

Question	Answer	Marks	Guidance
1	$(T-1)^4 / 2 = 8$	M1 A1	Equate radial acceln. to 8 at $t = T$ from v^2/r
	$T = 3$ (or $T - 1 = 2$)	A1	Hence find positive value of T (or of $T - 1$)
	$a_T = 2\,(T-1) = 4$ [m s^{-2}]	M1 A1	Find magnitude of transverse acceleration at $t = T$
		5	

Question	Answer	Marks	Guidance
2(i)	$R_E \times 3a = W\cos\theta \times 2a - W\sin\theta \times 2a$ or $R_E \times 3a = W \times 2a\,(1 - \tan\theta)\cos\theta$ or $R_E \times 3a = 2\sqrt{2}W\sin\left(\frac{\pi}{4} - \theta\right)$	M1 A1	Take moments about B
	$R_E = 4W / 3\sqrt{10}$	A1	Find normal reaction at E. AEF
	$R_B = W - R_E\cos\theta = 3W/5$	M1 A1	Find normal reaction at B by resolving forces vertically
		5	
2(ii)	$F_B = R_E\sin\theta = 2W/15$	M1 A1	Find friction at B by resolving forces horizontally
	$\mu = (2/15) / (3/5) = 2/9$	A1	Find μ from $F_B = \mu R_B$
		3	

Question	Answer	Marks	Guidance
3(i)	$5mv_A + 5mv_B = 5mu$ [$v_A + v_B = u$] and $v_B - v_A = e\,u$	M1	Use consvn. of momentum for A and B and use Newton's restitution law with consistent LHS signs. AEF
	$v_A = ½\,(1 - e)\,u$	A1	Combine to verify speed of A. AG
	$v_B = ½\,(1 + e)\,u$	A1	Find speed of B
		3	
3(ii)	$5mv_B' + 3mv_C = 5mv_B$ [$5v_B' + 3v_C = 5v_B$] $v_C - v_B' = ev_B$	M1	Use consvn. of momentum for B and C and use Newton's restitution law with consistent LHS signs. AEF
	$v_B' = (1/8)\,(5 - 3e)\,v_B$ [$v_C = (1/8)\,(5 + 5e)\,v_B$]	A1	Combine to find v_B' (v_C not reqd as B, C cannot collide again)
	$\frac{1}{2}(1 - e)\,u \leqslant (1/8)\,(5 - 3e) \times \frac{1}{2}\,(1 + e)\,u$	M1	Find condition on e using $v_A \leqslant v_B'$
	$3e^2 - 10e + 3 \leqslant 0$	A1	Simplify to a quadratic inequality
	$\frac{1}{3} \leqslant e$	A1	Solve to give a lower bound on e
	$\frac{1}{3} \leqslant e \leqslant 1$	A1	Non-strict inequality
		6	

Question	Answer	Marks	Guidance
4(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga\cos\theta$	**M1**	Use conservation of energy to slack point P_1
	$mv^2/a - mg\cos\theta = 0$	**M1** **A1**	Equate tension at P_1 to 0 by using $F = ma$ A1 if both eqns correct, with *m* included. AG
	$v^2 = 2ag - 2ag\cos\theta = ag\cos\theta$	**M1**	Combine to verify $\cos\theta$ using $u = \sqrt{(2ag)}$
	$\cos\theta = 2/3$	**A1**	
		5	
4(ii)	$v_V = v\sin\theta = \sqrt{(2ag/3)}\,(\sqrt{5}/3)$ or $v_V^2 = (10/27)\,ag$	**M1**	Find vertical speed v_V at P_1
	$h = v_V^2/2g = (5/27)\,a$ or $0{\cdot}185\,a$	**M1 A1**	Find height risen above P_1 by considering vertical motion
	$h + a\cos\theta = (23/27)\,a$ or $0{\cdot}852\,a$	**A1**	Find total height risen above level of *O*
		4	

