

Question	Answer	Marks	Guidance
1(i)	$-\sin y \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = -(\sin y)^{-1}$	M1 A1	Differentiates implicitly once.
	$\frac{d^2 y}{d^2 x} = (\sin y)^{-2} \cos y \left(\frac{dy}{dx} \right) = -\cot y \left(\frac{dy}{dx} \right)^2$	M1 A1	Differentiates again, AG.
		4	
1(ii)	$\frac{dy}{dx} = -\left(\sin \frac{\pi}{3} \right)^{-1} = -\frac{2}{\sqrt{3}}$	B1	
	$\frac{d^2 y}{d^2 x} = -\frac{1}{\sqrt{3}} \left(-\frac{2}{\sqrt{3}} \right)^2 = -\frac{4}{3\sqrt{3}} = -\frac{4}{9}\sqrt{3}$	B1	AEF, must be exact.
		2	

Question	Answer	Marks	Guidance
2(i)	$\frac{4\sin\left(n-\frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1)+\cos 1} = \frac{2(\cos(n-1)-\cos n)}{2\cos n \cos(n-1)}$	M1	Uses formulae for $\cos P \pm \cos Q$.
	$\frac{1}{\cos n} - \frac{1}{\cos(n-1)}$	A1	AG
		2	
2(ii)	$\sum_{n=1}^N \frac{4\sin\left(n-\frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1)+\cos 1} = \sum_{n=1}^N \frac{1}{\cos n} - \frac{1}{\cos(n-1)}$ $= \frac{1}{\cos 1} - \frac{1}{1} + \frac{1}{\cos 2} - \frac{1}{\cos 1} + \dots + \frac{1}{\cos N} - \frac{1}{\cos(N-1)}$	M1	Applies (i), shows enough terms and cancelation.
	$-1 + \frac{1}{\cos N}$	A1	
		2	
2(iii)	$\cos N$ oscillates as $N \rightarrow \infty$ so $u_1 + u_2 + u_3 + \dots$ does not converge.	B1	States “oscillates” or refers to diverging values of $\cos N$.
		1	

Question	Answer	Marks	Guidance
3	$\overrightarrow{OP} = \begin{pmatrix} 6+\lambda \\ 2+\lambda \\ 7 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 4 \\ 4-6\mu \\ \mu \end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} -2-\lambda \\ 2-\lambda-6\mu \\ -7+\mu \end{pmatrix}$	M1 A1	Finds $\overrightarrow{PQ}$.
	$\begin{pmatrix} -2-\lambda \\ 2-\lambda-6\mu \\ -7+\mu \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0$ Or $\begin{pmatrix} -2-\lambda \\ 2-\lambda-6\mu \\ -7+\mu \end{pmatrix} = k \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 0 \\ 0 & -6 & 1 \end{vmatrix} = k \begin{pmatrix} 1 \\ -1 \\ -6 \end{pmatrix}$	M1	Uses that dot product of $\overrightarrow{PQ}$ with line directions is 0. Or, alternatively, $\overrightarrow{PQ}$ is multiple of common perpendicular.
	$-2\lambda - 6\mu = 0$	A1	Deduces one equation. CWO.
	$\begin{pmatrix} -2-\lambda \\ 2-\lambda-6\mu \\ -7+\mu \end{pmatrix} \begin{pmatrix} 0 \\ -6 \\ 1 \end{pmatrix} = 0 \Rightarrow 6\lambda + 37\mu = 19$	A1	Deduces second equation. CWO.
	$\lambda = -3, \mu = 1$	M1 A1	Solves simultaneous equations.
	$\overrightarrow{OP} = \begin{pmatrix} 3 \\ -1 \\ 7 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$	A1	States $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.
		8	

