

Question	Answer	Marks	Guidance
1(i)	$a_R = 2 (d\theta/dt)^2 = 2 (2 \sin 2t)^2 = 2 (2 \sin \pi/3)^2$	M1	Verify radial acceleration a_R at $t = \pi/6$ from $r\omega^2$
	$= 2 (\sqrt{3})^2 = 6 \text{ [m s}^{-2}\text{]}$ AG	A1	
		2	
1(ii)	$a_T = 2 d^2\theta/dt^2 = 2 (4 \cos 2t) = 2 (4 \cos \pi/3)$	M1	Find transverse acceleration a_T at $t = \pi/6$ by differentiation
	$= 4 \text{ [m s}^{-2}\text{]}$	A1	
		2	

Question	Answer	Marks	Guidance
2(i)	$v_B = 2v_A, v_B^2 = 4 v_A^2, \omega^2 (a^2 - 1^2) = 4\omega^2 (a^2 - 3.5^2)$	M1	Find amplitude a : allow M1 for $v_A = 2 v_B$
	$3 a^2 = 48, a = [\pm] 4 \text{ [m]}$	A1	
	$1 = a\omega^2, \omega = 1/\sqrt{a} [= 1/2]$	B1	Find ω from given maximum acceleration
	$v_O = a\omega = \sqrt{a} = 2 \text{ [m s}^{-1}\text{]}$	B1	Find speed v_O at O
		4	
2(ii)	$\omega t_{AB} = \sin^{-1}(3.5/a) + \sin^{-1}(1/a)$ $= \sin^{-1} 0.875 + \sin^{-1} 0.25$	M1 A1	Find equation (AEF) for ωt_{AB} , combining t_A and t_B , using for example $x = a \sin \omega t$ allow sign errors for the M1
	$= 1.065 + 0.253 \text{ (or } t_{AB} = 2.131 + 0.505)$	A1	or $x = a \cos \omega t$
	$t_{AB} = 2 \times 1.318 = 2.64 \text{ [s]}$	A1	Hence find t_{AB}
	Alternative method for question 2(ii)		
	$\omega t_{AB} = \cos^{-1}(-1/a) - \cos^{-1}(3.5/a)$ $= \cos^{-1}(-0.25) - \cos^{-1} 0.875$ or $\pi - \cos^{-1} 0.25 - \cos^{-1} 0.875 \text{ (AEF)}$	M1 A1	or $x = a \cos \omega t$
	$= 1.823 - 0.505 \text{ (or } t_{AB} = 3.647 - 1.011)$	A1	
	$t_{AB} = 2 \times 1.318 = 2.64 \text{ [s]}$	A1	Hence find t_{AB}
		4	

Question	Answer	Marks	Guidance
3(i)	$2mv_A + 4mv_B = 4mu - 4mu$ $[v_A + 2v_B = 0]$ (AEF)	M1	Use conservation of momentum for A and B (m may be omitted)
	$v_B - v_A = e(2u + u)$ $[v_B - v_A = 3eu]$	M1	Use Newton's restitution law with consistent LHS signs
	$v_A = -2eu$ and $v_B = eu$	A1	Combine to find v_A and v_B (A0 if directions unclear)
		3	
3(ii)	$[4mv_B'] + mv_C = 4mv_B - (4/3)mu$ (AEF)	M1	Use conservation of momentum for B & C (m may be omitted)
	$v_C [-v_B'] = e(v_B + 4u/3)$	M1	Use Newton's restitution law
	$4v_B - (4/3)u = ev_B + 4eu/3, 4e - 4/3 = e^2 + 4e/3$	M1	Combine to find quadratic equation for e using $v_B' = 0$
	$3e^2 - 8e + 4 = 0, e = 2/3$ $[v_C = 4u/3]$	A1	Find value of e , (implicitly) rejecting $e = 2$
		4	

