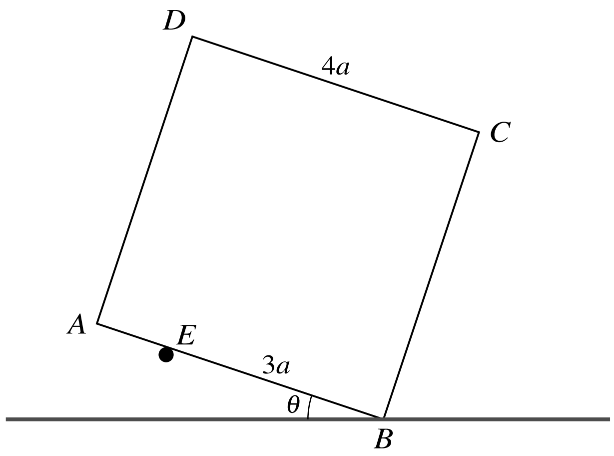


2



A uniform square lamina $ABCD$ of side $4a$ and weight W rests in a vertical plane with the edge AB inclined at an angle θ to the horizontal, where $\tan\theta = \frac{1}{3}$. The vertex B is in contact with a rough horizontal surface for which the coefficient of friction is μ. The lamina is supported by a smooth peg at the point E on AB, where $BE = 3a$ (see diagram).

(i) Find expressions in terms of W for the normal reaction forces at E and B. [5]

(ii) Given that the lamina is about to slip, find the value of μ. [3]

3 Three uniform small spheres A, B and C have equal radii and masses $5m$, $5m$ and $3m$ respectively. The spheres are at rest on a smooth horizontal surface, in a straight line, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Show that the speed of A after its collision with B is $\frac{1}{2}u(1-e)$ and find the speed of B. [3]

....

....

....

....

....

....

....

....

....

....

....

....

Sphere B now collides with sphere C. Subsequently there are no further collisions between any of the spheres.

(ii) Find the set of possible values of e. [6]

....

....

....

....

....

....

....

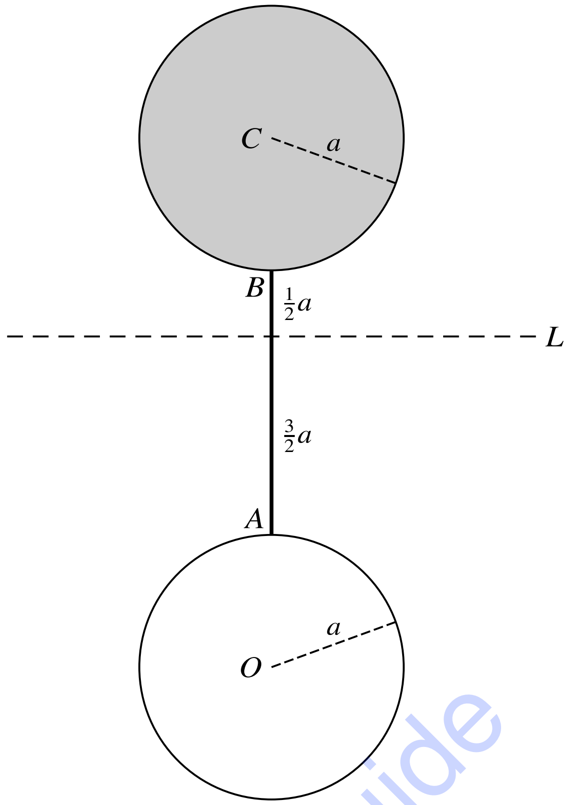
....

4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O and P is held with the string taut and horizontal. The particle P is projected vertically downwards with speed $\sqrt{(2ag)}$ so that it begins to move along a circular path. The string becomes slack when OP makes an angle θ with the upward vertical through O.

(i) Show that $\cos\theta = \frac{2}{3}$. [5]

(ii) Find the greatest height, above the horizontal through O, reached by P in its subsequent motion. [4]

5



A thin uniform rod AB has mass λM and length $2a$. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere (spherical shell) with centre O, mass $3M$ and radius a. The end B of the rod is rigidly attached to the surface of a uniform solid sphere with centre C, mass $5M$ and radius a. The rod lies along the line joining the centres of the spheres, so that $CBAO$ is a straight line. The horizontal axis L is perpendicular to the rod and passes through the point of the rod that is a distance $\frac{1}{2}a$ from B (see diagram). The object consisting of the rod and the two spheres can rotate freely about L.

(i) Show that the moment of inertia of the object about L is $\left(\frac{408 + 7\lambda}{12}\right)Ma^2$. [6]

....

