

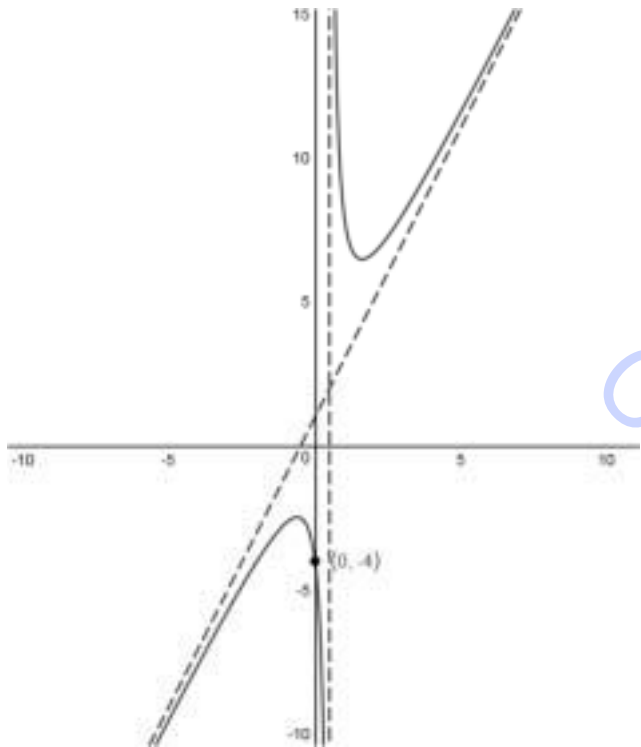
Question	Answer	Marks	Guidance
1	$A = \int_0^1 x^a dx = \left[ \frac{x^{a+1}}{a+1} \right]_0^1 = \frac{1}{a+1}$	<b>B1</b>	Finds area of region.
	$A\bar{x} = \int_0^1 xy dx = \int_0^1 x^{a+1} dx = \left[ \frac{x^{a+2}}{a+2} \right]_0^1 = \frac{1}{a+2}$	<b>M1 A1</b>	Finds $\int_0^1 xy dx$ .
	$2A\bar{y} = \int_0^1 y^2 dx = \int_0^1 x^{2a} dx = \left[ \frac{x^{2a+1}}{2a+1} \right]_0^1 = \frac{1}{2a+1}$	<b>M1 A1</b>	Finds $\int_0^1 y^2 dx$ .
	$(\bar{x}, \bar{y}) = \left( \frac{a+1}{a+2}, \frac{a+1}{2(2a+1)} \right)$	<b>A1</b>	Both coordinates correct.
		<b>6</b>	

Question	Answer	Marks	Guidance
2	$\frac{dy}{dx} = \frac{a}{ax+1} = (-1)^0 \frac{0!a^1}{(ax+1)^1}$ so true for $n = 1$ .	<b>M1 A1</b>	Proves base case.
	Assume that $\frac{d^k y}{dx^k} = (-1)^{k-1} \frac{(k-1)!a^k}{(ax+1)^k}$ for some positive integer $k$ .	<b>B1</b>	States inductive hypothesis.
	Then $\frac{d^{k+1} y}{dx^{k+1}} = -ka(-1)^{k-1} \frac{(k-1)!a^k}{(ax+1)^{k+1}} = (-1)^k \frac{k!a^{k+1}}{(ax+1)^{k+1}}$ so true for $n = k + 1$ .	<b>M1 A1</b>	Differentiates $k^{\text{th}}$ derivative.
	By induction, true for every positive integer $n$ .	<b>A1</b>	States conclusion.
		<b>6</b>	