Question	Answer	Marks	Guidance
4(i)	$\int_0^1 x^2 e^{x^3} dx = \frac{1}{3} [e^{x^3}]_0^1 = \frac{1}{3}(e-1)$	M1 A1	Must show working, AG.
		2	
4(ii)	$I_n = \int_0^1 x^{n-2} x^2 e^{x^3} dx = \left[\frac{1}{3} x^{n-2} e^{x^3} \right]_0^1 - \frac{n-2}{3} \int_0^1 x^{n-3} e^{x^3} dx$	M1 A1	Integrates by parts.
	$\frac{e}{3} - \frac{n-2}{3} I_{n-3} \Rightarrow 3I_n = e - (n-2)I_{n-3}$	A1	AG
		3	
4(iii)	$I_5 = \frac{1}{3}(e - 3I_2) = \frac{1}{3}(e - e + 1) = \frac{1}{3}$	M1 A1	Applies reduction formula once and uses (i).
	$I_8 = \frac{1}{3}(e - 6I_5) = \frac{1}{3}(e - 2).$	A1	Must be exact.
		3	

Question	Answer	Marks	Guidance
5(i)	$\frac{dx}{dt} = \frac{-2(e^t - e^{-t})}{(e^t + e^{-t})^2}$	B1	
	$\frac{dy}{dt} = \frac{(e^t + e^{-t})^2 - (e^t - e^{-t})^2}{(e^t + e^{-t})^2} = \frac{(2e^t)(2e^{-t})}{(e^t + e^{-t})^2} = \frac{4}{(e^t + e^{-t})^2}$	B1	Differentiates and simplifies. Accept $1 - \left(\frac{e^t - e^{-t}}{e^t + e^{-t}}\right)^2$.
	$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \frac{4(e^t - e^{-t})^2 + 16}{(e^t + e^{-t})^4} = \frac{4(e^t + e^{-t})^2}{(e^t + e^{-t})^4}$	M1 A1	Attempt at writing $\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$ as a square.
	$S = 2\pi \int_0^1 \left(\frac{e^t - e^{-t}}{e^t + e^{-t}}\right) \left(\frac{2}{e^t + e^{-t}}\right) dt = 4\pi \int_0^1 \frac{e^t - e^{-t}}{(e^t + e^{-t})^2} dt$	A1	Uses correct formula, simplifies to AG.
		5	
5(ii)	$S = 4\pi \int_2^{e+e^{-1}} u^{-2} du = 4\pi \left[-u^{-1}\right]_2^{e+e^{-1}}$	M1 A1	Applies given substitution.
	$= 4\pi \left(\frac{1}{2} - \frac{1}{e + e^{-1}}\right)$	A1	AEF, must be exact.
		3	

Question	Answer	Marks	Guidance
6(i)	$y = (y+1)^3$	M1	Obtains an equation in y not involving radicals.
	$y = y^3 + 3y^2 + 3y + 1 \Rightarrow y^3 + 3y^2 + 2y + 1 = 0$	A1	AG
	$S_3 = -3$	B1	
		3	
6(ii)	$S_{-3} = \frac{\alpha^3\beta^3 + \beta^3\gamma^3 + \alpha^3\gamma^3}{\alpha^3\beta^3\gamma^3} = \frac{2}{-1} = -2$	M1 A1	
		2	
6(iii)	$S_6 = (-3)^2 - 2(2) = 5$	M1 A1	Uses $(\sum \alpha)^2 = \sum \alpha^2 + 2 \sum_{\alpha \neq \beta} \alpha\beta$. AG.
	$S_9 = -3S_6 - 2S_3 - 3 = -3(5) - 2(-3) - 3 = -12$	M1 A1	Sums $y^3 - 3y^2 + 2y - 1 = 0$ for $y = \alpha^3, \beta^3, \gamma^3$.
		4	

