



## › Chapter 1

# Kinematics: describing motion

### LEARNING INTENTIONS

In this chapter you will learn how to:

- define and use displacement, speed and velocity
- draw and interpret displacement-time graphs
- describe laboratory methods for determining speed
- understand the differences between scalar and vector quantities and give examples of each
- use vector addition to add and subtract vectors that are in the same plane.

### BEFORE YOU START

- Do you know how to rearrange an equation that involves fractions? Choose an equation that you know from your previous physics course, such as  $P = \frac{V^2}{R}$ , and rearrange it to make  $R$  or  $V$  the subject of the formula.
- Can you write down a direction using compass bearings, for example, as  $014^\circ$ ,  $N14^\circ E$  or  $14^\circ$  east of north?

### DESCRIBING MOVEMENT

Our eyes are good at detecting movement. We notice even quite small movements out of the corners of our eyes. It's important for us to be able to judge movement – think about crossing the road, cycling or driving, or catching a ball.

Figure 1.1 shows a way in which movement can be recorded on a photograph. This is a stroboscopic photograph of a boy juggling three balls. As he juggles, a bright lamp flashes several times a second so that the camera records the positions of the balls at equal intervals of time.

How can the photograph be used to calculate the speed of the upper ball horizontally and vertically as it moves through the air? What other apparatus is needed? You can discuss this with someone else.



**Figure 1.1:** This boy is juggling three balls. A stroboscopic lamp flashes at regular intervals; the camera is moved to one side at a steady rate to show separate images of the boy.

# 1.1 Speed

We can calculate the average speed of something moving if we know the distance it moves and the time it takes:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}$$

In symbols, this is written as:

$$v = \frac{d}{t}$$

where  $v$  is the average speed and  $d$  is the distance travelled in time  $t$ .

If an object is moving at a constant speed, this equation will give us its speed during the time taken. If its speed is changing, then the equation gives us its **average speed**. Average speed is calculated over a period of time.

If you look at the speedometer in a car, it doesn't tell you the car's average speed; rather, it tells you its speed at the instant when you look at it. This is the car's **instantaneous speed**.

## KEY EQUATION

$$\begin{aligned}\text{average speed} &= \frac{\text{distance}}{\text{time}} \\ v &= \frac{d}{t}\end{aligned}$$

## Question

- 1 Look at Figure 1.2. The runner has just run 10 000 m in a time of 27 minutes 5.17 s. Calculate his average speed during the race.



**Figure 1.2:** England's Mo Farah winning his second gold medal at the Rio Olympics in 2016.

## Units

In the Système Internationale d'Unités (the SI system), distance is measured in metres (m) and time in seconds (s). Therefore, speed is in metres per second. This is written as  $\text{m s}^{-1}$  (or as  $\text{m/s}$ ). Here,  $\text{s}^{-1}$  is the same as  $1/\text{s}$ , or 'per second'.

There are many other units used for speed. The choice of unit depends on the situation. You would probably give the speed of a snail in different units from the speed of a racing car. Table 1.1 includes some alternative units of speed.

Note that in many calculations it is necessary to work in SI units ( $\text{m s}^{-1}$ ).

$\text{m s}^{-1}$	metres per second
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$\text{cm s}^{-1}$	centimetres per second
$\text{km s}^{-1}$	kilometres per second
$\text{km h}^{-1}$ or $\text{km/h}$	kilometres per hour
mph	miles per hour

**Table 1.1:** Units of speed.

## Questions

2 Here are some units of speed:

$\text{m s}^{-1}$   $\text{mm s}^{-1}$   $\text{km s}^{-1}$   $\text{km h}^{-1}$

Which of these units would be appropriate when stating the speed of each of the following?

- a a tortoise
- b a car on a long journey
- c light
- d a sprinter.

3 A snail crawls 12 cm in one minute. What is its average speed in  $\text{mm s}^{-1}$ ?

## Determining speed

You can find the speed of something moving by measuring the time it takes to travel between two fixed points. For example, some motorways have emergency telephones every 2000 m. Using a stopwatch you can time a car over this distance. Note that this can only tell you the car's average speed between the two points. You cannot tell whether it was increasing its speed, slowing down or moving at a constant speed.

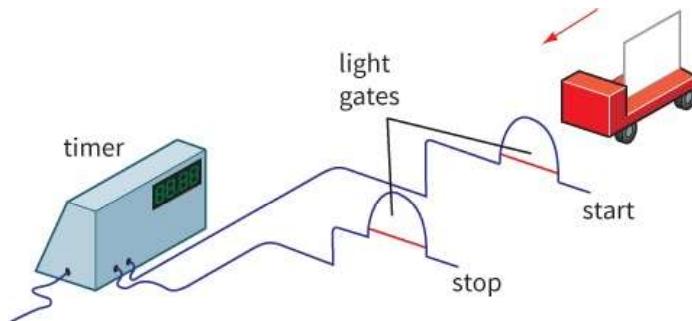
### PRACTICAL ACTIVITY 1.1

#### Laboratory measurements of speed

Here we describe four different ways to measure the speed of a trolley in the laboratory as it travels along a straight line. Each can be adapted to measure the speed of other moving objects, such as a glider on an air track or a falling mass.

#### Measuring speed using two light gates

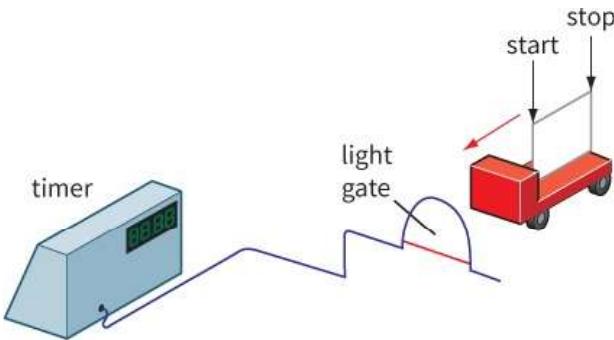
The leading edge of the card in Figure 1.3 breaks the light beam as it passes the first light gate. This starts the timer. The timer stops when the front of the card breaks the second beam. The trolley's speed is calculated from the time interval and the distance between the light gates.



