Chapter 13

Superposition of waves

LEARNING INTENTIONS

In this chapter you will learn how to:

- explain and use the principle of superposition
- explain the meaning of diffraction, interference, path difference and coherence
- understand experiments that demonstrate diffraction
- understand experiments that demonstrate two-source interference
- understand the conditions required if two-source interference fringes are to be observed
- recall and use $\lambda = \frac{ax}{D}$ for double-slit interference using light
- recall and use $d \sin \theta = n\lambda$ for a diffraction grating
- use a diffraction grating to determine the wavelength of light.

BEFORE YOU START

- Can you recall the general properties of waves, including electromagnetic waves? Write down as many properties as you can remember.
- Knowledge of phase difference is vital in understanding how waves combine in space–remind yourself by writing down the phase difference of two particles oscillating in step, and two particles oscillating in antiphase.

VIBRATIONS MAKING WAVES

High-level of noise would not be suitable in some jobs, such as working in a ship's engine room or looking after airplanes landing and lifting off at an airport. The simple solution would be to wear headphones. These will significantly reduce the intensity of the noise reaching the ears. Wearing noise-cancelling headphones will do a better job at protecting the ears. Electronics within such headphones create their own sound that is an exact copy of the incident noise, except it is always in antiphase (phase difference of 180°) with the noise. The addition of these two waves has the effect of reducing the intensity of the sound reaching the ears to almost zero.

Noise-cancelling headphones are useful in some situations, but they are not ideal if you are at a concert!

Can you think of other jobs where such headphones would be useful?

In this chapter, we will study how waves add-up and cancel-out. The principle of superposition of waves is an excellent starting point.

Figure 13.1: The headphones actively cancel out the noise – protecting the ears from damage.

13.1 The principle of superposition of waves

In Chapter 12, we studied the production of waves and the difference between longitudinal and transverse waves. In this chapter, we are going to consider what happens when two or more waves meet at a point in space and combine together (Figure 13.2).

So what happens when two waves arrive together at the same place? We can answer this from our everyday experience. What happens when the beams of light waves from two torches cross over? They pass straight through one another. Similarly, sound waves pass through one another, apparently without affecting each other. This is very different from the behaviour of **particles**. Two marbles meeting in mid-air would ricochet off one another in a very un-wave-like way. If we look carefully at how two sets of waves interact when they meet, we find some surprising results.

Figure 13.2: Ripples produced when drops of water fall into a swimming pool. The ripples overlap to produce a complex pattern of crests and troughs.

When two waves meet, they combine, with the displacements of the two waves adding together. Figure 13.3 shows the displacement–distance graphs for two sinusoidal waves (blue and green) of different wavelengths. It also shows the resultant wave (red), which comes from combining these two. How do we find this resultant displacement shown in red?

Consider position A. Here, the displacement of both waves is zero, and so the resultant displacement must also be zero. At position B, both waves have positive displacement. The resultant displacement is found by adding these together. At position C, the displacement of one wave is positive while the other is negative. The resultant displacement lies between the two displacements. In fact, the resultant displacement is the algebraic sum of the displacements of waves A and B; that is, their sum, taking account of their signs (positive or negative).

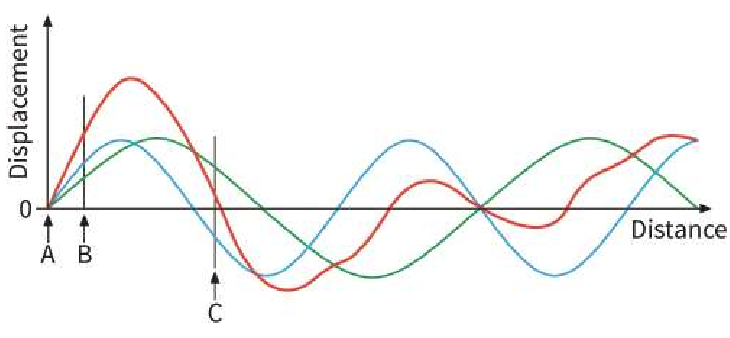

Figure 13.3: Adding two waves by the principle of superposition – the **red line** is the resultant wave.

We can work our way along the distance axis in this way, calculating the resultant of the two waves by algebraically adding them up at intervals. Notice that, for these two waves, the resultant wave is a rather complex wave with dips and bumps along its length.

The idea that we can find the resultant of two waves that meet at a point simply by adding up the displacements at each point is called the **principle of superposition** of waves. This principle can be applied to more than two waves and also to all types of waves. A statement of the principle of superposition is:

When two or more waves meet at a point, the resultant displacement is the algebraic sum of the displacements of the individual waves.

Question

1 On graph paper, draw two 'triangular' waves similar to those shown in Figure 13.4. (These are easier to work with than sinusoidal waves.) One should have wavelength 8.0 cm and amplitude 2.0 cm. The other should have wavelength 16.0 cm and amplitude 3.0 cm.

Use the principle of superposition of waves to determine the resultant displacement at suitable points along the waves, and draw the complete resultant wave.

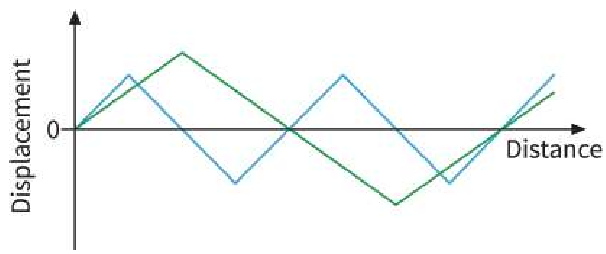

Figure 13.4: Two triangular waves.

13.2 Diffraction of waves

You should be aware that all waves (such as sound and light) can be reflected and refracted. Another wave phenomenon that applies to all waves is that they can be diffracted. **Diffraction** is the spreading of a wave as it passes through a gap or around an edge. It is easy to observe and investigate diffraction effects using water waves, as shown in Practical Activity 13.1.

Diffraction of sound and light

Diffraction effects are greatest when waves pass through a gap with a width roughly equal to their wavelength of the waves. This is useful in explaining why we can observe diffraction readily for some waves, but not for others. For example, sound waves in the audible range have wavelengths from a few centimetres to a few metres (see Table 12.1). So, we might expect to observe diffraction effects for sound in our environment. Sounds, for example, diffract as they pass through doorways. The width of a doorway is comparable to the wavelength of a sound and so a noise in one room spreads out into the next room.

Visible light has much shorter wavelengths (about 5×10^{-7} m). It is not diffracted noticeably by doorways because the width of the gap is a million times larger than the wavelength of light. However, we can observe diffraction of light by passing it through a very narrow slit or a very small hole. When laser light is directed onto a slit whose width is comparable to the wavelength of the incident light, it spreads out into the space beyond to form a smear on the screen (Figure 13.5). An adjustable slit allows you to see the effect of gradually narrowing the gap.

You can see the effects of diffraction for yourself by making a narrow slit with your two thumbs and looking through the slit at a distant light source (Figure 13.8). By gently pressing your thumbs together to narrow the gap between them, you can see the effect of narrowing the slit.

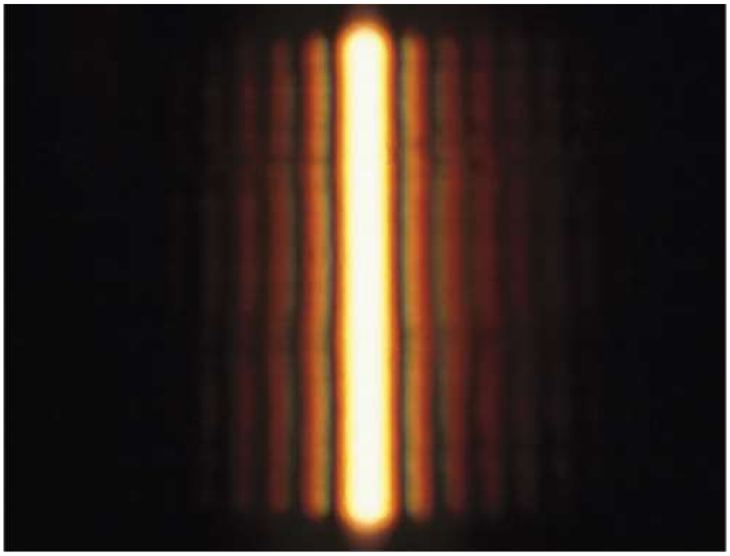

Figure 13.5: Light is diffracted as it passes through a very narrow slit.

