Chapter 12

# Waves

## LEARNING INTENTIONS

In this chapter you will learn how to:

- describe a progressive wave
- describe the motion of transverse and longitudinal waves
- describe waves in terms of their wavelength, amplitude, frequency, speed, phase difference and intensity
- use the time-base and *y*-gain of a cathode-ray oscilloscope (CRO) to determine frequency and amplitude
- use the wave equation $v = f\lambda$
- use the equations $\text{intensity} = \frac{\text{power}}{\text{area}}$ and intensity ∝ amplitude$^2$
- describe the Doppler effect for sound waves
- use the equation $f_0 = \frac{f_s v}{(v \pm v_s)}$
- describe and understand electromagnetic waves
- recall that wavelengths in the range 400–700 nm in free space are visible to the human eye
- describe and understand polarisation
- use Malus's law to determine the intensity of transmitted light through a polarising filter.

## BEFORE YOU START

- Write down definitions for displacement, speed and power.
- What do you know about the electromagnetic spectrum? Can you name any of the waves in this spectrum? Make a list to share with the class.

## SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

| I can | See topic... | Needs more work | Almost there | Ready to move on |
|---|---|---|---|---|
| understand the concept of internal resistance of a source of e.m.f. | 11.1 | | | |
| solve problems involving internal resistance and e.m.f. and the potential difference across the internal resistance. | 11.1 | | | |
| recognise a potential divider and solve problems using the equation: $V_{\text{out}} = \left(\frac{R_2}{R_1+R_2}\right) \times V_{\text{in}}$ | 11.2, 11.3 | | | |
| use a potentiometer to compare potential differences. | 11.4 | | | |

**b** When resistor $R_1$ alone is tested the length of resistance wire for balance is 15.4 cm. There is an uncertainty in measuring the beginning of the resistance wire of 0.1 cm, and in establishing the balance point of a further 0.1 cm.

**i** Determine the uncertainty in the balance length. **[1]**

When $R_1$ and $R_2$ are tested in series the balance length is 42.6 cm. There are similar uncertainties in measuring this balance length.

**ii** Calculate the ratio of $\frac{R_1}{(R_1+R_2)}$. **[1]**

**iii** Calculate the value of the ration of $\frac{R_1}{R_2}$. **[2]**

**iv** Calculate the uncertainty in the value of the ratio $\frac{R_1}{R_2}$. **[2]**

**[Total: 8]**

**a** Explain **one** advantage circuit 1 has over circuit 2. **[2]**

**b** **i** The lamp is rated at 60 W at 240 V. Calculate the resistance of the lamp filament at its normal operating temperature. **[2]**

**ii** State and explain how the resistance of the filament at room temperature would compare with the value calculated in **i**. **[2]**

**[Total: 6]**

**8** This circuit shows a potential divider. The battery has negligible internal resistance and the voltmeter has infinite resistance.

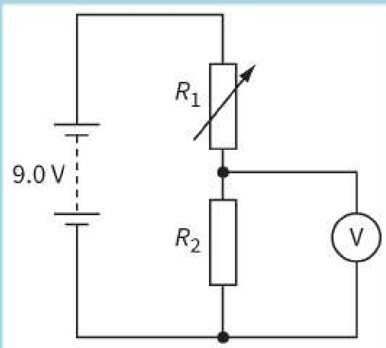



**Figure 11.17**

**a** State and explain how the reading on the voltmeter will change when the resistance of the variable resistor is increased. **[2]**

**b** Resistor $R_2$ has a resistance of 470 Ω. Calculate the value of the variable resistor when the reading on the voltmeter is 2.0 V. **[2]**

**c** The voltmeter is now replaced with one of resistance 2 kΩ. Calculate the reading on this voltmeter. **[2]**

**[Total: 6]**

**9** This is a potentiometer circuit.

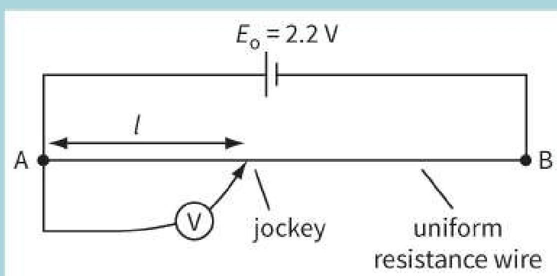



**Figure 11.18**

**a** **i** Sketch a graph of reading on the voltmeter against length, $l$, as the jockey is moved from point A to point B. **[2]**

**ii** State the readings on the voltmeter when the jockey is connected to A and when it is connected to B. (You may assume that the driver cell has negligible internal resistance.) **[1]**

**iii** Draw a circuit diagram to show how the potentiometer could be used to compare the e.m.f.s of two batteries. **[3]**

**b** When a pair of 4 Ω resistors are connected in series with a battery, there is a current of 0.60 A current through the battery. When the same two resistors are connected in parallel and then connected across the battery, there is a current of 1.50 A through it. Calculate the e.m.f. and the internal resistance of the battery. **[4]**

**[Total: 10]**

**10** A potentiometer, which consists of a driving cell connected to a resistance wire of length 100 cm, is used to compare the resistances of two resistors.

**a** Draw a diagram to show the circuits that are used to compare the two resistances. **[2]**

**Figure 11.15**

**a** **i** Explain why he is unable to find a balance point and state the change he must make in order to achieve balance. **[2]**

**ii** State how he would recognise the balance point. **[1]**

**b** He achieves balance when the distance AB is 22.5 cm. He repeats the experiment with a standard cell of e.m.f. of 1.434 V. The balance point using this cell is at 34.6 cm. Calculate the e.m.f. of the test cell. **[2]**

**[Total: 5]**

**5** **a** Explain what is meant by the **internal resistance** of a cell. **[2]**

**b** When a cell is connected in series with a resistor of 2.00 Ω there is a current of 0.625 A. If a second resistor of 2.00 Ω is put in series with the first, the current falls to 0.341 A.

Calculate:

**i** the internal resistance of the cell **[2]**

**ii** the e.m.f. of the cell. **[1]**

**c** A car battery needs to supply a current of 200 A to turn over the starter motor. Explain why a battery made of a series of cells of the type described **b** would not be suitable for a car battery. **[2]**

**[Total: 7]**

**6** **a** State what is meant by the term **e.m.f. of a cell**. **[2]**

A student connects a high-resistance voltmeter across the terminals of a battery and observes a reading of 8.94 V. He then connects a 12 Ω resistor across the terminals and finds that the potential difference falls to 8.40 V.

**b** Explain why the measured voltage falls. **[2]**

**c** **i** Calculate the current in the circuit. **[2]**

**ii** Calculate the internal resistance of the cell. **[2]**

**iii** State any assumptions you made in your calculations. **[1]**

**[Total: 9]**

**7** This diagram shows two circuits that could be used to act as a dimmer switch for a lamp.

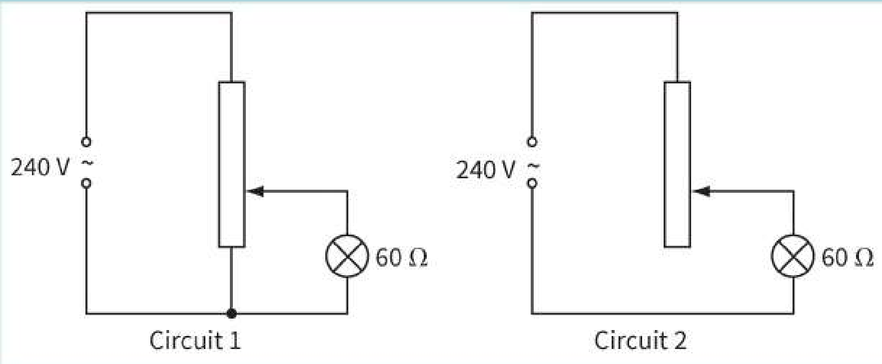



**Figure 11.16**

## EXAM-STYLE QUESTIONS

**1** A resistor of resistance 6.0 Ω and a second resistor of resistance 3.0 Ω are connected in parallel across a battery of e.m.f. 4.5 V and internal resistance 0.50 Ω.

What is the current in the battery? **[1]**

**A** 0.47 A

**B** 1.8 A

**C** 3.0 A

**D** 11 A

**2** This diagram shows a potential divider.

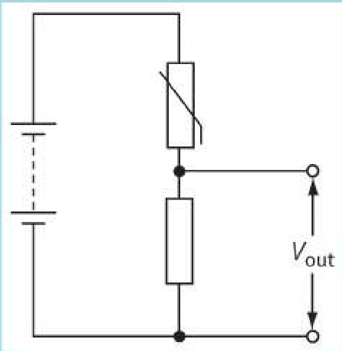



**Figure 11.14**

What happens when the temperature decreases? **[1]**

**A** The resistance of the thermistor decreases and $V_{out}$ decreases.

**B** The resistance of the thermistor decreases and $V_{out}$ increases.

**C** The resistance of the thermistor increases and $V_{out}$ decreases.

**D** The resistance of the thermistor increases and $V_{out}$ increases.

**3** A single cell of e.m.f. 1.5 V is connected across a 0.30 Ω resistor. The current in the circuit is 2.5 A.

**a** Calculate the terminal p.d. and explain why it is not equal to the e.m.f. of the cell. **[2]**

**b** Show that the internal resistance $r$ of the cell is 0.30 Ω. **[3]**

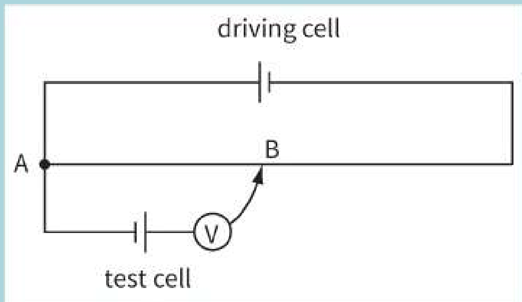
**c** It is suggested that the power dissipated in the external resistor is a maximum when its resistance $R$ is equal to the internal resistance $r$ of the cell.

**i** Calculate the power dissipated when $R = r$. **[1]**

**ii** Show that the power dissipated when $R = 0.50\ \Omega$ and $R = 0.20\ \Omega$ is less than that dissipated when $R = r$, as the statement suggests. **[4]**

**[Total: 10]**

**4** A student is asked to compare the e.m.f.s of a standard cell and a test cell. He sets up the circuit shown using the test cell.



## SUMMARY

A source of e.m.f., such as a battery, has an internal resistance. We can think of the source as having an internal resistance, *r*, in series with an e.m.f., *E*.

The terminal p.d. of a source of e.m.f. is less than the e.m.f. because of the potential difference across the internal resistor:

terminal p.d. = e.m.f. − p.d across the internal resistor

$$V = E - Ir$$

A potential divider circuit consists of two or more resistors connected in series to a supply. The output voltage $V_{\text{out}}$ across the resistor of resistance $R_2$ is given by:

$$V_{\text{out}} = \left(\frac{R_2}{R_1+R_2}\right) \times V_{\text{in}}$$

A potentiometer can be used to compare potential differences.

Since both resistors have the same current flowing through them, the ratio of the p.d.s is also the ratio of their resistances.

## Question

**13** To make a potentiometer, a driver cell of e.m.f. 4.0 V is connected across a 1.00 m length of resistance wire.

- **a** What is the potential difference across each 1 cm length of the wire? What length of wire has a p.d. of 1.0 V across it?
- **b** A cell of unknown e.m.f. $E$ is connected to the potentiometer and the balance point is found at a distance of 37.0 cm from the end of the wire to which the galvanometer is connected. Estimate the value of $E$. Explain why this can only be an estimate.
- **c** A standard cell of e.m.f. 1.230 V gives a balance length of 31.2 cm. Use this value to obtain a more accurate value for $E$.

### REFLECTION

A student sets up a potentiometer circuit to compare the e.m.f.s of two cells. The student is unable to find a balance point.

Discuss with a partner possible reasons for this. Consider using Kirchhoff's Laws as a way of exploring the reasons.

Did you do your work the way other people did theirs? In what ways did you do it differently? In what ways was your work or process similar?

resistance wire into two parts, equivalent to the two resistors of a potential divider.

## Comparing e.m.f.s with a potentiometer

When a potentiometer is balanced, no current flows from the cell being investigated. This means that its terminal p.d. is equal to its e.m.f.; we do not have to worry about the potential difference across the internal resistance. This is a great advantage that a potentiometer has over a voltmeter, which must draw a small current in order to work.

However, there is a problem: the driver cell is supplying current to the potentiometer, and so the p.d. between A and B will be less than the e.m.f. of the driver cell (some volts are lost because of its internal resistance). To overcome this problem, we use the potentiometer to **compare** p.d.s. Suppose we have two cells whose e.m.f.s $E_X$ and $E_Y$ we want to compare. Each is connected in turn to the potentiometer, giving balance points at C and D–see Figure 11.12. (In the diagram, you can see immediately that $E_Y$ must be greater than $E_X$ because D is further to the right than C.)

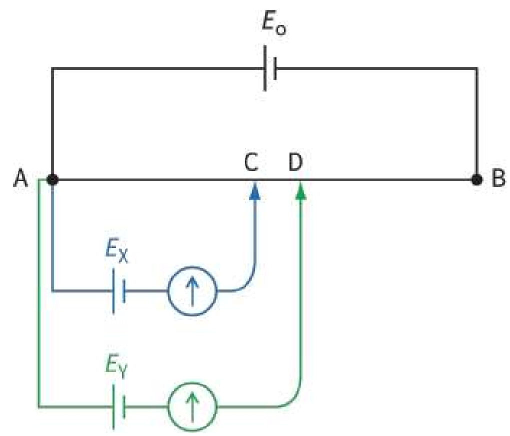



**Figure 11.12:** Comparing two e.m.f.s using a potentiometer.

The ratio of the e.m.f.s of the two cells will be equal to the ratio of the two lengths AC and AD:

$$\frac{E_X}{E_Y} = \frac{AC}{AD}$$

If one of the cells used has an accurately known e.m.f., the other can be calculated with the same degree of accuracy.

## Comparing p.d.s

The same technique can be used to compare potential differences. For example, two resistors could be connected in series with a cell (Figure 11.13). The p.d. across one resistor is first connected to the potentiometer and the balance length found. This is repeated with the other resistor and the new balance point is found. The ratio of the lengths is the ratio of the p.d.s.

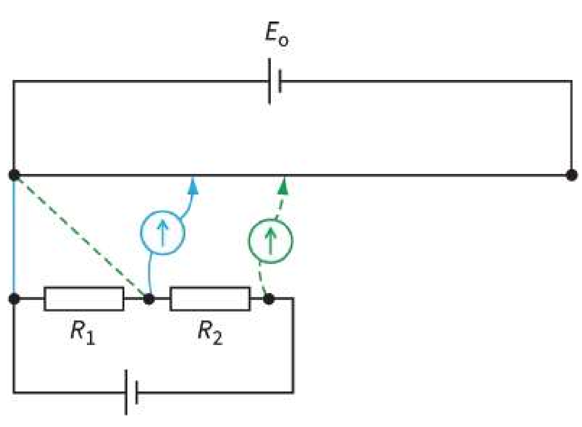



**Figure 11.13:** Comparing two potential differences using a potentiometer.

# 11.4 Potentiometer circuits

A **potentiometer** is a device used for comparing potential differences. For example, it can be used to measure the e.m.f. of a cell, provided you already have a source whose e.m.f. is known accurately. As we will see, a potentiometer can be thought of as a type of potential divider circuit.

A potentiometer consists of a piece of resistance wire, usually 1 m in length, stretched horizontally between two points. In Figure 11.11, the ends of the wire are labelled A and B. A **driver cell** is connected across the length of wire. Suppose this cell has an e.m.f. $E_o$ of 2.0 V. We can then say that point A is at a voltage of 2.0 V, B is at 0 V, and the midpoint of the wire is at 1.0 V. In other words, the voltage decreases steadily along the length of the wire.

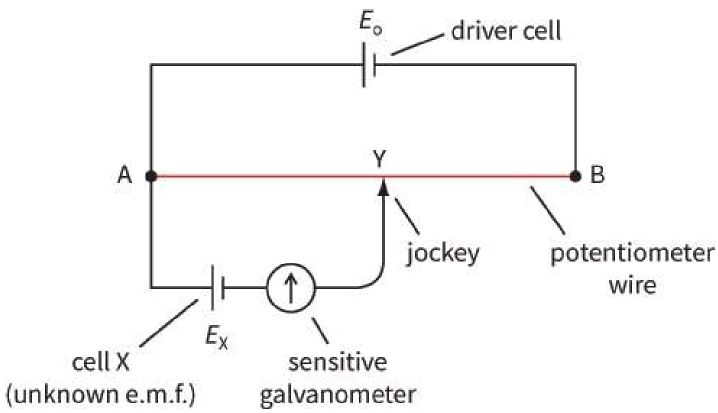



**Figure 11.11:** A potentiometer connected to measure the e.m.f. of cell X.

Now, suppose we wish to measure the e.m.f. $E_X$ of cell X (this must have a value less than that of the driver cell). The positive terminal of cell X is connected to point A. (Note that both cells have their positive terminals connected to A.) A lead from the negative terminal is connected to a sensitive **galvanometer** (such as a microammeter), and a lead from the other terminal of the galvanometer ends with a metal **jockey**. This is a simple connecting device with a very sharp edge that allows very precise positioning on the wire.

If the jockey is touched onto the wire close to point A, the galvanometer needle will deflect in one direction. If the jockey is touched close to B, the galvanometer needle will deflect in the opposite direction. Clearly, there must be some point Y along the wire that, when touched by the jockey, gives zero deflection – the needle moves neither to the left nor the right.

In finding this position, the jockey must be touched gently and briefly onto the wire; the deflection of the galvanometer shows whether the jockey is too far to the left or right. It is important not to slide the jockey along the potentiometer wire as this may scrape its surface, making it non-uniform so that the voltage does not vary uniformly along its length.

When the jockey is positioned at Y, the galvanometer gives zero deflection, showing that there is no current through it. This can only happen if the potential difference across the length of wire AY is equal to the e.m.f. of cell X. We can say that the potentiometer is balanced. If the balance point was exactly half-way along the wire, we would be able to say that the e.m.f. of X was half that of the driver cell. This technique – finding a point where there is a reading of zero – is known as a **null method**.

To calculate the unknown e.m.f. $E_X$ we measure the length AY. Then we have:

$$E_X = \frac{AY}{AB} \times E_o$$

where $E_o$ is the e.m.f. of the driver cell.

**KEY EQUATION**

To compare two e.m.f.s $E_X$ and $E_o$:

$$E_X = \frac{AY}{AB} \times E_o$$

The potentiometer can be thought of as a potential divider because the point of contact Y divides the

## Questions

**8** What is the voltage across the 3.0 kΩ resistor in Figure 11.9 when the light intensity is 10 lux?

**9** The circuit shown in Figure 11.8 produces a decreasing output voltage when the light intensity increases. How can the circuit be altered to produce an increasing output voltage as the light intensity increases?

