

1 A particle P moves along an arc of a circle with centre O and radius 2 m. At time t seconds, the angle POA is θ, where $\theta = 1 - \cos 2t$, and A is a fixed point on the arc of the circle.

(i) Show that the magnitude of the radial component of the acceleration of P when $t = \frac{1}{6}\pi$ is $6\,\mathrm{m\,s^{-2}}$. [2]

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(ii) Find the magnitude of the transverse component of the acceleration of P when $t = \frac{1}{6}\pi$. [2]

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2 A particle P moves on a straight line in simple harmonic motion. The centre of the motion is O. The points A and B are on the line on opposite sides of O such that $OA = 3.5$ m and $OB = 1$ m. The speed of P when it is at B is twice its speed when it is at A. The maximum acceleration of P is 1 m s^{-2}.

(i) Find the speed of P when it is at O. [4]

(ii) Find the time taken by P to travel directly from A to B. [4]

3 Three uniform small spheres A, B and C have equal radii and masses $2m$, $4m$ and m respectively. The spheres are moving in a straight line on a smooth horizontal surface, with B between A and C. The coefficient of restitution between each pair of spheres is e. Spheres A and B are moving towards each other with speeds $2u$ and u respectively. The first collision is between A and B.

(i) Find the velocities of A and B after this collision. [3]

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Sphere C is moving towards B with speed $\frac{4}{3}u$ and now collides with it. As a result of this collision, B is brought to rest.

(ii) Find the value of e. [4]

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(iii) Find the total kinetic energy lost by the three spheres as a result of the two collisions. [3]

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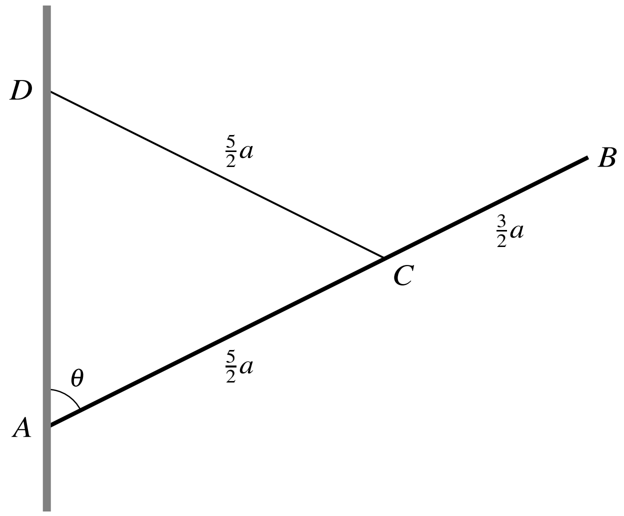

A uniform rod AB of length $4a$ and weight W rests with the end A in contact with a rough vertical wall. A light inextensible string of length $\frac{5}{2}a$ has one end attached to the point C on the rod, where $AC = \frac{5}{2}a$. The other end of the string is attached to a point D on the wall, vertically above A. The vertical plane containing the rod AB is perpendicular to the wall. The angle between the rod and the wall is θ, where $\tan\theta = 2$ (see diagram). The end A of the rod is on the point of slipping down the wall and the coefficient of friction between the rod and the wall is μ.

Find, in either order, the tension in the string and the value of μ. [10]

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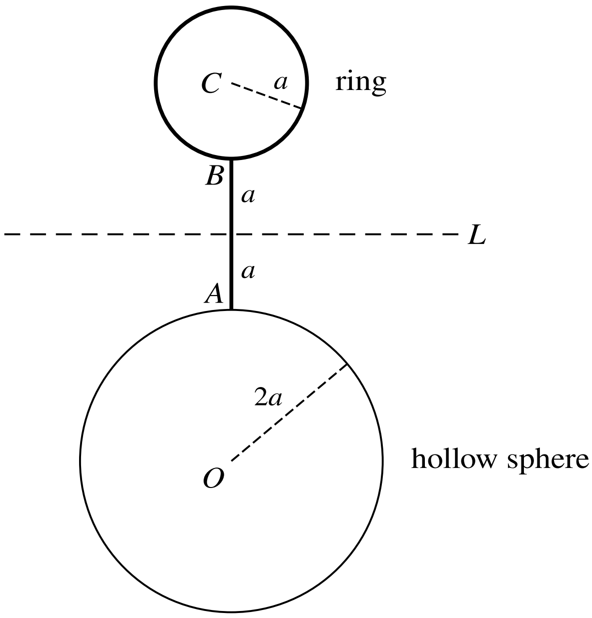

A thin uniform rod AB has mass kM and length $2a$. The end A of the rod is rigidly attached to the surface of a uniform hollow sphere with centre O, mass kM and radius $2a$. The end B of the rod is rigidly attached to the circumference of a uniform ring with centre C, mass M and radius a. The points C, B, A, O lie in a straight line. The horizontal axis L passes through the mid-point of the rod and is perpendicular to the rod and in the plane of the ring (see diagram). The object consisting of the rod, the ring and the hollow sphere can rotate freely about L.