**Figure 1.3:** Using two light gates to find the average speed of a trolley.

#### Measuring speed using one light gate

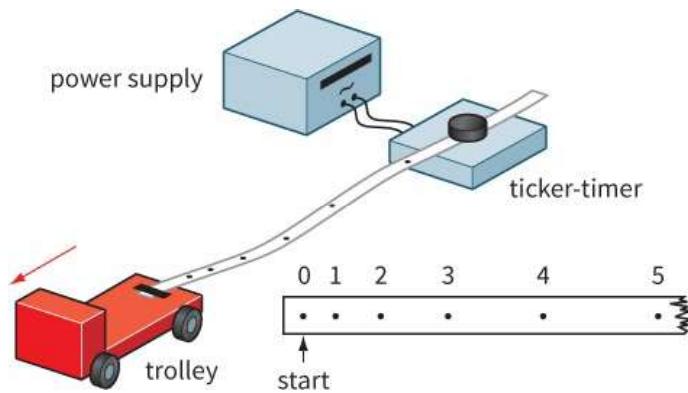
The timer in Figure 1.4 starts when the leading edge of the card breaks the light beam. It stops when the trailing edge passes through. In this case, the time shown is the time taken for the trolley to travel a distance equal to the length of the card. The computer software can calculate the speed directly by dividing the distance by the time taken.



**Figure 1.4:** Using a single light gate to find the average speed of a trolley.

### Measuring speed using a ticker-timer

The ticker-timer (Figure 1.5) marks dots on the tape at regular intervals, usually  $s$  (i.e. 0.02 s). (This is because it works with alternating current, and in most countries the frequency of the alternating mains is 50 Hz.) The pattern of dots acts as a record of the trolley's movement.



**Figure 1.5:** Using a ticker-timer to investigate the motion of a trolley.

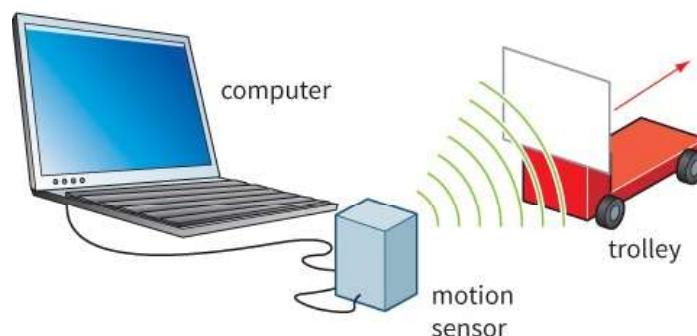
Start by inspecting the tape. This will give you a description of the trolley's movement. Identify the start of the tape. Then, look at the spacing of the dots:

- even spacing – constant speed
- increasing spacing – increasing speed.

Now you can make some measurements. Measure the distance of every fifth dot from the start of the tape. This will give you the trolley's distance at intervals of 0.10 s. Put the measurements in a table and draw a distance-time graph.

### Measuring speed using a motion sensor

The motion sensor (Figure 1.6) transmits regular pulses of ultrasound at the trolley. It detects the reflected waves and determines the time they took for the trip to the trolley and back. From this, the computer can deduce the distance to the trolley from the motion sensor. It can generate a distance-time graph. You can determine the speed of the trolley from this graph.



**Figure 1.6:** Using a motion sensor to investigate the motion of a trolley.

### Choosing the best method

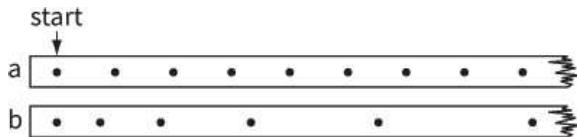
Each of these methods for finding the speed of a trolley has its merits. In choosing a method, you might think about the following points:

- Does the method give an average value of speed or can it be used to give the speed of the trolley at different points along its journey?
- How precisely does the method measure time to the nearest millisecond?
- How simple and convenient is the method to set up in the laboratory?

## Questions

4 A trolley with a 5.0 cm long card passed through a single light gate. The time recorded by a digital timer was 0.40 s. What was the average speed of the trolley in  $\text{m s}^{-1}$ ?

5 Figure 1.7 shows two ticker-tapes. Describe the motion of the trolleys that produced them.



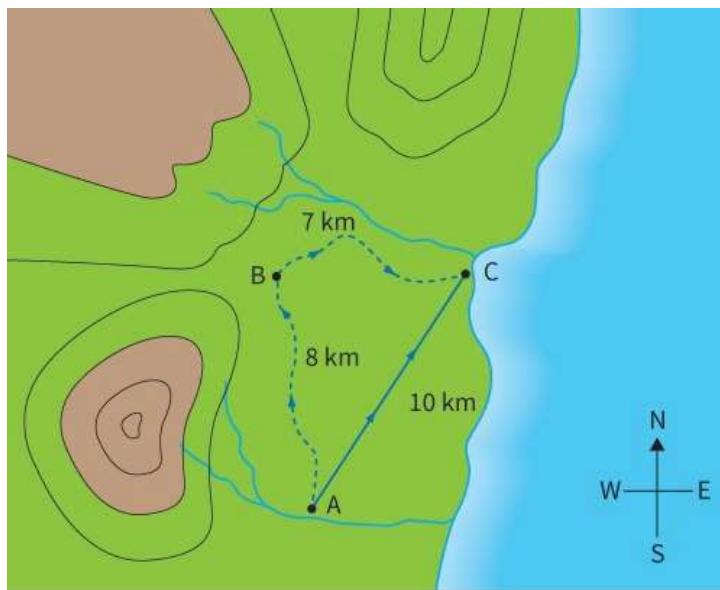
**Figure 1.7:** Two ticker-tapes. For Question 5.

