

Question	Answer	Marks	Guidance
1(i)	$a_R = 2\,(\mathrm{d}\theta/\mathrm{d}t)^2 = 2\,(2\sin 2t)^2 = 2\,(2\sin \pi/3)^2$	**M1**	Verify radial acceleration a_R at $t = \pi/6$ from $r\omega^2$
	$= 2\,(\sqrt{3})^2 = 6$ [m s^{-2}] AG	**A1**	
		2	
1(ii)	$a_T = 2\,\mathrm{d}^2\theta/\mathrm{d}t^2 = 2\,(4\cos 2t) = 2\,(4\cos \pi/3)$	**M1**	Find transverse acceleration a_T at $t = \pi/6$ by differentiation
	$= 4$ [m s^{-2}]	**A1**	
		2	

Question	Answer	Marks	Guidance
2(i)	$v_B = 2v_A$, $v_B^2 = 4\ v_A^2$, $\omega^2\,(a^2 - 1^2) = 4\omega^2\,(a^2 - 3{\cdot}5^2)$	**M1**	Find amplitude a: allow M1 for $v_A = 2\ v_B$
	$3\ a^2 = 48$, $a = [\pm]\ 4$ [m]	**A1**	
	$1 = a\omega^2$, $\omega = 1/\sqrt{a}$ [= ½]	**B1**	Find ω from given maximum acceleration
	$v_O = a\omega = \sqrt{a} = 2$ [m s^{-1}]	**B1**	Find speed v_O at O
		4	
2(ii)	$\omega\, t_{AB} = \sin^{-1}(3.5/a) + \sin^{-1}(1/a)$ $= \sin^{-1} 0{\cdot}875 + \sin^{-1} 0{\cdot}25$	**M1 A1**	Find equation (AEF) for $\omega\, t_{AB}$, combining t_A and t_B, using for example $x = a \sin\ \omega t$ allow sign errors for the M1
	$= 1{\cdot}065 + 0{\cdot}253$ (*or* $t_{AB} = 2{\cdot}131 + 0{\cdot}505$)	**A1**	*or* $x = a \cos\ \omega t$
	$t_{AB} = 2 \times 1{\cdot}318 = 2{\cdot}64$ [s]	**A1**	Hence find t_{AB}
	Alternative method for question 2(ii)		
	$\omega\, t_{AB} = \cos^{-1}(-1/a) - \cos^{-1}(3.5/a)$ $= \cos^{-1}(-0{\cdot}25) - \cos^{-1} 0{\cdot}875$ *or* $\pi - \cos^{-1} 0{\cdot}25 - \cos^{-1} 0{\cdot}875$ (AEF)	**M1 A1**	*or* $x = a \cos\ \omega t$
	$= 1{\cdot}823 - 0{\cdot}505$ (*or* $t_{AB} = 3{\cdot}647 - 1{\cdot}011$)	**A1**	
	$t_{AB} = 2 \times 1{\cdot}318 = 2{\cdot}64$ [s]	**A1**	Hence find t_{AB}
		4	

Question	Answer	Marks	Guidance
3(i)	$2mv_A + 4mv_B = 4mu - 4mu$ $[v_A + 2v_B = 0]$ (AEF)	**M1**	Use conservation of momentum for A and B (m may be omitted)
	$v_B - v_A = e\,(2u + u)$ $[v_B - v_A = 3eu]$	**M1**	Use Newton's restitution law with consistent LHS signs
	$v_A = -2eu$ *and* $v_B = eu$	**A1**	Combine to find v_A and v_B (A0 if directions unclear)
		3	
3(ii)	$[4mv_B'] + mv_C = 4mv_B - (4/3)\,mu$ (AEF)	**M1**	Use conservation of momentum for B & C (m may be omitted)
	$v_C\,[-\,v_B'] = e\,(v_B + 4u/3)$	**M1**	Use Newton's restitution law
	$4v_B - (4/3)\,u = ev_B + 4eu/3,\ 4e - 4/3 = e^2 + 4e/3$	**M1**	Combine to find quadratic equation for e using $v_B' = 0$
	$3e^2 - 8e + 4 = 0,\quad e = 2/3\quad [v_C = 4u/3]$	**A1**	Find value of e, (implicitly) rejecting $e = 2$
		4	