Question	Answer	Marks	Guidance
3(iii)	For A: Loss = $\frac{1}{2} 2m(2u)^2 - \frac{1}{2} 2m(4/3u)^2 = (20/9) mu^2$ For B: Loss = $\frac{1}{2} 4m u^2 = \frac{1}{2} mu^2$ For C: Loss = 0	M1	$v_A = -(4/3) u$, $[v_B = (2/3) u]$, $v_C = (4/3) u$ One correct
		M1	Other two correct
	$E_{initial} - E_{final}$ or $L_1 + L_2 = (38/9) mu^2$	A1	Hence find loss in KE
	Alternative method for question 3(iii)		
	$E_{initial} = \frac{1}{2} 2m(2u)^2 + \frac{1}{2} 4mu^2 + \frac{1}{2} m(4u/3)^2$ $= 4mu^2 + 2mu^2 + (8/9) mu^2 = (62/9) mu^2$	M1	Find initial KE of 3 particles in terms of m and u
	$E_{final} = \frac{1}{2} 2mv_A^2 [+ \frac{1}{2} 4mv_B'^2] + \frac{1}{2} mv_C^2$ $= (16/9) mu^2 + (8/9) mu^2 = (24/9) mu^2$	M1	Find final KE of 3 particles in terms of m and u
	$E_{initial} - E_{final}$ or $L_1 + L_2 = (38/9) mu^2$	A1	Hence find loss in KE
	Alternative method for question 3(iii)		
	$L_1 = \frac{1}{2} 2m(2u)^2 + \frac{1}{2} 4mu^2 - \frac{1}{2} 2mv_A'^2 - \frac{1}{2} 4mv_B'^2$ $= 4mu^2 + 2mu^2 - (16/9) mu^2 - (8/9) mu^2 = (30/9) mu^2$	M1	Find losses in KE in both collisions in terms of m and u
	$L_2 = \frac{1}{2} 4mv_B'^2 + \frac{1}{2} m(4u/3)^2 - [\frac{1}{2} 4mv_B'^2] - \frac{1}{2} mv_C'^2$ $= (8/9) mu^2 + (8/9) mu^2 - (8/9) mu^2 = (8/9) mu^2$	M1	
	$E_{initial} - E_{final}$ or $L_1 + L_2 = (38/9) mu^2$	A1	Hence find loss in KE
		3	

Question	Answer	Marks	Guidance
4	$A: T \times 5a/2 \sin(\pi - 2\theta) - W \times 2a \sin \theta = 0$ $C: F_A \times 5a/2 \sin \theta - R_A \times 5a/2 \cos \theta - W \times \frac{1}{2}a \sin \theta = 0$ $B: F_A \times 4a \sin \theta - R_A \times 4a \cos \theta - W \times 2a \sin \theta + T \times 3a/2 \sin(\pi - 2\theta) = 0$ $G: F_A \times 2a \sin \theta - R_A \times 2a \cos \theta - T \times \frac{1}{2}a \sin(\pi - 2\theta) = 0$ (G is mid-point of AB) $D: R_A \times 5a \cos \theta - W \times 2a \sin \theta = 0$	M1 A1	Take moments for rod about one chosen point [$\sin \theta = 2/\sqrt{5}$, $\cos \theta = 1/\sqrt{5}$, $\sin(\pi - 2\theta) = \sin 2\theta = 4/5$]
	$R_A = T \sin \theta$ $F_A = W - T \cos \theta$	B1	Find two more independent equations
		B1	e.g. resolution of forces on rod (a second moment equation may be used)
	$F_A = \mu R_A$	B1	Relate F_A and R_A (may be implied)
	$T = (2W \sin \theta) / (5/2 \sin 2\theta) = 2W / (5 \cos \theta)$	M1	Find T by any method (e.g. from moments about A)
	$= 2W/\sqrt{5}$ or $(2\sqrt{5}/5) W$ or $0.894 W$	A1	
	$F_A = (3/5) W$ and $R_A = (4/5) W$ $\mu = 3/4$ or 0.75	M1 A1	Find or imply F_A and R_A by any method (e.g. from resolutions) and hence μ
		A1	
		10	

