

Question	Answer	Marks	Guidance
1	$(T-1)^4/2 = 8$	M1 A1	Equate radial acceln. to 8 at $t = T$ from v^2/r
	$T = 3$ (or $T - 1 = 2$)	A1	Hence find positive value of T (or of $T - 1$)
	$a_T = 2(T-1) = 4$ [m s^{-2}]	M1 A1	Find magnitude of transverse acceleration at $t = T$
		5	

Question	Answer	Marks	Guidance
2(i)	$R_E \times 3a = W \cos \theta \times 2a - W \sin \theta \times 2a$ or $R_E \times 3a = W \times 2a (1 - \tan \theta) \cos \theta$ or $R_E \times 3a = 2\sqrt{2}W \sin\left(\frac{\pi}{4} - \theta\right)$	M1 A1	Take moments about B
	$R_E = 4W/3\sqrt{10}$	A1	Find normal reaction at E . AEF
	$R_B = W - R_E \cos \theta = 3W/5$	M1 A1	Find normal reaction at B by resolving forces vertically
		5	
2(ii)	$F_B = R_E \sin \theta = 2W/15$	M1 A1	Find friction at B by resolving forces horizontally
	$\mu = (2/15) / (3/5) = 2/9$	A1	Find μ from $F_B = \mu R_B$
		3	

Question	Answer	Marks	Guidance
3(i)	$5mv_A + 5mv_B = 5mu$ [$v_A + v_B = u$] and $v_B - v_A = e u$	M1	Use consvn. of momentum for A and B and use Newton's restitution law with consistent LHS signs. AEF
	$v_A = \frac{1}{2}(1 - e)u$	A1	Combine to verify speed of A . AG
	$v_B = \frac{1}{2}(1 + e)u$	A1	Find speed of B
		3	
3(ii)	$5mv_B' + 3mv_C = 5mv_B$ [$5v_B' + 3v_C = 5v_B$] $v_C - v_B' = ev_B$	M1	Use consvn. of momentum for B and C and use Newton's restitution law with consistent LHS signs. AEF
	$v_B' = (1/8)(5 - 3e)v_B$ [$v_C = (1/8)(5 + 5e)v_B$]	A1	Combine to find v_B' (v_C not reqd as B, C cannot collide again)
	$\frac{1}{2}(1 - e)u \leq (1/8)(5 - 3e) \times \frac{1}{2}(1 + e)u$	M1	Find condition on e using $v_A \leq v_B'$
	$3e^2 - 10e + 3 \leq 0$	A1	Simplify to a quadratic inequality
	$\frac{1}{3} \leq e$	A1	Solve to give a lower bound on e
	$\frac{1}{3} \leq e \leq 1$	A1	Non-strict inequality
		6	

Question	Answer	Marks	Guidance
4(i)	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 - mga \cos \theta$	M1	Use conservation of energy to slack point P_1
	$mv^2/a - mg \cos \theta = 0$	M1 A1	Equate tension at P_1 to 0 by using $F = ma$ A1 if both eqns correct, with m included. AG
	$v^2 = 2ag - 2ag \cos \theta = ag \cos \theta$	M1	Combine to verify $\cos \theta$ using $u = \sqrt{2ag}$
	$\cos \theta = 2/3$	A1	
		5	
4(ii)	$v_V = v \sin \theta = \sqrt{2ag/3} (\sqrt{5/3})$ or $v_V^2 = (10/27) ag$	M1	Find vertical speed v_V at P_1
	$h = v_V^2/2g = (5/27) a$ or $0.185 a$	M1 A1	Find height risen above P_1 by considering vertical motion
	$h + a \cos \theta = (23/27) a$ or $0.852 a$	A1	Find total height risen above level of O
		4	

