

1 A bullet of mass 0.2 kg is fired into a fixed vertical barrier. It enters the barrier horizontally with speed $250\,\mathrm{m\,s^{-1}}$ and emerges horizontally after a time T seconds with speed $40\,\mathrm{m\,s^{-1}}$. There is a constant horizontal resisting force of magnitude 1200 N. Find T. [4]

2 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is moving in a complete vertical circle about O. The points A and B are on the circle, at opposite ends of a diameter, and such that OA makes an acute angle α with the upward vertical through O. The speed of P as it passes through A is $\frac{3}{2}\sqrt{(ag)}$. The tension in the string when P is at B is four times the tension in the string when P is at A.

(i) Show that $\cos \alpha = \frac{3}{4}$. [6]

(ii) Find the tension in the string when P is at B. [2]

3 Three uniform small spheres A, B and C have equal radii and masses $3m$, m and m respectively. The spheres are at rest in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between each pair of spheres is e. Sphere A is projected directly towards B with speed u.

(i) Find, in terms of u and e, expressions for the speeds of A, B and C after the first two collisions. [6]

(ii) Given that A and C are moving with equal speeds after these two collisions, find the value of e. [3]

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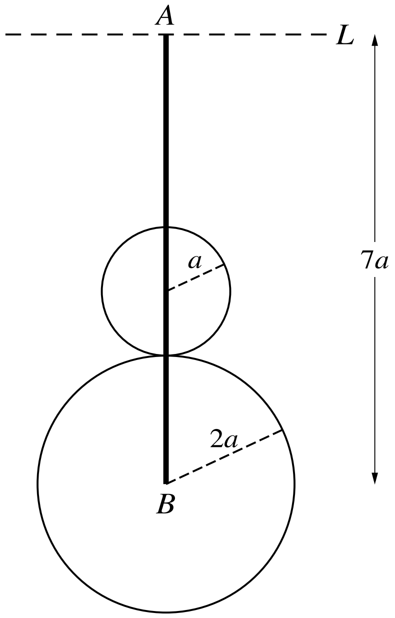

An object consists of two hollow spheres which touch each other, together with a thin uniform rod AB. The rod passes through small holes in the surfaces of the spheres. The rod is fixed to the spheres so that it passes through the centre of the smaller sphere. The end B of the rod is at the centre of the larger sphere. The larger sphere has radius $2a$ and mass M, the smaller sphere has radius a and mass kM, and the rod has length $7a$ and mass $5M$. A fixed horizontal axis L passes through A and is perpendicular to AB (see diagram).

(i) Find the moment of inertia of the object, consisting of the rod and two spheres, about L. [6]

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The object is pivoted at A so that it can rotate freely about L. The object is released from rest with the rod making an angle of 60° to the downward vertical. The greatest angular speed attained by the object in the subsequent motion is $\frac{9}{20}\sqrt{\left(\frac{g}{a}\right)}$.

(ii) Find the value of k. [5]

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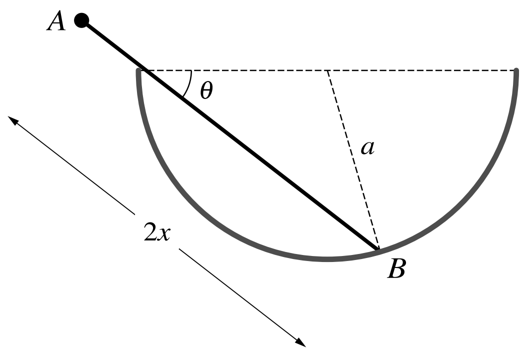

A uniform rod AB of length $2x$ and weight W rests on the smooth rim of a fixed hemispherical bowl of radius a. The end B of the rod is in contact with the rough inner surface of the bowl. The coefficient of friction between the rod and the bowl at B is $\frac{1}{3}$. A particle of weight $\frac{1}{4}W$ is attached to the end A of the rod. The end B is about to slip upwards when AB is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$ (see diagram).