Question	Answer	Marks	Guidance
5(i)	$I_{rod} = \frac{1}{3} kMa^2$	B1	Find or state MI of rod AB about axis L
	$I_{sphere} = \frac{2}{3} kM(2a)^2 + kM(3a)^2$ $[= (35k/3) Ma^2]$	M1 A1	M1 for one term correct, A1 for both terms correct
	$I_{ring} = \frac{1}{2} \times Ma^2 + M(2a)^2$ $[= (9/2) Ma^2]$	M1 A1	M1 for one term correct, A1 for both terms correct
	$I = (k/3 + 35k/3 + 9/2) Ma^2 = (3/2) (8k + 3) Ma^2$ AG	A1	MI of object about axis L
		6	
5(ii)	$[-] I d^2\theta/dt^2 = -kMg \times 3a \sin \theta + Mg \times 2a \sin \theta$ $[= - (3k - 2) Mga \sin \theta]$	M1 A1	Use equation of circular motion to find $d^2\theta/dt^2$ where θ is angle of rod with vertical
	$d^2\theta/dt^2 = - \{2g(3k - 2) / 3a(8k + 3)\} \theta$ (AEF)	M1	Approximate $\sin \theta$ by θ to give standard form of SHM equation (M0 if wrong sign or $\cos \theta \approx \theta$ used)
	SHM if $3k - 2 > 0$, $k > 2/3$	M1 A1	Find possible values of k
	$T = 2\pi \sqrt{\{3a(8k + 3) / 2g(3k - 2)\}}$ or $\pi \sqrt{\{6a(8k + 3) / g(3k - 2)\}}$	A1	Find period T
		6	

Question	Answer	Marks	Guidance
6(i)	Mean = 3	B1	State mean of X
		1	
6(ii)	$P(X=6) = q^5 p$ with $p = 1/3, q = 2/3$ $= 32/729$ or 0.0439 (AEF)	B1	Find probability of score of 3 or 4 on exactly 6 throws
		1	
6(iii)	$P(X > 4) = q^4 = 16/81$ or 0.1975 or 0.198	M1 A1	Find probability of score of 3 or 4 on more than 4 throws
		2	
6(iv)	$1 - q^{n-1} < 0.95$ (AEF)	M1	Formulate condition for n ($1 - q^n$ is M0)
	$0.05 < (2/3)^{n-1}, n-1 < \log 0.05 / \log 2/3$	M1	Set $q = 2/3$, rearrange and take logs (any base) to give bound
	$n-1 < 7.39, n_{\max} = 8$	A1	Find $n_{\max}$ ($>$ or $=$ can earn M1 M1 A0, max 2/3)
		3	

Question	Answer	Marks	Guidance
7(i)	$F(x) = \int f(x) dx = -3/(4x) + x/4 [+c]$	M1	Find or state distribution function $F(x)$ for $1 \leq x \leq 3$
	$= -3/(4x) + x/4 + 1/2$ or $1/4 (-3/x + x + 2)$	A1	using $F(1) = 0$ or $F(3) = 1$ to find c if necessary
	$F(x) = 0$ ($x < \text{or} \leq 1$), $F(x) = 1$ ($x > \text{or} \geq 3$)	A1	State $F(x)$ for other values of x
		3	
7(ii)	$\int_1^Q f(x) dx = -3/4Q + Q/4 + 1/2 = 1/4$ [or $3/4$] (AEF)	M1	Formulate equation for either quartile value Q
	$Q^2 + [\text{or} -] Q - 3 = 0$	A1	
	$Q_1 = 1/2 (-1 + \sqrt{13})$, $Q_3 = 1/2 (1 + \sqrt{13})$	A1 A1	Find lower quartile Q_1 and upper quartile Q_3
	$Q_3 - Q_1 = 1$	A1	Find interquartile range
		5	