Question	Answer	Marks	Guidance
3(iii)	*For A: Loss* $= \tfrac{1}{2}\, 2m(2u)^2 - \tfrac{1}{2}\, 2m(4/3u)^2 = (20/9)\, mu^2$ *For B: Loss* $= \tfrac{1}{2}\, 4m\, u^2 = \tfrac{1}{2}\, mu^2$ For C: Loss = 0	**M1**	$v_A = -(4/3)\,u$, $[v_B = (2/3)\,u]$, $v_C = (4/3)\,u$ One correct
		M1	Other two correct
	$E_{initial} - E_{final}$ *or* $L_1 + L_2 = (38/9)\, mu^2$	**A1**	Hence find loss in KE
	Alternative method for question 3(iii)		
	$E_{initial} = \tfrac{1}{2}\, 2m(2u)^2 + \tfrac{1}{2}\, 4mu^2 + \tfrac{1}{2}\, m(4u/3)^2$ $= 4mu^2 + 2mu^2 + (8/9)\, mu^2 = (62/9)\, mu^2$	**M1**	Find initial KE of 3 particles in terms of m and u
	$E_{final} = \tfrac{1}{2}\, 2mv_A{}^2\ [+ \tfrac{1}{2}\, 4mv_B'{}^2\,] + \tfrac{1}{2}\, mv_C{}^2$ $= (16/9)\, mu^2 + (8/9)\, mu^2 = (24/9)\, mu^2$	**M1**	Find final KE of 3 particles in terms of m and u
	$E_{initial} - E_{final}$ *or* $L_1 + L_2 = (38/9)\, mu^2$	**A1**	Hence find loss in KE
	Alternative method for question 3(iii)		
	$L_1 = \tfrac{1}{2}\, 2m(2u)^2 + \tfrac{1}{2}\, 4mu^2 - \tfrac{1}{2}\, 2mv_A{}^2 - \tfrac{1}{2}\, 4mv_B{}^2$ $= 4mu^2 + 2mu^2 - (16/9)\, mu^2 - (8/9)\, mu^2 = (30/9)\, mu^2$	**M1**	Find losses in KE in both collisions in terms of m and u
	$L_2 = \tfrac{1}{2}\, 4mv_B{}^2 + \tfrac{1}{2}\, m(4u/3)^2 - [\tfrac{1}{2}\, 4mv_B'{}^2\,] - \tfrac{1}{2}\, mv_C{}^2$ $= (8/9)\, mu^2 + (8/9)\, mu^2 - (8/9)\, mu^2 = (8/9)\, mu^2$	**M1**	
	$E_{initial} - E_{final}$ *or* $L_1 + L_2 = (38/9)\, mu^2$	**A1**	Hence find loss in KE
		3	

Question	Answer	Marks	Guidance
4	A: $T \times 5a/2 \sin(\pi - 2\theta) - W \times 2a \sin\theta = 0$ C: $F_A \times 5a/2 \sin\theta - R_A \times 5a/2 \cos\theta - W \times \frac{1}{2} a \sin\theta = 0$ B: $F_A \times 4a \sin\theta - R_A \times 4a \cos\theta - W \times 2a \sin\theta$ $+ T \times 3a/2 \sin(\pi - 2\theta) = 0$ G: $F_A \times 2a \sin\theta - R_A \times 2a \cos\theta - T \times \frac{1}{2} a \sin(\pi - 2\theta) = 0$ (G is mid-point of AB) D: $R_A \times 5a \cos\theta - W \times 2a \sin\theta = 0$	M1 A1	Take moments for rod about one chosen point [$\sin\theta = 2/\sqrt{5}$, $\cos\theta = 1/\sqrt{5}$, $\sin(\pi - 2\theta) = \sin 2\theta = 4/5$]
	$R_A = T\sin\theta$ $F_A = W - T\cos\theta$	B1	Find two more independent equations
		B1	e.g. resolution of forces on rod (a second moment equation may be used)
	$F_A = \mu R_A$	B1	Relate F_A and R_A (may be implied)
	$T = (2W\sin\theta) / (5/2 \sin 2\theta) = 2W / (5\cos\theta)$	M1	Find T by any method (e.g. from moments about A)
	$= 2W/\sqrt{5}$ *or* $(2\sqrt{5}/5)\, W$ *or* $0{\cdot}894\, W$	A1	
	$F_A = (3/5)\, W$ *and* $R_A = (4/5)\, W$ $\mu = \frac{3}{4}$ *or* $0{\cdot}75$	M1 A1	Find or imply F_A *and* R_A by any method (e.g. from resolutions) and hence μ
		A1	
		10	