PRACTICAL ACTIVITY 13.1

Observing diffraction in a ripple tank

A ripple tank can be used to show diffraction. Plane waves are generated using a vibrating bar, and **move towards** a gap in a barrier (Figure 13.6). Where the ripples strike the barrier, they are reflected back. Where they arrive at the gap, however, they pass through and spread out into the space beyond. It is this spreading out of waves as they travel through a gap (or past the edge of a barrier) that is called diffraction.

The extent to which ripples are diffracted depends on the width of the gap. This is illustrated in Figure 13.6. The lines in this diagram show the wavefronts. It is as if we are looking down on the ripples from above, and drawing lines to represent the tops of the ripples at some instant in time. The separation between adjacent wavefronts is equal to the wavelength λ of the ripples.

When the waves encounter a gap in a barrier, the amount of diffraction depends on the width of the gap. There is hardly any noticeable diffraction when the gap is very much larger than the wavelength.

As the gap becomes narrower, the diffraction effect becomes more noticeable. It is greatest when the width of the gap is roughly equal to the wavelength of the ripples.

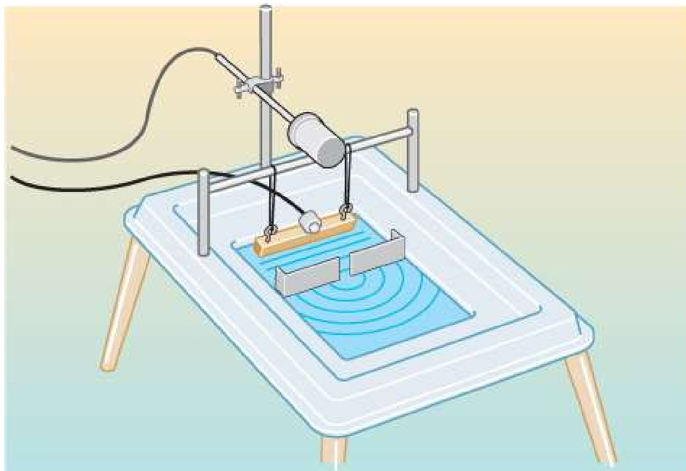

Figure 13.6: Ripples, initially straight, spread out into the space beyond the gap in the barrier.

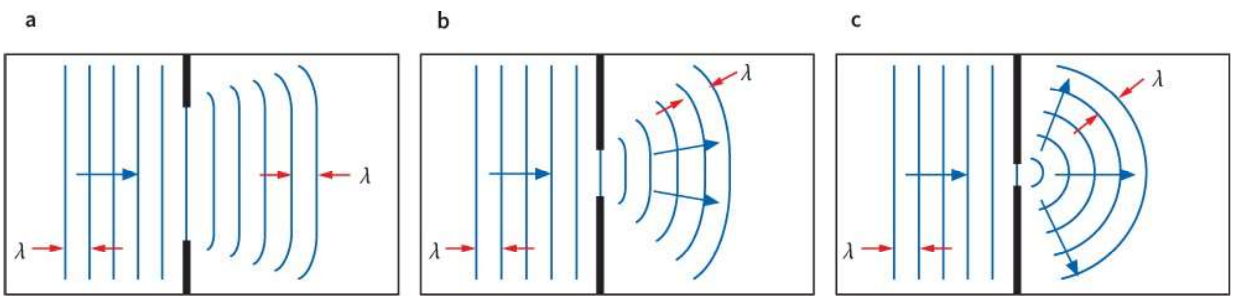

Figure 13.7: The extent to which ripples spread out depends on the relationship between their wavelength and the width of the gap. In **a**, the width of the gap is very much greater than the wavelength and there is hardly any noticeable diffraction. In **b**, the width of the gap is greater than the wavelength and there is limited diffraction. In **c**, the gap width is approximately equal to the wavelength and the diffraction effect is greatest.

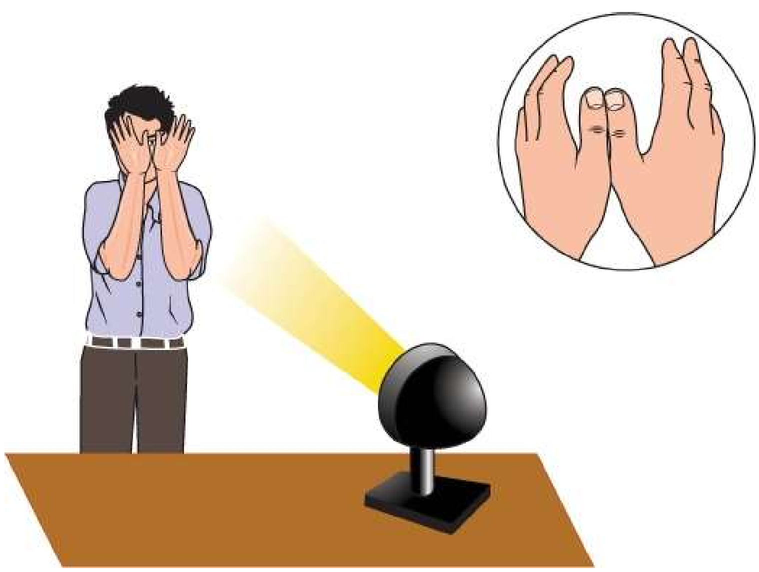

Figure 13.8: You can see the effects of diffraction by looking at a bright source (lamp) through a narrow slit. What happens when you make the slit narrower? What happens to the amount of diffraction when you put different coloured filters in front of the lamp? What does this tell you about the wavelengths of the different colours?

Diffraction of radio waves and microwaves

Radio waves can have wavelengths of the order of a kilometre. These waves are easily diffracted by gaps in the hills and by the tall buildings around our towns and cities. Microwaves, used by the mobile phone network, have wavelengths of about 10 cm. These waves are not easily diffracted (because their wavelengths are much smaller than the dimensions of the gaps) and mostly travel through space in straight lines.

Cars need external radio aerials because radio waves have wavelengths longer than the size of the windows, so they cannot diffract into the car. If you try listening to a radio in a train without an external aerial, you will find that FM signals can be picked up weakly (their wavelength is about 3 m), but AM signals, with longer wavelengths, cannot get in at all.

Question

2 A microwave oven (Figure 13.9) uses microwaves with a wavelength of 12.5 cm. The front door of the oven is made of glass with a metal grid inside; the gaps in the grid are a few millimetres across.

Explain how this design allows us to see the food inside the oven, while the microwaves are not allowed to escape into the kitchen (where they might harm us).

Figure 13.9: A microwave oven has a metal grid in the door to keep microwaves in and let light out

Explaining diffraction

Diffraction is a wave effect that can be explained by the principle of superposition. We have to think about what happens when a plane ripple reaches a gap in a barrier (Figure 13.10). Each point on the surface of the water in the gap is moving up and down. Each of these moving points can be thought of as a source of new ripples spreading out into the space beyond the barrier. Now we have a lot of new ripples, and we can use the principle of superposition to find their resultant effect. Without trying to calculate the effect of an infinite number of ripples, we can say that in some directions the ripples add together while in other directions they cancel out.

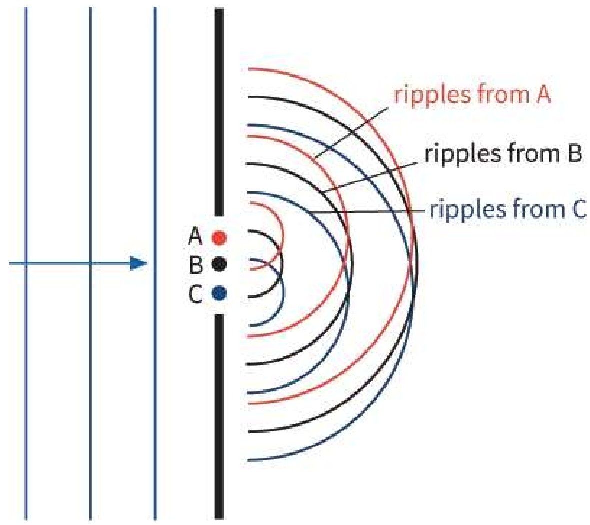

Figure 13.10: Ripples from all points across the gap contribute to the pattern in the space beyond.

13.3 Interference

Adding waves of different wavelengths and amplitudes results in complex waves – by complex, we really mean not sinusoidal. We can find some interesting effects if we consider what happens when two waves of the same type, and having the same wavelength, overlap at a point. Again, we will use the principle of superposition to explain what we observe.

PRACTICAL ACTIVITY 13.2

Observing interference

Interference of sound waves

A simple experiment shows what happens when two sets of sound waves meet. Two loudspeakers are connected to a single signal generator (Figure 13.11). They each produce sound waves of the same wavelength. Walk around in the space in front of the loudspeakers; you will hear the resultant effect.