Question	Answer	Marks	Guidance
3(i)	$I_{n+2} = \left[ \frac{x^{-n-1}}{-n-1} \sin \pi x \right]_{\frac{1}{2}}^1 - \pi \int_{\frac{1}{2}}^1 \frac{x^{-n-1}}{-n-1} \cos \pi x \, dx$	<b>M1 A1</b>	Integrates by parts.
	$= \frac{2^{n+1}}{n+1} + \frac{\pi}{n+1} \left( \left[ \frac{x^{-n}}{-n} \cos \pi x \right]_{\frac{1}{2}}^1 + \pi \int_{\frac{1}{2}}^1 \frac{x^{-n}}{-n} \sin \pi x \, dx \right)$	<b>M1</b>	Integrates by parts again.
	$= \frac{2^{n+1}}{n+1} + \frac{\pi}{n+1} \left( \frac{1}{n} - \frac{\pi}{n} I_n \right)$ $\Rightarrow (n+1)I_{n+2} = 2^{n+1} + \pi \left( \frac{1}{n} - \frac{\pi}{n} I_n \right)$	<b>M1</b>	Uses $I_n$ .
	$\Rightarrow n(n+1)I_{n+2} = 2^{n+1}n + \pi - \pi^2 I_n$	<b>A1</b>	AG
		<b>5</b>	
3(ii)	$2I_3 = 4 + \pi - \pi^2 I_1$ $12I_5 = 48 + \pi - \frac{\pi^2}{2} (4 + \pi - \pi^2 I_1)$	<b>M1</b>	Substitutes $I_3$ into reduction formula.
	$\Rightarrow I_5 = 4 + \frac{1}{24} (2\pi - 4\pi^2 - \pi^3 + \pi^4 I_1)$	<b>A1</b>	AEF, must be exact with fractions simplified.
		<b>2</b>	

Question	Answer	Marks	Guidance
4(i)	$x^2 + 1 = (ax + b)(2x + 1) + c$	<b>M1</b>	Uses that $2x + 1$ is the quotient.
	$\Rightarrow a = \frac{1}{2}, b = -\frac{1}{4}$	<b>A1 A1</b>	
		<b>3</b>	
4(ii)	$x = \frac{1}{2}$	<b>B1 FT</b>	
		<b>1</b>	

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4(iii)		<b>B1</b>	Intersection (0,-4) given and asymptotes drawn.
		<b>B1</b>	Left branch correct.
		<b>B1 FT</b>	Right branch correct.  Deduct at most one mark for poor forms at infinity.
		<b>3</b>	

Question	Answer	Marks	Guidance
5(i)	$\sum_{r=1}^N (5r+1)(5r+6) = 25 \sum_{r=1}^N r^2 + 35 \sum_{r=1}^N r + 6N$	<b>M1</b>	Expands.
	$25 \left( \frac{1}{6} N(N+1)(2N+1) \right) + 35 \left( \frac{1}{2} N(N+1) \right) + 6N$	<b>M1</b>	Substitutes formulae for $\sum r$ and $\sum r^2$ .
	$= N \left( \frac{25}{6} (2N^2 + 3N + 1) + \frac{35}{2} N + \frac{35}{2} + 6 \right) = \frac{1}{3} N (25N^2 + 90N + 83)$	<b>A1</b>	Simplifies to the given answer (AG).
		<b>3</b>	
5(ii)	$\frac{1}{(5r+1)(5r+6)} = \frac{1}{5} \left( \frac{1}{5r+1} - \frac{1}{5r+6} \right)$	<b>M1 A1</b>	Finds partial fractions.
	$T_N = \frac{1}{5} \left( \frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \dots + \frac{1}{5N+1} - \frac{1}{5N+6} \right)$	<b>M1</b>	Expresses terms as differences.
	$\frac{1}{5} \left( \frac{1}{6} - \frac{1}{5N+6} \right) = \frac{1}{30} - \frac{1}{5(5N+6)}$	<b>A1</b>	At least 3 terms including last.
		<b>4</b>	
5(iii)	$\frac{S_N}{N^3} T_N \rightarrow \frac{25}{3} \times \frac{1}{30} = \frac{5}{18}$	<b>M1 A1</b>	Divides $S_N$ by $N^3$ and takes limits as $N \rightarrow \infty$
		<b>2</b>	