Question	Answer	Marks	Guidance
5(i)	$I_{rod} = ⅓\ kMa^2$	**B1**	Find or state MI of rod AB about axis L
	$I_{sphere} = ⅔\ kM\ (2a)^2 + kM\ (3a)^2$ [= $(35k/3)\ Ma^2$]	**M1 A1**	M1 for one term correct, A1 for both terms correct
	$I_{ring} = ½ \times Ma^2 + M\ (2a)^2$ [= $(9/2)\ Ma^2$]	**M1 A1**	M1 for one term correct, A1 for both terms correct
	$I = (k/3 + 35k/3 + 9/2)\ Ma^2 = (3/2)\ (8k + 3)\ Ma^2$ AG	**A1**	MI of object about axis L
		6	
5(ii)	[–] $I\ d^2\theta/dt^2 = -\ kMg \times 3a \sin\theta + Mg \times 2a \sin\theta$ [= – $(3k - 2)\ Mga \sin\theta$]	**M1 A1**	Use equation of circular motion to find $d^2\theta/dt^2$ where θ is angle of rod with vertical
	$d^2\theta/dt^2 = -\ \{2g\ (3k - 2)\ /\ 3a(8k + 3)\}\ \theta$ (AEF)	**M1**	Approximate $\sin\theta$ by θ to give standard form of SHM equation (M0 if wrong sign or $\cos\theta \approx \theta$ used)
	SHM if $3k - 2 > 0$, $k > 2/3$	**M1 A1**	Find possible values of k
	$T = 2\pi\ \sqrt{\{3a(8k + 3)\ /\ 2g\ (3k - 2)\}}$ *or* $\pi\ \sqrt{\{6a(8k + 3)\ /\ g\ (3k - 2)\}}$	**A1**	Find period T
		6	

Question	Answer	Marks	Guidance
6(i)	Mean = 3	**B1**	State mean of X
		1	
6(ii)	$P(X = 6) = q^5 p$ with $p = 1/3$, $q = 2/3$ (AEF) $= 32/729$ or $0{\cdot}0439$	**B1**	Find probability of score of 3 or 4 on exactly 6 throws
		1	
6(iii)	$P(X > 4) = q^4 = 16/81$ *or* $0{\cdot}1975$ *or* $0{\cdot}198$	**M1 A1**	Find probability of score of 3 or 4 on more than 4 throws
		2	
6(iv)	$1 - q^{n-1} < 0{\cdot}95$ (AEF)	**M1**	Formulate condition for n ($1 - q^n$ is M0)
	$0{\cdot}05 < (2/3)^{n-1}$, $n - 1 < \log 0{\cdot}05 / \log 2/3$	**M1**	Set $q = 2/3$, rearrange and take logs (any base) to give bound
	$n - 1 < 7{\cdot}39$, $n_{\max} = 8$	**A1**	Find $n_{\max}$ (> or = can earn M1 M1 A0, max 2/3)
		3	

Question	Answer	Marks	Guidance
7(i)	$F(x) = \int f(x)\,dx = -3/(4x) + x/4$ [$+ c$]	**M1**	Find or state distribution function $F(x)$ for $1 \leqslant x \leqslant 3$
	$= -3/(4x) + x/4 + ½$ *or* $¼ (-3/x + x + 2)$	**A1**	using $F(1) = 0$ or $F(3) = 1$ to find c if necessary
	$F(x) = 0$ ($x <$ *or* $\leqslant 1$), $F(x) = 1$ ($x >$ *or* $\geqslant 3$)	**A1**	State $F(x)$ for other values of x
		3	
7(ii)	$\int_1^Q f(x)\,dx = -3/4Q + Q/4 + ½ = ¼$ [*or* ¾] (AEF)	**M1**	Formulate equation for either quartile value Q
	$Q^2 +$ [*or* $-$] $Q - 3 = 0$	**A1**	
	$Q_1 = ½ (-1 + \sqrt{13})$, $Q_3 = ½ (1 + \sqrt{13})$	**A1 A1**	Find lower quartile Q_1 and upper quartile Q_3
	$Q_3 - Q_1 = 1$	**A1**	Find interquartile range
		5	