## Thermistors as a sensors

The thermistors that we refer to in this course are known as **negative temperature coefficient** (NTC) thermistors. This means that, when the temperature rises, the resistance of the thermistor falls. This happens because the thermistor is made from a semiconductor material. One property of a semiconductor is that when the temperature rises the number of free electrons increases, and thus the resistance falls.

Figure 11.10 shows a graph of the resistance of a thermistor and the resistance of a metal wire plotted against temperature. You can see that the resistance of a metal wire increases with increase in temperature. A metal wire is not a negative temperature device, but it could be used as a sensing device. A thermistor is more useful than a metal wire because there is a much larger change in resistance with change in temperature. However, the change in resistance of a thermistor is not linear with temperature; indeed, it is likely to be an exponential decrease. This means that any device used to measure temperature electronically must be calibrated to take into account the resistance–temperature graph. The scale on an ordinary laboratory thermometer between 0 °C and 100 °C is divided up into 100 equal parts, each of which represents 1 °C. If the resistance of a thermistor were divided like this, the scale would be incorrect.

The thermistor can be used as a sensing device in the same way as an LDR. Instead of sensing a change in light level, it senses a change in temperature.

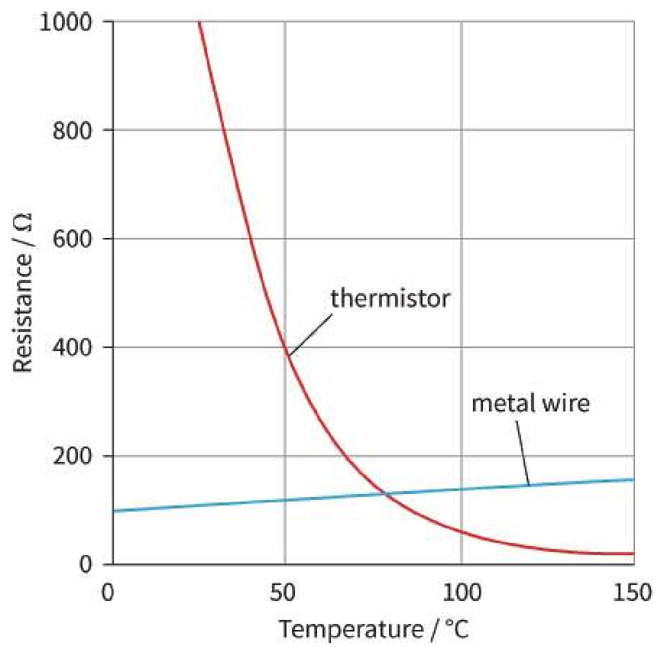



**Figure 11.10:** Variation of resistance with temperature.

## Questions

**10** Explain how a thermistor can be used as a transducer.

**11** State **two** similarities between an LDR and a thermistor.

**12** Design a circuit using the thermistor in Figure 11.10 that uses a cell of 10 V and produces an output voltage of 5 V at 50 °C. Explain whether the voltage output of your circuit increases or decreases as the temperature rises.

# 11.3 Sensors

## Light-dependent resistors as sensors

How is a light-dependent resistor (LDR) used as a **sensor** or **transducer**? A voltage is needed to drive the output device, such as a voltmeter, yet the LDR only produces a change in resistance. The sensor must use this change in resistance to generate the change in voltage. The solution is to place the LDR in series with a fixed resistor, as shown in Figure 11.8.

The voltage of the supply is shared between the two resistors in proportion to their resistance so, as the light level changes and the LDR's resistance changes, so does the voltage across each of the resistors. The two resistors form a potential divider whose output changes automatically with changing light intensities.

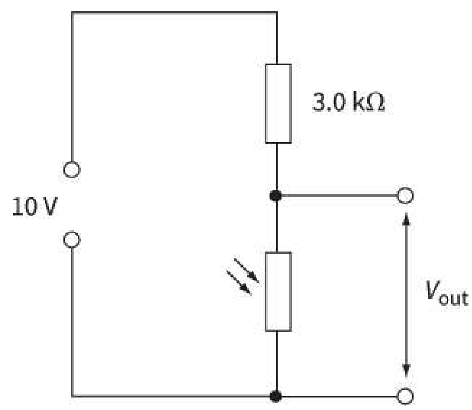



**Figure 11.8:** An LDR used as a sensor.

**WORKED EXAMPLE**

**2** Using the graph in Figure 11.9, calculate $V_{out}$ in Figure 11.8 when the light intensity is 60 lux.

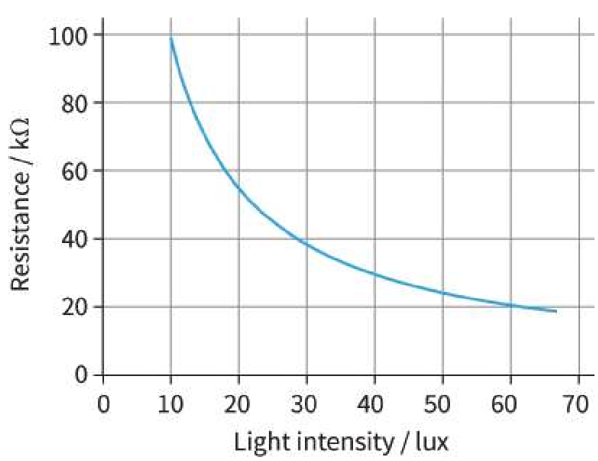



**Figure 11.9:** for Worked example 2 and question 8.

**Step 1** Find the resistance of the LDR at 60 lux.

$R_{LDR} = 20\ k\Omega$

**Step 2** Divide the total voltage of 10 V in the ratio 3 : 20. The total number of parts is 23 so:

$$V_{out} = \frac{20}{23} \times 10 = 8.70\text{V}$$

**Hint:** *The answer on your calculator might be 8.69565. When you give your answer to three significant figures, do not write 8.69 – you must round correctly.*

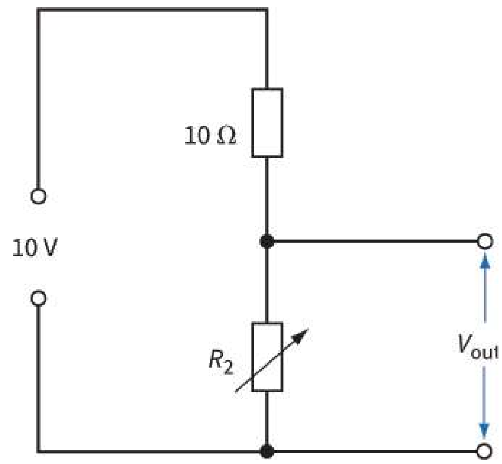



**Figure 11.7:** For Question 7.

# 11.2 Potential dividers

How can we get an output of 3.0 V from a battery of e.m.f. 6.0 V? Sometimes we want to use only part of the e.m.f. of a supply. To do this, we use an arrangement of resistors called a **potential divider** circuit.

Figure 11.6 shows two potential divider circuits, each connected across a battery of e.m.f. 6.0 V and of negligible internal resistance. The high-resistance voltmeter measures the voltage across the resistor of resistance $R_2$. We refer to this voltage as the output voltage, $V_{out}$, of the circuit. The first circuit, **a**, consists of two resistors of values $R_1$ and $R_2$. The voltage across the resistor of resistance $R_2$ is half of the 6.0 V of the battery. The second potential divider, **b**, is more useful. It consists of a single variable resistor. By moving the sliding contact, we can achieve any value of $V_{out}$ between 0.0 V (slider at the bottom) and 6.0 V (slider at the top).

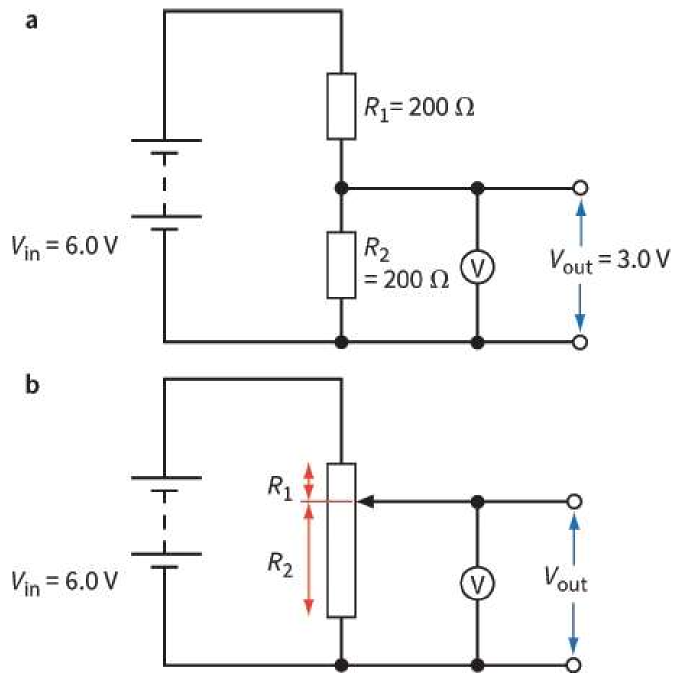



**Figure 11.6:** Two potential divider circuits.

The output voltage $V_{out}$ depends on the relative values of $R_1$ and $R_2$. You can calculate the value of $V_{out}$ using the **potential divider equation**:

$$V_{out} = \left(\frac{R_2}{R_1+R_2}\right) \times V_{in}$$

where $R_2$ is the resistance of the component over which the output is taken, $R_1$ is the resistance of the second component in the potential divider and $V_{in}$ is the p.d. across the two components.

**KEY EQUATION**

Potential divider equation:

$$V_{out} = \left(\frac{R_2}{R_1+R_2}\right) \times V_{in}$$

## Question

7 Determine the range of $V_{out}$ for the circuit in Figure 11.7 as the variable resistor $R_2$ is adjusted over its full range from 0 Ω to 40 Ω. (Assume the supply of e.m.f. 10 V has negligible internal resistance.)

You cannot ignore the effects of internal resistance. Consider a battery of e.m.f. 3.0 V and of internal resistance 1.0 Ω. The **maximum current** that can be drawn from this battery is when its terminals are shorted-out. (The external resistance $R \approx 0$.) The maximum current is given by:

$$\begin{aligned} \text{maximum current} &= \frac{E}{r} \\ &= \frac{3.0}{1.0} \\ &= 3.0\ \text{A} \end{aligned}$$

The **terminal p.d.** of the battery depends on the resistance of the external resistor. For an external resistor of resistance 1.0 Ω, the terminal p.d. is 1.5 V – half of the e.m.f. The terminal p.d. approaches the value of the e.m.f. when the external resistance $R$ is very much greater than the internal resistance of the battery. For example, a resistor of resistance 1000 Ω connected to the battery gives a terminal p.d. of 2.997 V. This is almost equal to the e.m.f. of the battery. The more current a battery supplies, the more its terminal p.d. will decrease. An example of this can be seen when a driver tries to start a car with the headlamps on. The starter motor requires a large current from the battery, the battery's terminal p.d. drops and the headlamps dim.

## Question

**6** A car battery has an e.m.f. of 12 V and an internal resistance of 0.04 Ω. The starter motor draws a current of 100 A.

- **a** Calculate the terminal p.d. of the battery when the starter motor is in operation.
- **b** Each headlamp is rated as '12 V, 36 W'. Calculate the resistance of a headlamp.
- **c** To what value will the power output of each headlamp decrease when the starter motor is in operation? (Assume that the resistance of the headlamp remains constant.)

**ii**

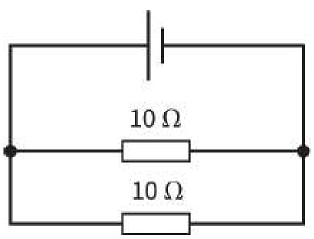



**Figure 11.4:** For Question 2.

**3** Four identical cells, each of e.m.f. 1.5 V and internal resistance 0.10 Ω, are connected in series. A lamp of resistance 2.0 Ω is connected across the four cells. Calculate the current in the lamp.

**PRACTICAL ACTIVITY 12.1**

### Determining e.m.f. and internal resistance

You can get a good idea of the e.m.f. of an isolated power supply or a battery by connecting a digital voltmeter across it. A digital voltmeter has a very high resistance (~$10^7$ Ω), so only a tiny current will pass through it. The potential difference across the internal resistance will then only be a tiny fraction of the e.m.f. If you want to determine the internal resistance $r$ as well as the e.m.f. $E$, you need to use a circuit like that shown in Figure 11.2. When the variable resistor is altered, the current in the circuit changes and measurements can be recorded of the circuit current $I$ and terminal p.d. $V$. The internal resistance $r$ can be found from a graph of $V$ against $I$ (Figure 11.5).

Compare the equation $V = E - Ir$ with the equation of a straight line $y = mx + c$. By plotting $V$ on the $y$-axis and $I$ on the $x$-axis, a straight line should result. The intercept on the $y$-axis is $E$, and the gradient is $-r$.

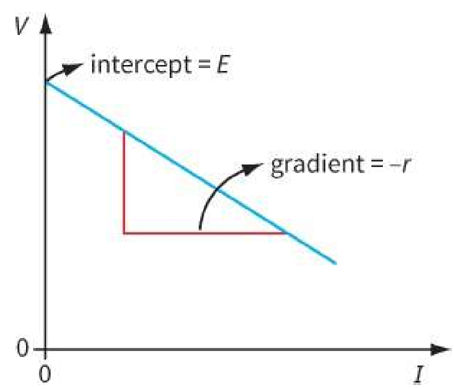



**Figure 11.5:** $E$ and $r$ can be found from this graph.

## Questions

**4** When a high-resistance voltmeter is placed across an isolated battery, its reading is 3.0 V. When a 10 Ω resistor is connected across the terminals of the battery, the voltmeter reading drops to 2.8 V. Use this information to determine the internal resistance of the battery.

**5** The results of an experiment to determine the e.m.f. $E$ and internal resistance $r$ of a power supply are shown in Table 11.1. Plot a suitable graph and use it to find $E$ and $r$.

| **V / V** | 1.43 | 1.33 | 1.18 | 1.10 | 0.98 |
|---|---|---|---|---|---|
| **I / A** | 0.10 | 0.30 | 0.60 | 0.75 | 1.00 |

**Table 11.1:** Results for Question 5.

## The effects of internal resistance

can see that $R$ and $r$ are in series with each other. The current $I$ is the same for both of these resistors. The combined resistance of the circuit is thus $R + r$, and we can write:

$$E = I(R + r) \quad \text{or} \quad E = IR + Ir$$

We cannot measure the e.m.f. $E$ of the cell directly, because we can only connect a voltmeter across its terminals. This **terminal p.d.** $V$ across the cell is always the same as the p.d. across the external resistor.

Therefore, we have:

$$V = IR$$

This will be less than the e.m.f. $E$ by an amount $Ir$. The quantity $Ir$ is the potential difference across the internal resistor. If we combine these two equations, we get:

$$V = E - Ir$$

where $E$ is the emf of the source, $I$ is the current in the source and $r$ is the internal resistance of the source.

or

terminal p.d. = e.m.f. − p.d across the internal resistance

The potential difference across the internal resistance indicates the energy transferred to the internal resistance of the supply. If you short-circuit a battery with a piece of wire, a large current will flow, and the battery will get warm as energy is transferred within it. This is also why you may damage a power supply by trying to make it supply a larger current than it is designed to give.

**KEY EQUATION**

Potential difference across a power source:

$$V = E - Ir$$

**WORKED EXAMPLE**

**1** There is a current of 0.40 A when a battery of e.m.f. 6.0 V is connected to a resistor of 13.5 Ω. Calculate the internal resistance of the cell.

**Step 1** Substitute values from the question in the equation for e.m.f.:

$$E = 6.0\text{ V}, \quad I = 0.40\text{ A}, \quad R = 13.5\,\Omega$$

$$E = IR + Ir$$

$$\begin{aligned} 6.0 &= 0.40 \times 13.5 + 0.40 \times r \\ &= 5.4 + 0.40r \end{aligned}$$

**Step 2** Rearrange the equation to make $r$ the subject and solve:

$$\begin{aligned} 6.0 - 5.4 &= 0.40r \\ 0.60 &= 0.40r \\ r &= \tfrac{0.60}{0.40} = 1.5\,\Omega \end{aligned}$$

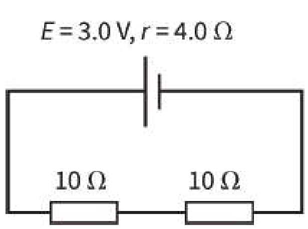
## Questions

**1** A battery of e.m.f. 5.0 V and internal resistance 2.0 Ω is connected to an 8.0 Ω resistor. Draw a circuit diagram and calculate the current in the circuit.

**2** **a** Calculate the current in each circuit in Figure 11.4.

**b** Calculate also the potential difference across the internal resistance for each cell, and the terminal p.d.

**i**



# 11.1 Internal resistance

You will have learnt that, when you use a power supply or other source of e.m.f., you cannot assume that it is providing you with the exact voltage across its terminals as suggested by the value of its e.m.f. There are several reasons for this. For example, the supply may not be made to a high degree of precision, or the batteries may have become flat, and so on. However, there is a more important factor, which is that all sources of e.m.f. have an **internal resistance**. For a power supply, this may be due to the wires and components inside, whereas for a cell the internal resistance is due to the chemicals within it. Experiments show that the voltage across the terminals of the power supply depends on the circuit of which it is part. In particular, the voltage across the power supply terminals decreases if it is required to supply more current.

Figure 11.2 shows a circuit you can use to investigate this effect, and a sketch graph showing how the voltage across the terminals of a power supply might decrease as the supplied current increases.

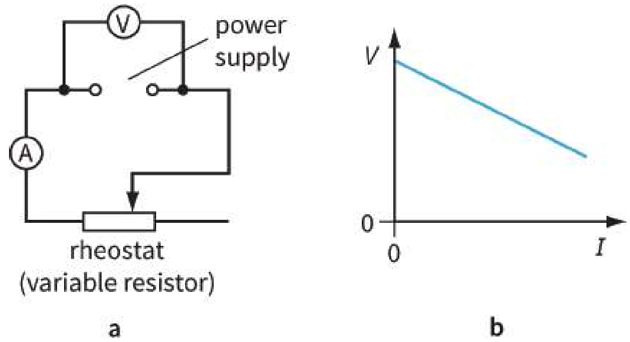



**Figure 11.2: a** A circuit for determining the e.m.f. and internal resistance of a supply; **b** typical form of results.

The charges moving round a circuit have to pass through the external components **and** through the internal resistance of the power supply. These charges gain electrical energy from the power supply. This energy is lost as thermal energy as the charges pass through the external components and through the internal resistance of the power supply. Power supplies and batteries get warm when they are being used. (Try using a cell to light a small torch bulb; feel the cell before connecting to the bulb, and then feel it again after the bulb has been lit for about 15 seconds.)

The reason for this heating effect is that some of the electrical potential energy of the charges is transformed to internal energy as they do work against the internal resistance of the cell.