Question	Answer	Marks	Guidance
6(i)	$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 2 \\ 7 \end{pmatrix}$	<b>B1</b>	
	$\overrightarrow{OC} \times \overrightarrow{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 1 & 2 & 7 \end{vmatrix} = \begin{pmatrix} -9 \\ -6 \\ 3 \end{pmatrix} = t \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$	<b>M1 A1</b>	Finds direction of common perpendicular.
	$\frac{\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{3^2 + 2^2 + 1^2}} = \frac{1}{\sqrt{14}} = 0.267$	<b>M1 A1</b>	Uses formula for shortest distance.
		<b>5</b>	
6(ii)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 3 & 2 & -1 \end{vmatrix} = t \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix}$	<b>M1 A1</b>	Finds normal to plane.
	$-(0) + 4(0) + 5(0) = 0$	<b>M1</b>	Uses point on plane.
	$-x + 4y + 5z = 0$	<b>A1</b>	AEF
		<b>4</b>	

Question	Answer	Marks	Guidance
7(i)	$\sqrt{-7y}(-7y) + 2(-7y) + \sqrt{-7y} + 7 = 0$ $\Rightarrow \sqrt{-7y}(-7y+1) = 14y-7 \Rightarrow -7y(-7y+1)^2 = (14y-7)^2$	<b>M1</b>	Uses given substitution and eliminates radical.
	$\Rightarrow 49y^3 + 14y^2 - 27y + 7 = 0$	<b>A1</b>	AG
	$y = \frac{x^2}{-7} = \frac{x^2}{\alpha\beta\gamma}$	<b>M1</b>	Uses $\alpha\beta\gamma = -7$ .
	So roots are $\frac{\alpha^2}{\alpha\beta\gamma} = \frac{\alpha}{\beta\gamma}$ , $\frac{\beta^2}{\alpha\beta\gamma} = \frac{\beta}{\alpha\gamma}$ , $\frac{\gamma^2}{\alpha\beta\gamma} = \frac{\gamma}{\alpha\beta}$	<b>A1</b>	AG
		<b>4</b>	
7(ii)	$\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta} = -\frac{2}{7}, \frac{1}{\gamma^2} + \frac{1}{\beta^2} + \frac{1}{\alpha^2} = -\frac{27}{49}$	<b>B1</b>	States sum of roots and $\alpha'\beta' + \alpha'\gamma' + \beta'\gamma'$ .
	$\frac{\alpha^2}{\beta^2\gamma^2} + \frac{\beta^2}{\alpha^2\gamma^2} + \frac{\gamma^2}{\alpha^2\beta^2} = \left(-\frac{2}{7}\right)^2 - 2\left(-\frac{27}{49}\right) = \frac{58}{49}$	<b>M1 A1</b>	Uses $\alpha'^2 + \beta'^2 + \gamma'^2 = (\alpha' + \beta' + \gamma')^2 - 2(\alpha'\beta' + \alpha'\gamma' + \beta'\gamma')$ AG
		<b>3</b>	
7(iii)	$49\left(\frac{\alpha^3}{\beta^3\gamma^3} + \frac{\beta^3}{\alpha^3\gamma^3} + \frac{\gamma^3}{\alpha^3\beta^3}\right) = -14\left(\frac{58}{49}\right) + 27\left(-\frac{2}{7}\right) - 21$	<b>M1</b>	Uses $49\alpha'^3 = -14\alpha'^2 + 27\alpha' - 7$ .
	$\Rightarrow \frac{\alpha^3}{\beta^3\gamma^3} + \frac{\beta^3}{\alpha^3\gamma^3} + \frac{\gamma^3}{\alpha^3\beta^3} = -\frac{317}{343}$	<b>A1</b>	
		<b>2</b>	