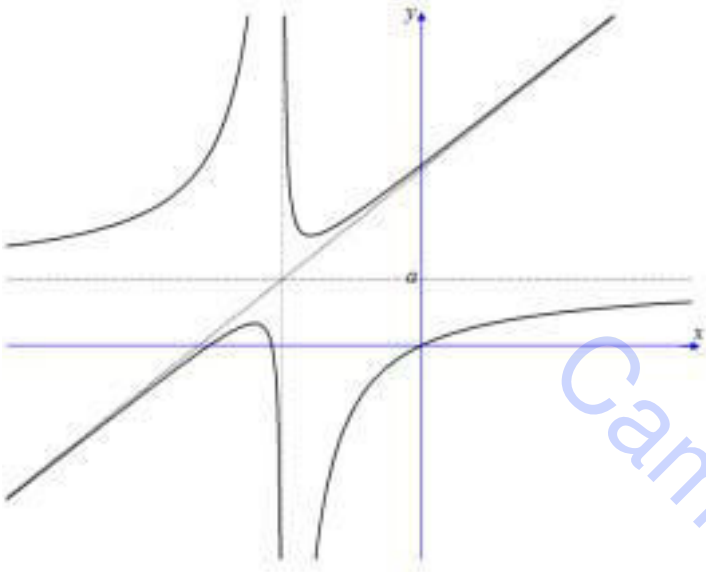
Question	Answer	Marks	Guidance
7	$10u^2 + 3u - 1 = 0 \Rightarrow (2u + 1)(5u - 1) = 0$	M1	Axillary equation
	CF: $x = Ae^{-\frac{1}{2}t} + Be^{\frac{1}{5}t}$	A1	
	PI: $x = p + qt \Rightarrow \ddot{x} = q \Rightarrow x = 0$	M1	Forms PI and differentiates.
	$3q - p - qt = t + 2 \Rightarrow q = -1, p = -5.$	M1 A1	Substitutes.
	GS: $x = Ae^{-\frac{1}{2}t} + Be^{\frac{1}{5}t} - t - 5$	A1	States general solution.
	$\dot{x} = -\frac{1}{2}Ae^{-\frac{1}{2}t} + \frac{1}{5}Be^{\frac{1}{5}t} - 1$	M1	Differentiates.
	$A + B = 5$ $-\frac{1}{2}A + \frac{1}{5}B = 1$	M1	Forms simultaneous equations.
	$A = 0, B = 5$	A1	
	$x = 5e^{\frac{1}{5}t} - t - 5$	A1	States PS.
		10	

Question	Answer	Marks	Guidance
8(i)	$1 = \frac{z^1 - 1}{z - 1}$ So true when $n = 1$.	B1	Shows base case.
	Assume that $1 + z + \dots + z^{k-1} = \frac{z^k - 1}{z - 1}$	B1	States inductive hypothesis.
	Then $1 + z + \dots + z^{k-1} + z^k = \frac{z^k - 1}{z - 1} + z^k = \frac{z^k - 1 + z^k(z - 1)}{z - 1} = \frac{z^{k+1} - 1}{z - 1},$ so true when $n = k + 1$	M1 A1	Combines fractions.
	$H_k \rightarrow H_{k+1}$ Hence, by induction, true for all positive integers.	A1	States conclusion.
		5	

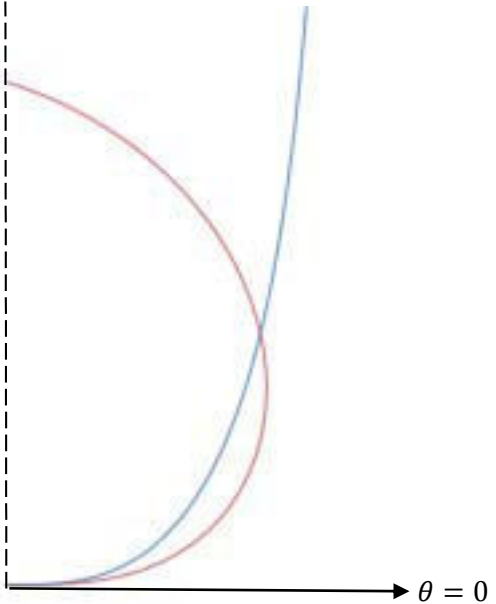
Question	Answer	Marks	Guidance
8(ii)	Since $ z < 1$, $\sum_{m=0}^{\infty} z^m = \frac{-1}{z-1}$	B1	States $ z < 1$ and uses formula for sum to infinity of geometric progression.
	$\sum_{m=1}^{\infty} 2^{-m} \sin m\theta = \operatorname{Im} \left(\sum_{m=0}^{\infty} z^m \right) = \operatorname{Im} \left(\frac{-1}{\frac{1}{2} \cos \theta + i \frac{1}{2} \sin \theta - 1} \right)$	M1 A1	Uses de Moivre's theorem.
	$\operatorname{Im} \left(\frac{-\left(\frac{1}{2} \cos \theta - 1 - i \frac{1}{2} \sin \theta\right)}{\frac{1}{4} \cos^2 \theta - \cos \theta + 1 + \frac{1}{4} \sin^2 \theta} \right)$	M1	Multiply numerator and denominator by conjugate.
	$\frac{\frac{1}{2} \sin \theta}{\frac{5}{4} - \cos \theta} = \frac{2 \sin \theta}{5 - 4 \cos \theta}$	A1	States imaginary part, AG.
		5	