(i) By resolving parallel to the rod, show that the normal component of the reaction of the bowl on the rod at B is $\frac{3}{4}W$. [5]

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(ii) Find, in terms of W, the reaction between the rod and the smooth rim of the bowl. [4]

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(iii) Find x in terms of a. [3]

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6 The random variable T is the lifetime, in hours, of a randomly chosen battery of a particular type. It is given that T has a negative exponential distribution with mean 400 hours.

(i) Write down the probability density function of T. [1]

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(ii) Find the probability that a battery of this type has a lifetime that is less than 500 hours. [2]

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(iii) Find the median of the distribution. [3]

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7 A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The number of throws taken is denoted by the random variable X.

(i) State the expected value of X. [1]

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(ii) Find the probability that exactly 3 throws are required to obtain a pair of tails. [2]

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(iii) Find the probability that fewer than 4 throws are required to obtain a pair of tails. [2]

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(iv) Find the least integer N such that the probability of obtaining a pair of tails in fewer than N throws is more than 0.95. [3]

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8 Two salesmen, A and B, work at a company that arranges different types of holidays: self-catering, hotel and cruise. The table shows, for a random sample of 150 holidays, the number of each type arranged by each salesman.

		Type of holiday		
		Self-catering	Hotel	Cruise
Salesman	A	25	38	21
	B	28	21	17

Test at the 10% significance level whether the type of holiday arranged is independent of the salesman. [8]

9 A farmer grows large amounts of a certain crop. On average, the yield per plant has been 0.75 kg. The farmer has improved the soil in which the crop grows, and she claims that the yield per plant has increased. A random sample of 10 plants grown in the improved soil is chosen. The yields, x kg, are summarised as follows.

$$\Sigma x = 7.85 \qquad \Sigma x^2 = 6.19$$

(i) Test at the 5% significance level whether the farmer's claim is justified, assuming a normal distribution. [7]

(ii) Find a 95% confidence interval for the population mean yield for plants grown in the new soil. [3]

10 The means and variances for a random sample of 8 pairs of values of x and y taken from a bivariate distribution are given in the following table.

	Mean	Variance
x	3.3125	3.3086
y	6.7375	7.9473

The product moment correlation coefficient for the sample is 0.5815, correct to 4 decimal places.

(i) Find the equation of the regression line of y on x. [6]

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(ii) Test at the 5% significance level whether there is evidence of positive correlation between x and y. [4]

(iii) Calculate an estimate of y when $x = 6.0$ and comment on the reliability of your estimate. [2]

11 Answer only **one** of the following two alternatives.

EITHER

A light spring has natural length a and modulus of elasticity kmg. The spring lies on a smooth horizontal surface with one end attached to a fixed point O. A particle P of mass m is attached to the other end of the spring. The system is in equilibrium with $OP = a$. The particle is projected towards O with speed u and comes to instantaneous rest when $OP = \frac{3}{4}a$.

(i) Use an energy method to show that $k = \dfrac{16u^2}{ag}$. [2]

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(ii) Show that P performs simple harmonic motion and find the period of this motion, giving your answer in terms of u and a. [4]

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(iii) Find, in terms of u and a, the time that elapses before P first loses 25% of its initial kinetic energy. [6]

OR

A company produces packets of sweets. Two different machines, A and B, are used to fill the packets. The manager decides to assess the performance of the two machines. He selects a random sample of 50 packets filled by machine A and a random sample of 60 packets filled by machine B. The masses of sweets, x kg, in packets filled by machine A and the masses of sweets, y kg, in packets filled by machine B are summarised as follows.

$$\Sigma x = 22.4 \qquad \Sigma x^2 = 10.1 \qquad \Sigma y = 28.8 \qquad \Sigma y^2 = 16.3$$

A test at the $\alpha\%$ significance level provides evidence that the mean mass of sweets in packets filled by machine A is less than the mean mass of sweets in packets filled by machine B. Find the set of possible values of α. [12]

