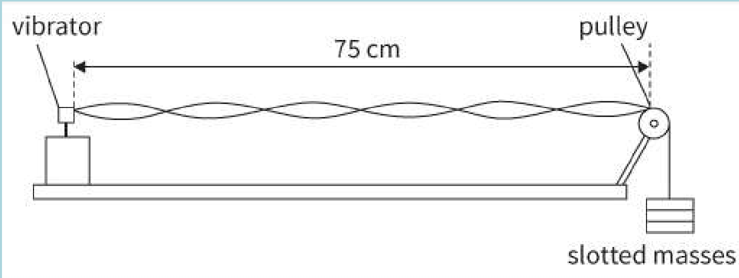

Figure 14.20

i On a copy of the diagram, mark a node (label it N) and an antinode (label it A). **[2]**

ii The frequency of the vibrator is 120 Hz. Calculate the speed at which a progressive wave would travel along the string. **[3]**

c The experiment is now repeated with the load on the string halved. In order to get a similar stationary wave the frequency has to be decreased to 30 Hz. Explain, in terms of the speed of the wave in the string, why the frequency must be adjusted. **[2]**

[Total: 11]

6 This diagram shows a stationary wave, of frequency 400 Hz, produced by a loudspeaker in a closed tube.

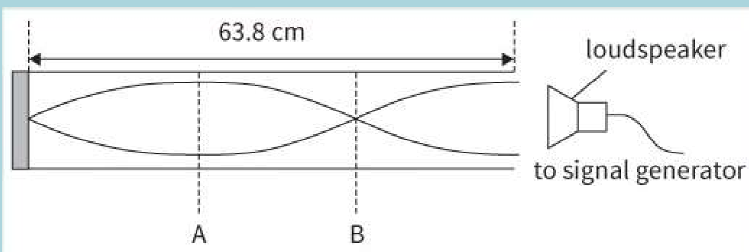

Figure 14.21

a Describe the movement of the air particles at:

i A **[2]**

ii B **[1]**

b The length the tube is 63.8 cm.

Calculate the speed of the sound. **[3]**

[Total: 6]

7 **a** Explain what is meant by:

i a **coherent** source of waves. **[2]**

ii **phase difference**. **[2]**

b A student, experimenting with microwaves, sets up the arrangement shown in this diagram.

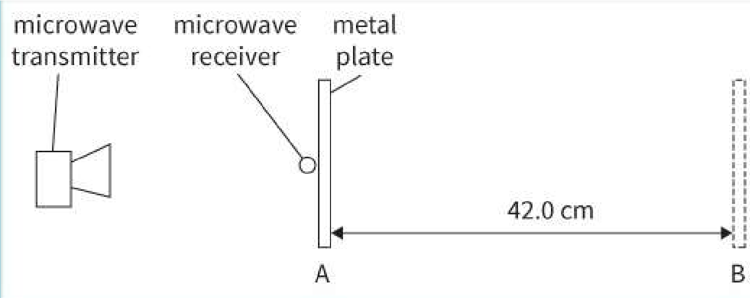

Figure 14.22

With the metal plate at position A there is a very small signal. He slowly moves the plate back, leaving the receiver in the same position. As he does so, he finds that the intensity initially rises until it becomes a maximum, then falls back to a minimum. This cycle repeats a total of five times until the plate reaches position B, where once again there is a minimum.

i Explain why a series of maxima and minima are heard. **[2]**

ii Determine the frequency of the microwaves. **[5]**

c Explain why there was a minimum when the plate was at position A, next to the detector. **[2]**

[Total: 13]

8 This diagram shows an experiment to measure the speed of sound in air.

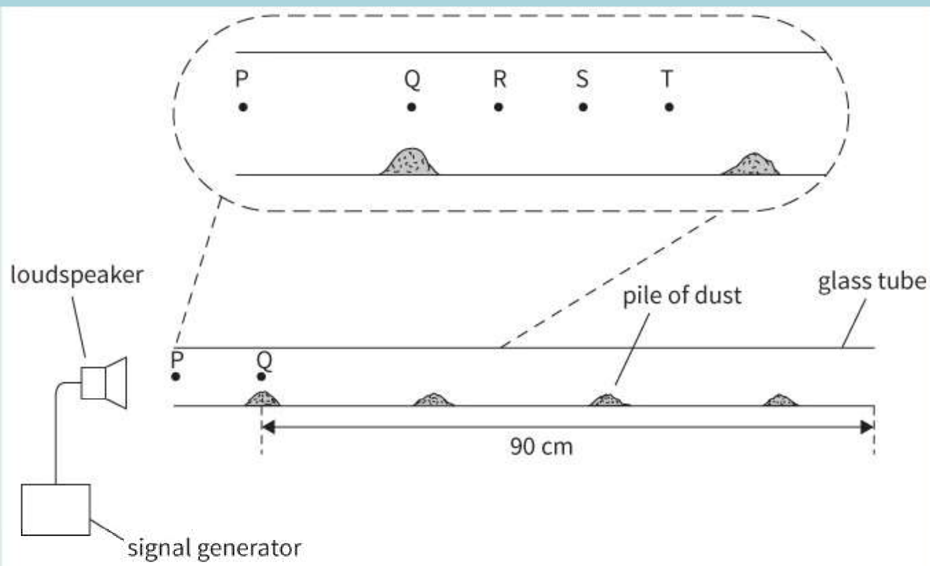

Figure 14.23

A small amount of dust is scattered along the tube. The loudspeaker is switched on. When the frequency is set at 512 Hz the dust collects in small piles as shown in the diagram.

a Determine the wavelength of the sound wave and calculate the speed of sound in the air in the tube. **[3]**

b On a copy of the diagram, show the movement of the air particles at positions P, Q, R, S and T. **[3]**

c Mark two points on your diagram where the movements of the air particles are 180° out of phase with each other. Label them A and B. **[1]**

[Total: 7]

9 The speed v of a transverse wave on a stretched wire is given by the expression $v \propto \sqrt{T}$

where T is the tension in the wire.

A length of wire is stretched between two fixed point. The tension in the wire is

T. The wire is gently plucked from the middle. A stationary wave, of fundamental frequency 210 Hz, is produced.

The tension in the wire is now increased to 1.4*T*. The percentage uncertainty in new tension is 8.0%. The length of the wire is unchanged.

Calculate the new value for the fundamental frequency when the wire is plucked in the middle. Your answer must include the absolute uncertainty written to an appropriate number of significant figures.

[4]