

1 A curve C has equation $\cos y = x$, for $-\pi < x < \pi$.

(i) Use implicit differentiation to show that

$$\frac{d^2y}{dx^2} = -\cot y \left(\frac{dy}{dx}\right)^2.$$

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(ii) Hence find the exact value of $\frac{d^2y}{dx^2}$ at the point $(\frac{1}{2}, \frac{1}{3}\pi)$ on C . [2]

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2 Let $u_n = \frac{4 \sin(n - \frac{1}{2}) \sin \frac{1}{2}}{\cos(2n - 1) + \cos 1}$.

(i) Using the formulae for $\cos P \pm \cos Q$ given in the List of Formulae MF10, show that

$$u_n = \frac{1}{\cos n} - \frac{1}{\cos(n - 1)}.$$

[2]

(ii) Use the method of differences to find $\sum_{n=1}^N u_n$. [2]

(iii) Explain why the infinite series $u_1 + u_2 + u_3 + \dots$ does not converge. [1]

- 3** The lines l_1 and l_2 have equations $\mathbf{r} = 6\mathbf{i} + 2\mathbf{j} + 7\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + \mu(-6\mathbf{j} + \mathbf{k})$ respectively. The point P on l_1 and the point Q on l_2 are such that PQ is perpendicular to both l_1 and l_2 . Find the position vectors of P and Q . [8]

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4 It is given that, for $n \geq 0$,

$$I_n = \int_0^1 x^n e^{x^3} \, dx.$$

(i) Show that $I_2 = \frac{1}{3}(e - 1)$. [2]

(ii) Show that, for $n \geq 3$, [3]

$$3I_n = e - (n - 2)I_{n-3}.$$

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(iii) Hence find the exact value of I_8 . [3]

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5 A curve C is defined parametrically by

$$x = \frac{2}{e^t + e^{-t}} \quad \text{and} \quad y = \frac{e^t - e^{-t}}{e^t + e^{-t}},$$

for $0 \leq t \leq 1$. The area of the surface generated when C is rotated through 2π radians about the x -axis is denoted by S .

(i) Show that $S = 4\pi \int_0^1 \frac{e^t - e^{-t}}{(e^t + e^{-t})^2} dt.$ [5]

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(ii) Using the substitution $u = e^t + e^{-t}$, or otherwise, find S in terms of π and e .

[3]

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6 The equation

$$x^3 - x + 1 = 0$$

has roots α, β, γ .

(i) Use the relation $x = y^{\frac{1}{3}}$ to show that the equation

$$y^3 + 3y^2 + 2y + 1 = 0$$

has roots $\alpha^3, \beta^3, \gamma^3$. Hence write down the value of $\alpha^3 + \beta^3 + \gamma^3$. [3]

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Let $S_n = \alpha^n + \beta^n + \gamma^n$.

(ii) Find the value of S_{-3} . [2]

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(iii) Show that $S_6 = 5$ and find the value of S_9 . [4]

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7 Find the particular solution of the differential equation

$$10 \frac{d^2x}{dt^2} + 3 \frac{dx}{dt} - x = t + 2,$$

given that when $t = 0$, $x = 0$ and $\frac{dx}{dt} = 0$.

[10]

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8 (i) Prove by mathematical induction that, for $z \neq 1$ and all positive integers n ,

$$1 + z + z^2 + \dots + z^{n-1} = \frac{z^n - 1}{z - 1}. \quad [5]$$

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$$\sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^m \sin m\theta = \frac{2 \sin \theta}{5 - 4 \cos \theta}. \quad [5]$$

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9 It is given that $\mathbf{e}$ is an eigenvector of the matrix $\mathbf{A}$, with corresponding eigenvalue λ .

(i) Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^2$, with corresponding eigenvalue λ^2 . [2]

The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{pmatrix} n & 1 & 3 \\ 0 & 2n & 0 \\ 0 & 0 & 3n \end{pmatrix} \text{ and } \mathbf{B} = (\mathbf{A} + n\mathbf{I})^2,$$

where $\mathbf{I}$ is the 3×3 identity matrix and n is a non-zero integer.

(ii) Find, in terms of n , a non-singular matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{B} = \mathbf{PDP}^{-1}$. [8]

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10 The curves C_1 and C_2 have equations

$$y = \frac{ax}{x+5} \quad \text{and} \quad y = \frac{x^2 + (a+10)x + 5a + 26}{x+5}$$

respectively, where a is a constant and $a > 2$.

(i) Find the equations of the asymptotes of C_1 . [2]

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(ii) Find the equation of the oblique asymptote of C_2 . [2]

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(iii) Show that C_1 and C_2 do not intersect. [2]

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(iv) Find the coordinates of the stationary points of C_2 . [3]

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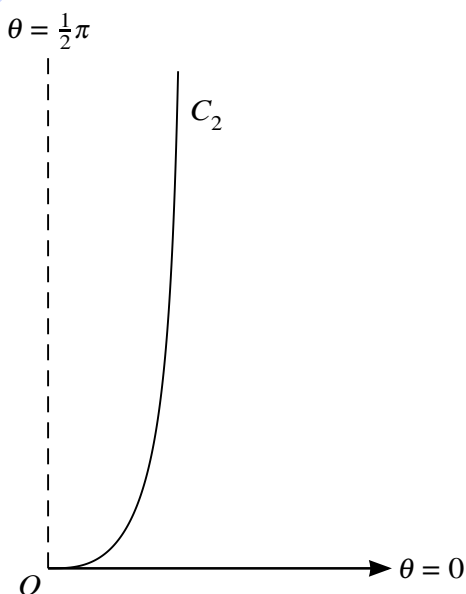
(v) Sketch C_1 and C_2 on a single diagram. [You do not need to calculate the coordinates of any points where C_2 crosses the axes.] [3]

The curve C_2 has polar equation $r^2 = \theta \sec^2 \theta$, for $0 \leq \theta < \frac{1}{2}\pi$. The curves C_1 and C_2 intersect at the pole, denoted by O , and at another point Q .

- (ii) Find the exact value of θ at Q . [2]

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- (iii) The diagram below shows the curve C_7 . Sketch C_1 on this diagram. [2]



(iv) Find, in exact form, the area of the region OPQ enclosed by C_1 and C_2 .

[5]

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OR

The linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} -1 & 2 & 3 & 4 \\ 1 & 0 & 1 & -1 \\ 1 & -2 & -3 & a \\ 1 & 2 & 5 & 2 \end{pmatrix}.$$

- (i) For $a \neq -4$, the range space of T is denoted by V .
- (a) Find the dimension of V and show that

$$\begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 0 \\ -2 \\ 2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 4 \\ -1 \\ a \\ 2 \end{pmatrix}$$

form a basis for V . [5]

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(b) Show that if $\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$ belongs to V then $x + 2y = t$. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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