6 Four methods for determining the speed of a moving trolley have been described. Each could be adapted to investigate the motion of a falling mass. Choose two methods that you think would be suitable, and write a paragraph for each to say how you would adapt it for this purpose.

## 1.2 Distance and displacement, scalar and vector

In physics, we are often concerned with the distance moved by an object in a particular direction. This is called its **displacement**.

Figure 1.8 illustrates the difference between distance and displacement. It shows the route followed by walkers as they went from town A to town C.



**Figure 1.8:** If you go on a long walk, the distance you travel will be greater than your displacement. In this example, the walkers travel a distance of 15 km, but their displacement is only 10 km, because this is the distance from the start to the finish of their walk.

Their winding route took them through town B, so that they covered a total distance of 15 km. However, their displacement was much less than this. Their finishing position was just 10 km from where they started. To give a complete statement of their displacement, we need to give both distance and direction:

$$\text{displacement} = 10 \text{ km at } 030^\circ \text{ or } 30^\circ \text{ E of N}$$

Displacement is an example of a **vector quantity**. A vector quantity has both magnitude (size) and direction. Distance, on the other hand, is a **scalar quantity**. Scalar quantities have magnitude only.

## 1.3 Speed and velocity

It is often important to know both the speed of an object and the direction in which it is moving.

Speed and direction are combined in another quantity, called **velocity**. The velocity of an object can be thought of as its speed in a particular direction. So, like displacement, velocity is a vector quantity. Speed is the corresponding scalar quantity, because it does not have a direction.

So, to give the velocity of something, we have to state the direction in which it is moving. For example, 'an aircraft flies with a velocity of  $300 \text{ m s}^{-1}$  due north'.

Since velocity is a vector quantity, it is defined in terms of displacement:

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

We can write the equation for velocity in symbols:

$$v = \frac{s}{t}$$

### KEY EQUATION

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

Alternatively, we can say that velocity is the *rate of change* of an object's displacement:

$$v = \frac{\Delta s}{\Delta t}$$

where the symbol  $\Delta$  (the Greek letter delta) means 'change in'. It does not represent a quantity (in the way that  $s$  and  $t$  do). Another way to write  $\Delta s$  would be  $s_2 - s_1$ , but this is more time-consuming and less clear.

From now on, you need to be clear about the distinction between velocity and speed, and between displacement and distance. Table 1.2 shows the standard symbols and units for these quantities.

Quantity	Symbol for quantity	Symbol for unit
distance	$d$	$\text{m}$
displacement	$s, x$	$\text{m}$
time	$t$	$\text{s}$
speed, velocity	$v$	$\text{m s}^{-1}$

**Table 1.2:** Standard symbols and units. (Take care not to confuse italic  $s$  for displacement with  $s$  for seconds. Notice also that  $v$  is used for both speed and velocity.)

### Question

7 Do these statements describe speed, velocity, distance or displacement? (Look back at the definitions of these quantities.)

- The ship sailed south-west for 200 miles.
- I averaged 7 mph during the marathon.
- The snail crawled at  $2 \text{ mm s}^{-1}$  along the straight edge of a bench.
- The sales representative's round trip was 420 km.

### Speed and velocity calculations

The equation for velocity,  $v = \frac{\Delta s}{\Delta t}$ , can be rearranged as follows, depending on which quantity we want to determine:

$$\text{change in displacement } \Delta s = v \times \Delta t$$

$$\text{change in time } \Delta t = \frac{\Delta s}{v}$$

Note that each of these equations is balanced in terms of units. For example, consider the equation for displacement. The units on the right-hand side are  $\text{m s}^{-1} \times \text{s}$ , which simplifies to  $\text{m}$ , the correct unit for displacement.

We can also rearrange the equation to find distance  $s$  and time  $t$ :

$$\Delta s = v \times t$$

$$t = \frac{\Delta s}{v}$$

### WORKED EXAMPLES

**1** A car is travelling at  $15 \text{ m s}^{-1}$ . How far will it travel in 1 hour?

**Step 1** It is helpful to start by writing down what you know and what you want to know:

$$v = 15 \text{ m s}^{-1}$$

$$t = 1 \text{ h} = 3600 \text{ s}$$

$$s = ?$$

**Step 2** Choose the appropriate version of the equation and substitute in the values. Remember to include the units:

$$s = v \times t$$

$$= 15 \times 3600$$

$$= 5.4 \times 10^4 \text{ m}$$

$$= 54 \text{ km}$$

The car will travel 54 km in 1 hour.

**2** The Earth orbits the Sun at a distance of 150 000 000 km. How long does it take light from the Sun to reach the Earth? (Speed of light in space =  $3.0 \times 10^8 \text{ m s}^{-1}$ .)

**Step 1** Start by writing what you know. Take care with units; it is best to work in m and s. You need to be able to express numbers in scientific notation (using powers of 10) and to work with these on your calculator.

$$v = 3.0 \times 10^8 \text{ m s}^{-1}$$

$$s = 150 000 000 \text{ km}$$

$$= 150 000 000 \text{ m}$$

$$= 1.5 \times 10^{11} \text{ m}$$

**Step 2** Substitute the values in the equation for time:

$$t = \frac{s}{v}$$

$$= \frac{1.5 \times 10^{11}}{3.0 \times 10^8}$$

$$= 500 \text{ s}$$

Light takes 500 s (about 8.3 minutes) to travel from the Sun to the Earth.