Question	Answer	Marks	Guidance
5(i)	$I_{rod} = \frac{1}{3} \lambda M a^2 + \lambda M (a/2)^2$ [= (7/12) $\lambda M a^2$]	B1	Find or state MI of rod AB about axis L
	$I_O = \frac{2}{3} 3M a^2 + 3M (5a/2)^2$ [= (83/4) $M a^2$]	M1 A1	Find MI of hollow sphere centre O about axis L
	$I_C = (2/5) 5M a^2 + 5M (3a/2)^2$ [= (53/4) $M a^2$]	M1 A1	Find MI of solid sphere centre C about axis L
	$I = (7\lambda/12 + 83/4 + 53/4) M a^2$ = ((7 λ + 408) / 12) $M a^2$	A1	Verify MI of object about axis L . AG
		6	
5(ii)	$[-] I d^2\theta/dt^2 = [- 3Mg \times (5a/2) \sin \theta + 5Mg \times (3a/2) \sin \theta]$ $- \lambda Mg \times (a/2) \sin \theta$	M1 A1	Use eqn of circular motion to find $d^2\theta/dt^2$ where θ is angle of rod with vertical. AEF
	$d^2\theta/dt^2 = - \{6g\lambda / (7\lambda + 408)a\} \theta$	M1*	Approximate $\sin \theta$ by θ to give standard form of SHM eqn
	$T = 2\pi \sqrt{\{(7\lambda + 408)a / 6g\lambda\}} = 5\pi\sqrt{(2a/g)}$	DM1A1	Find possible values of λ by equating period T to $5\pi\sqrt{(2a/g)}$. AEF
	$\lambda = 6$	A1	
		6	

Question	Answer	Marks	Guidance
6(i)	$t \sqrt{(s^2/9)} = \frac{1}{2} (1.85 - 1.65) [= 0.1]$	M1	Find estimate s^2 of population variance (must be t)
	$t_{8, 0.975} = 2.306$ (to 3 s.f.)	A1	Use of correct tabular t -value
	$s^2 = 9 \times 0.04337^2 = 0.0169$ or $0.130[1]^2$	A1	
		3	
6(ii)	$\bar{x} = \frac{1}{2} (1.65 + 1.85) = 1.75$ or $\Sigma x = 9 \times 1.75 = 15.75$	M1 A1	Find sample mean \bar{x}
	$s^2 = (\Sigma x^2 - 9 \times \bar{x}^2) / 8$ or $\{\Sigma x^2 - (\Sigma x)^2 / 9\} / 8$	M1	or Σx
	$\Sigma x^2 = 8 \times 0.0169 + 15.75^2/9 = 27.7$	A1	Find Σx^2 from s^2
		4	

Question	Answer	Marks	Guidance
7(i)	$a = 1/200$ or 0.005	B1	State a or find a by equating mean value to $1/a$
		1	
7(ii)	$p = P(T < 150) = F(150) = 1 - e^{-150a}$	M1	Find $P(T < 150)$
	$p = 1 - e^{-0.75} = 0.528$	A1	
		2	
7(iii)	$1 - p^n > 0.99$	M1	Formulate condition for n
	$0.01 > (1 - e^{-0.75})^n$ or $0.01 > 0.528^n$	A1	
	$n > \log 0.01 / \log 0.528$	M1	Rearrange and take logs to give bound
	$n > 7.20$ [or 7.21] so $n_{\min} = 8$	A1	Find n_{\min}
		4	

Question	Answer	Marks	Guidance
8	$\bar{x}_A = 32.4 / 8 = 4.05$	B1	Find sample mean for A
	$s_A^2 = (131.82 - 32.4^2/8) / 7$ $s_A^2 = 3/35$ (or 0.08571 or 0.2928 ² both to 3 s.f.)	M1	Estimate or imply popln. variance for A
	$H_0: \mu_A = \mu_B$, $H_1: \mu_A \neq \mu_B$	B1	State hypotheses. AEF
	$s^2 = (7 s_A^2 + 9 s_B^2) / 16 = 0.12497$ or 0.3535^2 or $\frac{3999}{32000}$	M1 A1	Estimate (pooled) common variance
	$t_{16, 0.95} = 1.746$	B1*	State or use correct tabular t value
	$[-] t = (x_A - \bar{x}_B) / s \sqrt{(1/8 + 1/10)}$	M1	
	$= 0.27 / 0.1677 = 1.61$	A1	Find value of t (or can compare $\bar{x}_A - \bar{x}_B = 0.27$ with 0.293)
	$t < 1.75$ so [accept H_0] mean masses are the same	DB1	Correct conclusion (FT on t , dep B1*). AEF
		9	