It can often help to solve problems if we show the internal resistance $r$ of a source of e.m.f. explicitly in circuit diagrams (Figure 11.3). Here, we are representing a cell as if it were a 'perfect' cell of e.m.f. $E$, together with a separate resistor of resistance $r$. The dashed line enclosing $E$ and $r$ represents the fact that these two are, in fact, a single component.

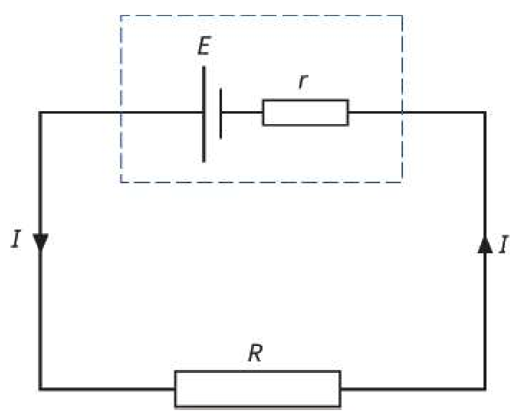



**Figure 11.3:** It can be helpful to show the internal resistance $r$ of a cell (or a supply) in a circuit diagram.

Now we can determine the current when this cell is connected to an external resistor of resistance $R$. You

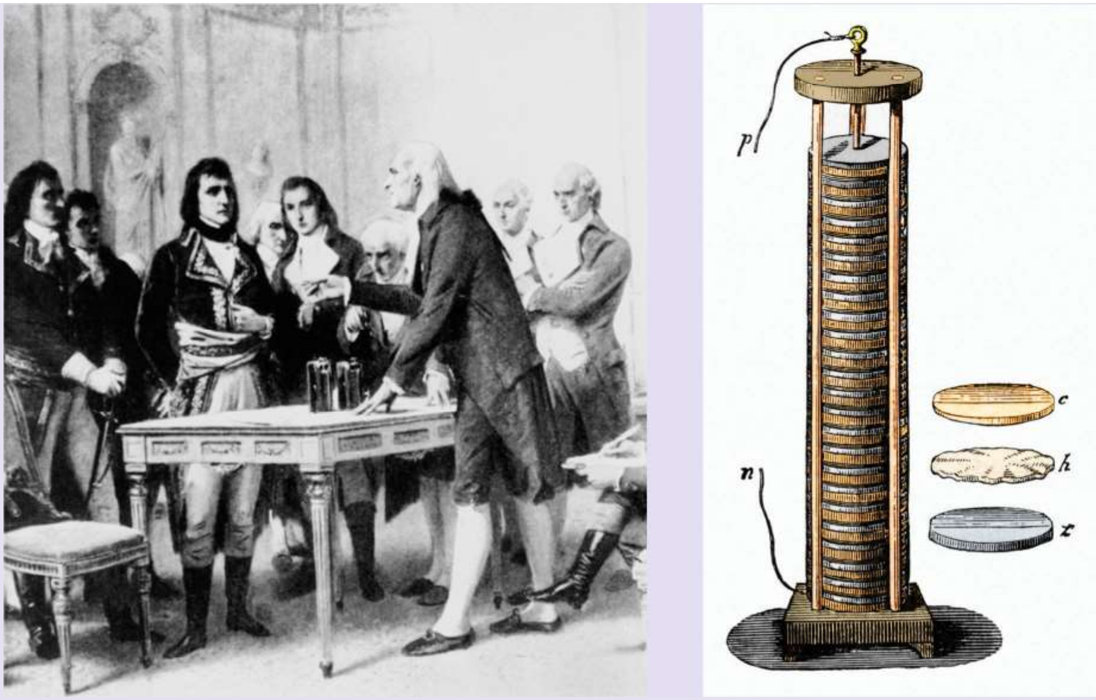



**Figure 11.1: a** Alessandro Volta demonstrating his newly invented pile (battery) to the French Emperor Napoleon. **b** Volta's pile, showing (top to bottom) discs of copper, wet felt and zinc.

However, there is evidence that earlier technologists may have beaten him by over 1000 years. In 1936, a small pot was discovered during an archaeological dig near Baghdad. The pot was sealed with pitch, and inside the pot there was a copper cylinder surrounding an iron rod. When filled with an acid, perhaps vinegar, a potential difference of around 1.5 volts could be produced between the copper and the iron.

It has been suggested that this battery might have been used to electroplate metal objects with gold. So, did Volta really invent the battery, or did he just rekindle an art that had been lost for more than a millennium?

Chapter 11

# Practical circuits

## LEARNING INTENTIONS

In this chapter you will learn how to:

- explain the effects of internal resistance on terminal p.d. and power output of a source of e.m.f.
- explain the use of potential divider circuits
- solve problems involving the potentiometer as a means of comparing voltages.

## BEFORE YOU START

How confident are you on the concepts of terminal potential difference and e.m.f.? Without looking at a textbook, either write down the meaning of each or discuss it with a partner. This will help you in the first part of this chapter, which further develops the idea of e.m.f. and illustrates why the terminal p.d. and the e.m.f. are different.

## THE FIRST ELECTRICAL CELL: AN HISTORICAL MYSTERY

The Italian Alessandro Volta (Figure 11.1a) is generally credited with inventing the first battery. He devised it after his friend and rival Luigi Galvani had shown that a (dead) frog's leg could be made to twitch if an electrically charged plate was connected to it. Volta's battery consisted of alternate discs of copper and zinc, separated by felt soaked in brine–see Figure 11.1b.

## SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

| I can | See topic... | Needs more work | Almost there | Ready to move on |
|---|---|---|---|---|
| state and understand Ohm's law | 10.2 | | | |
| recognise ohmic and non-ohmic components | 10.2, 10.3 | | | |
| recognise and understand the changes in the resistance of metals and thermistors when there is a change in their temperature | 10.3, 10.4 | | | |
| understand that a light-dependent resistor is a component whose resistance decreases as the light level increases | 10.3 | | | |
| understand that resistivity $\rho$ of a material is defined as: $$\rho = \frac{RA}{L}$$ where $R$ is the resistance of a wire of that material, $A$ is its cross-sectional area and $L$ is its length. The unit of resistivity is the ohm metre (Ω m). | 10.4 | | | |

**c** When the potential difference across the safety resistor is 1.4 V, the current in it is 20 mA. Calculate the resistance of the safety resistor. **[2]**

**[Total: 6]**

**11 a** Explain what is meant by an **ohmic conductor**. **[2]**

**b i** Sketch a graph of resistance *R* against voltage *V* for a wire of pure iron kept at constant temperature. Label this line X. **[1]**

**ii** Sketch a graph of resistance *R* against voltage *V* for a second wire of impure iron, of the same diameter and the same length, which is kept at the same temperature. Label this line Y. **[1]**

**iii** Explain how the graphs would change if the wires were kept at a higher, but still constant, temperature. **[1]**

**c** Deduce how the resistance of a wire made of pure iron would change if both the diameter and the length were doubled. **[3]**

**[Total: 8]**

**12** The readings in this table are recorded from an experiment to measure the resistivity of silver.

| | |
|---|---|
| Diameter of the wire | 0.40 ± 0.02 mm |
| Length of the wire | 2.25 ± 0.05 m |
| Resistance of the wire | 0.28 ± 0.01 Ω |

**Table 10.4**

**a** Calculate the resistivity of silver. **[2]**

**b i** Calculate the percentage uncertainty in each of the variables. **[2]**

**ii** Use your answers to **i** to calculate the absolute uncertainty in the value of the resistivity obtained in the experiment. **[2]**

**[Total: 6]**

**b** When switch S is closed the current in the ammeter increases to 0.72 A.

**i** Determine the current in the 6.4 Ω resistor. **[1]**

**ii** State the current in the thermistor. **[1]**

**c** State and explain how the reading on the ammeter changes when the temperature of the thermistor is increased. **[3]**

**[Total: 7]**

**8** **a** Explain why the resistance of a metal increases when its temperature increases. **[2]**

**b** State **two** other factors that determine the resistance of a stated length of wire. **[2]**

**c** When a potential difference of 1.5 V is applied across a 5.0 m length of insulated copper wire, a current of 0.24 A is measured in it.

**i** Calculate the resistance of the length of wire. **[2]**

**ii** The resistivity of copper is $1.69 \times 10^{-8}$ Ω m. Calculate the diameter of the wire. **[3]**

**d** The wire is now made into a tight bundle. State and explain how you would expect the current in it to change. **[3]**

**[Total: 12]**

**9** This diagram shows a piece of silicon of width 32 mm and length 36 mm. The resistance of the silicon between the points P and Q is 1.1 MΩ. Silicon has a resistivity of $2.3 \times 10^{3}$ Ω m.

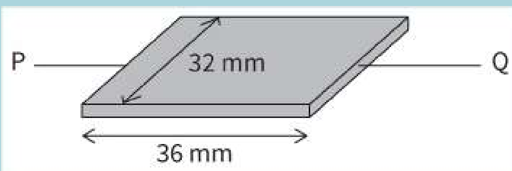



**Figure 10.16**

**a** Calculate the thickness of the piece of silicon. **[3]**

**b** Calculate the current that would pass through the silicon if a potential difference of 12 V were applied across P and Q. **[2]**

**c** Describe how the current would change if it were large enough to cause the silicon to become significantly warmer. **[3]**

**[Total: 8]**

**10** A student is investigating the properties of a semiconducting diode. This diagram shows the circuit she builds.

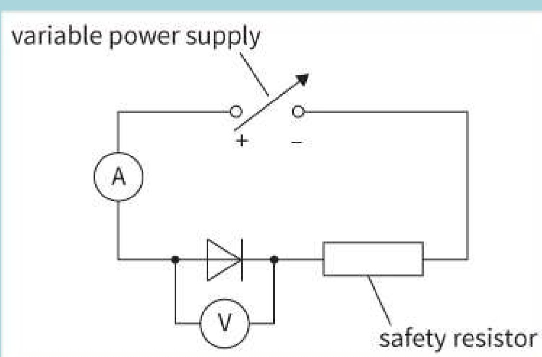



**Figure 10.16**

**a** **i** Sketch a graph to show how the current in the diode would vary as the voltage across it is increased from 0 V to 1.0 V. **[1]**

**ii** The supply is now connected in the reverse direction and once more the potential difference across the diode is increased from 0 V to 1.0 V. Complete the *I*–*V* graph. **[1]**

**b** Suggest why the safety resistor is required. **[2]**

**a** Calculate the resistance of the component when the potential difference across it is:

**i** 2.0 V **[2]**

**ii** 5.0 V. **[1]**

**b** Suggest what the component is. **[1]**

**[Total: 4]**

**4** A student connects a thermistor to a battery and an ammeter. He places the thermistor in a beaker of water and gradually heats the water from 10 °C to its boiling point, recording the value of the current as he does so. He then plots a graph of the current in the thermistor against the temperature of the water.

**a** Sketch the graph you would expect the student to obtain from the experiment. **[1]**

**b** Explain how the student could now use the thermistor as a thermometer. **[2]**

**[Total: 3]**

**5** **a** Describe the difference between the conduction processes in copper and in silicon, a semiconductor. **[3]**

**b** Explain why the resistance of a metallic conductor increases with temperature while that of a semiconductor decreases. **[3]**

**[Total: 6]**

**6** A nichrome wire has a length of 1.5 m and a cross-sectional area of 0.0080 $mm^2$. The resistivity of nichrome is $1.30 \times 10^{-8}$ Ω m.

**a** Calculate the resistance of the wire. **[2]**

**b** Calculate the length of this wire that would be needed to make an element of an electric heater of resistance 30 Ω. **[2]**

**[Total: 4]**

**7** This is a circuit.

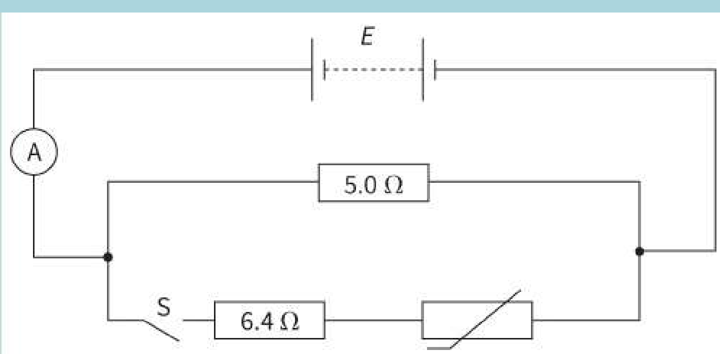



**Figure 10.15**

**a** When switch S is open the current in ammeter A is 0.48 A. Calculate the e.m.f. of the battery. You may assume the battery has negligible internal resistance. **[2]**

## EXAM-STYLE QUESTIONS

**1** An element of an electric fire is made up from a length of nichrome wire of diameter 0.40 mm and length 5.0 m.

The resistance of this element is $R_1$.

Another element, also made from nichrome, for a different electric fire, has a length of 2.0 m and a diameter of 0.20 mm. This element has a resistance of $R_2$.

What is the relationship between $R_1$ and $R_2$? **[1]**

**A** $R_2 = 0.80\ R_1$

**B** $R_2 = 1.6\ R_1$

**C** $R_2 = 5.0\ R_1$

**D** $R_2 = 10\ R_1$

**2** This is a circuit.

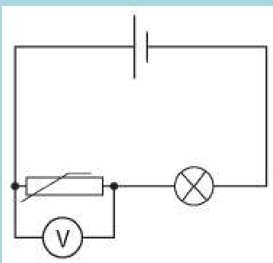



**Figure 10.13**

Which line in the table shows the changes to the lamp and the voltmeter reading when the temperature rises? **[1]**

| | **Lamp** | **Voltmeter reading** |
|---|---|---|
| A | gets brighter | decreases |
| B | gets brighter | increases |
| C | gets dimmer | decreases |
| D | gets dimmer | increases |

**Table 10.3**

**3** This shows the *I*–*V* characteristic of an electrical component.

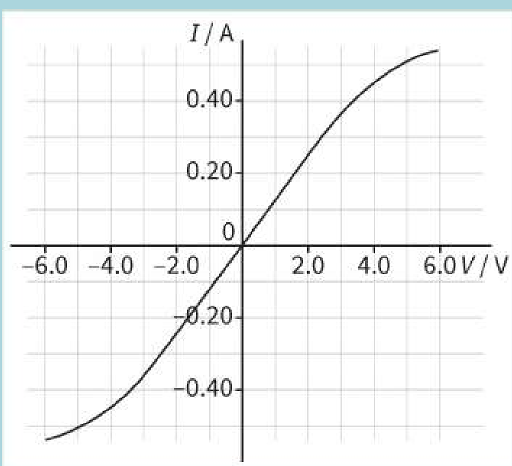



**Figure 10.14**

## SUMMARY

A conductor obeys Ohm's law if the current in it is directly proportional to the potential difference across its ends.

Ohmic components include a wire at constant temperature and a resistor.

Non-ohmic components include a filament lamp and a light-emitting diode.

A semiconductor diode allows current in one direction only.

As the temperature of a metal increases, so does its resistance.

A thermistor is a component that shows a rapid change in resistance over a narrow temperature range. The resistance of an NTC thermistor decreases as its temperature is increased.

The resistivity $\rho$ of a material is defined as:

$$\rho = \frac{RA}{L}$$

where $R$ is the resistance of a wire of that material, $A$ is its cross-sectional area and $L$ is its length. The unit of resistivity is the ohm metre (Ω m).

Resistivity, like resistance, depends on temperature. For a metal, resistivity increases with temperature. As we saw earlier, this is because there are more frequent collisions between the conduction electrons and the vibrating ions of the metal.

## Questions

**8** Use the resistivity value quoted in Table 10.2 to calculate the lengths of 0.50 mm diameter manganin wire needed to make resistance coils with resistances of:

- **a** 1.0 Ω
- **b** 5.0 Ω
- **c** 10 Ω.

**9** 1.0 $cm^3$ of copper is drawn out into the form of a long wire of cross-sectional area $4.0 \times 10^{-7}$ $m^2$. Calculate its resistance. (Use the resistivity value for copper from Table 10.2.)

**10** A 1.0 m length of copper wire has a resistance of 0.50 Ω.

- **a** Calculate the resistance of a 5.0 m length of the same wire.
- **b** What will be the resistance of a 1.0 m length of copper wire having half the diameter of the original wire?

**11** A piece of steel wire has a resistance of 10 Ω. It is stretched to twice its original length. Compare its new resistance with its original resistance.

**REFLECTION**

Imagine you are helping a younger cousin who is studying for her IGCSE (or similar course). She finds it difficult to understand why the resistivity does not change when the dimensions of a sample are changed, but resistance does.

Think about how you might help her understand.

Now that it is completed, what are your first thoughts about this activity? Are they mostly positive or negative?

$$\text{resitivity} = \frac{\text{resistance} \times \text{cross-sectional area}}{\text{length}}$$

$$\rho = \frac{RA}{L}$$

Values of the resistivities of some typical materials are shown in Table 10.2. Notice that the units of resistivity are ohm metres (Ω m); this is not the same as ohms per metre.

| **Material** | **Resistivity / Ω m** |
|---|---|
| silver | $1.60 \times 10^{-8}$ |
| copper | $1.69 \times 10^{-8}$ |
| nichrome[(a)] | $1.30 \times 10^{-8}$ |
| aluminium | $3.21 \times 10^{-8}$ |
| lead | $20.8 \times 10^{-8}$ |
| manganin[(b)] | $44.0 \times 10^{-8}$ |
| eureka[(c)] | $49.0 \times 10^{-8}$ |
| mercury | $69.0 \times 10^{-8}$ |
| graphite | $800 \times 10^{-8}$ |
| germanium | 0.65 |
| silicon | $2.3 \times 10^{3}$ |
| Pyrex glass | $10^{12}$ |
| PTFE[(d)] | $10^{13}$–$10^{16}$ |
| quartz | $5 \times 10^{16}$ |

[(a)] Nichrome – an alloy of nickel, copper and aluminium used in electric heaters because it does not oxidise at 1000 °C.

[(b)] Manganin – an alloy of 84% copper, 12% manganese and 4% nickel.

[(c)] Eureka (constantan) – an alloy of 60% copper and 40% nickel.

[(d)] PTFE – Poly(tetrafluoroethene) or Teflon.

**Table 10.2:** Resistivities of various materials at 20 °C.

**WORKED EXAMPLE**

**1** Find the resistance of a 2.6 m length of eureka wire with cross-sectional area $2.5 \times 10^{-7}$ m$^2$.

**Step 1** Use the equation for resistance:

$$\text{resistance} = \frac{\text{resistivity} \times \text{length}}{\text{cross-sectional area}}$$

$$R = \frac{\rho L}{A}$$

**Step 2** Substitute values from the question and use the value for $\rho$ from Table 10.2:

$$R = \frac{49.0 \times 10^{-8} \times 2.6}{2.5 \times 10^{-7}} = 5.1\Omega$$

So the wire has a resistance of 5.1 Ω.

