

- 2** It is given that $y = \ln(ax + 1)$, where a is a positive constant. Prove by mathematical induction that, for every positive integer n ,

$$\frac{d^n y}{dx^n} = (-1)^{n-1} \frac{(n-1)! a^n}{(ax+1)^n}. \quad [6]$$

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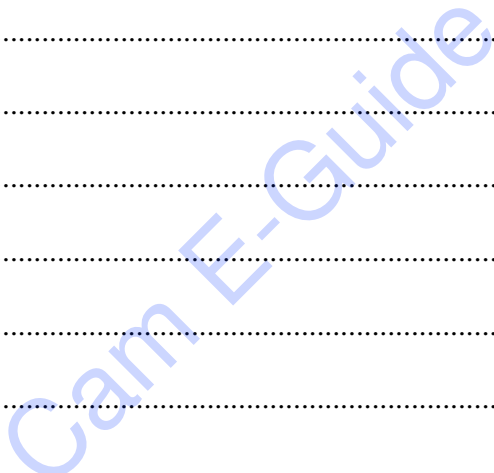
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3 The integral I_n , where n is a positive integer, is defined by

$$I_n = \int_{\frac{1}{2}}^1 x^{-n} \sin \pi x \, dx.$$

(i) Show that

$$n(n + 1)I_{n+2} = 2^{n+1}n + \pi - \pi^2 I_n. \tag{5}$$

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(ii) Find I_5 in terms of π and I_1 . [2]

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4 The line $y = 2x + 1$ is an asymptote of the curve C with equation

$$y = \frac{x^2 + 1}{ax + b}.$$

(i) Find the values of the constants a and b . [3]

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(ii) State the equation of the other asymptote of C . [1]

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(iii) Sketch C . [Your sketch should indicate the coordinates of any points of intersection with the y -axis. You do not need to find the coordinates of any stationary points.] [3]

5 Let $S_N = \sum_{r=1}^N (5r+1)(5r+6)$ and $T_N = \sum_{r=1}^N \frac{1}{(5r+1)(5r+6)}$.

(i) Use standard results from the List of Formulae (MF10) to show that

$$S_N = \frac{1}{3}N(25N^2 + 90N + 83). \quad [3]$$

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(ii) Use the method of differences to express T_N in terms of N . [4]

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(iii) Find $\lim_{N \rightarrow \infty} (N^{-3}S_N T_N)$. [2]

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- (ii) Find the cartesian equation of the plane containing the line OC and the common perpendicular of the lines OC and AB . [4]

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7 The equation $x^3 + 2x^2 + x + 7 = 0$ has roots α, β, γ .

(i) Use the relation $x^2 = -7y$ to show that the equation

$$49y^3 + 14y^2 - 27y + 7 = 0$$

has roots $\frac{\alpha}{\beta\gamma}, \frac{\beta}{\gamma\alpha}, \frac{\gamma}{\alpha\beta}$.

[4]

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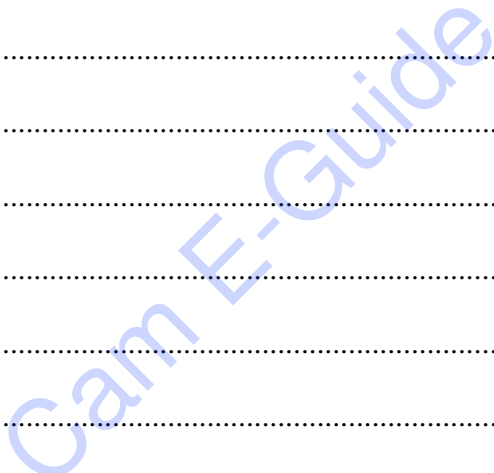
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(ii) Show that $\frac{\alpha^2}{\beta^2\gamma^2} + \frac{\beta^2}{\gamma^2\alpha^2} + \frac{\gamma^2}{\alpha^2\beta^2} = \frac{58}{49}$. [3]

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(iii) Find the exact value of $\frac{\alpha^3}{\beta^3\gamma^3} + \frac{\beta^3}{\gamma^3\alpha^3} + \frac{\gamma^3}{\alpha^3\beta^3}$. [2]

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8 The matrix M is defined by

$$M = \begin{pmatrix} 2 & m & 1 \\ 0 & m & 7 \\ 0 & 0 & 1 \end{pmatrix},$$

where $m \neq 0, 1, 2$.

- (i) Find a matrix P and a diagonal matrix D such that $M = PDP^{-1}$. [7]

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(ii) Find M^7P .

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10 The matrix \mathbf{A} is defined by

$$\mathbf{A} = \begin{pmatrix} 1 & 5 & 1 \\ 1 & -2 & -2 \\ 2 & 3 & \theta \end{pmatrix}.$$

(i) (a) Find the rank of \mathbf{A} when $\theta \neq -1$. [3]

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(b) Find the rank of \mathbf{A} when $\theta = -1$. [1]

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Consider the system of equations

$$\begin{aligned} x + 5y + z &= -1, \\ x - 2y - 2z &= 0, \\ 2x + 3y + \theta z &= \theta. \end{aligned}$$

(ii) Solve the system of equations when $\theta \neq -1$. [3]

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(iii) Find the general solution when $\theta = -1$. [3]

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(iv) Show that if $\theta = -1$ and $\phi \neq -1$ then $\mathbf{Ax} = \begin{pmatrix} -1 \\ 0 \\ \phi \end{pmatrix}$ has no solution. [2]

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11 Answer only **one** of the following two alternatives.

EITHER

It is given that $w = \cos y$ and

$$\tan y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 2 \tan y \frac{dy}{dx} = 1 + e^{-2x} \sec y.$$

(i) Show that

$$\frac{d^2w}{dx^2} + 2 \frac{dw}{dx} + w = -e^{-2x}. \quad [4]$$

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(ii) Find the particular solution for y in terms of x , given that when $x = 0$, $y = \frac{1}{3}\pi$ and $\frac{dy}{dx} = \frac{1}{\sqrt{3}}$. [10]

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(ii) Sketch C_1 and C_2 on the same diagram.

[3]

(iii) Find the area of the region enclosed by C_1 , C_2 and the initial line, giving your answer correct to 3 significant figures. [5]

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Additional Page

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