You may predict that we would hear a sound twice as loud as that from a single loudspeaker. However, this is not the case. At some points, the sound is louder than for a single loudspeaker. At other points, the sound is much quieter. The space around the two loudspeakers consists of a series of loud and quiet regions. We are observing the phenomenon known as **interference**. This phenomenon results in the formation of points of cancellation and reinforcement where two coherent waves pass through each other.

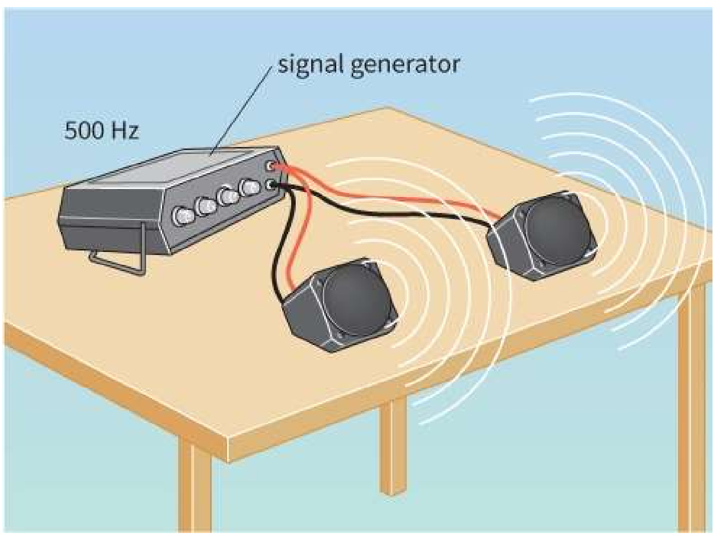

Figure 13.11: The sound waves from two loudspeakers combine to give an interference pattern. This experiment is best done outside so that reflections of sounds (or echoes) do not affect the results.

Interference in a ripple tank

Look at Figure 13.12. The two dippers in the ripple tank should be positioned so that they are just touching the surface of the water. When the bar vibrates, each dipper acts as a point-source of circular ripples spreading outwards. Where these sets of ripples overlap, we observe an interference pattern. Another way to observe interference in a ripple tank is to use plane waves passing through two gaps in a barrier. The water waves are diffracted at the two gaps and then interfere beyond the gaps. Figure 13.13 shows the interference pattern produced by two vibrating dippers in a ripple tank.

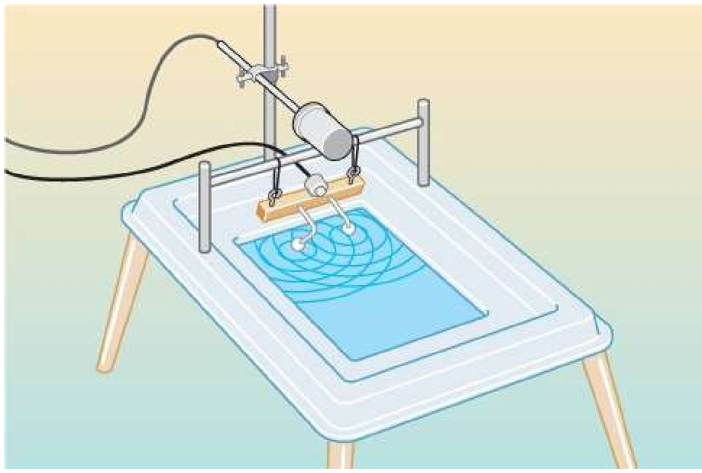

Figure 13.12: A ripple tank can be used to show how two sets of circular ripples combine.

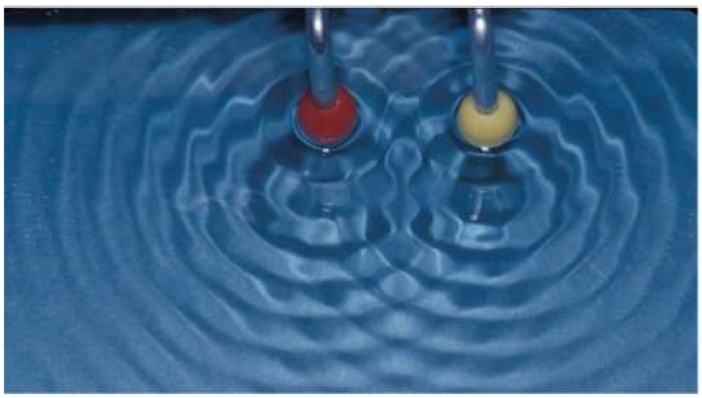

Figure 13.13: Ripples from two point-sources produce an interference pattern.

Explaining interference

Figure 13.14 shows how interference arises. The loudspeakers in Figure 13.11 (Practical Activity 13.2) are emitting waves that are in phase because both are connected to the same signal generator. At each point in front of the loudspeakers, waves are arriving from the two loudspeakers. At some points, the two waves arrive in phase (in step) with one another and with equal amplitude (Figure 13.14a). The principle of superposition predicts that the resultant wave has twice the amplitude of a single wave. We hear a louder sound.

At other points, something different happens. The two waves arrive **completely out of phase** or in antiphase (phase difference is 180°) with one another (Figure 13.14b). There is a cancelling out, and the resultant wave has zero amplitude. At this point, we would expect silence. At other points again, the waves are neither perfectly out of step nor perfectly in step, and the resultant wave has an amplitude less than that at the loudest point.

Where two waves arrive at a point in phase with one another so that they add up, we call this effect **constructive interference**. Where they cancel out, the effect is known as **destructive interference.** Where two waves have different amplitudes but are in phase (Figure 13.14c), constructive interference results in a wave whose amplitude is the sum of the two individual amplitudes.

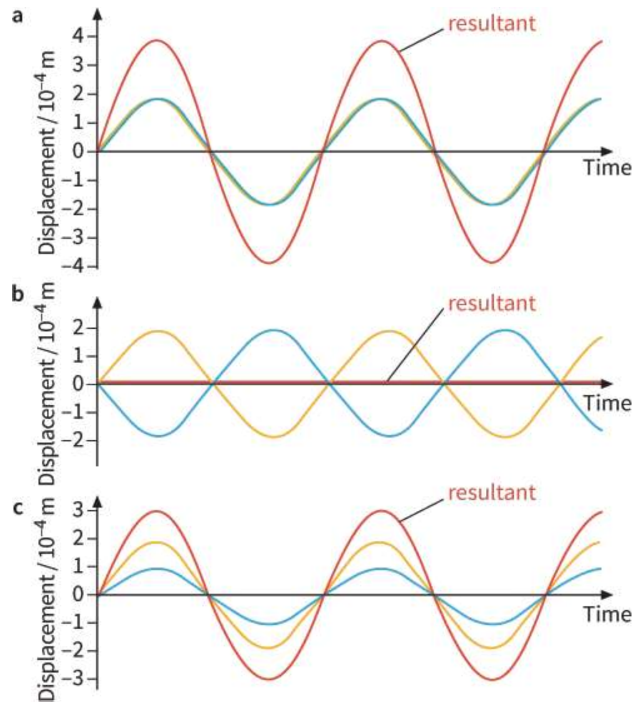

Figure 13.14: Adding waves by the principle of superposition. Blue and orange waves of the same amplitude may give **a** constructive or **b** destructive interference, depending on the phase difference between them. **c** Waves of different amplitudes can also interfere constructively.

Question

3 Explain why the two loudspeakers producing sounds of slightly different frequencies will not produce stable effects of interference.

How can we explain the interference pattern observed in a ripple tank (Practical Activity 13.2)? Look at Figure 13.15 and compare it to Figure 13.13. Figure 13.15 shows two sets of waves setting out from their sources. At a position such as A, ripples from the two sources arrive in phase with one another, and constructive interference occurs. At B, the two sets of ripples arrive in antiphase, and there is destructive interference. Although waves are arriving at B, the surface of the water remains approximately flat.

Whether the waves combine constructively or destructively at a point depends on the path difference of the waves from the two **coherent sources**. The **path difference** is defined as the extra distance travelled by one of the waves compared with the other.

At point A in Figure 13.15, the waves from the red source have travelled three whole wavelengths. The waves from the yellow source have travelled four whole wavelengths. The path difference between the two sets of waves is one wavelength. A path difference of one wavelength is equivalent to a phase difference of zero. This means that the two waves are in phase, so they interfere constructively.