## Resistivity and temperature

# 10.4 Resistivity

The resistance of a particular wire depends on its size and shape. A long wire has a greater resistance than a short one, provided it is of the same thickness and material. A thick wire has less resistance than a thin one. For a metal in the shape of a wire, $R$ depends on the following factors:

- length $L$
- cross-sectional area $A$
- the material the wire is made from
- the temperature of the wire.

At a constant temperature, the resistance is directly proportional to the length of the wire and inversely proportional to its cross-sectional area:

$$\text{resistance} \propto \text{length}$$

and

$$\text{resistance} \propto \frac{1}{\text{cross-sectional area}}$$

We can see how these relate to the formulae for adding resistors in series and in parallel:

- If we double the length of a wire it is like connecting two identical resistors in series; their resistances add to give double the resistance. The resistance is proportional to the length.
- Doubling the cross-sectional area of a wire is like connecting two identical resistors in parallel; their combined resistance is halved (since $\frac{1}{R_{\text{total}}} = \frac{1}{R} + \frac{1}{R}$).

  Hence the resistance is inversely proportional to the cross-sectional area.

Combining the two proportionalities for length and cross-sectional area, we get:

$$\text{resistance} \propto \frac{1}{\text{cross-sectional area}}$$

or

$$R \propto \frac{L}{A}$$

But the resistance of a wire also depends on the material it is made of. Copper is a better conductor than steel, steel is a better conductor than silicon, and so on. So if we are to determine the resistance $R$ of a particular wire, we need to take into account its length, its cross-sectional area and the material. The relevant property of the material is its **resistivity**, for which the symbol is $\rho$ (Greek letter **rho**).

The word equation for resistance is:

$$\begin{aligned} \text{resistance} &= \frac{\text{resistivity} \times \text{length}}{\text{cross-sectional area}} \\ R &= \frac{\rho L}{A} \end{aligned}$$

**KEY EQUATION**

$$\begin{aligned} \text{resistance} &= \frac{\text{resistivity} \times \text{length}}{\text{cross-sectional area}} \\ R &= \frac{\rho L}{A} \end{aligned}$$

We can rearrange this equation to give an equation for resistivity. The resistivity of a material is defined by the following word equation:

$$\begin{aligned} \text{resitivity} &= \frac{\text{resistance} \times \text{cross-sectional area}}{\text{length}} \\ \rho &= \frac{RA}{L} \end{aligned}$$

**KEY EQUATION**

- the temperature
- the presence of impurities.

Figure 10.12 shows a simple model that explains what happens in a metal when electrons flow through it.

In a metal, a current is due to the movement of free electrons. At low temperatures, they can move easily past the positive ions (Figure 10.12a). However, as the temperature is raised, the ions vibrate with larger amplitudes. The electrons collide more frequently with the vibrating ions, and this decreases their mean drift velocity. They lose energy to the vibrating ions (Figure 10.12b).

If the metal contains impurities, some of the atoms will be of different sizes (Figure 10.12c). Again, this disrupts the free flow of electrons. In colliding with impurity atoms, the electrons lose energy to the vibrating atoms.

You can see that electrons tend to lose energy when they collide with vibrating ions or impurity atoms. They give up energy to the metal, so it gets hotter. The resistance of the metal increases with the temperature of the wire because of the decrease in the mean drift velocity of the electrons.

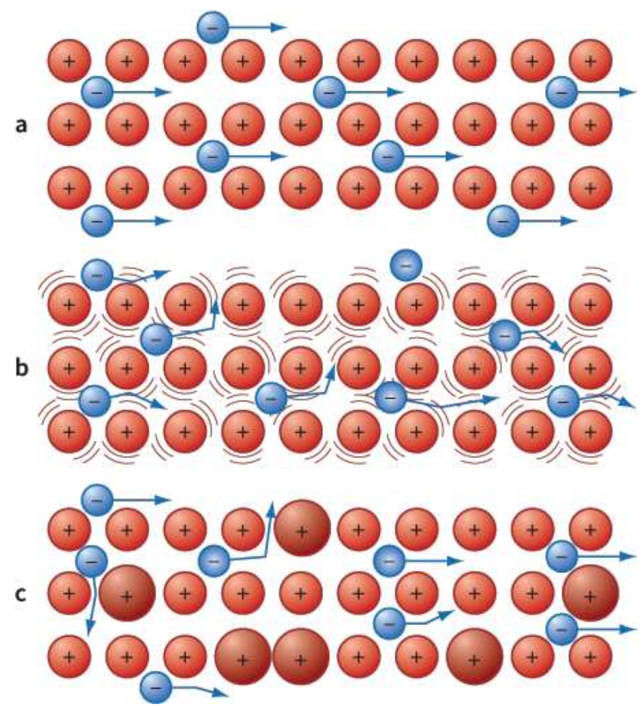



**Figure 10.12:** A model of the origins of resistance in a metal. **a:** At low temperatures, electrons flow relatively freely. **b:** At higher temperatures, the electrons are obstructed by the vibrating ions and they make very frequent collisions with the ions. **c:** Impurity atoms can also obstruct the free flow of electrons.

Conduction in semiconductors is different. At low temperatures, there are few **delocalised**, or free, electrons. For conduction to occur, electrons must have sufficient energy to free themselves from the atom they are bound to. As the temperature increases, a few electrons gain enough energy to break free of their atoms to become conduction electrons. The number of conduction electrons thus increases and so the material becomes a better conductor. At the same time, there are more electron–ion collisions, but this effect is small compared with the increase in the number of conduction electrons.

## Question

**7** The resistance of a metal wire changes with temperature. This means that a wire could be used to sense changes in temperature, in the same way that a thermistor is used.

- **a** Suggest **one** advantage a thermistor has over a metal wire for this purpose.
- **b** Suggest **one** advantage a metal wire has over a thermistor.

notice in the brightness of the lamp? Explain your answer.

## The light-dependent resistor (LDR)

A **light-dependent resistor (LDR)** is made of a high-resistance semiconductor. If light falling on the LDR is of a high enough frequency, photons are absorbed by the semiconductor. As some photons are absorbed, electrons are released from atoms in the semiconductor. The resulting free electrons conduct electricity and the resistance of the semiconductor is reduced.

The graph in Figure 10.10 shows the variation of the resistance of a typical LDR with light intensity. Only a narrow range of light intensity, measured in lux, is shown. A typical LDR will have a resistance of a few hundred ohms in sunlight, but in the dark its resistance will be millions of ohms.

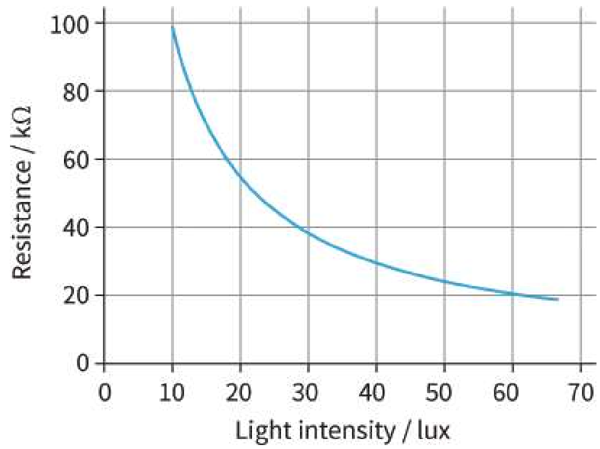



**Figure 10.10:** Resistance plotted against light intensity for an LDR.

## Understanding the origin of resistance

To understand a little more about the origins of resistance, it is helpful to look at how the resistance of a pure metal wire changes as its temperature is increased. This is shown in the graph in Figure 10.11. You will see that the resistance of the pure metal increases linearly as the temperature increases from 0 °C to 100 °C. Compare this with the graph in Figure 10.9 for an NTC thermistor; the thermistor's resistance decreases very dramatically over a narrow temperature range.

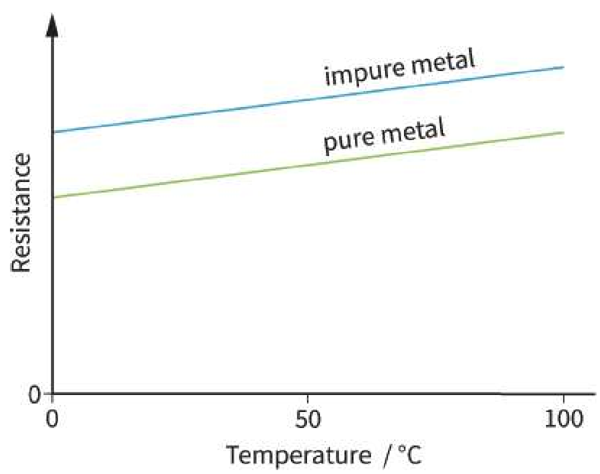



**Figure 10.11:** The resistance of a metal increases gradually as its temperature is increased. The resistance of an impure metal wire is greater than that of a pure metal wire of the same dimensions.

Figure 10.11 also shows how the resistance of the metal changes if it is slightly impure. The resistance of an impure metal is greater than that of the pure metal and follows the same gradual upward slope. The resistance of a metal changes in this gradual way over a wide range of temperatures–from close to absolute zero up to its melting point, which may be over 2000 °C.

This suggests there are two factors that affect the resistance of a metal:

LEDs have traditionally been used as indicator lamps to show when an appliance is switched on. Newer versions, some of which produce white light, are replacing filament lamps, for example, in traffic lights and torches (flashlights) – see Figure 10.8. Although they are more expensive to manufacture, they are more energy-efficient and hence cheaper to run, so that the overall cost is less.

The threshold voltage at which an LED starts to conduct and emit light is higher than 0.6 V and depends on the colour of light it emits, but may be taken to be about 2 V.

**Figure 10.8:** This torch has seven white LEDs, giving a brighter, whiter light than a traditional filament lamp.

## Questions

**5** The graph in Figure 10.9 was obtained by measuring the resistance $R$ of a particular thermistor as its temperature $\theta$ changed.

- **a** Determine its resistance at:
  - **i** 20 °C
  - **ii** 45 °C.
- **b** Determine the temperature when its resistance is:
  - **i** 5000 Ω
  - **ii** 2000 Ω.

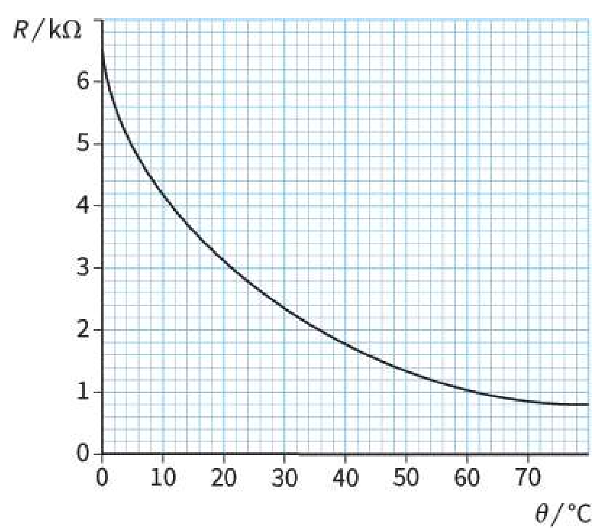



**Figure 10.9:** The resistance of an NTC thermistor decreases as the temperature increases. For Question 5.

**6** A student connects a circuit with an NTC thermistor, a filament lamp and a battery in series. The lamp glows dimly. The student warms the thermistor with a hair dryer. What change will the student

**b** State the voltage at which both have the same resistance.

**c** Determine the resistance at the voltage stated in part **b**.

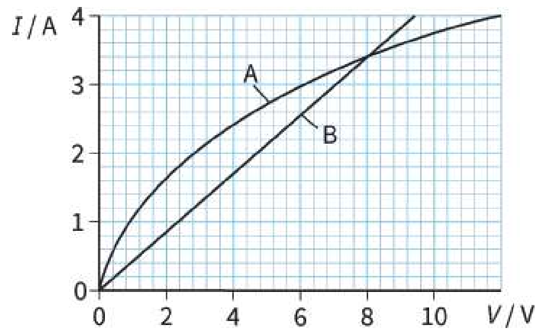



**Figure 10.6:** For Question 4.

## Diodes

The semiconductor diode is another example of a non-ohmic conductor. A diode is any component that allows electric current in only one direction. Most diodes are made of semiconductor materials. One type, the light-emitting diode or LED, gives out light when it conducts.

Figure 10.7 shows the *I*–*V* characteristic for a diode. There are some points you should notice about this graph.

- We have included positive and negative values of current and voltage. This is because, when connected one way round, forward-biased, the diode conducts and has a fairly low resistance. Connected the other way round, reverse-biased, it allows only a tiny current and has almost infinite resistance.
- For positive voltages less than about 0.6 V, the current is almost zero and hence the diode has almost infinite resistance. It starts to conduct suddenly at its **threshold voltage**. The resistance of the diode decreases dramatically for voltages greater than 0.6 V.

**KEY IDEA**

Most modern diodes are made from silicon and will start conducting when there is a potential difference of about 0.6 V across them. You need to remember this key 0.6 V value.

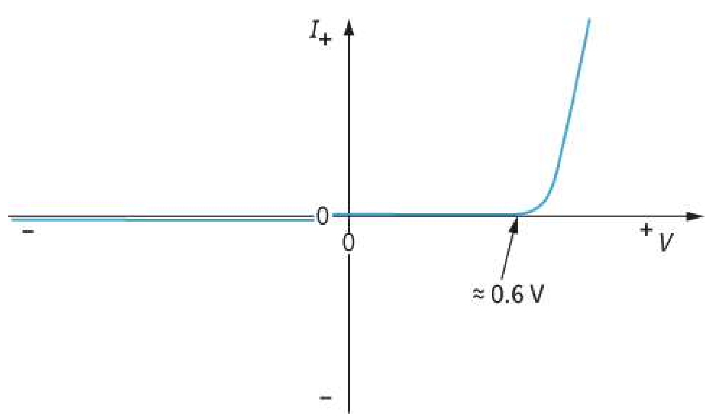



**Figure 10.7:** The current against potential difference (*I*–*V*) characteristic for a diode. The graph is not a straight line. A diode does not obey Ohm's law.

The resistance of a diode depends on the potential difference across it. From this we can conclude that it does not obey Ohm's law; it is a non-ohmic component.

Diodes are used as rectifiers. They allow current to pass in one direction only and so can be used to convert alternating current into direct current. (There is more about this in Chapter 27.) Most modern diodes are made from silicon and will start conducting when there is a potential difference of about 0.6 V across them. You need to remember this key 0.6 V value.

## Thermistors

Thermistors are components that are designed to have a resistance that changes rapidly with temperature. Thermistors ('**thermal** res**istors**') are made from metal oxides such as those of manganese and nickel.

There are two different types of thermistor:

- Negative temperature coefficient **(NTC) thermistors** – the resistance of this type of thermistor decreases with increasing temperature. Those commonly used for physics teaching may have a resistance of many thousands of ohms at room temperature, falling to a few tens of ohms at 100 °C. You should become familiar with the properties of NTC thermistors.
- Positive temperature coefficient (PTC) thermistors–the resistance of this type of thermistor rises abruptly at a definite temperature, usually around 100–150 °C.

In this course, you only need to know about NTC thermistors. So, whenever thermistors are mentioned, assume that it refers to an NTC thermistor.

The change in their resistance with temperature gives thermistors many uses. Examples include:

- water temperature sensors in cars and ice sensors on aircraft wings – if ice builds up on the wings, the thermistor 'senses' this temperature drop and a small heater is activated to melt the ice
- baby breathing monitors–the baby rests on an air-filled pad, and as he or she breathes, air from the pad passes over a thermistor, keeping it cool; if the baby stops breathing, the air movement stops, the thermistor warms up and an alarm sounds
- fire sensors – a rise in temperature activates an alarm
- overload protection in electric razor sockets – if the razor overheats, the thermistor's resistance decreases, the current increases rapidly and cuts off the circuit.

## Questions

**3** The two graphs in Figure 10.5 show the *I*–*V* characteristics of a metal wire at two different temperatures, $\theta_1$ and $\theta_2$.

**a** Calculate the resistance of the wire at each temperature.

**b** State which is the higher temperature, $\theta_1$ or $\theta_2$.

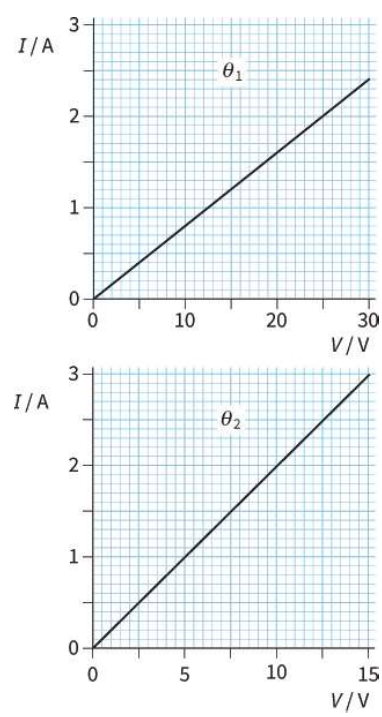



**Figure 10.5:** *I*–*V* graphs for a wire at two different temperatures. For Question 3.

**4** The graph in Figure 10.6 shows the *I*–*V* characteristics of two electrical components, a filament lamp and a length of steel wire.

**a** Identify which curve relates to each component.

# 10.3 Resistance and temperature

A conductor that does not obey Ohm's law is described as **non-ohmic**. An example is a filament lamp. Figure 10.3 shows such a lamp; you can clearly see the wire filament glowing as the current passes through it. Figure 10.4 shows the *I*–*V* characteristic for a similar lamp.

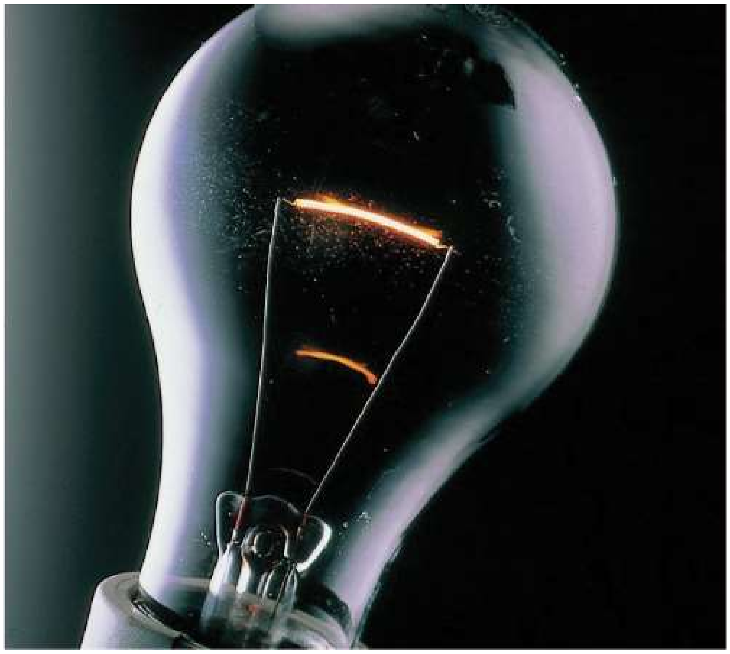


**Figure 10.3:** The metal filament in a lamp glows as the current passes through it. It also feels warm. This shows that the lamp produces both heat and light.