Question	Answer	Marks	Guidance
5(i)	$I_{rod} = \frac{1}{3}\ \lambda Ma^2 + \lambda M(a/2)^2$ [= (7/12) λMa^2]	**B1**	Find or state MI of rod *AB* about axis *L*
	$I_O = \frac{2}{3}\ 3M\,a^2 + 3M\,(5a/2)^2$ [= (83/4) Ma^2]	**M1 A1**	Find MI of hollow sphere centre *O* about axis *L*
	$I_C = (2/5)\ 5Ma^2 + 5M\,(3a/2)^2$ [= (53/4) Ma^2]	**M1 A1**	Find MI of solid sphere centre *C* about axis *L*
	$I = (7\lambda/12 + 83/4 + 53/4)\ Ma^2$ $= ((7\lambda + 408) / 12)\ Ma^2$	**A1**	Verify MI of object about axis *L*. AG
		6	
5(ii)	$[–]\ I\ \mathrm{d}^2\theta/\mathrm{d}t^2 = [–\ 3Mg \times (5a/2)\sin\theta + 5Mg \times (3a/2)\sin\theta]$ $–\ \lambda Mg \times (a/2)\sin\theta$	**M1 A1**	Use eqn of circular motion to find $\mathrm{d}^2\theta/\mathrm{d}t^2$ where θ is angle of rod with vertical. AEF
	$\mathrm{d}^2\theta/\mathrm{d}t^2 = –\ \{6g\lambda\ /\ (7\lambda + 408)a\}\ \theta$	**M1***	Approximate sin θ by θ to give standard form of SHM eqn
	$T = 2\pi\ \sqrt{\{(7\lambda + 408)a\ /\ 6g\lambda\}} = 5\pi\sqrt{(2a/g)}$	**DM1A1**	Find possible values of λ by equating period T to $5\pi\sqrt{(2a/g)}$.AEF
	$\lambda = 6$	**A1**	
		6	

Question	Answer	Marks	Guidance
6(i)	$t\sqrt{(s^2/9)} = ½ (1·85 – 1·65)$ [= 0·1]	**M1**	Find estimate s^2 of population variance (must be t)
	$t_{8,\ 0.975} = 2·306$ (to 3 s.f.)	**A1**	Use of correct tabular t-value
	$s^2 = 9 \times 0·04337^2 = 0·0169$ *or* $0·130[1]^2$	**A1**	
		3	
6(ii)	$\bar{x} = \frac{1}{2}(1·65 + 1·85) = 1·75$ or $\Sigma x = 9 \times 1·75 = 15·75$	**M1 A1**	Find sample mean $\bar{x}$
	$s^2 = (\Sigma x^2 - 9 \times \bar{x}^2) / 8$ or $\{\Sigma x^2 - (\Sigma x)^2 / 9\} / 8$	**M1**	*or* Σx
	$\Sigma x^2 = 8 \times 0·0169 + 15·75^2/9 = 27·7$	**A1**	Find Σx^2 from s^2
		4	

Question	Answer	Marks	Guidance
7(i)	$a = 1/200$ *or* $0{\cdot}005$	**B1**	State a or find a by equating mean value to $1/a$
		1	
7(ii)	$p = P(T < 150) = F(150) = 1 - e^{-150a}$	**M1**	Find $P(T < 150)$
	$p = 1 - e^{-0{\cdot}75} = 0{\cdot}528$	**A1**	
		2	
7(iii)	$1 - p^n > 0{\cdot}99$	**M1**	Formulate condition for n
	$0{\cdot}01 > (1 - e^{-0{\cdot}75})^n$ or $0{\cdot}01 > 0{\cdot}528^n$	**A1**	
	$n > \log 0{\cdot}01 / \log 0{\cdot}528$	**M1**	Rearrange and take logs to give bound
	$n > 7{\cdot}20$ [or $7{\cdot}21$] so $n_{min} = 8$	**A1**	Find n_{min}
		4	

Question	Answer	Marks	Guidance
8	$\bar{x}_A = 32{\cdot}4 / 8 = 4{\cdot}05$	**B1**	Find sample mean for *A*
	$s_A^2 = (131{\cdot}82 - 32{\cdot}4^2/8) / 7$ $s_A^2 = 3/35$ (or $0{\cdot}08571$ or $0{\cdot}2928^2$ both to 3 s.f.)	**M1**	Estimate or imply popln. variance for *A*
	H_0: $\mu_A = \mu_B$, H_1: $\mu_A \neq \mu_B$	**B1**	State hypotheses. AEF
	$s^2 = (7\ s_A^2 + 9\ s_B^2) / 16 = 0{\cdot}12497$ or $0{\cdot}3535^2$ or $\frac{3999}{32000}$	**M1 A1**	Estimate (pooled) common variance
	$t_{16,\ 0.95} = 1{\cdot}746$	**B1***	State or use correct tabular t value
	[–] $t = (x_A - \bar{x}_B) / s\ \sqrt{(1/8 + 1/10)}$	**M1**	
	$= 0{\cdot}27 / 0{\cdot}1677 = 1{\cdot}61$	**A1**	Find value of t (or can compare $\bar{x}_A - \bar{x}_B = 0{\cdot}27$ with $0{\cdot}293$)
	$t < 1{\cdot}75$ so [accept H_0] mean masses are the same	**DB1**	Correct conclusion (FT on t, dep B1*). AEF
		9	