**Hint:** When using a calculator, to calculate the time  $t$ , you press the buttons in the following sequence:

**[1.5] [10<sup>n</sup>] [11] [÷] [3] [10<sup>n</sup>] [8]**

## Making the most of units

In Worked example 1 and Worked example 2, units have been omitted in intermediate steps in the calculations. However, at times it can be helpful to include units as this can be a way of checking that you have used the correct equation; for example, that you have not divided one quantity by another when you should have multiplied them. The units of an equation must be balanced, just as the numerical values on each side of the equation must be equal.

If you take care with units, you should be able to carry out calculations in non-SI units, such as kilometres per hour, without having to convert to metres and seconds.

For example, how far does a spacecraft travelling at  $40 000 \text{ km h}^{-1}$  travel in one day? Since there are 24 hours in one day, we have:

$$\begin{aligned}\text{distance travelled} &= 40\ 000 \text{ km h}^{-1} \times 24 \text{ h} \\ &= 960\ 000 \text{ km}\end{aligned}$$

## Questions

- 8 A submarine uses sonar to measure the depth of water below it. Reflected sound waves are detected 0.40 s after they are transmitted. How deep is the water? (Speed of sound in water =  $1500 \text{ m s}^{-1}$ .)
- 9 The Earth takes one year to orbit the Sun at a distance of  $1.5 \times 10^{11} \text{ m}$ . Calculate its speed. Explain why this is its average speed and not its velocity.

## 1.4 Displacement-time graphs

We can represent the changing position of a moving object by drawing a displacement-time graph. The gradient (slope) of the graph is equal to its velocity (Figure 1.9). The steeper the slope, the greater the velocity. A graph like this can also tell us if an object is moving forwards or backwards. If the gradient is negative, the object's velocity is negative – it is moving backwards.

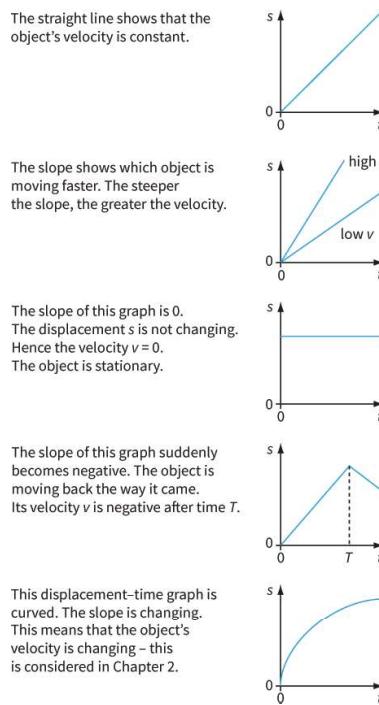
### Deducing velocity from a displacement-time graph

A toy car moves along a straight track. Its displacement at different times is shown in Table 1.3. This data can be used to draw a displacement-time graph from which we can deduce the car's velocity.

Displacement $s / \text{m}$	1.0	3.0	5.0	7.0	7.0	7.0
Time $t / \text{s}$	0.0	1.0	2.0	3.0	4.0	5.0

**Table 1.3:** Displacement  $s$  and time  $t$  data for a toy car.

It is useful to look at the data first, to see the pattern of the car's movement. In this case, the displacement increases steadily at first, but after 3.0 s it becomes constant. In other words, initially the car is moving at a steady velocity, but then it stops.



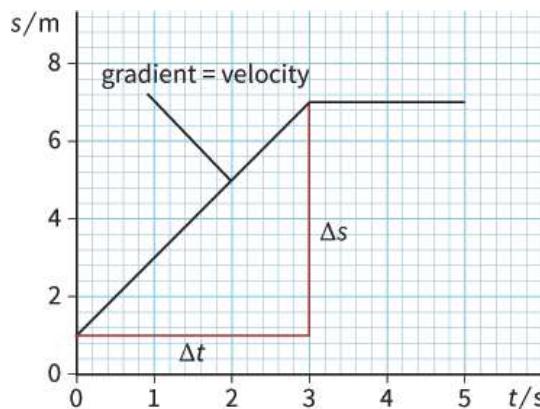
**Figure 1.9:** The slope of a displacement-time ( $s$ - $t$ ) graph tells us how fast an object is moving.

Now we can plot the displacement-time graph (Figure 1.10).

We want to work out the velocity of the car over the first 3.0 seconds. We can do this by working out the gradient of the graph, because:

$$\text{velocity} = \text{gradient of displacement-time graph}$$

We draw a right-angled triangle as shown. To find the car's velocity, we divide the change in displacement by the change in time. These are given by the two sides of the triangle labelled  $\Delta s$  and  $\Delta t$ .



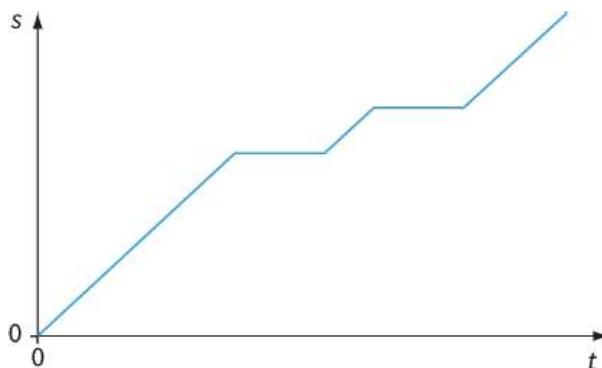
**Figure 1.10:** Displacement-time graph for a toy car; data as shown in Table 1.3.

$$\begin{aligned}
 \text{velocity} &= \frac{\text{change in displacement}}{\text{time taken}} \\
 &= \frac{\Delta s}{\Delta t} \\
 &= \frac{(7.0 - 1.0)}{(3.0 - 0)} \\
 &= \frac{6.0}{3.0} \\
 &= 2.0 \text{ ms}^{-1}
 \end{aligned}$$

If you are used to finding the gradient of a graph, you may be able to reduce the number of steps in this calculation.