Question	Answer	Marks	Guidance
8(i)	Eigenvalues of (upper diagonal matrix) $\mathbf{A}$ are $2, m$ and $1$ . (Or from characteristic equation: $(\lambda - 2)(\lambda - m)(\lambda - 1) = 0$ )	<b>B1</b>	
	$\lambda = 2: \mathbf{e}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & m-2 & 1 \\ 0 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 2-m \\ 0 \\ 0 \end{pmatrix} = t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	<b>M1 A1</b>	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda = m: \mathbf{e}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2-m & m & 1 \\ 0 & 0 & 7 \end{vmatrix} = \begin{pmatrix} 7m \\ 7(m-2) \\ 0 \end{pmatrix} = t \begin{pmatrix} m \\ m-2 \\ 0 \end{pmatrix}$	<b>A1</b>	
	$\lambda = 1: \mathbf{e}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & m & 1 \\ 0 & m-1 & 7 \end{vmatrix} = \begin{pmatrix} 6m+1 \\ -7 \\ m-1 \end{pmatrix} = t \begin{pmatrix} 6m+1 \\ -7 \\ m-1 \end{pmatrix}$	<b>A1</b>	
	Thus $\mathbf{P} = \begin{pmatrix} 1 & m & 6m+1 \\ 0 & m-2 & -7 \\ 0 & 0 & m-1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{pmatrix}$	<b>M1 A1 FT</b>	Or correctly matched permutations of columns. No follow through on two or more zero eigenvectors.
		<b>7</b>	

Question	Answer	Marks	Guidance
8(ii)	$\mathbf{M}^7 \mathbf{P} = \mathbf{P} \mathbf{D}^7 \mathbf{P}^{-1} \mathbf{P} = \mathbf{P} \mathbf{D}^7 = \begin{pmatrix} 1 & m & 6m+1 \\ 0 & m-2 & -7 \\ 0 & 0 & m-1 \end{pmatrix} \begin{pmatrix} 2^7 & 0 & 0 \\ 0 & m^7 & 0 \\ 0 & 0 & 1 \end{pmatrix}$	<b>M1 A1 FT</b>	Applies $\mathbf{M}^7 = \mathbf{P} \mathbf{D}^7 \mathbf{P}^{-1}$ .
	$= \begin{pmatrix} 2^7 & m^8 & 6m+1 \\ 0 & m^8 - 2m^7 & -7 \\ 0 & 0 & m-1 \end{pmatrix}$	<b>A1</b>	Order of columns might be swapped depending on $\mathbf{P}$ .
		<b>3</b>	

Question	Answer	Marks	Guidance
9(i)	Write $c = \cos \theta$ , $s = \sin \theta$ . $\cos 6\theta + i \sin 6\theta = (c + is)^6$	<b>M1</b>	Uses binomial theorem.
	$\Rightarrow \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	<b>A1</b>	
	$c^6 - 15c^4s^2 + 15c^2s^4 - s^6 = c^6 - 15c^4(1-c^2) + 15c^2(1-c^2)^2 - (1-c^2)^3$	<b>M1</b>	Uses $c^2 = 1 - s^2$ .
	$= c^6 - 15c^4(1-c^2) + 15c^2(1-2c^2+c^4) - (1-3c^2+3c^4-c^6)$	<b>A1</b>	
	$= 32c^6 - 48c^4 + 18c^2 - 1$	<b>M1</b>	Divides numerator and denominator by $c^6$ .
	$\Rightarrow \sec 6\theta = \frac{1}{32c^6 - 48c^4 + 18c^2 - 1} = \frac{\sec^6 \theta}{32 - 48\sec^2 \theta + 18\sec^4 \theta - \sec^6 \theta}$	<b>A1</b>	AG
		<b>6</b>	