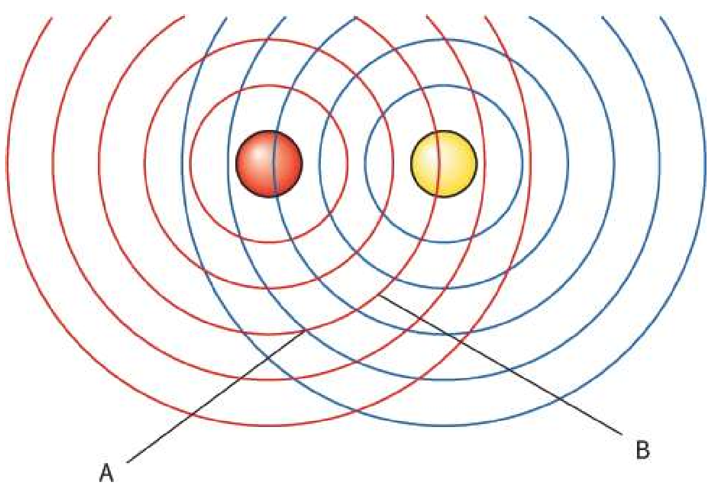

Figure 13.15: The result of interference depends on the path difference between the two waves.

Now think about destructive interference. At point B, the waves from the red source have travelled three wavelengths; the waves from the yellow source have travelled 2.5 wavelengths. The path difference between the two sets of waves is 0.5 wavelengths, which is equivalent to a phase difference of 180°. The waves interfere destructively because they are in antiphase. The conditions for constructive interference and destructive interference, in general, are outlined next. These conditions apply to **all** waves (water waves, light, microwaves, radio waves, sound and so on) that show interference effects. In the equations, *n* is an integer (any whole number, including zero).

For **constructive interference** the path difference is a whole number of wavelengths:

$$\text{path difference} = 0, \lambda, 2\lambda, 3\lambda, \text{ and so on}$$

or

$$\text{path difference} = n\lambda$$

For **destructive interference** the path difference is an odd number of half wavelengths:

$$\text{path difference} = \tfrac{1}{2}\lambda, 1\tfrac{1}{2}\lambda, 2\tfrac{1}{2}\lambda, \text{ and so on}$$

or

$$\text{path difference} = \left(n + \tfrac{1}{2}\right)\lambda$$

PRACTICAL ACTIVITY 13.3

Interference of radiation

Interference of light

Here is one way to show the interference effects produced by light. A simple arrangement involves directing the light from a laser at a double-slit (Figure 13.16). The slits are two clear lines on a black slide, separated by a fraction of a millimetre. Where the light falls on the screen, a series of equally spaced dots of light are seen (see Figure 13.21). These bright dots are referred to as **interference maxima** or ‘fringes’, and they are regions where light waves from the two slits are arriving in phase with each other; in other words, there is constructive interference. The dark regions in between are the result of destructive interference.

If you carry out experiments using a laser, you should follow correct safety procedures. In particular, you should wear eye protection and avoid allowing the beam to enter your eye directly.

These bright and dark fringes are the equivalent of the loud and quiet regions that you detected if you investigated the interference pattern of sounds from the two loudspeakers described in Practical Activity 13.2. Bright fringes correspond to loud sound, and dark fringes to quiet sound or silence.

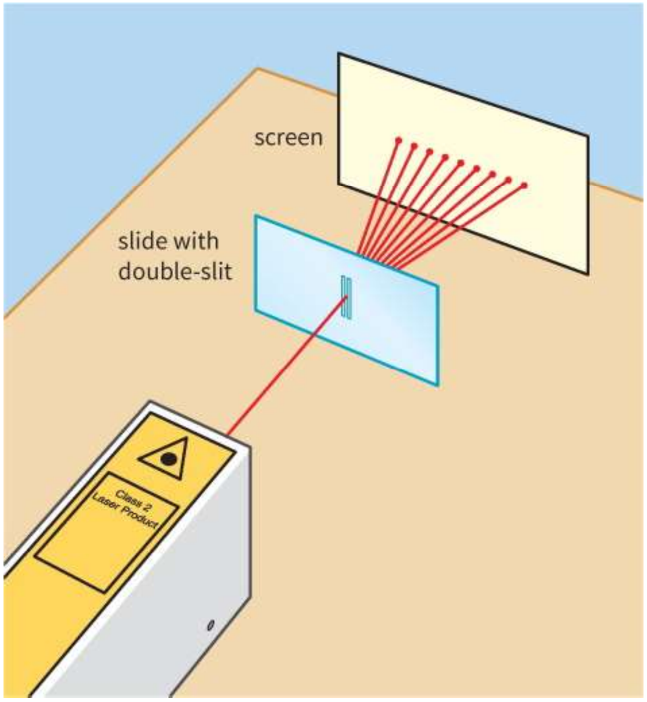

Figure 13.16: Laser light passing through the two slits show interference effects in the space beyond.

You can check that light is indeed reaching the screen from both slits as follows. Mark a point on the screen where there is a dark fringe. Now carefully cover up one of the slits so that light from the laser is only passing through one slit. You should find that the pattern of interference fringes disappears. Instead, a broad band of light appears across the screen. This broad band of light is the diffraction pattern produced by a single slit. The point that was dark is now light. Cover up the other slit instead, and you will see the same effect. You have now shown that light is arriving at the screen from both slits, but at some points (the dark fringes) the two beams of light cancel each other out.

You can achieve similar results with a bright light bulb rather than a laser, but a laser is much more convenient because the light is concentrated into a narrow, more intense beam. This famous experiment is called the Young double-slit experiment (discussed in more detail later in this chapter), although Thomas Young had no laser available to him when he first demonstrated it in 1801.

Interference of microwaves

Using 2.8 cm wavelength microwave equipment (Figure 13.17), you can observe an interference pattern. The microwave transmitter is directed towards the double gap in a metal barrier. The microwaves are diffracted at the two gaps so that they spread out into the region beyond, where they can be detected using the microwave probe (receiver). By moving the probe around, it is possible to detect regions of high intensity (constructive interference) and low intensity (destructive interference). The probe may be connected to a meter, or to an audio amplifier and loudspeaker to give an audible output.

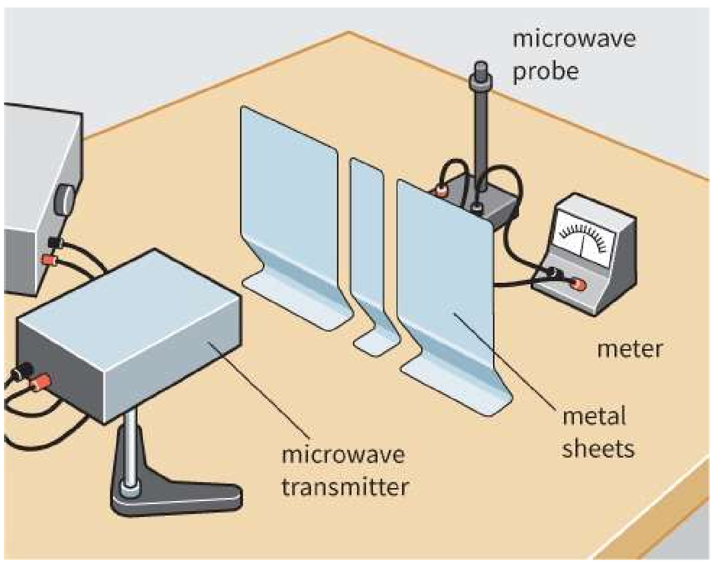

Figure 13.17: Microwaves also show interference effects.

Question

4 Look at the experimental arrangement shown in Figure 13.17. Suppose that the microwave probe is placed at a point of low intensity in the interference pattern.

Suggest what will happen if one of the gaps in the barrier is now blocked.

Coherence

We are surrounded by many types of wave – light, infrared radiation, radio waves, sound and so on. There are waves coming at us from all directions. So why do we not observe interference patterns all the time? Why do we need special equipment in a laboratory to observe these effects?

In fact, we can see interference of light occurring in everyday life. For example, you may have noticed haloes of light around street lamps or the Moon on a foggy night. You may have noticed light and dark bands of light if you look through fabric at a bright source of light. These are all examples of interference effects.

We usually need specially arranged conditions to produce interference effects that we can measure. Think about the demonstration with two loudspeakers. If they were connected to different signal generators with slightly different frequencies, the sound waves might start off in phase with one another, but they would soon go out of phase (Figure 13.18). We would hear loud, then soft, then loud again. The interference pattern would keep shifting around the room – there would be no stable interference pattern of loud and quiet regions.

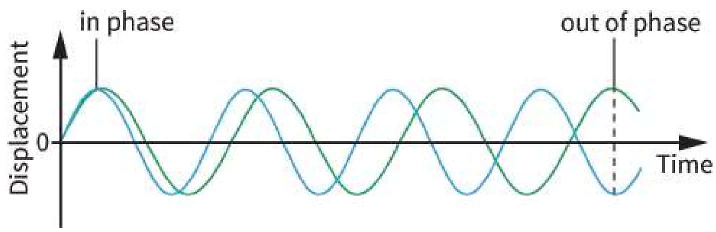

Figure 13.18: Waves of slightly different frequencies (and therefore wavelengths) move in and out of phase with one another.