(i) Show that the moment of inertia of the object about L is $\frac{3}{2}(8k+3)Ma^2$. [6]

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The object performs small oscillations about L, with the ring above the sphere as shown in the diagram.

(ii) Find the set of possible values of k and the period of these oscillations in terms of k. [6]

6 A fair six-sided die is thrown until a 3 or a 4 is obtained. The number of throws taken is denoted by the random variable X.

(i) State the mean value of X. [1]

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(ii) Find the probability that obtaining a 3 or a 4 takes exactly 6 throws. [1]

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(iii) Find the probability that obtaining a 3 or a 4 takes more than 4 throws. [2]

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(iv) Find the greatest integer n such that the probability of obtaining a 3 or a 4 in fewer than n throws is less than 0.95. [3]

7 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \dfrac{3}{4x^2} + \dfrac{1}{4} & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

(ii) Find the exact value of the interquartile range of X. [5]

8 A large number of runners are attending a summer training camp. A random sample of 6 runners is chosen and their times to run 1500 m at the beginning of the camp and at the end of the camp are recorded. Their times, in minutes, are shown in the following table.

Runner	*A*	*B*	*C*	*D*	*E*	*F*
Time at beginning of camp	3.82	3.62	3.55	3.71	3.75	3.92
Time at end of camp	3.72	3.55	3.52	3.68	3.54	3.73

The organiser of the training camp claims that a runner's time will improve by more than 0.05 minutes between the beginning and end of the camp. Assuming that differences in time over the two runs are normally distributed, test at the 10% significance level whether the organiser's claim is justified. [8]

9 A random sample of 50 observations of the continuous random variable X was taken and the values are summarised in the following table.

Interval	$0 \leqslant x < 0.8$	$0.8 \leqslant x < 1.6$	$1.6 \leqslant x < 2.4$	$2.4 \leqslant x < 3.2$	$3.2 \leqslant x < 4$
Observed frequency	18	16	8	6	2

It is required to test the goodness of fit of the distribution with probability density function f given by

$$f(x) = \begin{cases} \frac{3}{16}(4-x)^{\frac{1}{2}} & 0 \leqslant x < 4, \\ 0 & \text{otherwise.} \end{cases}$$

The relevant expected frequencies, correct to 2 decimal places, are given in the following table.

Interval	$0 \leqslant x < 0.8$	$0.8 \leqslant x < 1.6$	$1.6 \leqslant x < 2.4$	$2.4 \leqslant x < 3.2$	$3.2 \leqslant x < 4$
Expected frequency	14.22	12.54	10.59	8.18	4.47

(i) Show how the expected frequency for $1.6 \leqslant x < 2.4$ is obtained. [3]

(ii) Carry out a goodness of fit test at the 5% significance level. [7]

10 The values from a random sample of five pairs (x, y) taken from a bivariate distribution are shown below.

x	3	4	4	6	8
y	5	7	q	6	7

The equation of the regression line of x on y is given by $x = \frac{5}{4}y + c$.

(i) Given that q is an integer, find its value. [5]

(ii) Find the value of c. [3]

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(iii) Find the value of the product moment correlation coefficient. [3]

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11 Answer only **one** of the following two alternatives.

EITHER

A particle P, of mass m, is able to move in a vertical circle on the smooth inner surface of a sphere with centre O and radius a. Points A and B are on the inner surface of the sphere and AOB is a horizontal diameter. Initially, P is projected vertically downwards with speed $\sqrt{\left(\frac{21}{2}ag\right)}$ from A and begins to move in a vertical circle. At the lowest point of its path, vertically below O, the particle P collides with a stationary particle Q, of mass $4m$, and rebounds. The speed acquired by Q, as a result of the collision, is just sufficient for it to reach the point B.

(i) Find the speed of P and the speed of Q immediately after their collision. [7]

In its subsequent motion, P loses contact with the inner surface of the sphere at the point D, where the angle between OD and the upward vertical through O is θ.

(ii) Find $\cos\theta$. [5]

OR

A farmer grows two different types of cherries, Type A and Type B. He assumes that the masses of each type are normally distributed. He chooses a random sample of 8 cherries of Type A. He finds that the sample mean mass is 15.1 g and that a 95% confidence interval for the population mean mass, μ g, is $13.5 \leqslant \mu \leqslant 16.7$.

(i) Find an unbiased estimate for the population variance of the masses of cherries of Type A. [3]

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The farmer now chooses a random sample of 6 cherries of Type B and records their masses as follows.

12.2 13.3 16.4 14.0 13.9 15.4

(ii) Test at the 5% significance level whether the mean mass of cherries of Type B is less than the mean mass of cherries of Type A. You should assume that the population variances for the two types of cherry are equal. [9]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