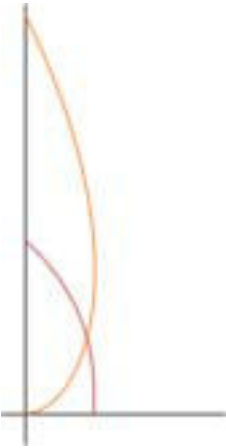
Question	Answer	Marks	Guidance
9(ii)	$x^6 = 2(32 - 48x^2 + 18x^4 - x^6) \Rightarrow \frac{x^6}{32 - 48x^2 + 18x^4 - x^6} = 2$	<b>M1 A1</b>	Relates with equation in part (i).
	$\sec 6\theta = 2 \Rightarrow \cos 6\theta = \frac{1}{2}$	<b>M1</b>	Solves $\cos 6\theta = \frac{1}{2}$ .
	$x = \sec \frac{\pi}{18}$	<b>A1</b>	Gives one correct solution.
	$x = \sec q\pi, q = \frac{5}{18}, \frac{7}{18}, \frac{11}{18}, \frac{13}{18}, \frac{17}{18}$	<b>A1</b>	Gives five other solutions. Allow different values of $q$ as long as all six solutions are found.
		<b>5</b>	

Question	Answer	Marks	Guidance
10(i)	$\begin{pmatrix} 1 & 5 & 1 \\ 1 & -2 & -2 \\ 2 & 3 & \theta \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 5 & 1 \\ 0 & -7 & -3 \\ 0 & 0 & \theta+1 \end{pmatrix}$	<b>M1 A1</b>	Reduces to echelon form. At least one row operation for M1.
	$r(\mathbf{A}) = 3$ if $\theta \neq -1$	<b>A1</b>	
	$r(\mathbf{A}) = 2$ if $\theta = -1$	<b>B1</b>	
		<b>4</b>	
10(ii)	$\begin{aligned} x + 5y + z &= -1 \\ -7y - 3z &= 1 \\ (\theta + 1)z &= (\theta + 1) \end{aligned}$	<b>M1</b>	Uses reduced form of augmented matrix or eliminates variables from scratch.
	$z = 1, y = -\frac{4}{7}, x = \frac{6}{7}$	<b>A1</b> <b>A1</b>	One correct. All three correct.
		<b>3</b>	
10(iii)	$\begin{aligned} x + 5y + z &= -1 \\ -7y - 3z &= 1 \\ (\theta + 1)z &= (\theta + 1) \end{aligned}$		
	$z = t$	<b>M1</b>	Uses parameter.
	$y = -\frac{3t+1}{7}, x = \frac{8t-2}{7}$	<b>A1 A1</b>	
		<b>3</b>	

Question	Answer	Marks	Guidance
10(iv)	$x + 5y + z = -1,$ $-7y - 3z = 1,$ $(\theta + 1)z = \phi + 1$	<b>M1</b>	Uses reduced form of augmented matrix or eliminates variables from scratch
	$\theta = -1 \Rightarrow \phi = -1$ so no solution (inconsistent).	<b>A1</b>	
		<b>2</b>	

Question	Answer	Marks	Guidance
11E(i)	$w = \cos y \Rightarrow \frac{dw}{dx} = -\sin y \frac{dy}{dx}$	<b>B1</b>	
	$\frac{d^2w}{dx^2} = -\sin y \frac{d^2y}{dx^2} - \cos y \left(\frac{dy}{dx}\right)^2$	<b>B1</b>	
	$\frac{d^2w}{dx^2} + 2\frac{dw}{dx} + w = -\sin y \frac{d^2y}{dx^2} - \cos y \left(\frac{dy}{dx}\right)^2 - 2\sin y \frac{dy}{dx} + \cos y$	<b>M1</b>	Uses substitution to obtain $w - x$ equation, AG.
	$= -\cos y (e^{-2x} \sec y) = -e^{-2x}$	<b>A1</b>	
		<b>4</b>	