By connecting the two loudspeakers to the **same** signal generator, we can be sure that the sound waves that they produce are constantly in phase with one another. We say that they act as two **coherent sources** of sound waves (coherent means sticking together). The sound waves from the loudspeakers has **coherence**. Coherent sources emit waves that have a **constant phase difference**. Note that the two waves can only have a constant phase difference if their frequency is the same and remains constant.

Now think about the laser experiment. Could we have used two lasers producing exactly the same frequency and hence wavelength of light? Figure 13.19a represents the light from a laser. We can think of it as being made up of many separate bursts of light. We cannot guarantee that these bursts from two lasers will always be in phase with one another.

This problem is overcome by using a single laser and dividing its light using the two slits (Figure 13.19b). The slits act as two coherent sources of light. They are constantly in phase with one another (or there is a constant phase difference between them).

If they were not coherent sources, the interference pattern would be constantly changing, far too fast for our eyes to detect. We would simply see a uniform band of light, without any definite bright and dark regions. From this you should be able to see that, in order to observe interference, we need two coherent sources of waves.

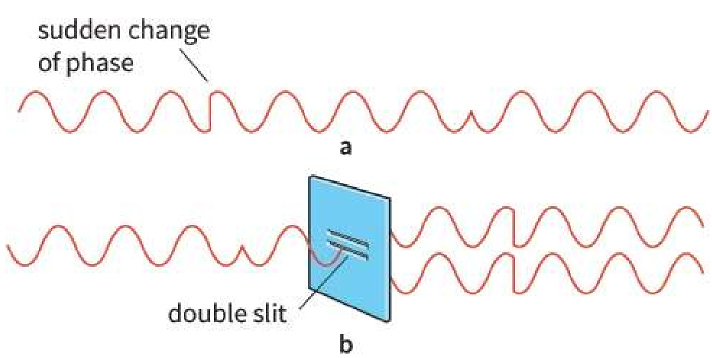

Figure 13.19: Waves must be coherent if they are to produce a clear interference pattern.

Question

5 Draw sketches of displacement against time to illustrate the following:

- **a** two waves having the same amplitude and in phase with one another
- **b** two waves having the same amplitude and with a phase difference of 90°
- **c** two waves initially in phase but with slightly different wavelengths.

13.4 The Young double-slit experiment

Now we will take a close look at a famous experiment that Thomas Young performed in 1801. He used this experiment to show the wave-nature of light. A beam of light is shone on a pair of parallel slits placed at right angles to the beam. Light diffracts and spreads outwards from each slit into the space beyond. The light from the two slits overlaps on a screen. An interference pattern of light and dark bands called 'fringes' is formed on the screen.

Explaining the experiment

In order to observe interference, we need two sets of waves. The sources of the waves must be coherent– the phase difference between the waves emitted at the sources must remain constant. This also means that the waves must have the same wavelength. Today, this is readily achieved by passing a single beam of laser light through the two slits. A laser produces intense coherent light. As the light passes through the slits, it is diffracted so that it spreads out into the space beyond (Figure 13.20). Now we have two overlapping sets of waves, and the pattern of fringes on the screen shows us the result of their interference (Figure 13.21).

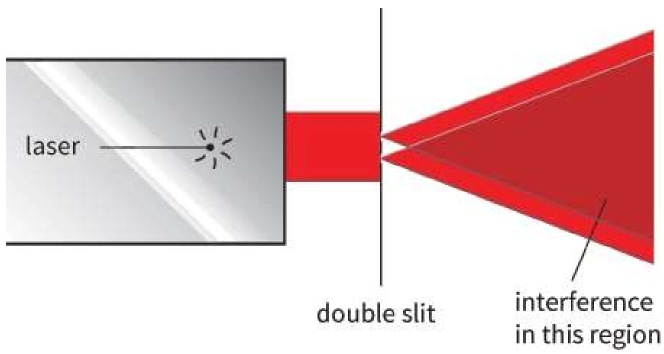

Figure 13.20: Interference occurs where diffracted beams from the two slits overlap. How does this pattern arise? We will consider three points on the screen (Figure 13.22), and explain what we would expect to observe at each.

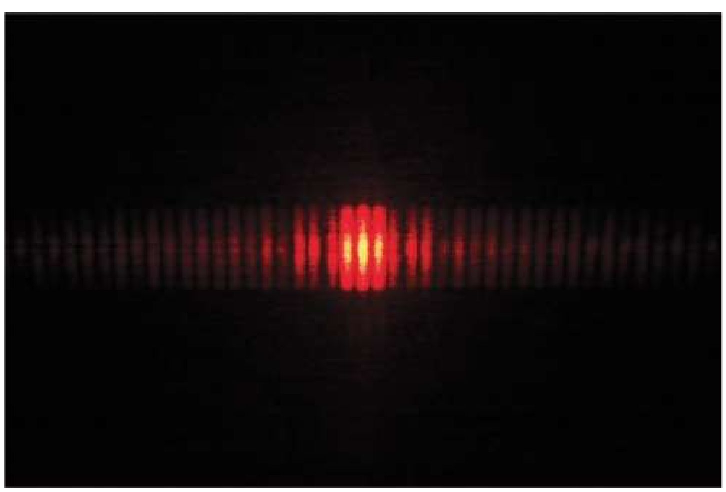

Figure 13.21: Interference fringes obtained using a laser and a double-slit.

Point A

This point is directly opposite the midpoint of the slits. Two rays of light arrive at A, one from slit 1 and the other from slit 2. Point A is equidistant from the two slits, and so the two rays of light have travelled the same distance. The path difference between the two rays of light is zero. If we assume that they were in phase (in step) with each other when they left the slits, then they will be in phase when they arrive at A. Hence they will interfere constructively, and we will observe a bright fringe at A.

Point B

This point is slightly to the side of point A, and is the midpoint of the first dark fringe. Again, two rays of light arrive at B, one from each slit. The light from slit 1 has to travel slightly further than the light from slit 2, and so the two rays are no longer in step. Since point B is at the midpoint of the dark fringe, the two rays must be in antiphase (phase difference of 180°). The path difference between the two rays of light must be half a wavelength, and so the two rays interfere destructively.

Point C

This point is the midpoint of the next bright fringe, with AB = BC. Again, ray 1 has travelled further than ray 2; this time, it has travelled an extra distance equal to a whole wavelength λ. The path difference between the rays of light is now a whole wavelength. The two rays are in phase at the screen. They interfere constructively, and we see a bright fringe.

The complete interference pattern (Figure 13.21) can be explained in this way.

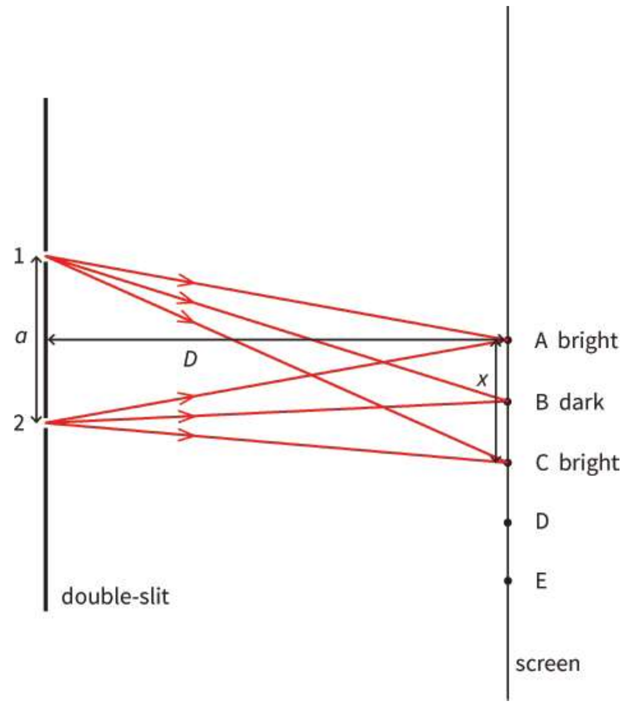

Figure 13.22: The type of interference, and hence whether a bright or a dark fringe is seen on the screen, depends on the path difference between the rays of light arriving at the screen from the double-slit.

Question

6 Consider points D and E on the screen in Figure 13.22, where BC = CD = DE. State and explain what you would expect to observe at D and E.

