

Question	Answer	Marks	Guidance
1(i)	$-\sin y\dfrac{\mathrm{d}y}{\mathrm{d}x}=1\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}x}=-(\sin y)^{-1}$	**M1 A1**	Differentiates implicitly once.
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}^2x}=(\sin y)^{-2}\cos y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)=-\cot y\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$	**M1 A1**	Differentiates again, AG.
		4	
1(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\left(\sin\dfrac{\pi}{3}\right)^{-1}=-\dfrac{2}{\sqrt{3}}$	**B1**	
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}^2x}=-\dfrac{1}{\sqrt{3}}\left(-\dfrac{2}{\sqrt{3}}\right)^2=-\dfrac{4}{3\sqrt{3}}=-\dfrac{4}{9}\sqrt{3}$	**B1**	AEF, must be exact.
		2	

Question	Answer	Marks	Guidance
2(i)	$\dfrac{4\sin\left(n-\frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1)+\cos 1}=\dfrac{2\left(\cos(n-1)-\cos n\right)}{2\cos n\cos(n-1)}$	M1	Uses formulae for $\cos P \pm \cos Q$.
	$\dfrac{1}{\cos n}-\dfrac{1}{\cos(n-1)}$	A1	AG
		2	
2(ii)	$\displaystyle\sum_{n=1}^{N}\frac{4\sin\left(n-\frac{1}{2}\right)\sin\frac{1}{2}}{\cos(2n-1)+\cos 1}=\sum_{n=1}^{N}\frac{1}{\cos n}-\frac{1}{\cos(n-1)}$ $=\dfrac{1}{\cos 1}-\dfrac{1}{1}+\dfrac{1}{\cos 2}-\dfrac{1}{\cos 1}+\ldots+\dfrac{1}{\cos N}-\dfrac{1}{\cos(N-1)}$	M1	Applies (i), shows enough terms and cancelation.
	$-1+\dfrac{1}{\cos N}$	A1	
		2	
2(iii)	$\cos N$ oscillates as $N\to\infty$ so $u_1+u_2+u_3+\ldots$ does not converge.	B1	States "oscillates" or refers to diverging values of $\cos N$.
		1	

Question	Answer	Marks	Guidance
3	$\overrightarrow{OP}=\begin{pmatrix}6+\lambda\\2+\lambda\\7\end{pmatrix},\overrightarrow{OQ}=\begin{pmatrix}4\\4-6\mu\\\mu\end{pmatrix}\Rightarrow\overrightarrow{PQ}=\begin{pmatrix}-2-\lambda\\2-\lambda-6\mu\\-7+\mu\end{pmatrix}$	**M1 A1**	Finds $\overrightarrow{PQ}$.
	$\begin{pmatrix}-2-\lambda\\2-\lambda-6\mu\\-7+\mu\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}=0$ **Or** $\begin{pmatrix}-2-\lambda\\2-\lambda-6\mu\\-7+\mu\end{pmatrix}=k\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\1&1&0\\0&-6&1\end{vmatrix}=k\begin{pmatrix}1\\-1\\-6\end{pmatrix}$	**M1**	Uses that dot product of $\overrightarrow{PQ}$ with line directions is 0. Or, alternatively, $\overrightarrow{PQ}$ is multiple of common perpendicular.
	$-2\lambda-6\mu=0$	**A1**	Deduces one equation. CWO.
	$\begin{pmatrix}-2-\lambda\\2-\lambda-6\mu\\-7+\mu\end{pmatrix}\begin{pmatrix}0\\-6\\1\end{pmatrix}=0\Rightarrow 6\lambda+37\mu=19$	**A1**	Deduces second equation. CWO.
	$\lambda=-3,\ \mu=1$	**M1 A1**	Solves simultaneous equations.
	$\overrightarrow{OP}=\begin{pmatrix}3\\-1\\7\end{pmatrix},\overrightarrow{OQ}=\begin{pmatrix}4\\-2\\1\end{pmatrix}$	**A1**	States $\overrightarrow{OP}$ and $\overrightarrow{OQ}$.
		8	

Question	Answer	Marks	Guidance
4(i)	$\int_0^1 x^2 e^{x^3}\ \mathrm{d}x = \frac{1}{3}\left[e^{x^3}\right]_0^1 = \frac{1}{3}(e-1)$	**M1 A1**	Must show working, AG.
		2	
4(ii)	$I_n = \int_0^1 x^{n-2} x^2 e^{x^3}\ \mathrm{d}x = \left[\frac{1}{3}x^{n-2}e^{x^3}\right]_0^1 - \frac{n-2}{3}\int_0^1 x^{n-3}e^{x^3}\ \mathrm{d}x$	**M1 A1**	Integrates by parts.
	$\frac{e}{3} - \frac{n-2}{3}I_{n-3} \Rightarrow 3I_n = e - (n-2)I_{n-3}$	**A1**	AG
		3	
4(iii)	$I_5 = \frac{1}{3}(e-3I_2) = \frac{1}{3}(e-e+1) = \frac{1}{3}$	**M1 A1**	Applies reduction formula once and uses **(i)**.
	$I_8 = \frac{1}{3}(e-6I_5) = \frac{1}{3}(e-2).$	**A1**	Must be exact.
		3	