Question	Answer	Marks	Guidance
8	$H_0: \mu_b - \mu_e = 0.05, H_1: \mu_b - \mu_e > 0.05$ (AEF)	B1	State both hypotheses (B0 for $\bar{x} \dots$)
	$d_i: 0.10 \quad 0.07 \quad 0.03 \quad 0.03 \quad 0.21 \quad 0.19$ (or in sec)	M1	Consider differences d_i from e.g. $x_b - y_e$
	$\bar{d} = 0.63 / 6 = 0.105$ (or 6.3 sec)	B1	Find sample mean
	$s^2 = (0.0969 - 0.63^2/6) / 5$ [= 123/20 000 or 0.00615 or 0.0784 ²](or 22.14)	M1	Estimate population variance (allow biased here: [41/8000 or 0.005125 or 0.0716 ²])
	$t_{5, 0.9} = 1.476$ (to 3 sf)	B1	State or use correct tabular t -value
	$t = (\bar{d} - 0.05) / (s/\sqrt{6}) = 1.72$	M1 A1	Find value of t (or compare $\bar{d} - 0.05 = 0.055$ with $(t_{5, 0.9})s/\sqrt{6} = 0.0473$)
	[Reject H_0 :] Evidence for organiser's belief or times improve by more than 0.05 min (AEF)	B1	FT on both t -values Consistent conclusion
			SC Wrong type of hypothesis test can earn only B1 for hypotheses B1FT for conclusion (max 2/8)
		8	

Question	Answer	Marks	Guidance
9 (i)	$E_3 = (3/16) \int_{1.6}^{2.4} (4-x)^{1/2} dx$ $= (3/16) [- (2/3) (4-x)^{3/2}]_{1.6}^{2.4}$	M1	State or imply expression for required expected value E_3 of X
	$= (2.4^{3/2} - 1.6^{3/2}) / 8 = 1.694/8 \text{ or } 0.2118$	A1	Find expected value E_3 (may be implied in finding 50 E_3) (M1 A1 requires adequate explicit working)
	50 $E_3 = 10.59$ AG	A1	Hence verify corresponding expected frequency
		3	
9(ii)	H_0 : Distribution fits data (AEF)	B1	State (at least) null hypothesis in full
	O_i 18 16 8 <u>8</u>	M1	Combine values consistent with all exp. values ≥ 5
	E_i : 14.22 12.54 10.59 <u>12.65</u>	M1 A1	Find value of X^2 from $\Sigma (E_i - O_i)^2 / E_i$ [or $\Sigma O_i^2 / E_i - n$]
	$X^2 = 1.005 + 0.955 + 0.633 + 1.709 = 4.30$		
	No. n of cells: 5 <u>4</u> 3 $\chi_{n-1, 0.95}^2$: 9.488 <u>7.815</u> 5.991 (to 3 s.f.)	B1	FT on number, n , of cells used to find X^2 State or use consistent tabular value $\chi_{n-1, 0.95}^2$
	Accept H_0 if $X^2 < \text{tabular value}$ (AEF)	M1	State or imply valid method for conclusion
	$4.30 [\pm 0.01] < 7.81[5]$ so distribution fits [data] or distribution is a suitable model (AEF)	A1	Conclusion (requires both values approx. correct)
		7	

Question	Answer	Marks	Guidance
10(i)	$\Sigma x = 25, \Sigma y = 25 + q, \Sigma xy = 135 + 4q,$ $\Sigma x^2 = 141, \Sigma y^2 = 159 + q^2$	B1	Find required values
	$S_{xy} = 135 + 4q - 25(25 + q)/5 = [10 - q]$ (AEF) [$S_{xx} = 141 - 25^2/5 = 16$] $S_{yy} = (159 + q^2) - (25 + q)^2/5 [= 34 - 10q + 4q^2/5]$	M1 A1	(S_{xy}, S_{xx}, S_{yy} may be scaled by the same constant)
	$40 - 4q = 170 - 50q + 4q^2, 2q^2 - 23q + 65 = 0$ $(2q - 13)(q - 5) = 0, q = 5$	M1 A1	Equate gradient 5/4 in line of x on y to S_{xy} / S_{yy} and solve quadratic to find integer value of q
		5	
10(ii)	$c = 25/5 - (5/4)(25 + q)/5 = -(5 + q) / 4$	M1 A1	Find c from $\bar{x} - (5/4) \bar{y}$
	$= -5/2$ or -2.5	A1	
		3	
10(iii)	$r = S_{xy} / \sqrt{(S_{xx} S_{yy})} = 5 / \sqrt{(16 \times 4)}$	M1 A1	Find correlation coefficient r
	$= 5/8$ or 0.625	A1	
		3	