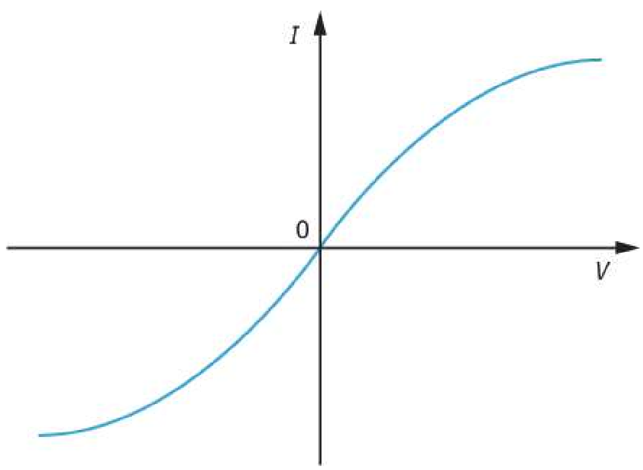



**Figure 10.4:** The *I*–*V* characteristic for a filament lamp.

There are some points you should notice about the graph in Figure 10.4:

- The line passes through the origin (as for an ohmic component).
- For very small currents and voltages, the graph is roughly a straight line.
- At higher voltages, the line starts to curve. The current is a bit less than we would have expected from a straight line. This suggests that the lamp's resistance has increased. You can also tell that the resistance has increased because the ratio $\frac{V}{I}$ is larger for higher voltages than for low voltages.

The graph of Figure 10.4 is not a straight line–this shows that the resistance of the lamp depends on the temperature of its filament. Its resistance may increase by a factor as large as ten between when it is cold and when it is brightest (when its temperature may be as high as 1750 °C).

# 10.2 Ohm's law

For the metallic conductor whose *I*–*V* characteristic is shown in Figure 10.2, the current in it is directly proportional to the p.d. across it. This means that its resistance is independent of both the current and the p.d.

This is because the ratio $\frac{V}{I}$ is a constant. Any component that behaves like this is described as an **ohmic** component, and we say that it obeys **Ohm's law**. The statement of Ohm's law is very precise and you must not confuse this with the equation $\frac{V}{I} = R$.

A conductor obeys Ohm's law if the current in it is directly proportional to the potential difference across its ends.

## Question

**2** An electrical component allows a current of 10 mA through it when a voltage of 2.0 V is applied. When the voltage is increased to 8.0 V, the current becomes 60 mA. Does the component obey Ohm's law? Give numerical values for the resistance to justify your answer.

resistances, so the gradient of the *I*–*V* graph will be different for different resistors.

## Question

1 Table 10.1 shows the results of an experiment to measure the resistance of a carbon resistor whose resistance is given by the manufacturer as 47 Ω ± 10%.

**a** Plot a graph to show the *I*–*V* characteristic of this resistor.

**b** Do the points appear to fall on a straight line that passes through the origin of the graph?

**c** Use the graph to determine the resistance of the resistor.

**d** Does the value of the resistance fall within the range given by the manufacturer?

| Potential difference / V | Current / A |
|---|---|
| 2.1 | 0.040 |
| 4.0 | 0.079 |
| 6.3 | 0.128 |
| 7.9 | 0.192 |
| 10.0 | 0.202 |
| 12.1 | 0.250 |

**Table 10.1:** Potential difference V and current I

# 10.1 The *I–V* characteristic for a metallic conductor

In Chapter 8, we saw how we could measure the resistance of a resistor using a voltmeter and ammeter. In this topic we are going to investigate the variation of the current – and, therefore, resistance – as the potential difference across a conductor changes.

The potential difference across a metal conductor can be altered using a variable power supply or by placing a variable resistor in series with the conductor. This allows us to measure the current at different potential differences across the conductor. The results of such a series of measurements are shown graphically in Figure 10.2.

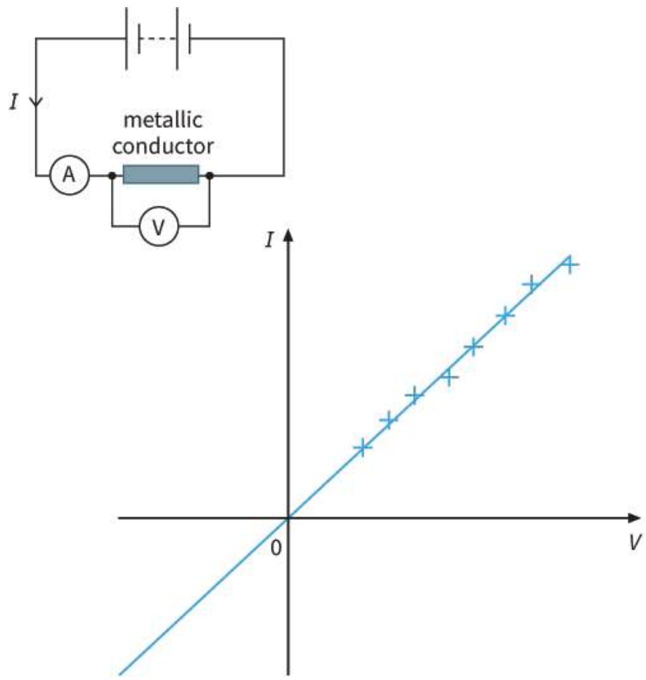



**Figure 10.2:** To determine the resistance of a component, you need to measure both current and potential difference.

Look at the graph of Figure 10.2. Such a graph is known as an **I–V characteristic**. The points are slightly scattered, but they clearly lie on a straight line. A line of best fit has been drawn. You will see that it passes through the origin of the graph. In other words, the current $I$ is directly proportional to the voltage $V$.

The straight-line graph passing through the origin shows that the resistance of the conductor remains constant. If you double the current, the voltage will also double. However, its resistance, which is the ratio of the voltage to the current, remains the same. Instead of using:

$$R = \frac{V}{I}$$

to determine the resistance, for a graph of $I$ against $V$ that is a straight line passing through the origin, you can also use:

$$\text{resistance} = \frac{1}{\text{gradient of graph}}$$

(This will give a more accurate value for $R$ than if you were to take a single experimental data point. Take care! You can only find resistance from the gradient if the $I$–$V$ graph is a straight line through the origin.)

By reversing the connections to the resistor, the p.d. across it will be reversed (in other words, it becomes negative). The current will be in the opposite direction – it is also negative. The graph is symmetrical, showing that if a p.d. of, say, 2.0 V produces a current of 0.5 A, then a p.d. of −2.0 V will produce a current of −0.5 A. This is true for most simple metallic conductors but is not true for some electronic components, such as diodes.

You get results similar to those shown in Figure 10.2 for a commercial **resistor**. Resistors have different helium that was required to cool the superconductors is very expensive to produce. In 1986, it was discovered that particular ceramics became superconducting at much higher temperatures – above 77 K, the boiling point of liquid nitrogen. This meant that liquid nitrogen, which is readily available, could be used to cool the superconductors and expensive liquid helium was no longer needed. Consequently, superconductor technology became a feasible proposition.

## Uses of superconductors

The JR-Maglev train in Japan's Yamanashi province floats above the track using superconducting magnets (Figure 10.1). This means that not only is the heating effect of the current in the magnet coils reduced to zero – it also means that the friction between the train and the track is eliminated and that the train can reach incredibly high speeds of up to 580 km $h^{-1}$.

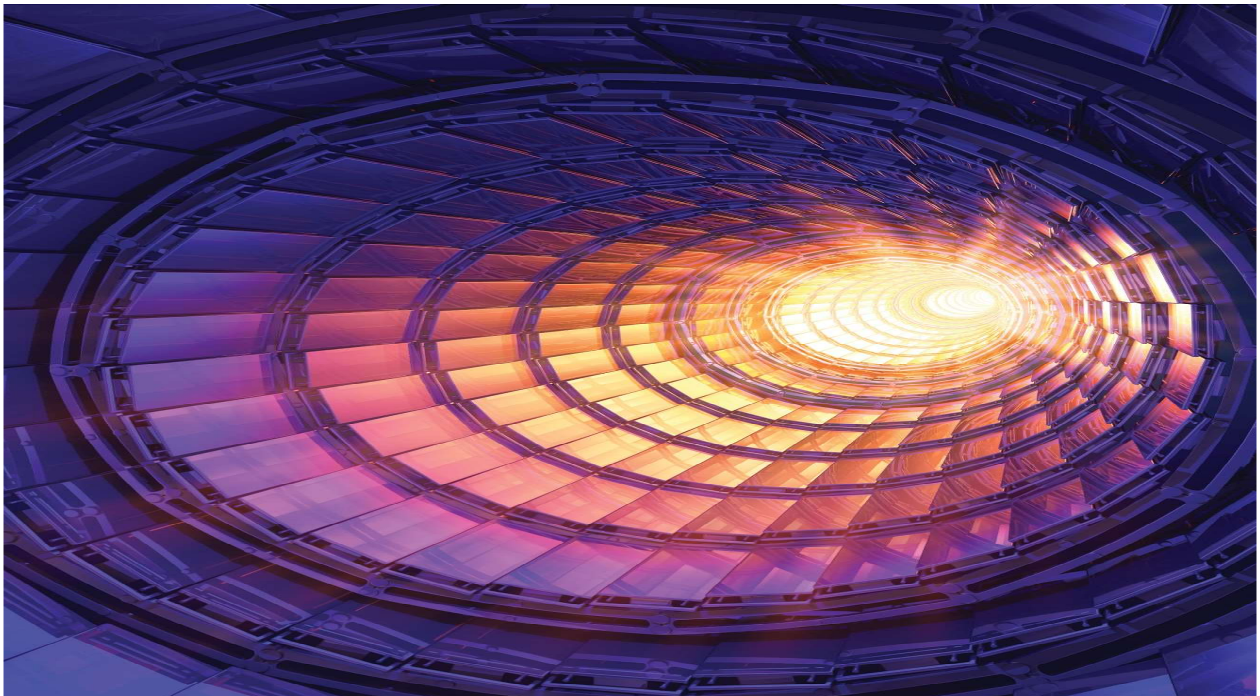
Particle accelerators, such as the Large Hadron Collider (LHC) at the CERN research facility in Switzerland, accelerate beams of charged particles to very high energies by making them orbit around a circular track many times. The particles are kept moving in the circular path by very strong magnetic fields produced by electromagnets whose coils are made from superconductors. Much of our understanding of the fundamental nature of matter is from doing experiments in which beams of these very high speed particles are made to collide with each other.

**Figure 10.1:** The Japanese JR-Maglev train, capable of speeds approaching 600 km $h^{-1}$.

Magnetic resonance imaging (MRI) was developed in the 1940s. It is used by doctors to examine internal organs without invasive surgery.

Superconducting magnets can be made much smaller than conventional magnets, and this has enabled the magnetic fields produced to be much more precise, resulting in better imaging.

Imagine you are a scientific consultant for a new science fiction film. You have been instructed to find a use of a superconductor to enable the hero to escape from a villain who is about to destroy the world. What use would you come up with?



## Chapter 10

# Resistance and resistivity

**LEARNING INTENTIONS**

In this chapter you will learn how to:

- state Ohm's law
- sketch and explain the *I*–*V* characteristics for various components
- sketch the temperature characteristic for an NTC thermistor
- solve problems involving the resistivity of a material.

**BEFORE YOU START**

- Do you understand the terms introduced in Chapters 8 and 9: current, charge, potential difference, e.m.f., resistance and their relationships to one another?
- What are their units?
- Take turns in challenging a partner to define a term or to write down an equation linking different terms. Do not use the textbook or your notes to look up the terms.

**SUPERCONDUCTIVITY**

As metals are cooled, their resistance decreases. It was discovered as long ago as 1911 that when mercury was cooled using liquid helium to 4.1 K (4.1 degrees above absolute zero), its resistance suddenly fell to zero. This phenomenon was named **superconductivity**. Other metals, such as lead at 7.2 K, also become superconductors.

When charge flows in a superconductor, it can continue in that superconductor without the need for any potential difference and without dissipating any energy. This means that large currents can occur without the unwanted heating effect that would occur in a normal metallic or semiconducting conductor.

Initially, superconductivity was only of scientific interest and had little practical use, as the liquid

## SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

| I can | See topic… | Needs more work | Almost there | Ready to move on |
|---|---|---|---|---|
| state and use Kirchhoff's first law | 9.1, 9.3 | | | |
| state and use Kirchhoff's second law that states that the sum of the e.m.f.s around any loop in a circuit is equal to the sum of the p.d.s around the loop | 9.2, 9.3 | | | |
| calculate the total resistance of two or more resistors in series | 9.4 | | | |
| calculate the resistance of two or more resistors in parallel | 9.4 | | | |
| understand that ammeters have a low resistance and are connected in series in a circuit | 9.4 | | | |
| understand that voltmeters have a high resistance and are connected in parallel in a circuit. | 9.4 | | | |

**10 a** Explain what is meant by the resistance of a resistor. **[1]**

**b** This diagram shows a network of resistors connected to a cell of e.m.f. 6.0 V.

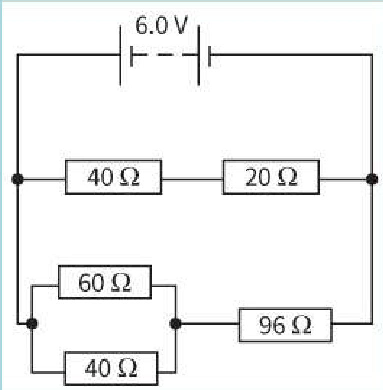



**Figure 9.31**

Show that the resistance of the network of resistors is 40 Ω. **[3]**

**c** Calculate the current in the 60 Ω resistor. **[3]**

**[Total: 7]**

The current in the resistor X is 2.0 A and the current in the 6.0 Ω resistor is 0.5 A. Calculate:

**a** the current in resistor Y **[1]**

**b** the resistance of resistor Y **[2]**

**c** the resistance of resistor X. **[2]**

**[Total: 5]**

**8** **a** Explain the difference between the terms e.m.f. and potential difference. **[2]**

**b** This circuit contains batteries and resistors. You may assume that the batteries have negligible internal resistance.

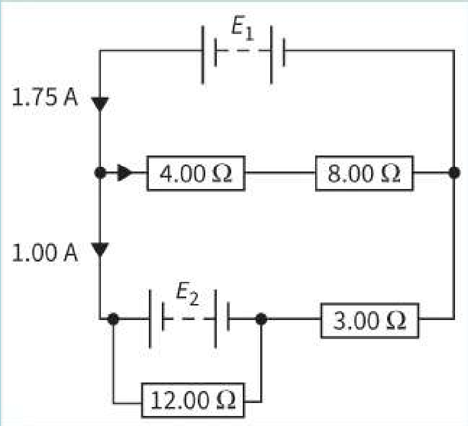



**Figure 9.29**

**i** Use Kirchhoff's first law to find the current in the 4.00 Ω and 8.00 Ω resistors. **[1]**

**ii** Calculate the e.m.f. of $E_1$. **[2]**

**iii** Calculate the value of $E_2$. **[2]**

**iv** Calculate the current in the 12.00 Ω resistor. **[2]**

**[Total: 9]**

**9** **a** Explain why an ammeter is designed to have a low resistance. **[1]**

A student builds the circuit, as shown, using a battery of negligible internal resistance. The reading on the voltmeter is 9.0 V.

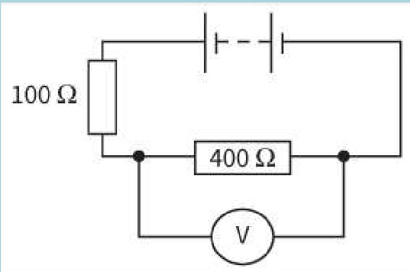



**Figure 9.30**

**b** **i** The voltmeter has a resistance of 1200 Ω. Calculate the e.m.f. of the battery. **[4]**

**ii** The student now repeats the experiment using a voltmeter of resistance 12 kΩ. Show that the reading on this voltmeter would be 9.5 V. **[3]**

**iii** Refer to your answers to **i** and **ii** and explain why a voltmeter should have as high a resistance as possible. **[2]**

**[Total: 10]**

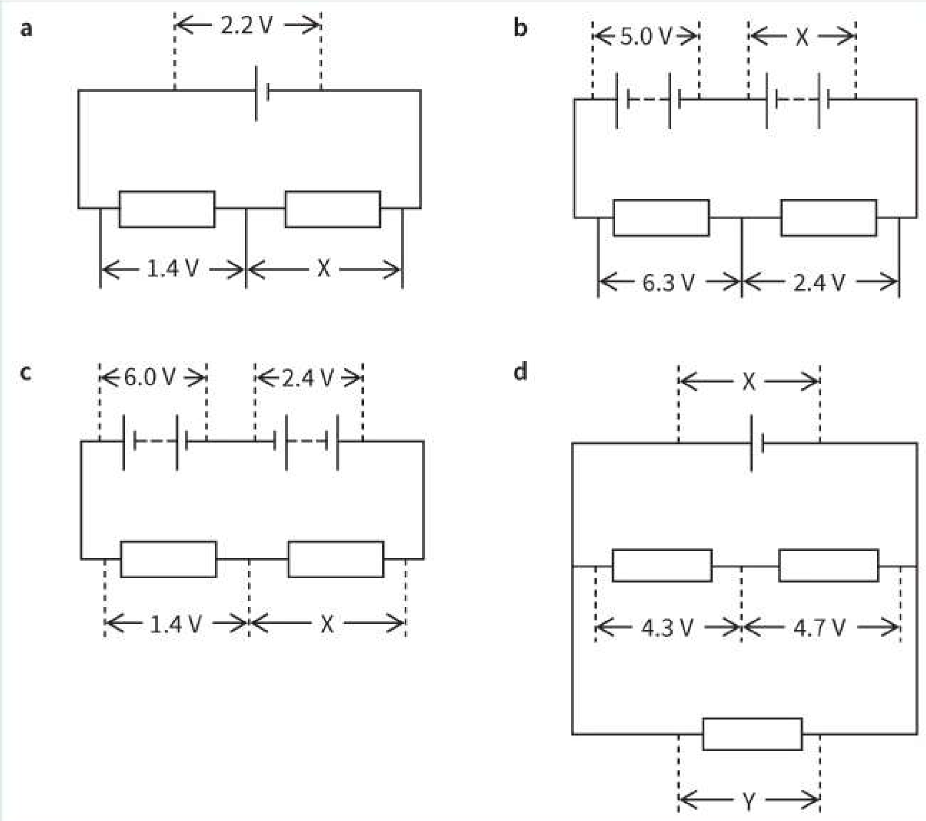



**Figure 9.27**

Determine the unknown potential difference (or differences) in each case. **[5]**

**6** A filament lamp and a 220 Ω resistor are connected in series to a battery of e.m.f. 6.0 V. The battery has negligible internal resistance. A high-resistance voltmeter placed across the resistor measures 1.8 V.