Determining wavelength λ

The double-slit experiment can be used to determine the wavelength λ of monochromatic light. The following three quantities have to be measured:

- **Slit separation *a*** – This is the distance between the centres of the slits, which is the distance between slits 1 and 2 in Figure 13.22.
- **Fringe separation *x*** – This is the distance between the centres of adjacent bright (or dark) fringes, which is the distance AC in Figure 13.22.
- **Slit-to-screen distance *D*** – This is the distance from the midpoint of the slits to the central fringe on the screen.

Once these three quantities have been determined, the wavelength λ of the light can be found using the relationship:

$$\lambda = \frac{ax}{D}$$

KEY EQUATION

The double-slit equation:

$$\lambda = \frac{ax}{D}$$

where λ is the wavelength of the monochromatic light incident normally at the double-slit. a is the separation between the centres of the slits, x is the separation between the centres of adjacent bright (or dark) fringes and D is distance between the slits and the screen.

WORKED EXAMPLE

1 In a double-slit experiment using light from a helium–neon laser, a student obtained the following results:

width of 10 fringes $10x = 1.5$ cm

separation of slits $a = 1.0$ mm

slit-to-screen distance $D = 2.40$ m

Calculate the wavelength of the light in nm.

Step 1 Work out the fringe separation x in metres (m):

$$\text{fringe sepration, } x = \frac{1.5 \times 10^{2}}{10} = 1.5 \times 10^{-3} \text{ m}$$

Step 2 Substitute the values of a, x and D (all in metres) into the equation and then calculate the wavelength λ:

$$\lambda = \frac{ax}{D}$$

$$= \frac{1.0 \times 10^{-3} \times 1.5 \times 10^{-3}}{2.40}$$

$$= 6.25 \times 10^{-7}\text{m} \approx 6.30 \times 10^{-7}\text{m}$$

1 nm = 10^{-9} m

Therefore:

λ = 630 nm

Question

7 The student in Worked example 1 moved the screen to a distance of 4.8 m from the slits. Determine the fringe separation x now.

PRACTICAL ACTIVITY 13.4

Using Young's slits to determine λ

The Young double-slit experiment can be used to determine the wavelength λ of monochromatic light. Here, we look at a number of practical features of the experiment and consider how the percentage uncertainty in the value of λ can be reduced.

One way to carry out the double-slit experiment is shown in Figure 13.23. Here, a white light source is used, rather than a laser. A monochromatic filter allows only one wavelength of light to pass through. A single slit diffracts the light. This diffracted light arrives in phase at the double slit, which ensures that the two parts of the double slit behave as coherent sources of light. The double slit is placed a centimetre or two beyond the single slit, and the fringes are observed on a screen a metre or so away. The experiment has to be carried out in a darkened room, as the intensity of the bright fringes is low – making them hard to see.

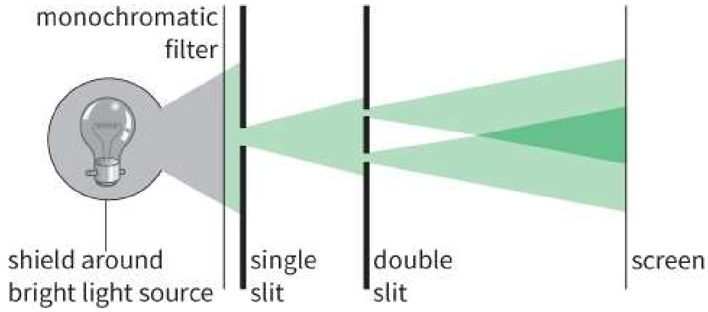

Figure 13.23: Arrangement for seeing fringes using a white light source.

There are three important factors involved in the way the equipment is set up:

- All slits are a fraction of a millimetre in width. Since the wavelength of light is less than a micrometre (10^{-6} m), this gives a small amount of diffraction in the space beyond. If the slits were narrower, the intensity of the light would be too low for visible fringes to be achieved.
- The double slits are about a millimetre apart. If they were much further apart, the fringes would be too close together to be distinguishable.
- The screen is about a metre from the slits. The fringes produced are clearly separated without being too dim.

Measuring a, x and D

Measuring slit separation a: a travelling microscope is suitable for measuring a. It is difficult to judge the position of the centre of a slit. If the slits are the same width, the separation of their left-hand edges is the same as the separation of their centres.

Measuring fringe width x: it is best to measure across several fringes (say, ten) and then to calculate the average separation later. A 30 cm ruler or a travelling microscope can be used.

Measuring the slit-to-screen distance D: this can be measured using a metre rule or a tape measure.

Reducing percentage errors

Why use a laser rather than white light? With a laser, the light beam is more concentrated, and the initial single slit is not necessary. The greater intensity of the beam means that the screen can be further from the slits, so that the fringes are further apart. This reduces the percentage uncertainties in measurements of x and D. Consequently, the overall percentage uncertainty in the calculated value for the wavelength λ will be smaller.

A laser has a second advantage. The light from a laser is monochromatic; that is, it consists of a single wavelength. This makes the fringes very clear, and they are present in large numbers across the screen. With white light, a range of wavelengths is present. Different wavelengths form fringes at different points across the screen, smearing them out so that they are not as clear.

Using white light with no filter results in a central fringe that is white (because all wavelengths are in phase here), but the other fringes show coloured effects, as the different wavelengths interfere constructively at different points. In addition, only a few fringes are visible in the interference pattern.

Questions

8 Use the equation $\lambda = \frac{ax}{D}$ to explain the following observations:

a With the slits closer together, the fringes are further apart.

b Interference fringes for blue light are closer together than for red light.

c In an experiment to measure the wavelength of light, it is desirable to have the screen as far from the slits as possible.

9 Yellow light from a sodium source is used in the double-slit experiment. This yellow light has wavelength 589 nm. The slit separation is 0.20 mm, and the screen is placed 1.20 m from the slits.

Calculate the separation between adjacent bright fringes formed on the screen.

10 In a double-slit experiment, filters were placed in front of a white light source to investigate the effect of changing the wavelength of the light. At first, a red filter was used instead (λ = 600 nm) and the fringe separation was found to be 2.4 mm. A blue filter was then used instead (λ = 450 nm).

Calculate the fringe separation with the blue filter.

13.5 Diffraction gratings

A **transmission** diffraction grating is similar to the slide used in the double-slit experiment, but with many more slits than just two. It consists of a large number of equally spaced lines ruled on a glass or plastic slide. Each line is capable of diffracting the incident light. There may be as many as 10 000 lines per centimetre. When light is shone through this grating, a pattern of interference fringes is seen.

A **reflection** diffraction grating consists of lines made on a reflecting surface so that light is both reflected and diffracted by the grating. The shiny surface of a CD (compact disc), or a DVD (digital versatile disc), is an everyday example of a reflection diffraction grating. Hold a CD in your hand so that you are looking at the reflection of light from a lamp. You will observe coloured bands (Figure 13.24). A CD has thousands of equally spaced lines of microscopic pits on its surface; these carry the digital information. It is the diffraction from these lines that produces the coloured bands of light from the surface of the CD.

Figure 13.24: A CD acts as a reflection diffraction grating. White light is reflected and diffracted at its surface, producing a display of spectral colours.

Observing diffraction with a transmission grating

In Figure 13.25, monochromatic light from a laser is incident normally on a transmission diffraction grating. In the space beyond, interference fringes are formed. These can be observed on a screen, as with the double slit. However, it is usual to measure the angle θ at which they are formed, rather than measuring their separation. With double slits, the fringes are equally spaced and the angles are very small. With a diffraction grating, the angles are much greater and the fringes are not equally spaced.

The bright fringes are also referred to as **maxima**. The central fringe is called the zeroth-order maximum, the next fringe is the first-order maximum, and so on. The pattern is symmetrical, so there are two first-order maxima, two second-order maxima, and so on.

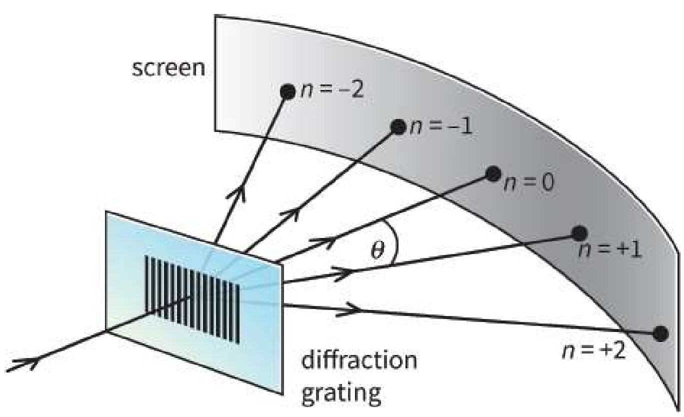

Figure 13.25: A laser beam passing through a diffraction grating produces a symmetrical pattern of maxima on a screen.