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11E(i)	$\frac{1}{2}mu_P^2 = \frac{1}{2}m(21ag/2) + mga \quad [u_P^2 = (25/2)ag]$	M1 A1	u_P is speed of P at lowest point, v_Q is speed of Q immediately after collision. Apply conservation of energy at lowest point (A0 if no m)
	$\frac{1}{2}4mv_Q^2 = 4mag$	M1	Find speed v_Q at lowest point by conservation of energy (A0 if no m)
	$v_Q = \sqrt{2ag}$ or $1.41\sqrt{ag}$ or $4.47\sqrt{a}$	A1	
	$mu_P = [\pm] mv_P + 4mv_Q$	M1	Find v_P using conservation of momentum (m may be omitted)
	$v_P = [\pm] (-5/\sqrt{2} + 4\sqrt{2})\sqrt{ag}$	A1	
	$ v_P = (3/\sqrt{2})\sqrt{ag}$ or $2.12\sqrt{ag}$ or $0.671\sqrt{a}$ (AEF)	A1	Hence find speed of P
		7	
11E(ii)	V_P is speed of P when it loses contact $\frac{1}{2}mV_P^2 = \frac{1}{2}mv_P^2 - mga(1 + \cos \alpha)$ $[V_P^2 = (9/2)ag - 2ga(1 + \cos \alpha) = (5/2 - 2 \cos \alpha)ag]$	M1 A1	Apply conservation of energy at D (A0 if no m)
	$[R_D =] mV_P^2/a - mg \cos \alpha = 0 \quad [V_P^2 = ag \cos \alpha]$	M1 A1	Apply $F = ma$ radially at D with reaction = 0
	$(5/2 - 2 \cos \alpha)ag = ag \cos \alpha, \cos \alpha = 5/6$ or 0.833	A1	Combine to find $\cos \alpha$
		5	

Question	Answer	Marks	Guidance
11O(i)	$ts_A / \sqrt{8} = \frac{1}{2} (16.7 - 13.5) [= 1.6]$	M1	Relate s_A to semi-width of confidence interval
	$t_{7, 0.975} = 2.365$ (to 3 s.f.)	A1	State or use correct tabular t value
	$[s_A = \sqrt{8} \times 1.6 / 2.365 = 1.9135], s_A^2 = 3.66[16]$	A1	Hence find unbiased estimate of A 's population variance
		3	
11O(ii)	$H_0: \mu_A = \mu_B, H_1: \mu_A > \mu_B$ (AEF)	B1	State hypotheses (B0 for $\bar{x} \dots$)
	$[\bar{x}_A = 15.1], \bar{x}_B = 85.2 / 6 = 14.2$	B1	Find sample mean for B
	$s_B^2 = (1221.06 - 85.2^2/6) / 5$ $= 561/250$ or 2.244 or 1.498^2 (all to 3 s.f.)	M1	Estimate or imply population variance for B (allow biased here: 1.87 or 1.367^2)
	$s^2 = (7 s_A^2 + 5 s_B^2) / 12 = 3.0709$ or 1.752^2	M1 A1	Estimate (pooled) common variance (s_B^2 not needed explicitly)
	$t_{12, 0.95} = 1.782$	B1	State or use correct tabular t value
	$[-] t = (\bar{x}_A - \bar{x}_B) / (s \sqrt{1/8 + 1/6}) = 0.951$ $t < 1.78$ so [accept H_0]	M1 A1	Find value of t (or can compare $\bar{x}_A - \bar{x}_B = 0.9$ with 1.69) Correct conclusion
	mean mass of B not less than mean mass of A (AEF)	B1	
		9	
			SC1: Implicitly taking s_A^2, s_B^2 as unequal population variances (may also earn first B1 B1 M1) $z = (\bar{x}_A - \bar{x}_B) / \sqrt{(s_A^2/8 + s_B^2/6)}$ $= 0.9 / \sqrt{(0.8317)} = 0.987$ $z < 1.645$ so
			DepSC1: mean mass of B not less than mean mass of A (AEF) Comparison with $z_{0.95}$ and conclusion (FT on z) (can earn at most 5/9)