## Questions

**10** The displacement-time sketch graph in Figure 1.11 represents the journey of a bus. What does the graph tell you about the journey?



**Figure 1.11:** For Question 10.

**11** Sketch a displacement-time graph to show your motion for the following event. You are walking at a constant speed across a field after jumping off a gate. Suddenly you see a horse and stop. Your friend says there's no danger, so you walk on at a reduced constant speed. The horse neighs, and you run back to the gate. Explain how each section of the walk relates to a section of your graph.

**12** Table 1.4 shows the displacement of a racing car at different times as it travels along a straight track during a speed trial.

- Determine the car's velocity.
- Draw a displacement-time graph and use it to find the car's velocity.

<b>Displacement / m</b>	0	85	170	255	340
<b>Time / s</b>	0	1.0	2.0	3.0	4.0

**Table 1.4:** Displacement  $s$  and time  $t$  data for Question 12.

13 An old car travels due south. The distance it travels at hourly intervals is shown in Table 1.5.

- Draw a distance-time graph to represent the car's journey.
- From the graph, deduce the car's speed in  $\text{km h}^{-1}$  during the first three hours of the journey.
- What is the car's average speed in  $\text{km h}^{-1}$  during the whole journey?

<b>Time / h</b>	0	1	2	3	4
<b>Distance / km</b>	0	23	46	69	84

**Table 1.5:** Data for Question 13.

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## 1.5 Combining displacements

The walkers shown in Figure 1.12 are crossing difficult ground. They navigate from one prominent point to the next, travelling in a series of straight lines. From the map, they can work out the distance that they travel and their displacement from their starting point:

$$\text{distance travelled} = 25 \text{ km}$$



**Figure 1.12:** In rough terrain, walkers head straight for a prominent landmark.

(Lay thread along route on map; measure thread against map scale.)

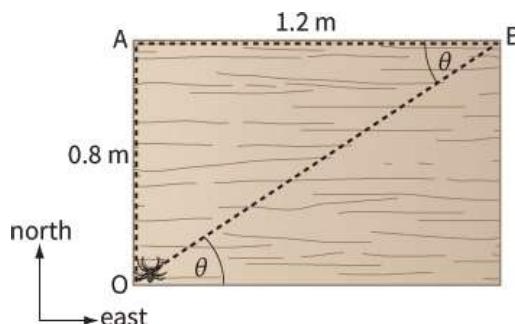
displacement = 15 km in the direction 045°, N45° E or north-east

(Join starting and finishing points with straight line; measure line against scale.)

A map is a scale drawing. You can find your displacement by measuring the map. But how can you **calculate** your displacement? You need to use ideas from geometry and trigonometry. Worked examples 3 and 4 show how.

### WORKED EXAMPLES

**3** A spider runs along two sides of a table (Figure 1.13). Calculate its final displacement.



**Figure 1.13:** The spider runs a distance of 2.0 m. For Worked example 3.

**Step 1** Because the two sections of the spider's run (OA and AB) are at right angles, we can **add** the two displacements using Pythagoras's theorem:

$$\begin{aligned} OB^2 &= OA^2 + AB^2 \\ &= 0.8^2 + 1.2^2 = 2.08 \\ OB &= \sqrt{2.08} = 1.44 \text{ m} \approx 1.4 \text{ m} \end{aligned}$$

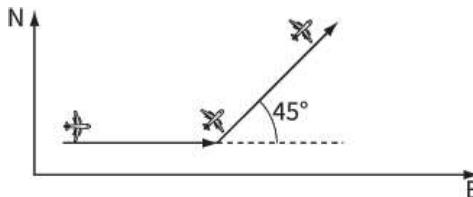
**Step 2** Displacement is a vector. We have found the **magnitude** of this vector, but now we have to

find its direction. The angle  $\theta$  is given by:

$$\begin{aligned}\tan \theta &= \frac{\text{opp}}{\text{adj}} = \frac{0.8}{1.2} \\ &= 0.667 \\ \theta &= \tan^{-1}(0.667) \\ &= 33.7^\circ \approx 34^\circ\end{aligned}$$

So the spider's displacement is 1.4 m at  $056^\circ$  or N $56^\circ$ E or at an angle of  $34^\circ$  north of east.

4 An aircraft flies 30 km due east and then 50 km north-east (Figure 1.14). Calculate the final displacement of the aircraft.



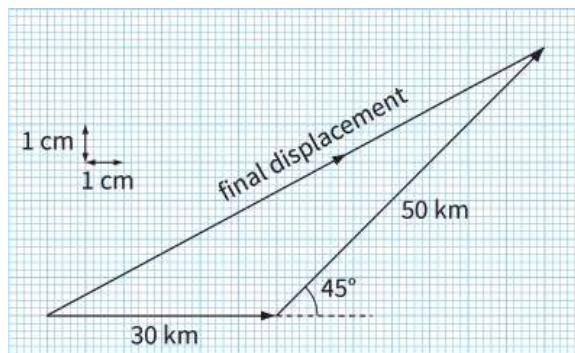
**Figure 1.14:** For Worked example 4.