Question	Answer	Marks	Guidance
9(i)	$\mathbf{A}^2\mathbf{e} = \mathbf{A}(\mathbf{Ae}) = \lambda\mathbf{Ae} = \lambda^2\mathbf{e}$	M1 A1	AG
		2	
9(ii)	Eigenvalues of $\mathbf{A}$ are $n, 2n$ and $3n$.	B1	
	$\lambda = n: \mathbf{e}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 3 \\ 0 & n & 0 \end{vmatrix} = \begin{pmatrix} -3n \\ 0 \\ 0 \end{pmatrix} = t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1 A1	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda = 2n: \mathbf{e}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -n & 1 & 3 \\ 0 & 0 & n \end{vmatrix} = \begin{pmatrix} n \\ n^2 \\ 0 \end{pmatrix} = t \begin{pmatrix} 1 \\ n \\ 0 \end{pmatrix}$	A1	
	$\lambda = 3n: \mathbf{e}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2n & 1 & 3 \\ 0 & -n & 0 \end{vmatrix} = \begin{pmatrix} 3n \\ 0 \\ 2n^2 \end{pmatrix} = t \begin{pmatrix} 3 \\ 0 \\ 2n \end{pmatrix}$	A1	
	Eigenvalues of $\mathbf{A} + n\mathbf{I}$ are $2n, 3n$ and $4n$	B1	
	Thus $\mathbf{P} = \begin{pmatrix} 1 & 1 & 3 \\ 0 & n & 0 \\ 0 & 0 & 2n \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} (2n)^2 & 0 & 0 \\ 0 & (3n)^2 & 0 \\ 0 & 0 & (4n)^2 \end{pmatrix}$	M1 A1	Or correctly matched permutations of columns.
		8	

Question	Answer	Marks	Guidance
10(i)	$x = -5$ and $y = a$	B1 B1	
		2	
10(ii)	$x^2 + (a + 10)x + 5a + 26 = (x + 5)(x + a + 5) + 1$	M1	By inspection or long division.
	oblique asymptote is $y = x + a + 5$	A1	
		2	
10(iii)	$x^2 + 10x + 5a + 26 = 0$	M1	Puts y -values equal and forms quadratic equation.
	$10^2 - 4(5a + 26) = -4 - 20a < 0$ so no intersection point	A1	Correct discriminant and conclusion.
		2	
10(iv)	$(x + 5)(2x + a + 10) - x^2 - ax - 10x - 5a - 26 = 0$	M1	Differentiates and forms quadratic equation.
	$x^2 + 10x + 24 = 0$	A1	
	Stationary points are $(-4, a + 2)$ and $(-6, a - 2)$	A1	Must have both points.
		3	

Question	Answer	Marks	Guidance
10(v)		B1	Asymptotes drawn, intersection correct.
		B1	C_1 correct.
		B1	C_2 correct.
		3	

Question	Answer	Marks	Guidance
11E(i)	$x = \sqrt{2}\theta^{\frac{1}{2}} \cos \theta$	M1	Uses $x = r \cos \theta$.
	$\frac{d}{d\theta} \left(\sqrt{2}\theta^{\frac{1}{2}} \cos \theta \right) = \sqrt{2} \left(-\theta^{\frac{1}{2}} \sin \theta + \frac{1}{2}\theta^{-\frac{1}{2}} \cos \theta \right) = 0$	M1 A1	Sets derivative of $r \cos \theta$ equal to zero.
	$-\theta^{\frac{1}{2}} \sin \theta + \frac{1}{2}\theta^{-\frac{1}{2}} \cos \theta = 0 \Rightarrow \cos \theta = 2\theta \sin \theta \Rightarrow 2\theta \tan \theta = 1$	A1	AG
	$2(0.6) \tan(0.6) - 1 = -0.179$ and $2(0.7) \tan(0.7) - 1 = 0.179$	B1	Shows sign change.
		5	