Question	Answer	Marks	Guidance
11E(ii)	$m^2 + 2m + 1 = 0 \Rightarrow m = -1$	<b>M1</b>	Finds CF.
	CF: $w = (Ax + B)e^{-x}$	<b>A1</b>	
	PI: $w = ke^{-2x} \Rightarrow w' = -2ke^{-2x} \Rightarrow w'' = 4ke^{-2x}$	<b>M1</b>	Forms PI and differentiates.
	$4k - 4k + k = -1 \Rightarrow k = -1$	<b>A1</b>	
	$w = (Ax + B)e^{-x} - e^{-2x}$	<b>A1</b>	States general solution.
	$x = 0, y = \frac{1}{3}\pi, w = \frac{1}{2} \Rightarrow B = \frac{3}{2}$	<b>B1</b>	Uses initial conditions to find constants.
	$w' = -(Ax + B)e^{-x} + Ae^{-x} + 2e^{-2x}$	<b>M1</b>	Differentiates general solution.
	$x = 0, y = \frac{1}{3}\pi, y' = \frac{\sqrt{3}}{3}, w' = -\frac{1}{2} \Rightarrow -\frac{1}{2} = -\frac{3}{2} + A + 2 \Rightarrow A = -1$	<b>M1 A1</b>	Substitutes initial conditions.
	$y = \cos^{-1}\left(\left(\frac{3}{2} - x\right)e^{-x} - e^{-2x}\right)$	<b>A1</b>	States particular solution for $y$ in terms of $x$ .
		<b>10</b>	

Question	Answer	Marks	Guidance
110(i)	$e^{2\alpha} - e^{-2\alpha} = 2(e^{\alpha} + e^{-\alpha}) \Rightarrow e^{\alpha} - e^{-\alpha} = 2$	<b>M1</b>	Sets equations equal and divides by $e^{\alpha} + e^{-\alpha}$ .
	$e^{2\alpha} - 2e^{\alpha} - 1 = 0 \Rightarrow e^{\alpha} = 1 + \sqrt{2}$	<b>M1 A1</b>	Forms quadratic in $e^{\alpha}$ , AG.
	$\alpha = \ln(1 + \sqrt{2})$	<b>A1</b>	Must be exact.
	$r = 2(1 + \sqrt{2} + \sqrt{2} - 1) = 4\sqrt{2}$	<b>M1 A1</b>	Substitutes to find $r$ .
		<b>6</b>	
110(ii)		<b>B1</b>	$C_1$ has correct shape.
		<b>B1</b>	$C_2$ has correct shape.
		<b>B1</b>	Intersection points positioned correctly.
		<b>3</b>	

Question	Answer	Marks	Guidance
11O(iii)	$2 \int_0^{\ln(1+\sqrt{2})} (e^\theta + e^{-\theta})^2 d\theta - \frac{1}{2} \int_0^{\ln(1+\sqrt{2})} (e^{2\theta} - e^{-2\theta})^2 d\theta$ $= \int_0^{\ln(1+\sqrt{2})} 5 + 2e^{2\theta} + 2e^{-2\theta} - \frac{1}{2}e^{4\theta} - \frac{1}{2}e^{-4\theta} d\theta$	<b>M1 A1</b>	Uses $\frac{1}{2} \int r^2 d\theta$ to formulate correct area.
	$= \left[ 5\theta + e^{2\theta} - e^{-2\theta} - \frac{1}{8}e^{4\theta} + \frac{1}{8}e^{-4\theta} \right]_0^{\ln(1+\sqrt{2})}$	<b>M1 A1</b>	Expands and integrates.
	$= 5\ln(1+\sqrt{2}) + (1+\sqrt{2})^2 - (1+\sqrt{2})^{-2} - \frac{1}{8} \left( (1+\sqrt{2})^4 - (1+\sqrt{2})^{-4} \right) = 5.82$	<b>A1</b>	
		<b>5</b>	