Question	Answer	Marks	Guidance
8	H$_0$: $\mu_b - \mu_e = 0{\cdot}05$, H$_1$: $\mu_b - \mu_e > 0{\cdot}05$ (AEF)	**B1**	State both hypotheses (B0 for $\overline{x}$ …)
	d_i: 0·10 0·07 0·03 0·03 0·21 0·19 (or in sec)	**M1**	Consider differences d_i from e.g. $x_b - y_e$
	$\overline{d} = 0{\cdot}63 / 6 = 0{\cdot}105$ (or 6·3 sec)	**B1**	Find sample mean
	$s^2 = (0{\cdot}0969 - 0{\cdot}63^2/6) / 5$ [= 123/20 000 *or* 0·00615 *or* $0{\cdot}0784^2$](or 22·14)	**M1**	Estimate population variance (allow biased here: [41/8000 *or* 0·005125 *or* $0{\cdot}0716^2$])
	$t_{5,\,0.9} = 1{\cdot}476$ (to 3 sf)	**B1**	State or use correct tabular t-value
	$t = (\overline{d} - 0{\cdot}05) / (s/\sqrt{6}) = 1{\cdot}72$	**M1 A1**	Find value of t (or compare $\overline{d} - 0{\cdot}05 = 0{\cdot}055$ with $(t_{5,\,0.9})s/\sqrt{6} = 0{\cdot}0473$)
	[Reject H$_0$:] Evidence for organiser's belief *or* times improve by more than 0·05 min (AEF)	**B1**	**FT** on both t-values Consistent conclusion
			SC Wrong type of hypothesis test can earn only B1 for hypotheses B1FT for conclusion (max 2/8)
		8	

Question	Answer	Marks	Guidance
9 (i)	$E_3 = (3/16)\int_{1.6}^{2.4}(4-x)^{1/2}\,dx$ $= (3/16)\left[-(2/3)(4-x)^{3/2}\right]_{1.6}^{2.4}$	M1	State or imply expression for required expected value E_3 of X
	$= (2{\cdot}4^{3/2} - 1{\cdot}6^{3/2})/8 = 1{\cdot}694/8$ *or* $0{\cdot}2118$	A1	Find expected value E_3 (may be implied in finding 50 E_3) (M1 A1 requires adequate explicit working)
	$50\,E_3 = 10{\cdot}59$ AG	A1	Hence verify corresponding expected frequency
		3	
9(ii)	H_0: Distribution fits data (AEF)	B1	State (at least) null hypothesis in full
	O_i: 18, 16, 8, 8 E_i: 14·22, 12·54, 10·59, 12·65 $X^2 = 1{\cdot}005 + 0{\cdot}955 + 0{\cdot}633 + 1{\cdot}709 = 4{\cdot}30$	M1	Combine values consistent with all exp. values ≥ 5
		M1 A1	Find value of X^2 from $\Sigma\,(E_i - O_i)^2/E_i$ [or $\Sigma\,O_i^2/E_i - n$]
	No. n of cells: 5, 4, 3 $\chi_{n-1,\,0.95}^{\;2}$: 9·488, 7·815, 5·991 (to 3 s.f.)	B1	**FT** on number, n, of cells used to find X^2 State or use consistent tabular value $\chi_{n-1,\,0.95}^{\;2}$
	Accept H_0 if X^2 < tabular value (AEF)	M1	State or imply valid method for conclusion
	4·30 [± 0·01] < 7·81[5] so distribution fits [data] *or* distribution is a suitable model (AEF)	A1	Conclusion (requires both values approx. correct)
		7	

Question	Answer	Marks	Guidance
10(i)	$\Sigma x = 25$, $\Sigma y = 25 + q$, $\Sigma xy = 135 + 4q$, $\Sigma x^2 = 141$, $\Sigma y^2 = 159 + q^2$	B1	Find required values
	$S_{xy} = 135 + 4q - 25(25 + q)/5 = [10 - q]$ (AEF) [$S_{xx} = 141 - 25^2/5 = 16$] $S_{yy} = (159 + q^2) - (25 + q)^2/5$ [$= 34 - 10q + 4q^2/5$]	M1 A1	(S_{xy}, S_{xx}, S_{yy} may be scaled by the same constant)
	$40 - 4q = 170 - 50q + 4q^2$, $2q^2 - 23q + 65 = 0$ $(2q - 13)(q - 5) = 0$, $q = 5$	M1 A1	Equate gradient 5/4 in line of x on y to S_{xy} / S_{yy} and solve quadratic to find integer value of q
		5	
10(ii)	$c = 25/5 - (5/4)(25 + q)/5 = -(5 + q) / 4$	M1 A1	Find c from $\bar{x} - (5/4)\ \bar{y}$
	$= -5/2$ *or* $-2{\cdot}5$	A1	
		3	
10(iii)	$r = S_{xy} / \sqrt{(S_{xx} S_{yy})} = 5 / \sqrt{(16 \times 4)}$	M1 A1	Find correlation coefficient r
	$= 5/8$ or $0{\cdot}625$	A1	
		3	