Explaining the experiment

The principle is the same as for the double-slit experiment, but here we have light passing through many slits. As it passes through each slit, it diffracts into the space beyond. So now we have many overlapping beams of light, and these interfere with one another.

There is a bright fringe, the zeroth-order maximum, in the straight-through direction ($\theta = 0$). This is because all of the rays here are travelling parallel to one another and in phase, so the interference is constructive (Figure 13.26a).

Imagine if you could look through the diffraction grating at the source of light. Your eye would be focused on the light source, which is far away. All the rays with $\theta = 0$ come together at the back of your eye, where an image is formed. It is here that interference occurs.

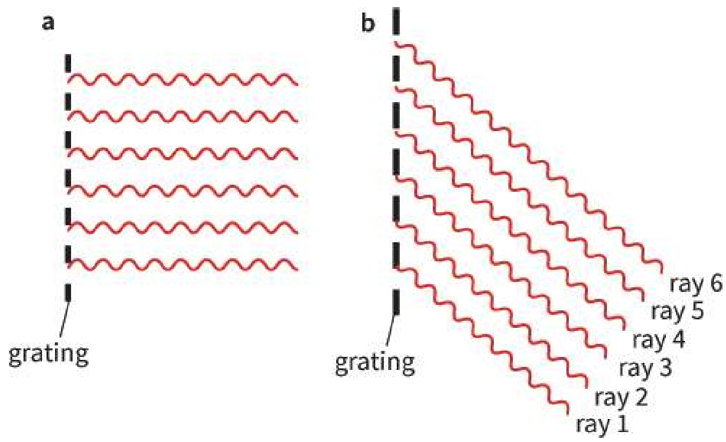

Figure 13.26: a Waves from each slit are in phase in the straight-through direction. **b** In the direction of the first-order maximum, the waves are in phase, but each one has travelled one wavelength further than the one below it.

The first-order maximum forms in a specific direction as follows. Diffraction occurs at all of the slits. Rays of light emerge from all of the slits to form a bright fringe – all the rays must be in phase. In the direction of the first-order maximum, ray 1 has travelled the smallest distance (Figure 13.26b). Ray 2 has travelled an extra distance equal to one whole wavelength and is therefore in phase with ray 1. The path difference between ray 1 and ray 2 is equal to one wavelength λ. Ray 3 has travelled two extra wavelengths and is in phase with rays 1 and 2. In fact, the rays from all of the slits are in step in this direction, and a bright fringe results.

Question

11 Explain how the second-order maximum arises in terms of path difference.

Determining wavelength λ with a diffraction grating

By measuring the angles at which the maxima occur, we can determine the wavelength λ of the incident monochromatic light. The wavelength λ is related to the angle θ by the equation:

$$d \sin\theta = n\lambda$$

KEY EQUATION

$$d \sin\theta = n\lambda$$

WORKED EXAMPLE

2 Monochromatic light is incident normally on a diffraction grating having 300 lines mm^{-1}. The angle θ between the zeroth- and first-order maxima is measured to be 10.0°.

Calculate the wavelength of the incident light.

Step 1 Calculate the distance between the adjacent lines (grating spacing) d. Since there are 3000 lines mm^{-1}, d must be:

$$d = \frac{1\text{ mm}}{300}$$
$$= 3.33 \times 10^{-3}\text{mm}$$
$$= 3.33 \times 10^{-6}\text{m}$$

Step 2 Rearrange the equation $d\sin\theta = n\lambda$ and substitute values:

$$\theta = 10.0^\circ, n = 1\,(\text{first order})$$
$$\lambda = \frac{d\sin\theta}{n}$$
$$= \frac{3.33\times10^{-6}\times\sin 10.0^\circ}{1}$$
$$= 5.8 \times 10^{-7}\text{m}$$

This is the same as 580 nm. (1 nm = 10^{-9} m.)

where d is the separation between adjacent lines of the grating, θ is the angle for the n^{th}-order maximum and λ is the wavelength of the monochromatic light incident normally at the diffraction grating. n is known as the **order** of the maximum; n can only have integer values 0, 1, 2, 3 and so on. The distance d is also known as the grating element or grating spacing.

Worked example 2 shows how you can determine λ.

Questions

12 **a** For the case described in Worked example 2, with λ = 580 nm, calculate the angle θ for the second-order maximum.

b Repeat the calculation of θ for n = 3, 4, and so on. Determine how many maxima can be seen. Explain your answer.

13 Consider the equation $d\sin\theta = n\lambda$. State and explain how the interference pattern would change when:

a the wavelength of the incident light is increased for the same grating

b the grating is changed for one with more lines per cm for the same incident light.

14 A student is trying to make an accurate measurement of the wavelength of green light from a mercury lamp. The wavelength λ of this light is 546 nm. Using a double-slit of separation 0.50 mm, the student can see 10 clear bright fringes on a screen at a distance of 0.80 m from the slits. The student can measure their overall width to within ±1 mm using a ruler.

The student then tries an alternative experiment using a diffraction grating with 3000 lines cm^{-1}. The angle between the two second-order maxima can be measured to within ±0.1°.

a Determine the width of the 10 fringes that the student can measure in the first experiment.

b Determine the angle of the second-order maximum that the student can measure in the second experiment.

c Based on your answers to parts *a* and *b*, suggest which experiment you think will give the more accurate value of λ.

Diffracting white light

A diffraction grating can be used to split white light up into its component colours. This splitting of light is known as **dispersion**, shown in Figure 13.27. A beam of white light is shone onto the grating. A zeroth-order, white maximum is observed at θ = 0°, because all waves of each colour are in phase in this direction.

On either side, a series of spectra appear, with violet closest to the centre and red furthest away. We can see why different wavelengths have their maxima at different angles if we rearrange the equation $d\sin\theta = n\lambda$ to give:

$$\sin\theta = \frac{n\lambda}{d}$$

From this it follows that the greater the wavelength λ, the greater the value of sin θ and hence the greater the angle θ. Red light is at the long wavelength end of the visible spectrum, and so it appears at the greatest angle.

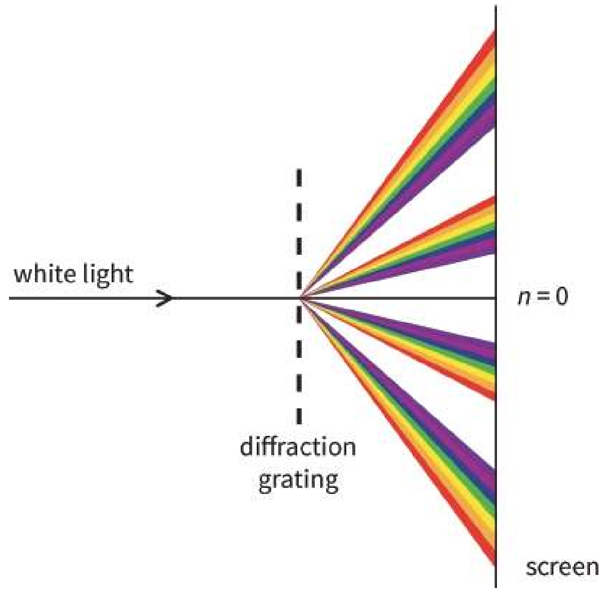

Figure 13.27: A diffraction grating is a simple way of separating white light into its constituent wavelengths.

PRACTICAL ACTIVITY 13.5

Diffraction gratings versus double-slit

It is worth comparing the use of a diffraction grating to determine wavelength with the Young two-slit experiment.

- With a diffraction grating, the maxima are very **sharp**.
- With a diffraction grating, the maxima are also very **bright**. This is because rather than contributions from only two slits, there are contributions from a thousand or more slits.
- With double-slit, there may be a large uncertainty in the measurement of the slit separation **a**. The fringes are close together, so their separation may also be measured imprecisely.
- With a diffraction grating the maxima are widely separated, the angle θ can be measured to a high degree of precision. So, an experiment with a diffraction grating can be expected to give a value for the wavelength to a much higher degree of precision than a simple double-slit arrangement.

Question

15 White light is incident normally on a diffraction grating with a slit-separation d of 2.00×10^{-6} m. The visible spectrum has wavelengths between 400 nm and 700 nm.

a Calculate the angle between the red and violet ends of the first-order spectrum.

b Explain why the second- and third-order spectra overlap.

REFLECTION

Make a short list of everyday items that would diffract sound, then do the same for light.

Summarise two experiments for your fellow learners for determining the wavelength of visible light. What did you learn about yourself as you worked on this summary?

SUMMARY

The principle of superposition states that when two or more waves meet at a point, the resultant displacement is the algebraic sum of the displacements of the individual waves.