Question	Answer	Marks	Guidance
9(i)	$\Sigma x = 15, \Sigma y = 7 + p + q, \Sigma xy = 17 + 2p + 3q$ $\Sigma x^2 = 55, [\Sigma y^2 = 21 + p^2 + q^2]$	M1	Find required summations
	$S_{xx} = 55 - 15^2/5 = 10$ and $S_{xy} = 17 + 2p + 3q - 15 \times (7 + p + q) / 5 = -4 - p$	M1 A1	
	$-0.5 = S_{xy} / S_{xx} = (-4 - p) / 10 \quad p = 1$	M1 A1	Find p from gradient in eqn. of regression line
	$(7 + p + q) / 5 = -0.5 \times 15/5 + 3.5 \quad q = 2$	M1 A1	Find q from means and regression line
		7	
9(ii)	$\Sigma y = 10, \Sigma y^2 = 26, S_{yy} = 26 - 10^2/5 = 6$	M1	Find S_{yy}
	$r = S_{xy} / \sqrt{(S_{xx}S_{yy})} = -5 / \sqrt{(10 \times 6)}$	M1	Find correlation coefficient r
	$r = -0.645[5]$ [allow -0.646]	A1	
		3	

Question	Answer	Marks	Guidance
10(i)	$F(x) = \int f(x) dx = (1/30)(-8/x + x^3 - 14x) [+ c]$	M1	Find or state distribution function $F(x)$ for $2 \leq x \leq 4$
	$F(x) = (1/30)(-8/x + x^3 - 14x + 24)$	M1	Using $F(2) = 0$ or $F(4) = 1$ to find c if necessary. AEF
	$F(x) = 0$ ($x < \text{or} \leq 2$), $F(x) = 1$ ($x > \text{or} \geq 4$)	A1	State $F(x)$ for other values of x
		3	
10(ii)	$G(y) = P(Y < y) = P(X^2 < y)$ $G(y) = P(X < \sqrt{y}) = F(\sqrt{y})$ $G(y) = (1/30)\left(-8/y^{\frac{1}{2}} + y^{\frac{3}{2}} - 14y^{\frac{1}{2}} + 24\right)$	M1 A1	Find or state $G(y)$ for $2 \leq x \leq 4$ from $Y = X^2$ (allow $<$ or \leq throughout)
	Alternative method for question 10(ii)		
	Use $x = y^{\frac{1}{2}}$ to find $f(x) = (1/30)(8/y + 3y - 14)$, $\frac{dx}{dy} = -\frac{1}{2}y^{-\frac{1}{2}}$	(M1 A1)	Find $f(x)$ and $\frac{dx}{dy}$ for use in $g(y) = f(x) \times \left \frac{dy}{dx} \right $
	$g(y) [= G'(y)] = (1/30)\left(4/y^{\frac{3}{2}} + (3/2)y^{\frac{1}{2}} - 7/y^{\frac{1}{2}}\right)$ for $4 \leq y \leq 16$ [$g(y) = 0$ otherwise]	A1 A1	Find $g(y)$. AEF State corresponding range of y for $G(y)$ or $g(y)$
		4	
10(iii)	$(1/30)\left(-8/y^{\frac{1}{2}} + y^{\frac{3}{2}} - 14y^{\frac{1}{2}} + 24\right) = 0.8$	M1	Set $G(y) = 0.8$
	$-8 + y^2 - 14y = 0$, $y = 7 + \sqrt{57}$ or $14.5[5]$ [rejecting $7 - \sqrt{57}$; allow 14.6]	M1 A1	Rearrange to give quadratic in y and solve to find value of y
		3	