Calculate:

**a** the current drawn from the battery **[1]**

**b** the p.d. across the lamp **[1]**

**c** the total resistance of the circuit **[1]**

**d** the number of electrons passing through the battery in a time of 1.0 minute. **[4]**

(The elementary charge is $1.6 \times 10^{-19}$ C.)

**[Total: 7]**

**7** The circuit diagram shows a 12 V power supply connected to some resistors.

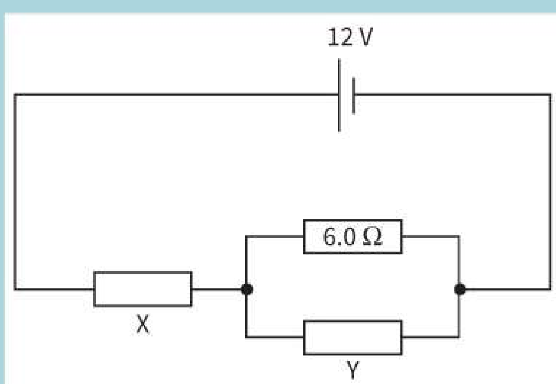



**Figure 9.28**

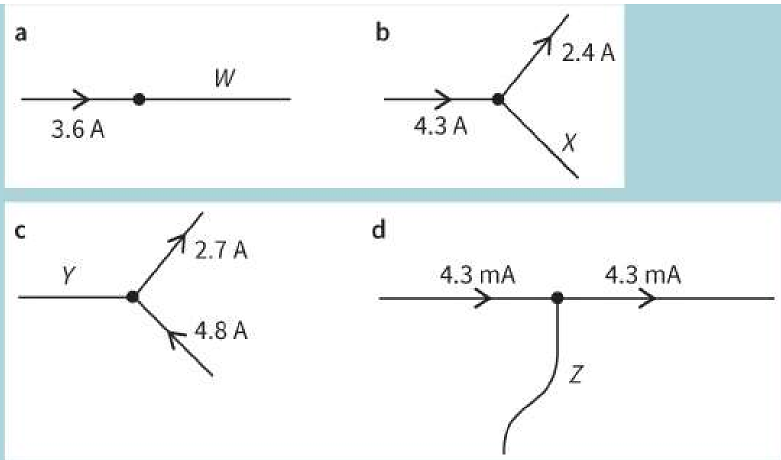



**Figure 9.25**

For each example, state the direction of the current. **[4]**

**4** This diagram shows a part of a circuit.

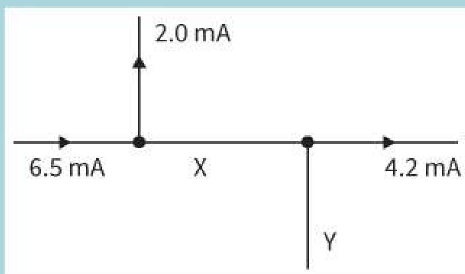



**Figure 9.26**

Copy the circuit and write in the currents at X and at Y, and show their directions. **[2]**

**5** Look at these four circuits.

## EXAM-STYLE QUESTIONS

**1** Which row in this table is correct? **[1]**

| | | |
|---|---|---|
| A | Kirchhoff's first law is an expression of the conservation of charge. | Kirchhoff's second law is an expression of the conservation of charge. |
| B | Kirchhoff's first law is an expression of the conservation of charge. | Kirchhoff's second law is an expression of the conservation of energy. |
| C | Kirchhoff's first law is an expression of the conservation of energy. | Kirchhoff's second law is an expression of the conservation of charge. |
| D | Kirchhoff's first law is an expression of the conservation of energy. | Kirchhoff's second law is an expression of the conservation of energy. |

**Table 9.1**

**2** What is the current $I_1$ in this circuit diagram? **[1]**

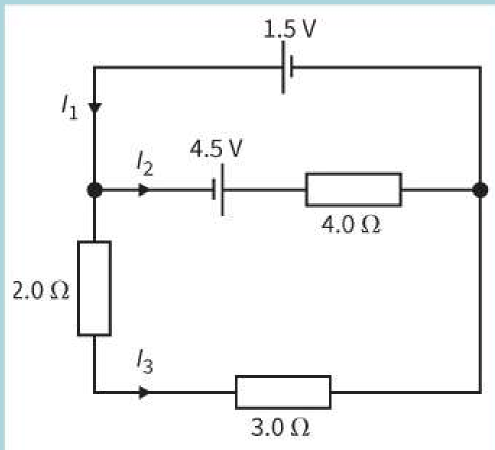



**Figure 9.24**

**A** −0.45 A

**B** +0.45 A

**C** +1.2 A

**D** +1.8 A

**3** Use Kirchhoff's first law to calculate the unknown currents in these examples.

## SUMMARY

| |
|---|
| Kirchhoff's first law states that the sum of the current currents entering any point in a circuit is equal to the sum of the currents leaving that point. |
| Kirchhoff's second law states that the sum of the e.m.f.s around any loop in a circuit is equal to the sum of the p.d.s around the loop. |
| The combined resistance of resistors in series is given by the formula: $$R = R_1 + R_2 + \ldots$$ |
| The combined resistance of resistors in parallel is given by the formula: $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots.$$ |
| Ammeters have a low resistance and are connected in series in a circuit. |
| Voltmeters have a high resistance and are connected in parallel in a circuit. |

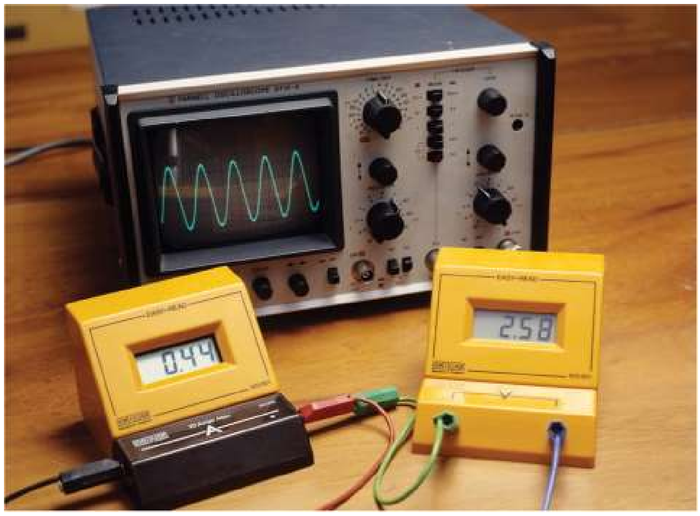



**Figure 9.23:** Electrical measuring instruments: an ammeter, a voltmeter and an oscilloscope. The oscilloscope can display rapidly changing voltages.

## Question

**23 a** A 10 V power supply of negligible internal resistance is connected to a 100 Ω resistor. Calculate the current in the resistor.

**b** An ammeter is now connected in the circuit, to measure the current. The resistance of the ammeter is 5.0 Ω. Calculate the ammeter reading.

### REFLECTION

Kirchhoff's Laws formalise facts that you might already have been familiar with.

Make a list of the main points that these laws have helped clarify in your mind.

Compare your list with two or three other people's lists.

Are they identical?

Thinking back on this chapter, what things might you want more help with?

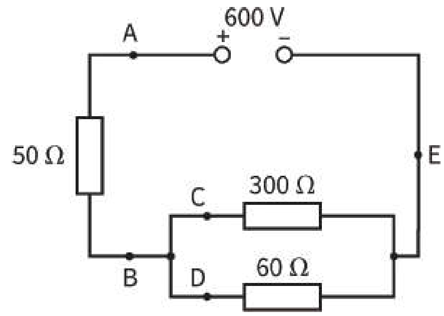



**Figure 9.21:** For Question 22.

## PRACTICAL ACTIVITY 10.1

### Ammeters and voltmeters

Ammeters and voltmeters are connected differently in circuits (Figure 9.22). Ammeters are always connected in series, since they measure the current in a circuit. For this reason, an ammeter should have as low a resistance as possible so that as little energy as possible is dissipated in the ammeter itself. Inserting an ammeter with a higher resistance could significantly reduce the current flowing in the circuit. The ideal resistance of an ammeter is zero. Digital ammeters have very low resistances.

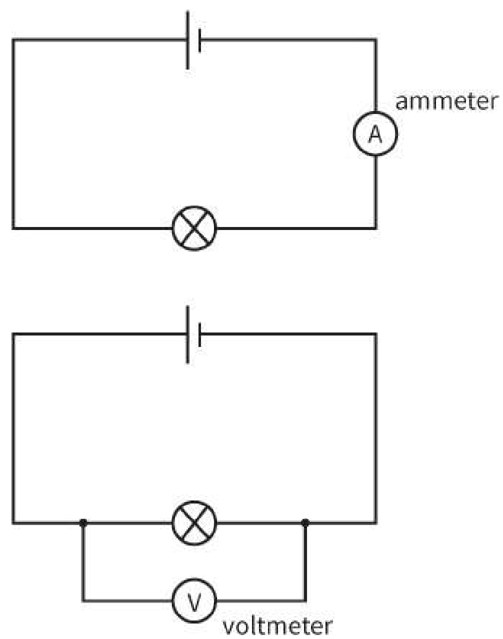



**Figure 9.22:** How to connect up an ammeter and a voltmeter.

Voltmeters measure the potential difference between two points in the circuit. For this reason, they are connected in parallel (i.e., between the two points), and they should have a very high resistance to take as little current as possible. The ideal resistance of a voltmeter would be infinite. In practice, voltmeters have typical resistance of about 1 MΩ. A voltmeter with a resistance of 10 MΩ measuring a p.d. of 2.5 V will take a current of $2.5 \times 10^{-7}$ A and dissipate just 0.625 μJ of heat energy from the circuit every second.

Figure 9.23 shows some measuring instruments.

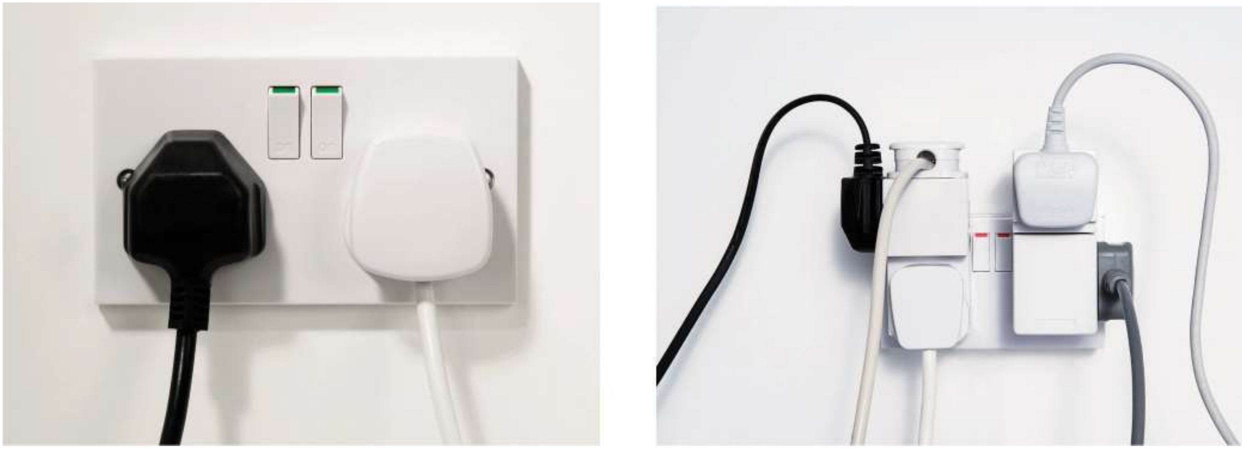


**Figure 9.19: a** Correct use of an electrical socket. **b** Here, too many appliances (resistances) are connected in parallel. This reduces the total resistance and increases the current drawn, to the point where it becomes dangerous.

## Questions

**17** Three resistors of resistances 20 Ω, 30 Ω and 60 Ω are connected together in parallel. Select which of the following gives their combined resistance:

**110** Ω,   50 Ω,   20 Ω,   10 Ω

(No need to do the calculation!)

**18** In the circuit in Figure 9.20 the battery of e.m.f. 10 V has negligible internal resistance. Calculate the current in the 20 Ω resistor shown in the circuit.

**19** Determine the current drawn from the battery in Figure 9.20.

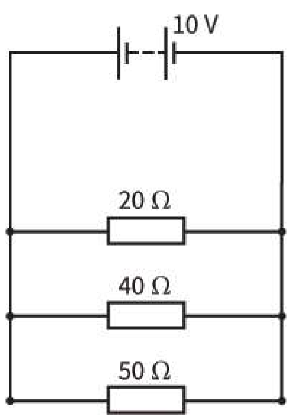



**Figure 9.20:** Circuit diagram for Questions 18 and 19.

**20** What value of resistor must be connected in parallel with a 20 Ω resistor so that their combined resistance is 10 Ω?

**21** You are supplied with a number of 100 Ω resistors. Describe how you could combine the minimum number of these to make a 250 Ω resistor.

**22** Calculate the current at each point (A–E) in the circuit shown in Figure 9.21.

**Step 1** We have $R_1 = R_2 = 10\ \Omega$, so:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$$

**Step 2** Inverting both sides of the equation gives:

$R = 5\ \Omega$

**Hint**: *Take care not to forget this step! Nor should you write* $\frac{1}{R} = \frac{1}{5} = 5\ \Omega$*, as then you are saying* $\frac{1}{5} = 5$).

You can also determine the resistance as follows:

$$\begin{aligned} R &= \left(R_1{}^{-1} + R_2{}^{-1}\right)^{-1} \\ &= \left(10^{-1} + 10^{-1}\right)^{-1} = 5\Omega \end{aligned}$$

## Questions

**13** Calculate the total resistance of four 10 Ω resistors connected in parallel.

**14** Calculate the resistances of the following combinations:

- **a** 100 Ω and 200 Ω in series
- **b** 100 Ω and 200 Ω in parallel
- **c** 100 Ω and 200 Ω in series and this in parallel with 200 Ω.

**15** Calculate the current drawn from a 12 V battery of negligible internal resistance connected to the ends of the following:

- **a** 500 Ω resistor
- **b** 500 Ω and 1000 Ω resistors in series
- **c** 500 Ω and 1000 Ω resistors in parallel.

**16** You are given one 200 Ω resistor and two 100 Ω resistors. What total resistances can you obtain by connecting some, none, or all of these resistors in various combinations?

## Solving problems with parallel circuits

Here are some useful ideas that may help when you are solving problems with parallel circuits (or checking your answers to see whether they seem reasonable).

- When two or more resistors are connected in parallel, their combined resistance is smaller than any of their individual resistances. For example, three resistors of 2 Ω, 3 Ω and 6 Ω connected together in parallel have a combined resistance of 1 Ω. This is less than the smallest of the individual resistances. This comes about because, by connecting the resistors in parallel, you are providing extra pathways for the current. Since the combined resistance is lower than the individual resistances, it follows that connecting two or more resistors in parallel will increase the current drawn from a supply. Figure 9.19 shows a hazard that can arise when electrical appliances are connected in parallel.
- When components are connected in parallel, they all have the same p.d. across them. This means that you can often ignore parts of the circuit that are not relevant to your calculation.
- Similarly, for resistors in parallel, you may be able to calculate the current in each one individually, then add them up to find the total current. This may be easier than working out their combined resistance using the reciprocal formula. (This is illustrated in Question 19.)

**a** 7.5 V
**b** 1.5 V
**c** 4.5 V?

## Resistors in parallel

For two resistors of resistances $R_1$ and $R_2$ connected in parallel (Figure 9.18), we have a situation where the current divides between them. Hence, using Kirchhoff's first law, we can write:

$$I = I_1 + I_2$$

If we apply Kirchhoff's second law to the loop that contains the two resistors, we have:

$$I_1R_1 - I_2R_2 = 0\text{ V}$$

(because there is no source of e.m.f. in the loop).

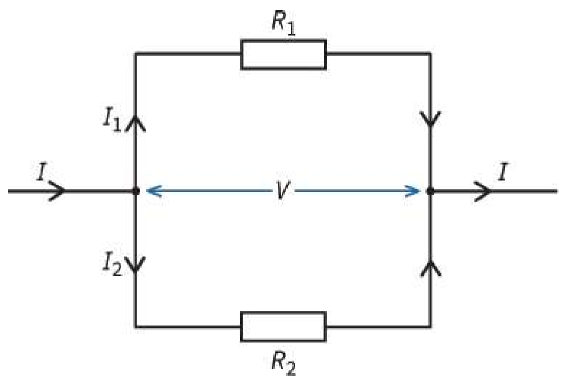



**Figure 9.18:** Resistors connected in parallel.

This equation states that the two resistors have the same p.d. $V$ across them. Hence we can write:

$$I = \frac{V}{R}$$
$$I_1 = \frac{V}{R_1}$$
$$I_2 = \frac{V}{R_2}$$

Substituting in $I = I_1 + I_2$ and cancelling the common factor $V$ gives:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

For three or more resistors, the equation for total resistance $R$ becomes:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

**KEY EQUATION**

Total resistance $R$ of three or more resistors in parallel is given by the equation $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$

You must learn how to derive this equation using Kirchhoff's laws.

To summarise, when components are connected in parallel:

- all have the same p.d. across their ends
- the current is shared between them
- we use the reciprocal formula to calculate their combined resistance.

**WORKED EXAMPLE**

**3** Two 10 Ω resistors are connected in parallel. Calculate the total resistance.

# 9.4 Resistor combinations

You are already familiar with the formulae used to calculate the combined resistance $R$ of two or more resistors connected in series or in parallel. To derive these formulae we have to use Kirchhoff's laws.

## Resistors in series

Take two resistors of resistances $R_1$ and $R_2$ connected in series (Figure 9.16). According to Kirchhoff's first law, the current in each resistor is the same. The p.d. $V$ across the combination is equal to the sum of the p.d.s across the two resistors:

$$V = V_1 + V_2$$

Since $V = IR$, $V_1 = IR_1$ and $V_2 = IR_2$, we can write:

$$IR = IR_1 + IR_2$$

Cancelling the common factor of current $I$ gives:

$$R = R_1 + R_2$$

**KEY EQUATION**

Total resistance $R$ of three or more resistors in series = $R_1 + R_2 + R_3 + \ldots$

For three or more resistors, the equation for total resistance $R$ becomes:

$$R = R_1 + R_2 + R_3 + \ldots$$

You must learn how to derive this equation using Kirchhoff's laws.

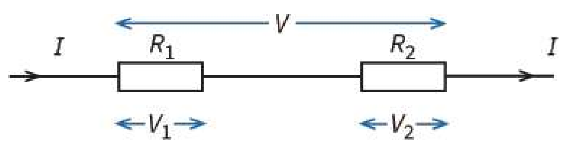



**Figure 9.16:** Resistors in series.

## Questions

**10** Calculate the combined resistance of two 5 Ω resistors and a 10 Ω resistor connected in series.

**11** The cell shown in Figure 9.17 provides an e.m.f. of 2.0 V. The p.d. across one lamp is 1.2 V. Determine the p.d. across the other lamp.