Here, the two displacements are not at  $90^\circ$  to one another, so we can't use Pythagoras's theorem. We can solve this problem by making a scale drawing, and measuring the final displacement. (However, you could solve the same problem using trigonometry.)

**Step 1** Choose a suitable scale. Your diagram should be reasonably large; in this case, a scale of 1 cm to represent 5 km is reasonable.

**Step 2** Draw a line to represent the first vector. North is at the top of the page. The line is 6 cm long, towards the east (right).

**Step 3** Draw a line to represent the second vector, starting at the end of the first vector. The line is 10 cm long, and at an angle of  $45^\circ$  (Figure 1.15).



**Figure 1.15:** Scale drawing for Worked example 4. Using graph paper can help you to show the vectors in the correct directions.

**Step 4** To find the final displacement, join the start to the finish. You have created a **vector triangle**. Measure this displacement vector, and use the scale to convert back to kilometres:

$$\text{length of vector} = 14.8 \text{ cm}$$

$$\text{final displacement} = 14.8 \times 5 = 74 \text{ km}$$

**Step 5** Measure the angle of the final displacement vector:

$$\text{angle} = 28^\circ \text{ N of E}$$

Therefore the aircraft's final displacement is 74 km at  $28^\circ$  north of east,  $062^\circ$  or N $62^\circ$ E.

## Questions

**14** You walk 3.0 km due north, and then 4.0 km due east.

- Calculate the total distance in km you have travelled.
- Make a scale drawing of your walk, and use it to find your final displacement. Remember to give both the magnitude and the direction.
- Check your answer to part **b** by calculating your displacement.

**15** A student walks 8.0 km south-east and then 12 km due west.

- Draw a vector diagram showing the route. Use your diagram to find the total displacement. Remember to give the scale on your diagram and to give the direction as well as the magnitude of your answer.
- Calculate the resultant displacement. Show your working clearly.

This process of adding two displacements together (or two or more of any type of vector) is known as vector addition. When two or more vectors are added together, their combined effect is known as the **resultant** of the vectors.

# 1.6 Combining velocities

Velocity is a vector quantity and so two velocities can be combined by vector addition in the same way that we have seen for two or more displacements.

Imagine that you are attempting to swim across a river. You want to swim directly across to the opposite bank, but the current moves you sideways at the same time as you are swimming forwards. The outcome is that you will end up on the opposite bank, but downstream of your intended landing point. In effect, you have two velocities:

- the velocity due to your swimming, which is directed straight across the river
- the velocity due to the current, which is directed downstream, at right angles to your swimming velocity.

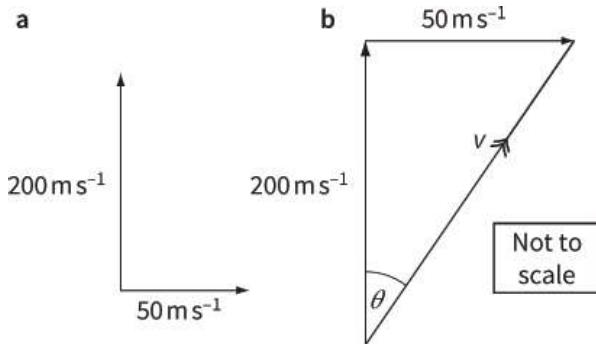
These combine to give a resultant (or net) velocity, which will be diagonally downstream. In order to swim directly across the river, you would have to aim upstream. Then your resultant velocity could be directly across the river.

## WORKED EXAMPLE

5 An aircraft is flying due north with a velocity of  $200 \text{ m s}^{-1}$ . A side wind of velocity  $50 \text{ m s}^{-1}$  is blowing due east. What is the aircraft's resultant velocity (give the magnitude and direction)?  
Here, the two velocities are at  $90^\circ$ . A sketch diagram and Pythagoras's theorem are enough to solve the problem.

**Step 1** Draw a sketch of the situation – this is shown in Figure 1.16a.

**Step 2** Now sketch a vector triangle. Remember that the second vector starts where the first one ends. This is shown in Figure 1.16b.



**Figure 1.16:** Finding the resultant of two velocities. For Worked example 5.

**Step 3** Join the start and end points to complete the triangle.

**Step 4** Calculate the magnitude of the resultant vector  $v$  (the hypotenuse of the right-angled triangle).

$$v^2 = 200^2 + 50^2 = 40\,000 + 2500 = 42\,500$$

$$v = \sqrt{42\,500} \approx 206 \text{ m s}^{-1}$$

**Step 5** Calculate the angle  $\theta$ :

$$\begin{aligned}\tan \theta &= \frac{50}{200} \\ &= 0.25\end{aligned}$$

$$\theta = \tan^{-1}(0.25) \approx 14^\circ$$

So the aircraft's resultant velocity is  $206 \text{ m s}^{-1}$  at  $14^\circ$  east of north,  $076^\circ$  or  $\text{N}76^\circ\text{E}$ .

## Questions

16 A swimmer can swim at  $2.0 \text{ m s}^{-1}$  in still water. She aims to swim directly across a river that is flowing at  $0.80 \text{ m s}^{-1}$ . Calculate her resultant velocity. (You must give both the magnitude and the

direction.)

17 A stone is thrown from a cliff and strikes the surface of the sea with a vertical velocity of  $18 \text{ m s}^{-1}$  and a horizontal velocity  $v$ . The resultant of these two velocities is  $25 \text{ m s}^{-1}$ .

- a Draw a vector diagram showing the two velocities and the resultant.
- b Use your diagram to find the value of  $v$ .
- c Use your diagram to find the angle between the stone and the vertical as it strikes the water.