Question	Answer	Marks	Guidance
11E(i)	$\frac{1}{2}mu_P^2 = \frac{1}{2}m\,(21ag/2) + mga$ $\quad [u_P^2 = (25/2)\,ag]$	M1 A1	u_P is speed of P at lowest point, v_Q is speed of Q immediately after collision. Apply conservation of energy at lowest point (A0 if no m)
	$\frac{1}{2}\,4mv_Q^2 = 4mag$	M1	Find speed v_Q at lowest point by conservation of energy (A0 if no m)
	$v_Q = \sqrt{(2ag)}$ *or* $1{\cdot}41\sqrt{(ag)}$ *or* $4{\cdot}47\sqrt{a}$	A1	
	$mu_P = [\pm]\,mv_P + 4mv_Q$	M1	Find v_P using conservation of momentum (m may be omitted)
	$v_P = [\pm]\,(-5/\sqrt{2} + 4\sqrt{2})\,\sqrt{(ag)}$	A1	
	$\lvert v_P \rvert = (3/\sqrt{2})\,\sqrt{(ag)}$ *or* $2{\cdot}12\,\sqrt{(ag)}$ *or* $0{\cdot}671\sqrt{a}$ (AEF)	A1	Hence find speed of P
		7	
11E(ii)	V_P is speed of P when it loses contact $\frac{1}{2}mV_P^2 = \frac{1}{2}mv_P^2 - mga\,(1 + \cos\alpha)$ $[V_P^2 = (9/2)ag - 2ga\,(1 + \cos\alpha) = (5/2 - 2\cos\alpha)\,ag]$	M1 A1	Apply conservation of energy at D (A0 if no m)
	$[R_D =]\ mV_P^2/a - mg\cos\alpha = 0$ $\quad [V_P^2 = ag\cos\alpha]$	M1 A1	Apply $F = ma$ radially at D with reaction $= 0$
	$(5/2 - 2\cos\alpha)\,ag = ag\cos\alpha$, $\cos\alpha = 5/6$ *or* $0{\cdot}833$	A1	Combine to find $\cos\alpha$
		5	

Question	Answer	Marks	Guidance
11O(i)	$ts_A / \sqrt{8} = ½ (16{\cdot}7 – 13{\cdot}5)$ [= 1·6]	M1	Relate s_A to semi-width of confidence interval
	$t_{7,\,0.975} = 2{\cdot}365$ (to 3 s.f.)	A1	State or use correct tabular t value
	[$s_A = \sqrt{8} \times 1{\cdot}6 / 2{\cdot}365 = 1{\cdot}9135$], $s_A{}^2 = 3{\cdot}66$[16]	A1	Hence find unbiased estimate of A's population variance
		3	
11O(ii)	H_0: $\mu_A = \mu_B$, H_1: $\mu_A > \mu_B$ (AEF)	B1	State hypotheses (B0 for $\bar{x}$ …)
	[$\bar{x}_A = 15.1$], $\bar{x}_B = 85.2 / 6 = 14.2$	B1	Find sample mean for B
	$s_B{}^2 = (1221{\cdot}06 – 85{\cdot}2^2/6) / 5$ $= 561/250$ *or* $2{\cdot}244$ *or* $1{\cdot}498^2$ (all to 3 s.f.)	M1	Estimate or imply population variance for B (allow biased here: $1{\cdot}87$ *or* $1{\cdot}367^2$)
	$s^2 = (7\, s_A{}^2 + 5\, s_B{}^2) / 12 = 3{\cdot}0709$ *or* $1{\cdot}752^2$	M1 A1	Estimate (pooled) common variance ($s_B{}^2$ not needed explicitly)
	$t_{12,\,0.95} = 1{\cdot}782$	B1	State or use correct tabular t value
	[-] $t = (\bar{x}_A - \bar{x}_B) / (s \sqrt{(1/8 + 1/6)}) = 0{\cdot}951$ $t < 1{\cdot}78$ so [accept H_0]	M1 A1	Find value of t (or can compare $\bar{x}_A - \bar{x}_B = 0{\cdot}9$ with $1{\cdot}69$) Correct conclusion
	mean mass of B not less than mean mass of A (AEF)	B1	
		9	
			SC1: Implicitly taking $s_A{}^2$, $s_B{}^2$ as unequal population variances (may also earn first B1 B1 M1) $z = (\bar{x}_A - \bar{x}_B) / \sqrt{(s_A{}^2/8 + s_B{}^2/6)}$ $= 0{\cdot}9 / \sqrt{(0{\cdot}8317)} = 0{\cdot}987$ $z < 1{\cdot}645$ so
			DepSC1: mean mass of B not less than mean mass of A (AEF) Comparison with $z_{0.95}$ and conclusion (FT on z) (can earn at most 5/9)