Question	Answer	Marks	Guidance
9(i)	$\Sigma x = 15$, $\Sigma y = 7 + p + q$, $\Sigma xy = 17 + 2p + 3q$ $\Sigma x^2 = 55$, $[\Sigma y^2 = 21 + p^2 + q^2]$	**M1**	Find required summations
	$S_{xx} = 55 - 15^2 / 5 = 10$ and $S_{xy} = 17 + 2p + 3q - 15 \times (7 + p + q) / 5 = -4 - p$	**M1 A1**	
	$-0{\cdot}5 = S_{xy} / S_{xx} = (-4 - p) / 10$ $\quad p = 1$	**M1 A1**	Find p from gradient in eqn. of regression line
	$(7 + p + q) / 5 = -0{\cdot}5 \times 15/5 + 3{\cdot}5$ $\quad q = 2$	**M1 A1**	Find q from means and regression line
		7	
9(ii)	$\Sigma y = 10$, $\Sigma y^2 = 26$, $S_{yy} = 26 - 10^2/5 = 6$	**M1**	Find S_{yy}
	$r = S_{xy} / \sqrt{(S_{xx} S_{yy})} = -5 / \sqrt{(10 \times 6)}$	**M1**	Find correlation coefficient r
	$r = -0{\cdot}645[5]$ [allow $-0{\cdot}646$]	**A1**	
		3	

Question	Answer	Marks	Guidance
10(i)	$F(x) = \int f(x)\,dx = (1/30)(-8/x + x^3 - 14x)\ [+ c]$	M1	Find or state distribution function $F(x)$ for $2 \leqslant x \leqslant 4$
	$F(x) = (1/30)(-8/x + x^3 - 14x + 24)$	M1	Using $F(2) = 0$ or $F(4) = 1$ to find c if necessary. AEF
	$F(x) = 0$ ($x <$ *or* $\leqslant 2$), $F(x) = 1$ ($x >$ *or* $\geqslant 4$)	A1	State $F(x)$ for other values of x
		3	
10(ii)	$G(y) = P(Y < y) = P(X^2 < y)$ $G(y) = P(X < \sqrt{y}) = F(\sqrt{y})$ $G(y) = (1/30)\left(-8/y^{\frac{1}{2}} + y^{\frac{3}{2}} - 14y^{\frac{1}{2}} + 24\right)$	M1 A1	Find or state $G(y)$ for $2 \leqslant x \leqslant 4$ from $Y = X^2$ (allow $<$ or $\leqslant$ throughout)
	Alternative method for question 10(ii)		
	Use $x = y^{\frac{1}{2}}$ to find $f(x) = (1/30)(8/y + 3y - 14), \dfrac{dx}{dy} = -\dfrac{1}{2}y^{-\frac{1}{2}}$	(M1 A1)	Find $f(x)$ *and* $\dfrac{dx}{dy}$ for use in $g(y) = f(x) \times \left\lvert\dfrac{dy}{dx}\right\rvert$
	$g(y)[= G'(y)] = (1/30)\left(4/y^{\frac{3}{2}} + (3/2)y^{\frac{1}{2}} - 7/y^{\frac{1}{2}}\right)$ for $4 \leqslant y \leqslant 16$ [$g(y) = 0$ otherwise]	A1 A1	Find $g(y)$. AEF State corresponding range of y for $G(y)$ *or* $g(y)$
		4	
10(iii)	$(1/30)\left(-8/y^{\frac{1}{2}} + y^{\frac{3}{2}} - 14y^{\frac{1}{2}} + 24\right) = 0.8$	M1	Set $G(y) = 0{\cdot}8$
	$-8 + y^2 - 14y = 0$, $y = 7 + \sqrt{57}$ *or* 14·5[5] [rejecting $7 - \sqrt{57}$; allow 14·6]	M1 A1	Rearrange to give quadratic in y and solve to find value of y
		3	