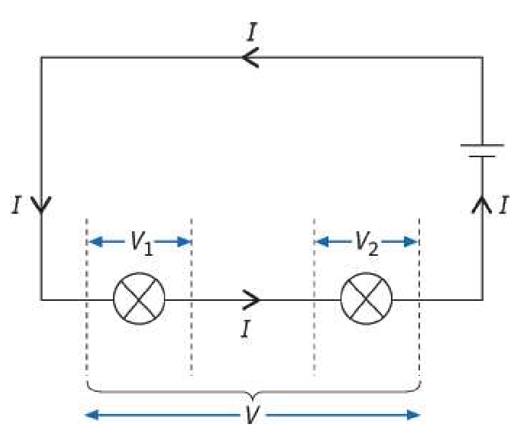



**Figure 9.17:** A series circuit for Question 11.

**12** You have five 1.5 V cells. How would you connect all five of them to give an e.m.f. of:

**Figure 9.14:** For Question 7.

## Conservation of energy

Kirchhoff's second law is a consequence of the principle of conservation of energy. If a charge, say 1 C, moves around the circuit, it **gains** energy as it moves through each source of e.m.f. and loses energy as it passes through each p.d. If the charge moves all the way round the circuit so that it ends up where it started, it must have the same energy at the end as at the beginning. (Otherwise we would be able to create energy from nothing simply by moving charges around circuits.)

So:

energy gained passing through sources of e.m.f. = energy lost passing through components with p.d.s

You should recall that an e.m.f. in volts is simply the energy gained per 1 C of charge as it passes through a source. Similarly, a p.d. is the energy lost per 1 C as it passes through a component.

1 volt = 1 joule per coulomb

Hence, we can think of Kirchhoff's second law as:

energy gained per coulomb around loop = energy lost per coulomb around loop

Here is another way to think of the meaning of e.m.f. A 1.5 V cell gives 1.5 J of energy to each coulomb of charge that passes through it. The charge then moves round the circuit, transferring the energy to components in the circuit. The consequence is that, by driving 1 C of charge around the circuit, the cell transfers 1.5 J of energy. Hence, the e.m.f. of a source simply tells us the amount of energy (in joules) transferred by the source in driving unit charge (1 C) around a circuit.

## Questions

**8** Use the idea of the energy gained and lost by a 1 C charge to explain why two 6 V batteries connected together in series can give an e.m.f. of 12 V or 0 V, but connected in parallel they give an e.m.f. of 6 V.

**9** Apply Kirchhoff's laws to the circuit shown in Figure 9.15 to determine the current that will be shown by the ammeters $A_1$, $A_2$ and $A_3$.

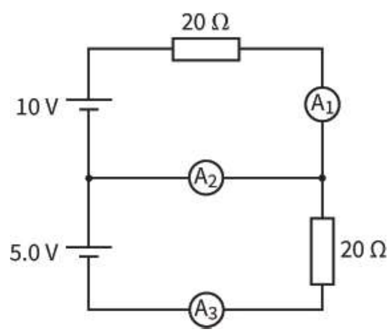



**Figure 9.15:** Kirchhoff's laws make it possible to deduce the ammeter readings.

**Step 1** Mark the currents. The diagram shows $I_1$, $I_2$ and $I_3$.

**Hint**: *It does not matter if we mark these in the wrong directions, as they will simply appear as negative quantities in the solutions.*

**Step 2** Apply Kirchhoff's first law. At point P, this gives:

$$I_1 + I_2 = I_3 \qquad (1)$$

**Step 3** Choose a loop and apply Kirchhoff's second law. Around the upper loop, this gives:

$$6.0 = (I_3 \times 30) + (I_1 \times 10) \qquad (2)$$

**Step 4** Repeat step 3 around other loops until there are the same number of equations as unknown currents. Around the lower loop, this gives:

$$2.0 = I_3 \times 30 \qquad (3)$$

We now have three equations with three unknowns (the three currents).

**Step 5** Solve these equations as simultaneous equations. In this case, the situation has been chosen to give simple solutions. Equation 3 gives $I_3 = 0.067$ A, and substituting this value in Equation 2 gives $I_1 = 0.400$ A. We can now find $I_2$ by substituting in equation 1:

$$I_2 = I_3 - I_1 = 0.067 - 0.400 = -0.333 \text{ A} \approx -0.33 \text{ A}$$

Thus $I_2$ is negative–it is in the opposite direction to the arrow shown in Figure 9.11.

Note that there is a third 'loop' in this circuit; we could have applied Kirchhoff's second law to the outermost loop of the circuit. This would give a fourth equation:

$$6 - 2 = I_1 \times 10$$

However, this is not an independent equation; we could have arrived at it by subtracting equation 3 from equation 2.

## Questions

**6** You can use Kirchhoff's second law to find the current $I$ in the circuit shown in Figure 9.13. Choosing the best loop can simplify the problem.

**a** Which loop in the circuit should you choose?

**b** Calculate the current $I$.

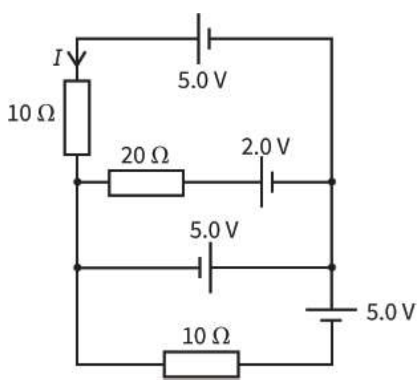
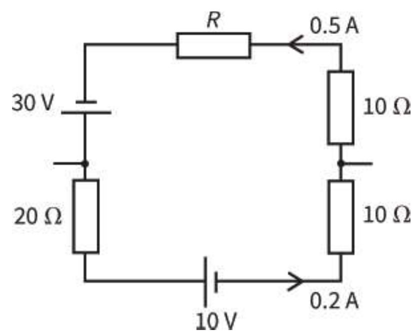



**Figure 9.13:** Careful choice of a suitable loop can make it easier to solve problems like this. For Question 6.

**7** Use Kirchhoff's second law to deduce the resistance $R$ of the resistor shown in the circuit loop of Figure 9.14.



# 9.3 Applying Kirchhoff's laws

Figure 9.11 shows a more complex circuit, with more than one 'loop'. Again, there are two batteries and two resistors. The problem is to find the current in each resistor. There are several steps in this; Worked example 2 shows how such a problem is solved.

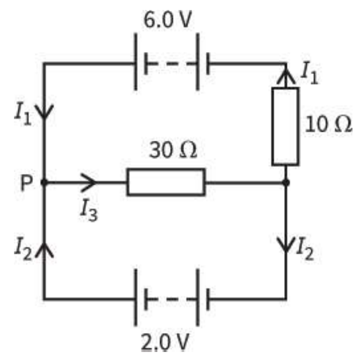



**Figure 9.11:** Kirchhoff's laws are needed to determine the currents in this circuit.

## Signs and directions

Caution is necessary when applying Kirchhoff's second law. You need to take account of the ways in which the sources of e.m.f. are connected and the directions of the currents. Figure 9.12 shows one loop from a larger complicated circuit to illustrate this point. Only the components and currents in this particular loop are shown.

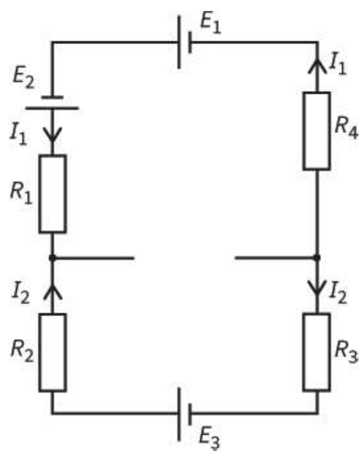



**Figure 9.12:** A loop extracted from a complicated circuit.

### e.m.f.s

Starting with the cell of e.m.f. $E_1$ and working **anticlockwise** around the loop (because $E_1$ is 'pushing current' anticlockwise):

$$\text{sum of e.m.f.s} = E_1 + E_2 - E_3$$

Note that $E_3$ is opposing the other two e.m.f.s.

### p.d.s

Starting from the same point, and working **anticlockwise** again:

$$\text{sum of p.d.s} = I_1R_1 - I_2R_2 - I_2R_3 + I_1R_4$$

Note that the direction of current $I_2$ is clockwise, so the p.d.s that involve $I_2$ are negative.

**WORKED EXAMPLE**

**2** Calculate the current in each of the resistors in the circuit shown in Figure 9.11.

can be applied in general.

## Question

**5** Use Kirchhoff's second law to deduce the p.d. across the resistor of resistance $R$ in the circuit shown in Figure 9.10, and hence find the value of $R$. (Assume the battery of e.m.f. 10 V has negligible internal resistance.)

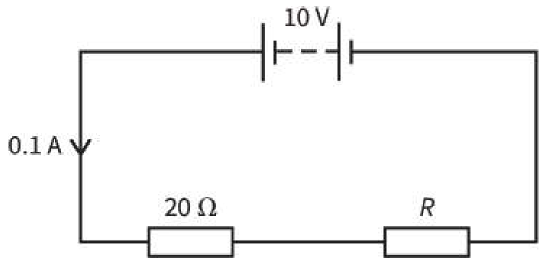



**Figure 9.10:** Circuit for Question 5.

## An equation for Kirchhoff's second law

In a similar manner to the formal statement of the first law, the second law can be written as an equation:

$$\Sigma E = \Sigma V$$

where $\Sigma E$ is the sum of the e.m.f.s and $\Sigma V$ is the sum of the potential differences.

**KEY EQUATION**

Kirchhoff's second law:

$$\Sigma E = \Sigma V$$

# 9.2 Kirchhoff's second law

This law deals with e.m.f.s and voltages in a circuit. We will start by considering a simple circuit that contains a cell and two resistors of resistances $R_1$ and $R_2$ (Figure 9.8). Since this is a simple series circuit, the current $I$ must be the same all the way around, and we need not concern ourselves further with Kirchhoff's first law. For this circuit, we can write the following equation:

$$E = IR_1 + IR_2$$

e.m.f. of battery = sum of p.d.s across the resistors

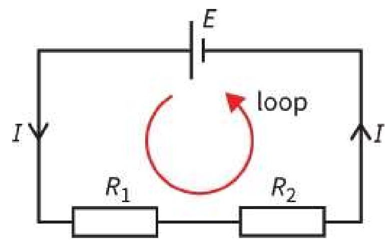



**Figure 9.8:** A simple series circuit.

You should not find these equations surprising. However, you may not realise that they are a consequence of applying **Kirchhoff's second law** to the circuit. This law states that the sum of the e.m.f.s around any loop in a circuit is equal to the sum of the p.d.s around the loop.

**WORKED EXAMPLE**

**1** Use Kirchhoff's laws to find the current in the circuit in Figure 9.9.

This is a series circuit so the current is the same all the way round the circuit.

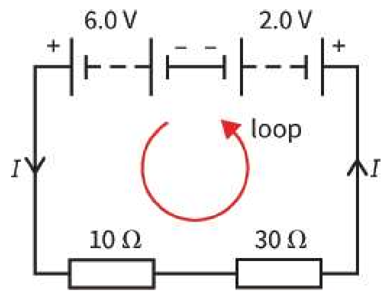



**Figure 9.9:** A circuit with two opposing batteries.

**Step 1** We calculate the sum of the e.m.f.s:

sum of e.m.f.s = 6.0 V − 2.0 V = 4.0 V

The batteries are connected in opposite directions so we must consider one of the e.m.f.s as negative.

**Step 2** We calculate the sum of the p.d.s.

sum of p.d.s = $(I \times 10) + (I \times 30) = 40\,I$

**Step 3** We equate these:

$4.0 = 40\,I$

and so $I = 0.1$ A

No doubt, you could have solved this problem without formally applying Kirchhoff's second law, but you will find that in more complex problems the use of these laws will help you to avoid errors.

You will see later that Kirchhoff's second law is an expression of the conservation of energy. We shall look at another example of how this law can be applied, and then look at how it

Here, the symbol Σ (Greek letter sigma) means 'the sum of all', so $\Sigma I_{in}$ means 'the sum of all currents entering into a point' and $\Sigma I_{out}$ means 'the sum of all currents leaving that point'. This is the sort of equation that a computer program can use to predict the behaviour of a complex circuit.

**KEY EQUATIONS**

Kirchhoff's first law:

$$\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$$

## Questions

**3** Calculate $\Sigma I_{in}$ and $\Sigma I_{out}$ in Figure 9.6. Is Kirchhoff's first law satisfied?

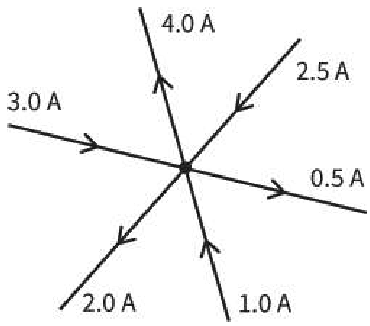



**Figure 9.6:** For Question 3.

**4** Use Kirchhoff's first law to deduce the value and direction of the current $I$ in Figure 9.7.

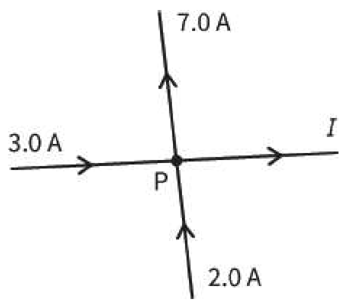



**Figure 9.7:** For Question 4.

# 9.1 Kirchhoff's first law

You will have learnt that current may divide up where a circuit splits into two separate branches. For example, a current of 5.0 A may split at a junction or a point in a circuit into two separate currents of 2.0 A and 3.0 A. The total amount of current remains the same after it splits. We would not expect some of the current to disappear, or extra current to appear from nowhere. This is the basis of **Kirchhoff's first law**, which states that the sum of the currents entering any point in a circuit is equal to the sum of the currents leaving that same point.

This is illustrated in Figure 9.3. In the first part, the current into point P must equal the current out, so:

$$I_1 = I_2$$

In the second part of the figure, we have one current coming into point Q, and two currents leaving. The current divides at Q. Kirchhoff's first law gives:

$$I_1 = I_2 + I_3$$

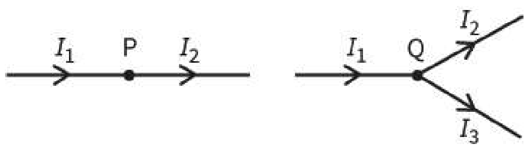



**Figure 9.3:** Kirchhoff's first law: current is conserved because charge is conserved.

Kirchhoff's first law is an expression of the **conservation of charge**. The idea is that the total amount of charge entering a point must exit the point. To put it another way, if a billion electrons enter a point in a circuit in a time interval of 1.0 s, then one billion electrons must exit this point in 1.0 s. The law can be tested by connecting ammeters at different points in a circuit where the current divides. You should recall that an ammeter must be connected in series so the current to be measured passes through it.

## Questions

**1** Use Kirchhoff's first law to deduce the value of the current *I* in Figure 9.4.

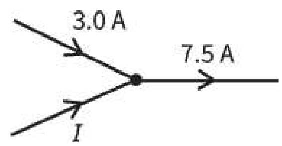



**Figure 9.4:** For Question 1.

**2** In Figure 9.5, calculate the current in the wire X. State the direction of this current (towards P or away from P).

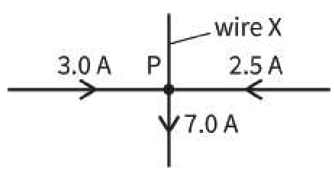



**Figure 9.5:** For Question 2.

## Formal statement of Kirchhoff's first law

We can write Kirchhoff's first law as an equation:

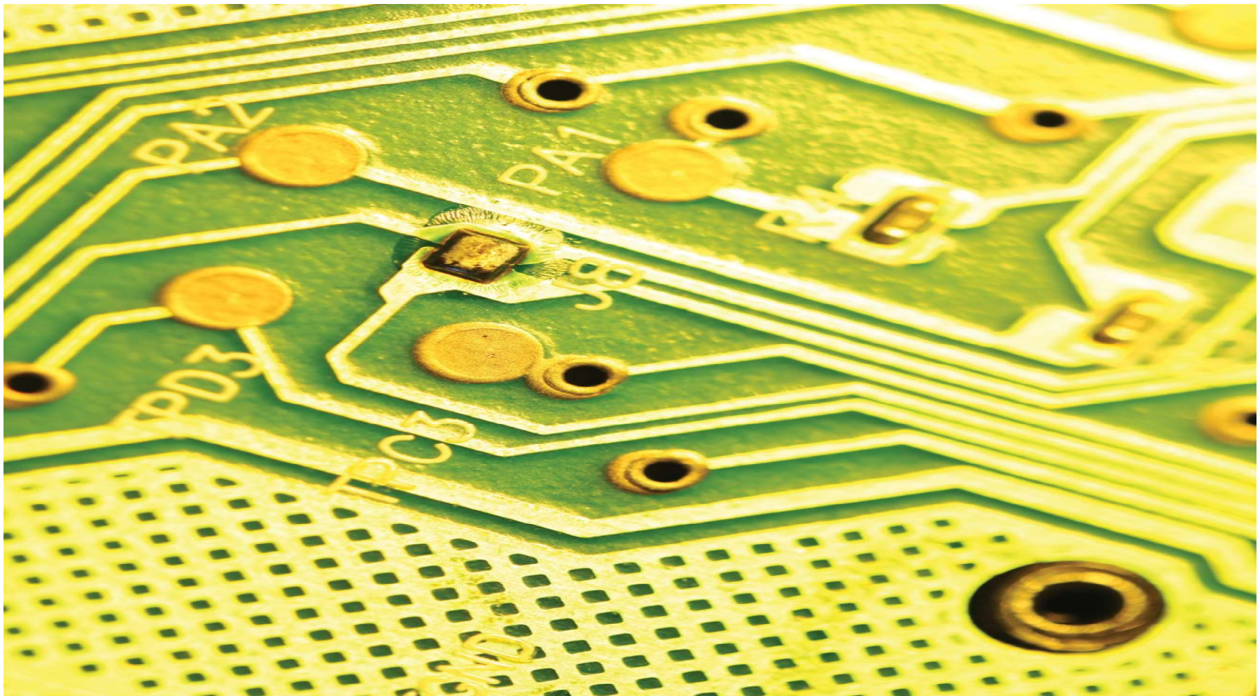
$$\Sigma I_{\text{in}} = \Sigma I_{\text{out}}$$

Instead, electronics engineers (Figure 9.2) rely on computer-based design software that can work out the effect of any combination of components. This is only possible because computers can be programmed with the equations that describe how current and voltage behave in a circuit. These equations, which include Ohm's law and Kirchhoff's two laws, were established in the 18th century, but they have come into their own in the 21st century through their use in computer-aided design (CAD) systems.