Question	Answer	Marks	Guidance
11A(i)	$10 (AP - 0.6) / 0.6 = 20 (1.2 - AP - 0.4) / 0.4$	M1 A1	Verify AP by equating equilibrium tensions. AEF
	$4 AP - 2.4 = 9.6 - 12 AP$ $AP = 0.75$ [m]	A1	AG
		3	
11A(ii)	$m \frac{d^2x}{dt^2} = -10 (0.15 + x) / 0.6 + 20 (0.05 - x) / 0.4$ or $m \frac{d^2x}{dt^2} = +10 (0.15 - x) / 0.6 - 20 (0.05 + x) / 0.4$	M1 A1 A1	Apply Newton's law at $0.75 + x$ or $0.75 - x$ from A (M1 requires LHS and 2 tensions: A1 for each correct tension)
	$\frac{2}{3} \frac{d^2x}{dt^2} = - (80 / 1.2) x$, $\frac{d^2x}{dt^2} = -100 x$	M1 A1	Simplify to give SHM eqn. in standard form
	$T = 2\pi/\omega = 2\pi/10 = \pi/5$ or 0.628 [s]	DB1	State the period T with FT on ω from SHM eqn.
		6	
11A(iii)	$(a = 0.75 - 0.7 = 0.05)$ $v_{\max} = \omega \times a$	M1	Find speed at equilibrium position from ωa
	$v_{\max} = 0.5$ [m s ⁻¹]	A1	
		2	
11A(iv)	$x = a/2 = 0.025$	M1	Find value of x giving half max. acceln.
	$v = \omega \sqrt{(a^2 - x^2)} = 10 \sqrt{(0.05^2 - 0.025^2)}$	M1	
	$v = 0.433$ [m s ⁻¹]	A1	Find corresponding speed
		3	

Question	Answer	Marks	Guidance
11B(i)	$\bar{x} = (1/40) \sum x f(x) = 68/40 = 1.7$	B1	Find mean of sample
	$(1/40) \sum x^2 f(x) = 178/40 = 4.45$, $\text{Var} = 4.45 - 1.7^2 = 1.56$	B1	Find variance of sample
	Mean and variance are similar so Poisson may be suitable	B1	State valid comment
		3	
11B(ii)	$a = 40 \times 1.6^5 e^{-1.6} / 5! = 40 \times 0.01764$	M1	AG
	$a = 0.706$	A1	Verify a from Poisson term
	$b = 40 - 39.758 = 0.242$	B1*	Find b
		3	
11B(iii)	H_0 : Distribution fits/models data	B1	State (at least) null hypothesis in full
	O_i : 6 15 9 <u>10</u> E_i : 8.076 12.921 10.337 <u>8.666</u>	DM1 A1	Combine values consistent with all exp. values ≥ 5 (FT on b , dep B1*)
	$X^2 = 0.5337 + 0.3345 + 0.1729 + 0.2053 = 1.25$	M1	Find value of X^2 from $\sum (E_i - O_i)^2 / E_i$ [or $\sum O_i^2 / E_i - n$]
	$X^2 = 1.25$	A1	
	No. n of cells: 7 6 5 <u>4</u> 3 $\chi_{n-1, 0.9}^2$: 10.64 9.236 7.779 <u>6.251</u> 4.605	DB1	State or use consistent tabular value $\chi_{n-1, 0.9}^2$ (to 3 s.f.) [FT on number, n , of cells used to find X^2]
	Accept H_0 if $X^2 <$ tabular value (using their values)	M1	AEF
	1.25 [± 0.1] $<$ 6.25 so distn. fits [data] or distn. is a suitable model	A1	Conclusion (requires both values approx. correct). AEF
		8	