# 1.7 Subtracting vectors

Sometimes, vectors need to be subtracted rather than added. For example, if you are in a car moving at  $2.0 \text{ m s}^{-1}$  and another car on the same road is moving in the same direction at  $5.0 \text{ m s}^{-1}$ , then you approach the car at  $5.0 - 2.0 = 3.0 \text{ m s}^{-1}$ . You are subtracting two velocity vectors.

Subtraction of vectors can be done using the formula:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

where  $\mathbf{A}$  and  $\mathbf{B}$  are vectors.

## KEY IDEA

To subtract a vector, add on the vector to be subtracted in the opposite direction.

So, to subtract, just add the negative vector.

But first you have to understand what the negative of vector  $\mathbf{B}$  means. The negative of vector  $\mathbf{B}$  is another vector of the same size as  $\mathbf{B}$  but in the opposite direction.

This is straightforward if the velocities are in the same direction. For example, to subtract a velocity of  $4 \text{ m s}^{-1}$  north from a velocity of  $10 \text{ m s}^{-1}$  north, you start by drawing a vector  $10 \text{ m s}^{-1}$  north and then add a vector of  $4 \text{ m s}^{-1}$  south. The answer is  $6 \text{ m s}^{-1}$  north.

It is less straightforward if the velocities are in the opposite direction. For example, to subtract a velocity of  $4 \text{ m s}^{-1}$  south from a velocity of  $10 \text{ m s}^{-1}$  north, you start by drawing a vector  $10 \text{ m s}^{-1}$  north and then add a vector of  $4 \text{ m s}^{-1}$  north. The answer is  $14 \text{ m s}^{-1}$  north.

The example in Figure 1.17 shows how to find  $\mathbf{A} - \mathbf{B}$  and  $\mathbf{A} + \mathbf{B}$  when the vectors are along different directions.

## Question

18 A velocity of  $5.0 \text{ m s}^{-1}$  is due north. Subtract from this velocity another velocity that is:

- a  $5.0 \text{ m s}^{-1}$  due south
- b  $5.0 \text{ m s}^{-1}$  due north

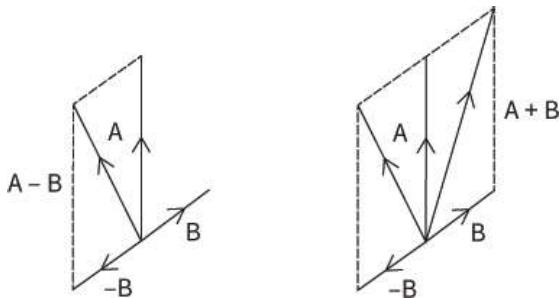


Figure 1.17: Subtracting and adding two vectors  $\mathbf{A}$  and  $\mathbf{B}$  in different directions.

- c  $5.0 \text{ m s}^{-1}$  due west
- d  $5.0 \text{ m s}^{-1}$  due east

(You can do a scale drawing or make a calculation but remember to give the direction of your answers as well as their size.)

## 1.8 Other examples of scalar and vector quantities

Direction matters when vectors are combined. You can use this to decide whether a quantity is a vector or a scalar. For example, if you walk for 3 minutes north and then 3 minutes in another direction, the total time taken is 6 minutes whatever direction you choose. A vector of 3 units added to another vector of 3 units can have any value between 0 and 6 but two scalars of 3 units added together always make six units. So, time is a scalar.

Mass and density are also both scalar quantities.

Force and acceleration, as you will see in later chapters, are both vector quantities. This is because, if an object is pushed with the same force in two opposite directions, the forces cancel out.

Work and pressure, which you will also study in later chapters, both involve force. However, work and pressure are both scalar quantities. For example, if you pull a heavy case along the floor north and then the same distance south, the total work done is clearly not zero. You just add scalar quantities even if they are in the opposite direction.

### REFLECTION

- Write down anything that you found interesting or challenging in this chapter.
- Look at your notes later when you revise this topic.

## SUMMARY

Displacement is the distance travelled in a particular direction.

Velocity is defined by the word equation:

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

The gradient of a displacement-time graph is equal to velocity:

$$v = \frac{\Delta s}{\Delta t}$$

Distance, speed, mass and time are scalar quantities. A scalar quantity has only magnitude.

Displacement and velocity are vector quantities. A vector quantity has both magnitude and direction.

Vector quantities may be combined by vector addition to find their resultant. The second vector can be subtracted from the first by adding the negative of the second vector, which acts in the opposite direction.

## EXAM-STYLE QUESTIONS

1 Which of the following pairs contains one vector and one scalar quantity? [1]

A displacement : mass  
B displacement : velocity  
C distance : speed  
D speed : time

2 A vector **P** of magnitude 3.0 N acts towards the right and a vector **Q** of magnitude 4.0 N acts upwards. [1]

What is the magnitude and direction of the vector  $(\mathbf{P} - \mathbf{Q})$ ?

A 1.0 N at an angle of  $53^\circ$  downwards to the direction of **P**  
B 1.0 N at an angle of  $53^\circ$  upwards to the direction of **P**  
C 5.0 N at an angle of  $53^\circ$  downwards to the direction of **P**  
D 5.0 N at an angle of  $53^\circ$  upwards to the direction of **P**

3 A car travels one complete lap around a circular track at a constant speed of  $120 \text{ km h}^{-1}$ .

a If one lap takes 2.0 minutes, show that the length of the track is 4.0 km. [2]

b Explain why values for the average speed and average velocity are different. [1]

c Determine the magnitude of the displacement of the car in a time of 1.0 min. [2]

(The circumference of a circle =  $2\pi R$ , where  $R$  is the radius of the circle.)