**Figure 9.1:** A complex electronic circuit – this is the circuit board that controls a computer's hard drive.

Think about other areas of industry. How have computers changed those industrial practices in the last 30 years?

**Figure 9.2:** A computer engineer uses a computer-aided design (CAD) software tool to design a circuit that will form part of a microprocessor, the device at the heart of every computer.




## Chapter 9

# Kirchhoff's laws

### LEARNING INTENTIONS

In this chapter you will learn how to:

- recall and apply Kirchhoff's laws
- use Kirchhoff's laws to derive the formulae for the combined resistance of two or more resistors in series and in parallel
- recognise that ammeters are connected in series within a circuit and therefore should have low resistance
- recognise that voltmeters are connected in parallel across a component, or components, and therefore should have high resistance.

### BEFORE YOU START

- Write down the name(s) of the meters you use to measure current in a component and potential difference across it.
- Draw a circuit diagram showing a circuit in which a battery is used to drive a current through a variable resistor in series with a lamp. Show on your circuit how you would connect the meters named in your list.
- Try to draw a circuit diagram to measure the potential difference of a component and the current in it. Swap with a classmate to check.

### CIRCUIT DESIGN

Over the years, electrical circuits have become increasingly complex, with more and more components combining to achieve very precise results (Figure 9.1). Such circuits typically include power supplies, sensing devices, potential dividers and output devices. At one time, circuit designers would start with a simple circuit and gradually modify it until the desired result was achieved. This is impossible today when circuits include many hundreds or thousands of components.

## SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

| I can | See topic... | Needs more work | Almost there | Ready to move on |
|---|---|---|---|---|
| understand of the nature of electric current | 8.2 | | | |
| understand the term charge and recognise its unit, the coulomb | 8.2 | | | |
| understand that charge is quantised | 8.2 | | | |
| solve problems using the equation $\Delta Q = I\Delta t$ | 8.2 | | | |
| solve problems using the formula $I = nAve$ | 8.3 | | | |
| solve problems involving the mean drift velocity of charge carriers | 8.3 | | | |
| understand the terms potential difference, e.m.f. and the volt | 8.4 | | | |
| use energy considerations to distinguish between p.d. and e.m.f. | 8.4 | | | |
| define resistance and recognise its unit, the ohm | 8.5 | | | |
| solve problems using the formula $V = IR$ | 8.5 | | | |
| solve problems concerning energy and power in electric circuits. | 8.6 | | | |

**a** Calculate the charge passing through the ammeter in 3 minutes. **[3]**

**b** Calculate the number of electrons that hit the anode in 3 minutes. **[3]**

**c** The potential difference between the cathode and the anode is 75 V. Calculate the energy gained by an electron as it travels from the cathode to the anode. **[2]**

**[Total: 8]**

**12** A length of copper track on a printed circuit board has a cross-sectional area of $5.0 \times 10^{-8}$ m$^2$. The current in the track is 3.5 mA. You are provided with some useful information about copper:

1 m$^3$ of copper has a mass of $8.9 \times 10^3$ kg

54 kg of copper contains $6.0 \times 10^{26}$ atoms

In copper, there is roughly one electron liberated from each copper atom.

**a** Show that the electron number density $n$ for copper is about $10^{29}$ m$^{-3}$. **[2]**

**b** Calculate the mean drift velocity of the electrons. **[3]**

**[Total: 5]**

**13** **a** Explain the difference between **potential difference** and **e.m.f.** **[2]**

**b** A battery has negligible internal resistance, an e.m.f. of 12.0 V and a capacity of 100 A h (ampere-hours). Calculate:

**i** the total charge that it can supply **[2]**

**ii** the total energy that it can transfer. **[2]**

**c** The battery is connected to a 27 W lamp. Calculate the resistance of the lamp. **[2]**

**[Total: 8]**

**14** Some electricity-generating companies use a unit called the kilowatt-hour (kW h) to calculate energy bills. 1 kWh is the energy a kilowatt appliance transfers in 1 hour.

**a** Show that 1 kWh is equal to 3.6 MJ. **[2]**

**b** An electric shower heater is rated at 230 V, 9.5 kW.

**i** Calculate the current it will take from the mains supply. **[2]**

**ii** Suggest why the shower requires a separate circuit from other appliances. **[1]**

**iii** Suggest a suitable current rating for the fuse in this circuit. **[1]**

**c** Calculate the energy transferred when a boy uses the shower for 5 minutes. **[2]**

**[Total: 8]**

**15** A student is measuring the resistance per unit length of a resistance wire. He takes the following measurements.

| **Quantity** | **Value** | **Uncertainty** |
|---|---|---|
| length of wire | 80 mm | ±2% |
| current in the wire | 2.4 A | ±0.1 A |
| potential difference across the wire | 8.9 V | ±5% |

**a** Calculate the percentage uncertainty in the measurement of the current. **[1]**

**b** Calculate the value of the resistance per unit length of the wire. **[1]**

**c** Calculate the absolute uncertainty of the resistance per unit length of the wire. **[2]**

**[Total: 4]**

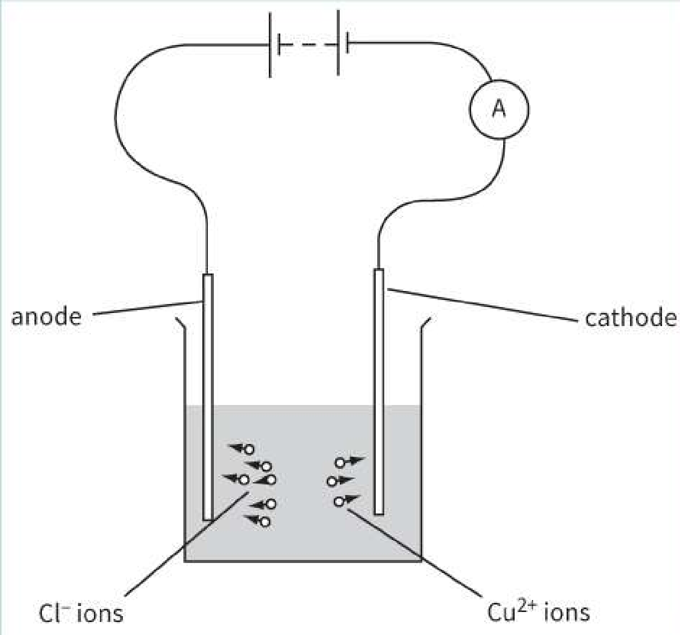



**Figure 8.16**

**a** **i** On a copy of the diagram, mark the direction of the conventional current in the electrolyte. Label it conventional current. **[1]**

**ii** Mark the direction of the electron flow in the connecting wires. Label this electron flow. **[1]**

**b** In a time period of 8 minutes, $3.6 \times 10^{16}$ chloride ($Cl^-$) ions are neutralised and liberated at the anode and $1.8 \times 10^{16}$ copper ($Cu^{2+}$) ions are neutralised and deposited on the cathode.

**i** Calculate the total charge passing through the electrolyte in this time. **[2]**

**ii** Calculate the current in the circuit. **[2]**

**[Total: 6]**

**11** This diagram shows an electron tube. Electrons moving from the cathode to the anode constitute a current. The current in the ammeter is 4.5 mA.

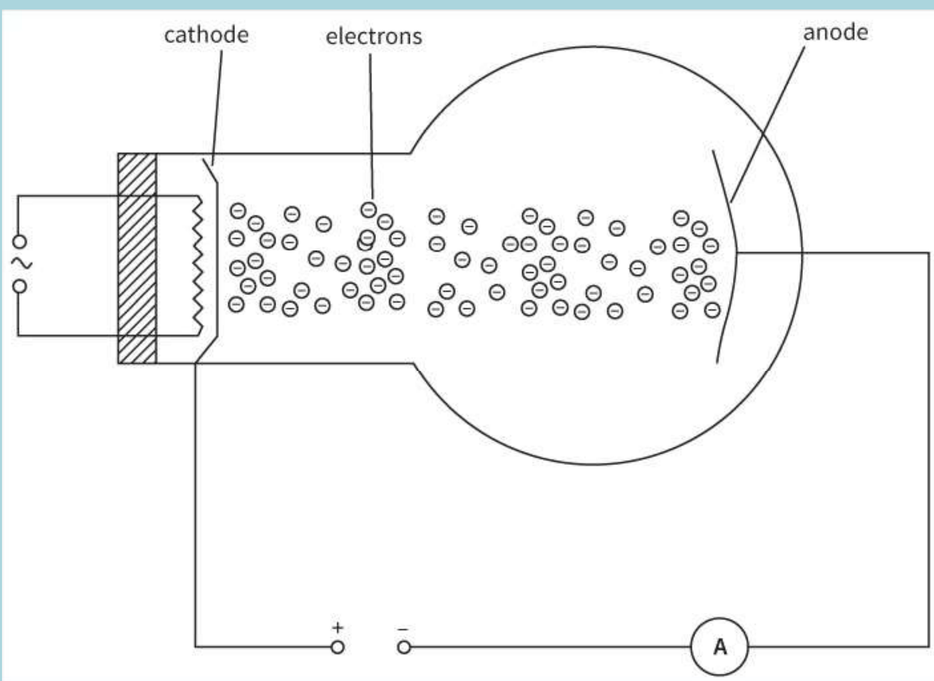



**Figure 8.17**

## EXAM-STYLE QUESTIONS

**1** A small immersion heater is connected to a power supply of e.m.f. of 12 V for a time of 150 s. The output power of the heater is 100 W.

What charge passes through the heater? **[1]**

**A** 1.4 C

**B** 8.0 C

**C** 1250 C

**D** 1800 C

**2** Which statement defines e.m.f.? **[1]**

**A** The e.m.f. of a source is the energy transferred when charge is driven through a resistor.

**B** The e.m.f. of a source is the energy transferred when charge is driven round a complete circuit.

**C** The e.m.f. of a source is the energy transferred when unit charge is driven round a complete circuit.

**D** The e.m.f. of a source is the energy transferred when unit charge is driven through a resistor.

**3** Calculate the charge that passes through a lamp when there is a current of 150 mA for 40 minutes. **[3]**

**4** A generator produces a current of 40 A. Calculate how long will it take for a total of 2000 C to flow through the output. **[2]**

**5** In a lightning strike there is an average current of 30 kA, which lasts for 2000 μs. Calculate the charge that is transferred in this process. **[3]**

**6** **a** A lamp of resistance 15 Ω is connected to a battery of e.m.f. 4.5 V. Calculate the current in the lamp. **[2]**

**b** Calculate the resistance of the filament of an electric heater that takes a current of 6.5 A when it is connected across a mains supply of 230 V. **[2]**

**c** Calculate the voltage that is required to drive a current of 2.4 A through a wire of resistance 3.5 Ω. **[2]**

**[Total: 6]**

**7** A battery of e.m.f. 6 V produces a steady current of 2.4 A for 10 minutes. Calculate:

**a** the charge that it supplied **[2]**

**b** the energy that it transferred. **[2]**

**[Total: 4]**

**8** Calculate the energy gained by an electron when it is accelerated through a potential difference of 50 kV. (Charge on the electron = $-1.6 \times 10^{-19}$ C.) **[2]**

**9** A woman has available 1 A, 3 A, 5 A, 10 A and 13 A fuses. Explain which fuse she should use for a 120 V, 450 W hairdryer. **[3]**

**10** This diagram shows the electrolysis of copper chloride.

## SUMMARY

Electric current is the rate of flow of charge. In a metal, the charge is electrons; in an electrolyte, it is both positive and negative ions.

The direction of conventional current is from positive to negative; because electrons are negative, they move in the opposite direction.

The SI unit of charge is the coulomb (C). One coulomb is the charge passing a point when there is a current of one ampere at that point for one second:

$$\text{charge} = \text{current} \times \text{time}\,(\Delta Q = I\Delta t)$$

The current $I$ in a conductor of cross-sectional area $A$ depends on the mean drift velocity ($v$) of the charge carriers and the number density ($n$):

$$I = nAvQ$$

The term potential difference is used when charge transfers energy to the component or the surroundings. It is defined as energy transferred per unit charge:

$$V = \frac{\Delta W}{\Delta Q} \text{ or } \Delta W = V\Delta Q$$

The term electromotive force is used when describing the maximum energy per unit charge that a source can provide:

$$E = \frac{\Delta W}{\Delta Q} \text{ or } \Delta W = E\Delta Q$$

A volt is a joule per coulomb (1 J $C^{-1}$).

Power is the energy transferred per unit time. There are three formulae to calculate power used according to the quantities that are given:

$$P = VI \text{ or } P = I^2R \text{ or } P = \frac{V^2}{R}$$

Resistance is the ratio of voltage to current:

$$R = \frac{V}{I}$$

The resistance of a component is 1 ohm when the voltage of 1 V produces a current of 1 ampere.

Energy transferred in the circuit in a time $\Delta t$ is given by the equation:

$$PV = \frac{\Delta W}{\Delta Q} \text{ or } \Delta W = V\Delta Q$$

$$W = IV\Delta t$$

**Figure 8.15:** A power station and electrical transmission lines. How much electrical power is lost as heat in these cables? (See Worked examples 7a and 7b.)

## Questions

**19** A calculator is powered by a 3.0 V battery. The calculator's resistance is 20 kΩ. Calculate the power transferred to the calculator.

**20** An energy-efficient light bulb is labelled '230 V, 15 W'. This means that when connected to the 230 V mains supply it is fully lit and changes electrical energy to heat and light at the rate of 15 W. Calculate:

- **a** the current in the bulb when fully lit
- **b** its resistance when fully lit.

**21** Calculate the resistance of a 100 W light bulb that draws a current of 0.43 A from a power supply.

## Calculating energy

We can use the relationship for power as energy transferred per unit time and the equation for electrical power to find the energy transferred in a circuit.

Since:

$$\text{power} = \text{current} \times \text{voltage}$$

and:

$$\text{energy} = \text{power} \times \text{time}$$

we have:

$$\begin{aligned} \text{energy transferred} &= \text{current} \times \text{voltage} \times \text{time} \\ W &= IV\Delta t \end{aligned}$$

Working in SI units, this gives energy transferred in joules.

## Questions

**22** A 12 V car battery can supply a current of 10 A for 5.0 hours. Calculate how many joules of energy the battery transfers in this time.

**23** A lamp is operated for 20 s. The current in the lamp is 10 A. In this time, it transfers 400 J of energy to the lamp. Calculate:

- **a** how much charge flows through the lamp
- **b** how much energy each coulomb of charge transfers to the lamp
- **c** the p.d. across the lamp.

**REFLECTION**

Without referring back to your textbook, explain to a classmate the difference between potential difference and electromotive force.

A common error is to think that the higher the resistance between two points, the greater the power output. Explain to someone, without using mathematics, why this is incorrect.

As you look at this activity, what is one thing you would like to change?

**b** The grid cables are 15 km long, with a resistance per unit length of 0.20 Ω $km^{-1}$. How much power is wasted as heat in these cables?

**Step 1** First, we must calculate the resistance of the cables:

resistance $R$ = 15 km × 0.20 Ω $km^{-1}$ = 3.0 Ω

**Step 2** Now we know $I$ and $R$ and we want to find $P$. We can use $P = I^2R$:

power wasted as heat,

$$\begin{aligned} P &= I^2R = (100)^2 \times 3.0 \\ &= 3.0 \times 10^4 \text{W} \\ &= 30 \text{ k W} \end{aligned}$$

Hence, of the 20 MW of power produced by the power station, 30 kW is wasted – just 0.15%.

**8** A bathroom heater, when connected to a 230 V supply has an output power of 1.0 kW. Calculate the resistance of the heater.

**Step 1** We have $P$ and $V$ and have to find $R$, so we can use $P = \frac{V^2}{R}$

**Step 2** Rearrange the equation and substitute in the known values:

resistance $R = \frac{V^2}{P} = \frac{230}{1000}^2 = 53\Omega$

***Note:*** *The kilowatts were converted to watts in a similar way to the previous example.*

## Questions

**17** Calculate the current in a 60 W light bulb when it is connected to a 230 V power supply.

**18** A power station supplies electrical energy to the grid at a voltage of 25 kV. Calculate the output power of the station when the current it supplies is 40 kA.

## Power and resistance

A current $I$ in a resistor of resistance $R$ transfers energy to it. The resistor dissipates energy heating the resistor and the surroundings.. The p.d. $V$ across the resistor is given by $V = IR$. Combining this with the equation for power, $P = VI$, gives us two further forms of the equation for power dissipated in the resistor:

$$P = I^2R$$

$$P = \frac{V^2}{R}$$

Which form of the equation we use in any particular situation depends on the information we have available to us. This is illustrated in Worked examples 7a and 7b, which relate to a power station and to the grid cables that lead from it (Figure 8.15).

# 8.6 Electrical power

The rate at which energy is transferred is known as power. Power $P$ is measured in watts (W). (If you are not sure about this, refer back to Chapter 5, where we looked at the concept of power in relation to forces and work done.)

$$\text{power} = \frac{\text{energy transferred}}{\text{time taken}} \equiv P = \frac{\Delta W}{\Delta t}$$

where $P$ is the power and $\Delta W$ is the energy transferred in a time $\Delta t$.

Take care not to confuse $W$ for energy transferred or work done with W for watts.

Refer back to the equation derived from the definition of potential difference:

$$V = \frac{\Delta W}{\Delta Q}$$

This can be rearranged as:

$$\Delta W = V\Delta Q$$

Thus:

$$P = \frac{W}{\Delta t} = \frac{V\Delta Q}{\Delta t} = V\left(\frac{\Delta Q}{\Delta t}\right)$$

The ratio of charge to time, $\frac{\Delta Q}{\Delta t}$, is the current $I$ in the component. Therefore:

$$P = VI$$

By substituting from the resistance equation $V = IR$, we get the alternative equations for power:

$$P = I^2R \text{ and } P = \frac{V^2}{R}$$

**KEY EQUATIONS**

Equations for power:

$$P = VI$$
$$P = I^2R$$
$$P = \frac{V^2}{R}$$

**WORKED EXAMPLE**

**6** Calculate the rate at which energy is transferred by a 230 V mains supply that provides a current of 8.0 A to an electric heater.

**Step 1** Use the equation for power:

$P = VI$

with $V = 230$ V and $I = 8.0$ A.

**Step 2** Substitute values:

$P = 8 \times 230 = 1840$ W (1.84 kW)

**7 a** A power station produces 20 MW of power at a voltage of 200 kV. Calculate the current supplied to the grid cables.

**Step 1** Here we have $P$ and $V$ and we have to find $I$, so we can use $P = VI$.

**Step 2** Rearranging the equation and substituting the values we know gives:

$$\text{current } I = \frac{P}{V} = \frac{20\times10^6}{200\times10^3} = 100 \text{ A}$$

**Hint**: *Remember to convert megawatts into watts and kilovolts into volts.*

So, the power station supplies a current of 100 A.

## Question

**16** In Figure 8.14 the reading on the ammeter is 2.4 A and the reading on the voltmeter is 6.0 V. Calculate the resistance of the metallic conductor.