When waves pass through a slit, they may be diffracted so that they spread out into the space beyond. The diffraction effect is greatest when the wavelength of the waves is similar to the width of the gap.

Interference is the superposition of two or more waves from coherent sources.

Two sources are coherent when they emit waves that have a **constant phase difference**. (This can only happen if the waves have the same frequency or wavelength.)

Path difference is the extra distance travelled by one of the waves compared with the other.

For **constructive interference**, the path difference is a whole number of wavelengths $(0, \lambda, 2\lambda, 3\lambda, \text{ and so on})$, or simply path difference = $n\lambda$.

For constructive interference, the waves are always in phase (phase difference = 0°).

For **destructive interference**, the path difference is an odd number of half wavelengths $\left(\frac{1}{2}\lambda, 1\frac{1}{2}\lambda, 2\frac{1}{2}\lambda \text{ and so on}\right)$ or simply $\text{path difference} = \left(n + \frac{1}{2}\right)\lambda$.

For destructive interference, the waves are completely out of phase (e.g. phase difference = 180°).

When light passes through a double-slit, it is diffracted at each slit and an interference pattern of equally spaced light and dark fringes is observed. This can be used to determine the wavelength of light using the equation:

$$\lambda = \frac{ax}{D}$$

This equation can be used for all waves, including sound and microwaves.

A diffraction grating diffracts light at its many slits or lines. The diffracted light interferes in the space beyond the grating.

The equation for a diffraction grating is:

$$d \sin\theta = n\lambda$$

where d is the distance between adjacent lines of the grating θ is the angle between the zeroth order and n^{th}-order maximum, and λ is the wavelength of the light incident normally at the grating.

EXAM-STYLE QUESTIONS

1 Rays of light from two coherent sources produces constructive interference.

Which of the following **cannot** be the phase difference between these two rays? **[1]**

A 0°

B 270°

C 360°

D 720°

2 **a** Copy the waves shown in the diagram onto a sheet of graph paper and use the principle of superposition to sketch the resultant wave. **[2]**

b **Compare** the wavelength of the resultant wave with that of the component waves. **[1]**

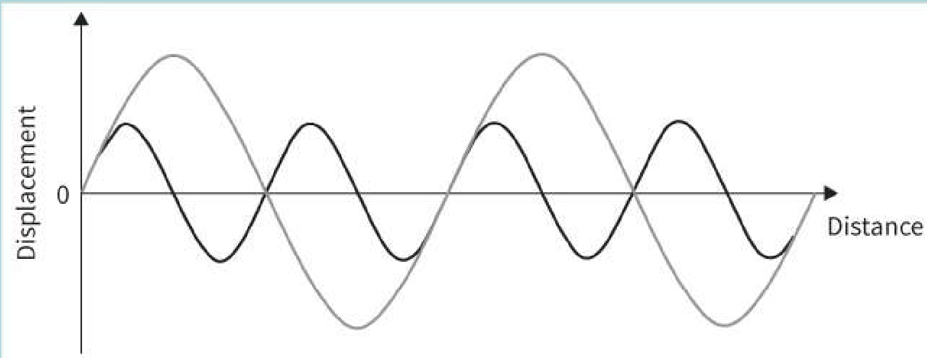

Figure 13.28

[Total: 3]

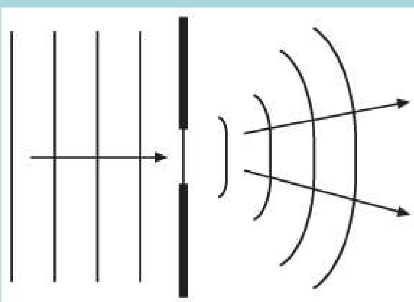

Figure 13.29

3 State how the diffracted pattern will change when:

a the wavelength of the incident wave is increased **[1]**

b the wavelength of the incident wave is decreased. **[1]**

[Total: 2]

4 Explain why, in remote mountainous regions, such as the Hindu Kush, radio signals from terrestrial transmitters can be received, but television reception can only be received from satellite transmissions. **[2]**

5 A constant frequency signal from a signal generator is fed to two loudspeakers placed 1.5 m apart. A student, who is 8.0 m away from the loudspeakers, walks across in a line parallel to the line between the loudspeakers. The student measures the distance between successive spots of loudness to be 1.2 m.

Calculate:

a the wavelength of the sound **[2]**

b the frequency of the sound (assume the speed of sound is 330 m s^{-1}) **[2]**

[Total: 4]

6 Two signal generators feed signals with slightly different frequencies to two separate loudspeakers. Suggest why a sound of continuously rising and falling loudness is heard. **[3]**

7 One of the spectral lines from a hydrogen discharge lamp has wavelength 656 nm. This light is incident normally at a diffraction grating with 5000 lines cm $^{-1}$.

Calculate the angles for the first- and second-order maxima for this light. **[5]**

8 **a** Explain what is meant by the term superposition. **[2]**

b In a double-slit experiment, yellow light of wavelength 590 nm from a sodium discharge tube is used. A student sets up a screen 1.8 m from the double-slit. The distance between 12 bright fringes is measured to be 16.8 mm.

Calculate the separation of the slits. **[3]**

c Describe the effect of:

i using slits of narrower width, but with the same separation **[2]**

ii using slits with a smaller separation, but of the same width. **[2]**

[Total: 9]

9 **a** A laser light is described as producing light that is both highly coherent and highly monochromatic.
Explain what is meant by the terms **coherent** and monochromatic. **[2]**

b This diagram shows the experimental setup (left) used to analyse the spectrum of a sodium discharge lamp with a diffraction grating with 500 lines mm^{-1}, and the spectral lines observed (right) in the developed photographic film.

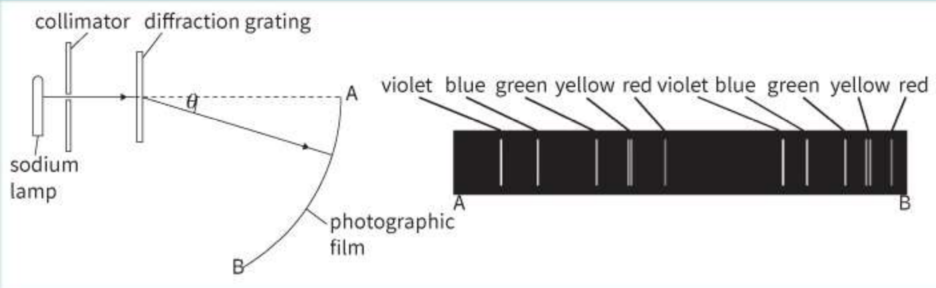

Figure 13.30

i Explain why two spectra are observed. **[2]**

ii Describe **two** differences between these two spectra. **[2]**

iii The green maximum near end A is at an angle θ of 19.5°.

Calculate the wavelength of the green light. **[3]**

iv Calculate the angle produced by the second green line. **[2]**

[Total: 11]

10 **a** Explain what is meant by **destructive interference**. **[2]**

b A student sets up an experiment to investigate the interference pattern formed by microwaves of wavelength 1.5 cm. The apparatus is set up as in Figure 13.17. The distance between the centres of the two slits is 12.5 cm. The detector is centrally placed 1.2 m from the metal plates where it detects a maximum. The student moves the detector 450 cm across the bench parallel to the plates.

Calculate how many maxima the detector will be moved through. **[3]**

c Calculate the frequency of these microwaves. **[2]**

[Total: 7]

11 **a** Explain what is meant by the **diffraction** of a wave. **[2]**

b This diagram shows waves, in a ripple tank, spreading out from two slits.

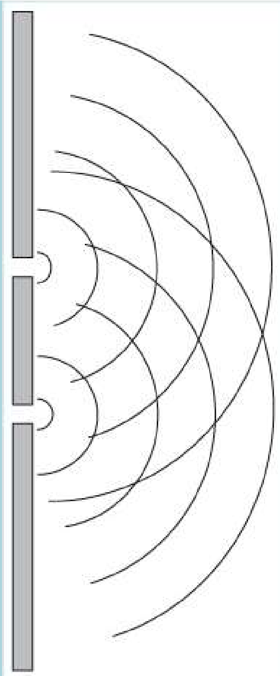

Figure 13.31

Copy this diagram. On your diagram, sketch:

i a line showing points along the central maximum–label this line **0** **[1]**

ii a line showing the points along first maximum–label this line **1** **[1]**

iii a line showing points along one of the first minima–label this line **min**. **[1]**

c The centres of the slits are 12 cm apart. At a distance of 60 cm from the barrier, the first maxima are 18 cm either side of the central maximum. Calculate the wavelength of the waves. (You may assume that the double-slit equation developed for light is applicable to ripples.) **[3]**

[Total: 8]