4 A boat leaves point A and travels in a straight line to point B. The journey takes 60 s.

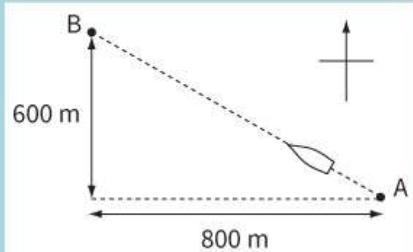


Figure 1.18

**Calculate:**

a the distance travelled by the boat [2]  
b the total displacement of the boat [2]  
c the average velocity of the boat. [2]

Remember that each vector quantity must be given a direction as well as a magnitude.

5 A boat travels at  $2.0 \text{ m s}^{-1}$  east towards a port, 2.2 km away. When the boat reaches the port, the passengers travel in a car due north for 15 minutes at  $60 \text{ km h}^{-1}$ .

Calculate:

a the total distance travelled [2]  
b the total displacement [3]  
c the total time taken [2]  
d the average speed in  $\text{m s}^{-1}$  [2]  
e the magnitude of the average velocity. [2]

6 A river flows from west to east with a constant velocity of  $1.0 \text{ m s}^{-1}$ . A boat

[Total: 6]

[Total: 11]

leaves the south bank heading due north at  $2.4 \text{ m s}^{-1}$ . Find the resultant velocity of the boat.

[3]

7 a **Define** displacement.

[1]

b Use the definition of displacement to explain how it is possible for an athlete to run round a track yet have no displacement.

[2]

[Total: 6]

8 A girl is riding a bicycle at a constant velocity of  $3.0 \text{ m s}^{-1}$  along a straight road. At time  $t = 0$ , she passes her brother sitting on a stationary bicycle. At time  $t = 0$ , the boy sets off to catch up with his sister. His velocity increases from time  $t = 0$  until  $t = 5.0 \text{ s}$ , when he has covered a distance of 10 m. He then continues at a constant velocity of  $4.0 \text{ m s}^{-1}$ .

[1]

a Draw the displacement-time graph for the girl from  $t = 0$  to  $t = 12 \text{ s}$ .

[1]

b On the same graph axes, draw the displacement-time graph for the boy.

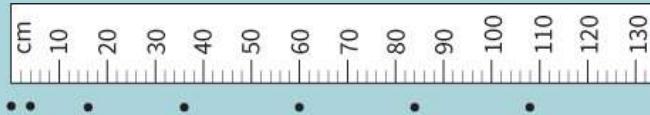
[2]

c Using your graph, determine the value of  $t$  when the boy catches up with his sister.

[1]

[Total: 4]

9 A student drops a small black sphere alongside a vertical scale marked in centimetres. A number of flash photographs of the sphere are taken at  $0.10 \text{ s}$  intervals:



This diagram is shown sideways – the first black dot is at 0 cm and the next at 4 cm.

**Figure 1.19**

The first photograph is taken with the sphere at the top at time  $t = 0 \text{ s}$ .

a Explain how Figure 1.19 shows that the sphere reaches a constant speed.

[2]

b Determine the constant speed reached by the sphere.

[2]

c Determine the distance that the sphere has fallen when  $t = 0.80 \text{ s}$ .

[2]

d In a real photograph, each image of the sphere appears slightly blurred because each flash is not instantaneous and takes a time of  $0.0010 \text{ s}$ .

Determine the absolute uncertainty that this gives in the position of each position of the black sphere when it is travelling at the final constant speed.

**Suggest** whether this should be observable on the diagram.

[2]

[Total: 8]

10 a **State one** difference between a scalar quantity and a vector quantity and give an example of each.

[3]

b A plane has an air speed of  $500 \text{ km h}^{-1}$  due north. A wind blows at  $100 \text{ km h}^{-1}$  from east to west.

Draw a vector diagram to calculate the resultant velocity of the plane. Give the direction of travel of the plane with respect to north.

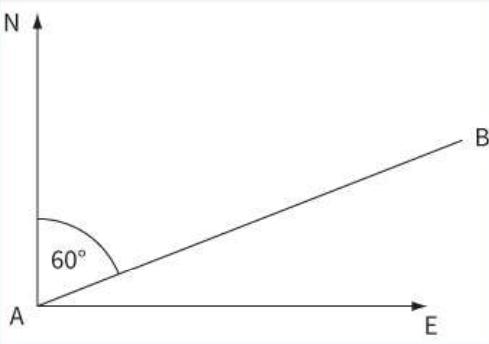
[4]

c The plane flies for 15 minutes. Calculate the displacement of the plane in this time.

[1]

[Total: 8]

11 A small aircraft for one person is used on a short horizontal flight. On its journey from A to B, the resultant velocity of the aircraft is  $15 \text{ m s}^{-1}$  in a direction  $60^\circ$  east of north and the wind velocity is  $7.5 \text{ m s}^{-1}$  due north.



**Figure 1.20**

a **Show that** for the aircraft to travel from A to B it should be pointed due east. [2]

b After flying 5 km from A to B, the aircraft returns along the same path from B to A with a resultant velocity of  $13.5 \text{ m s}^{-1}$ . Assuming that the time spent at B is negligible, calculate the average speed for the complete journey from A to B and back to A. [3]

**[Total: 5]**

## SELF-EVALUATION CHECKLIST

After studying the chapter, complete a table like this:

I can	See topic...	Needs more work	Almost there	Ready to move on
define and use displacement, speed and velocity	<a href="#">1.1</a> , <a href="#">1.2</a> , <a href="#">1.3</a>			
draw and interpret displacement-time graphs	<a href="#">1.4</a>			
describe laboratory methods for determining speed	<a href="#">1.1</a>			
understand the differences between scalar and vector quantities and give examples of each	<a href="#">1.2</a>			
use vector addition to add and subtract vectors that are in the same plane.	<a href="#">1.6</a> , <a href="#">1.7</a>			