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11E(ii)	$2\theta = \theta \sec^2 \theta \Rightarrow \theta^{\frac{1}{2}} (\sec \theta - \sqrt{2}) = 0 \Rightarrow \theta = \frac{\pi}{4}$	M1 A1	Finds value of θ .
		2	
11E(iii)		B1	Correct shape.
		B1	Intersection correct.
		2	

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11E(iv)	$\frac{1}{2} \int_0^{\frac{\pi}{4}} 2\theta d\theta - \frac{1}{2} \int_0^{\frac{\pi}{4}} \theta \sec^2 \theta d\theta = \frac{1}{2} \int_0^{\frac{\pi}{4}} (2 - \sec^2 \theta) d\theta$	M1	Forms correct integral.
	$\frac{1}{2} [\theta(2\theta - \tan \theta)]_0^{\frac{\pi}{4}} - \frac{1}{2} \int_0^{\frac{\pi}{4}} (2\theta - \tan \theta) d\theta$	M1 A1	Integrates by parts.
	$\frac{1}{2} [\theta(2\theta - \tan \theta)]_0^{\frac{\pi}{4}} - \frac{1}{2} [\theta^2 + \ln \cos \theta]_0^{\frac{\pi}{4}}$	A1	
	$\frac{\pi}{8} \left(\frac{\pi}{2} - 1 \right) - \frac{1}{2} \left(\left(\frac{\pi}{4} \right)^2 + \frac{1}{2} \ln 2 \right) = \frac{1}{4} \ln 2 + \frac{\pi}{8} \left(\frac{\pi}{4} - 1 \right).$	A1	AEF, must be exact.
		5	
11O(i)(a)	$\begin{pmatrix} -1 & 2 & 3 & 4 \\ 1 & 0 & 1 & -1 \\ 1 & -2 & -3 & a \\ 1 & 2 & 5 & 2 \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} -1 & 2 & 3 & 4 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 0 & a+4 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\begin{matrix} v_1 & v_2 & v_3 & v_4 \\ v_1' & v_2' & v_3' & v_4' \end{matrix}$	M1A1	Reduces M or M^T to echelon form.
	$\dim V = \text{rank} = 3.$	A1	
	$c_1 v_1' + c_2 v_2' + c_3 v_4' = 0 \Rightarrow c_1 = c_2 = c_3 = 0$	M1	Shows v_1', v_2', v_4' are linearly independent.
	Thus v_1, v_2, v_4 are linearly independent (and so form a basis for V).	A1	
		5	

Question	Answer	Marks	Guidance
11O(i)(b)	$\begin{pmatrix} -1 & 2 & 4 & x \\ 1 & 0 & -1 & y \\ 1 & -2 & a & z \\ 1 & 2 & 2 & t \end{pmatrix} \rightarrow \dots \rightarrow \begin{pmatrix} -1 & 2 & 4 & x \\ 0 & 2 & 3 & y+x \\ 0 & 0 & a+4 & z+x \\ 0 & 4 & 6 & t+x \end{pmatrix}$	M1 A1	$x = -\alpha + 2\beta + 4\gamma$ $y = \alpha - \gamma$ Uses row operations or $z = \alpha - 2\beta + a\gamma$ $t = \alpha + 2\beta + 2\gamma$
	System is consistent when $t + x = 2(y + x) \Rightarrow x + 2y = t$ Or $(-\alpha + 2\beta + 4\gamma) + 2(\alpha - \gamma) = \alpha + 2\beta + 2\gamma = t$	M1 A1	AG
		4	
11O(ii)	$-x + 2y + 3z + 4t = 0$ $2y + 4z + 3t = 0$	M1	Finds basis for null space.
	$t = \mu, \quad z = \lambda, \quad y = -2\lambda - \frac{3}{2}\mu, \quad x = -\lambda + \mu$	A1	
	A basis is $\left\{ \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 0 \\ 2 \end{pmatrix} \right\} = \{\mathbf{e}_1, \mathbf{e}_2\}$	A1	AEF
	$\mathbf{M} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \text{ so particular solution is } \mathbf{e} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	B1	Finds particular solution.
	General solution is $\mathbf{x} = \mathbf{e} + \lambda \mathbf{e}_1 + \mu \mathbf{e}_2$	A1	FT Accept <i>their</i> basis. Must have correct particular solution.
		5	