Question	Answer	Marks	Guidance
11A(i)	$10\ (AP - 0{\cdot}6) / 0{\cdot}6 = 20\ (1{\cdot}2 - AP - 0{\cdot}4) / 0{\cdot}4$	**M1 A1**	Verify *AP* by equating equilibrium tensions. AEF
	$4\ AP - 2{\cdot}4 = 9{\cdot}6 - 12\ AP$ $AP = 0{\cdot}75$ [m]	**A1**	AG
		3	
11A(ii)	$m\ d^2x/dt^2 = -\ 10\ (0{\cdot}15 + x) / 0{\cdot}6 + 20\ (0{\cdot}05 - x) / 0{\cdot}4$ or $m\ d^2x/dt^2 = +\ 10\ (0{\cdot}15 - x) / 0{\cdot}6 - 20\ (0{\cdot}05 + x) / 0{\cdot}4$	**M1** **A1 A1**	Apply Newton's law at $0{\cdot}75 + x$ or $0{\cdot}75 - x$ from *A* (M1 requires LHS and 2 tensions: A1 for each correct tension)
	$\frac{2}{3}\ d^2x/dt^2 = -\ (80 / 1{\cdot}2)\ x$, $d^2x/dt^2 = -\ 100\ x$	**M1 A1**	Simplify to give SHM eqn. in standard form
	$T = 2\pi/\omega = 2\pi/10 = \pi/5$ *or* $0{\cdot}628$ [s]	**DB1**	State the period *T* with FT on ω from SHM eqn.
		6	
11A(iii)	$(a = 0{\cdot}75 - 0{\cdot}7 = 0{\cdot}05)$ $v_{max} = \omega \times a$	**M1**	Find speed at equilibrium position from ωa
	$v_{max} = 0{\cdot}5$ [m s^{-1}]	**A1**	
		2	
11A(iv)	$x = a/2 = 0{\cdot}025$	**M1**	Find value of *x* giving half max. acceln.
	$v = \omega\ \sqrt{(a^2 - x^2)} = 10\ \sqrt{(0{\cdot}05^2 - 0{\cdot}025^2)}$	**M1**	
	$v = 0{\cdot}433$ [m s^{-1}]	**A1**	Find corresponding speed
		3	

Question	Answer	Marks	Guidance
11B(i)	$\overline{x} = (1/40)\ \Sigma\ x\ \mathrm{f}(x) = 68/40 = 1{\cdot}7$	**B1**	Find mean of sample
	$(1/40)\ \Sigma\ x^2\ \mathrm{f}(x) = 178/40 = 4{\cdot}45$, Var $= 4{\cdot}45 - 1{\cdot}7^2 = 1{\cdot}56$	**B1**	Find variance of sample
	Mean and variance are similar so Poisson may be suitable	**B1**	State valid comment
		3	
11B(ii)	$a = 40 \times 1{\cdot}6^5\ \mathrm{e}^{-1{\cdot}6}/5! = 40 \times 0{\cdot}01764$	**M1**	AG
	$a = 0{\cdot}706$	**A1**	Verify a from Poisson term
	$b = 40 - 39{\cdot}758 = 0{\cdot}242$	**B1***	Find b
		3	
11B(iii)	H_0: Distribution fits/models data	**B1**	State (at least) null hypothesis in full
	O_i: 6 15 9 <u>10</u> E_i: 8·076 12·921 10·337 <u>8·666</u>	**DM1** **A1**	Combine values consistent with all exp. values $\geqslant 5$ (FT on b, dep B1*)
	$X^2 = 0{\cdot}5337 + 0{\cdot}3345 + 0{\cdot}1729 + 0{\cdot}2053 = 1{\cdot}25$	**M1**	Find value of X^2 from $\Sigma\ (E_i - O_i)^2 / E_i$ [or $\Sigma\ O_i^2/E_i - n$]
	$X^2 = 1{\cdot}25$	**A1**	
	No. n of cells: 7 6 5 <u>4</u> 3 $\chi_{n-1,\ 0.9}{}^2$: 10·64 9·236 7.779 <u>6·251</u> 4·605	**DB1**	State or use consistent tabular value $\chi_{n-1,\ 0.9}{}^2$ (to 3 s.f.) [FT on number, n, of cells used to find X^2]
	Accept H_0 if $X^2 <$ tabular value (using their values)	**M1**	AEF
	$1{\cdot}25\ [\pm\ 0{\cdot}1] < 6{\cdot}25$ so distn. fits [data] or distn. is a suitable model	**A1**	Conclusion (requires both values approx. correct). AEF
		8	