....

....

....

....

....

....

....

....

The period of small oscillations of the object about L is $5\pi\sqrt{\left(\frac{2a}{g}\right)}$.

(ii) Find the value of λ. [6]

6 A random sample of 9 members is taken from the large number of members of a sports club, and their heights are measured. The heights of all the members of the club are assumed to be normally distributed. A 95% confidence interval for the population mean height, μ metres, is calculated from the data as $1.65 \leqslant \mu \leqslant 1.85$.

(i) Find an unbiased estimate for the population variance. [3]

....

....

....

....

....

....

....

....

(ii) Denoting the height of a member of the club by x metres, find Σx^2 for this sample of 9 members. [4]

....

....

....

....

....

....

....

....

....

....

....

....

....

7 The time, T days, before an electrical component develops a fault has distribution function F given by

$$F(t) = \begin{cases} 1 - e^{-at} & t \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant. The mean value of T is 200.

(i) Write down the value of a. [1]

....

....

(ii) Find the probability that an electrical component of this type develops a fault in less than 150 days. [2]

....

....

....

....

....

A piece of equipment contains n of these components, which develop faults independently of each other. The probability that, after 150 days, at least one of the n components has not developed a fault is greater than 0.99.

(iii) Find the smallest possible value of n. [4]

....

....

....

....

....

....

....

....

....

....

8 A random sample of 8 elephants from region A is taken and their weights, x tonnes, are recorded. (1 tonne = 1000 kg.) The results are summarised as follows.

$$\Sigma x = 32.4 \qquad \Sigma x^2 = 131.82$$

A random sample of 10 elephants from region B is taken. Their weights give a sample mean of 3.78 tonnes and an unbiased variance estimate of 0.1555 tonnes^2. The distributions of the weights of elephants in regions A and B are both assumed to be normal with the same population variance. Test at the 10% significance level whether the mean weight of elephants in region A is the same as the mean weight of elephants in region B. [9]

9 A random sample of five pairs of values of x and y is taken from a bivariate distribution. The values are shown in the following table, where p and q are constants.

x	1	2	3	4	5
y	4	p	q	2	1

The equation of the regression line of y on x is $y = -0.5x + 3.5$.

(i) Find the values of p and q. [7]

(ii) Find the value of the product moment correlation coefficient. [3]

10 The random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{30}\left(\frac{8}{x^2} + 3x^2 - 14\right) & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

....

....

....

....

....

....

....

....

....

....

....

....

The random variable Y is defined by $Y = X^2$.

(ii) Find the probability density function of Y. [4]

....

....

....

....

....

....

....

....

(iii) Find the value of y such that $P(Y < y) = 0.8$. [3]

11 Answer only **one** of the following two alternatives.

EITHER

The points A and B are a distance 1.2 m apart on a smooth horizontal surface. A particle P of mass $\frac{2}{3}$ kg is attached to one end of a light spring of natural length 0.6 m and modulus of elasticity 10 N. The other end of the spring is attached to the point A. A second light spring, of natural length 0.4 m and modulus of elasticity 20 N, has one end attached to P and the other end attached to B.

(i) Show that when P is in equilibrium $AP = 0.75$ m. [3]

....

....

....

....

....

....

....

....

The particle P is displaced by 0.05 m from the equilibrium position towards A and then released from rest.

(ii) Show that P performs simple harmonic motion and state the period of the motion. [6]

....

....

....

....

....

....

....

....

....

....

(iii) Find the speed of P when it passes through the equilibrium position. [2]

(iv) Find the speed of P when its acceleration is equal to half of its maximum value. [3]

OR

The number of puncture repairs carried out each week by a small repair shop is recorded over a period of 40 weeks. The results are shown in the following table.

Number of repairs in a week	0	1	2	3	4	5	$\geqslant 6$
Number of weeks	6	15	9	6	3	1	0

(i) Calculate the mean and variance for the number of repairs in a week and comment on the possible suitability of a Poisson distribution to model the data. [3]

....

....

....

....

....

....

....

Records over a longer period of time indicate that the mean number of repairs in a week is 1.6. The following table shows some of the expected frequencies, correct to 3 decimal places, for a period of 40 weeks using a Poisson distribution with mean 1.6.

Number of repairs in a week	0	1	2	3	4	5	$\geqslant 6$
Expected frequency	8.076	12.921	10.337	5.513	2.205	a	b

(ii) Show that $a = 0.706$ and find the value of the constant b. [3]

....

....

....

....

....

....

....

....

(iii) Carry out a goodness of fit test of a Poisson distribution with mean 1.6, using a 10% significance level. [8]