Question	Answer	Marks	Guidance
5(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{-2\left(\mathrm{e}^t-\mathrm{e}^{-t}\right)}{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^2}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^2-\left(\mathrm{e}^t-\mathrm{e}^{-t}\right)^2}{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^2}=\frac{\left(2\mathrm{e}^t\right)\left(2\mathrm{e}^{-t}\right)}{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^2}=\frac{4}{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^2}$	**B1**	Differentiates and simplifies. Accept $1-\left(\frac{\mathrm{e}^t-\mathrm{e}^{-t}}{\mathrm{e}^t+\mathrm{e}^{-t}}\right)^2$.
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2=\frac{4\left(\mathrm{e}^t-\mathrm{e}^{-t}\right)^2+16}{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^4}=\frac{4\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^2}{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^4}$	**M1 A1**	Attempt at writing $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$ as a square.
	$S=2\pi\int_0^1\left(\frac{\mathrm{e}^t-\mathrm{e}^{-t}}{\mathrm{e}^t+\mathrm{e}^{-t}}\right)\left(\frac{2}{\mathrm{e}^t+\mathrm{e}^{-t}}\right)\mathrm{d}t=4\pi\int_0^1\frac{\mathrm{e}^t-\mathrm{e}^{-t}}{\left(\mathrm{e}^t+\mathrm{e}^{-t}\right)^2}\mathrm{d}t$	**A1**	Uses correct formula, simplifies to AG.
		5	
5(ii)	$S=4\pi\int_2^{e+e^{-1}}u^{-2}\,\mathrm{d}u=4\pi\left[-u^{-1}\right]_2^{\mathrm{e}+\mathrm{e}^{-1}}$	**M1 A1**	Applies given substitution.
	$=4\pi\left(\frac{1}{2}-\frac{1}{\mathrm{e}+\mathrm{e}^{-1}}\right)$.	**A1**	AEF, must be exact.
		3	

Question	Answer	Marks	Guidance
6(i)	$y=(y+1)^3$	**M1**	Obtains an equation in y not involving radicals.
	$y=y^3+3y^2+3y+1 \Rightarrow y^3+3y^2+2y+1=0$	**A1**	AG
	$S_3=-3$	**B1**	
		3	
6(ii)	$S_{-3}=\dfrac{\alpha^3\beta^3+\beta^3\gamma^3+\alpha^3\gamma^3}{\alpha^3\beta^3\gamma^3}=\dfrac{2}{-1}=-2$	**M1 A1**	
		2	
6(iii)	$S_6=(-3)^2-2(2)=5$	**M1 A1**	Uses $\left(\sum\alpha\right)^2=\sum\alpha^2+2\sum_{\alpha\neq\beta}\alpha\beta$. AG.
	$S_9=-3S_6-2S_3-3=-3(5)-2(-3)-3=-12$	**M1 A1**	Sums $y^3-3y^2+2y-1=0$ for $y=\alpha^3,\beta^3,\gamma^3$.
		4	

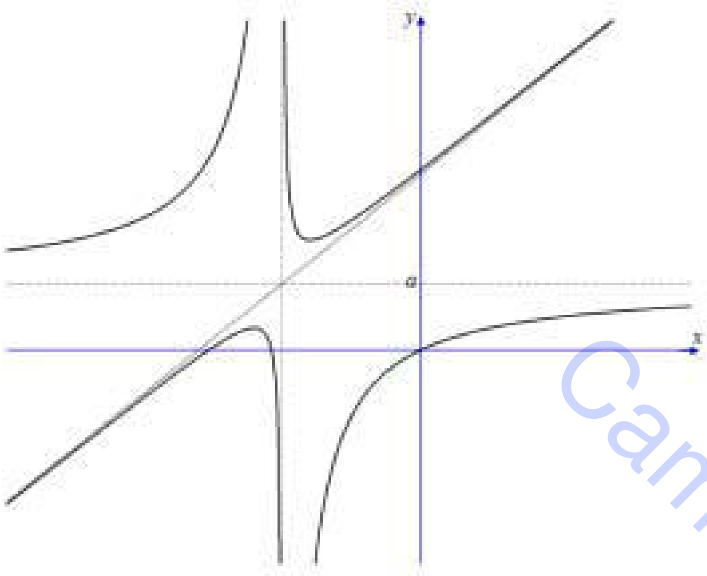
Question	Answer	Marks	Guidance
7	$10u^2+3u-1=0 \Rightarrow (2u+1)(5u-1)=0$	**M1**	Axillary equation
	CF: $x=Ae^{-\frac{1}{2}t}+Be^{\frac{1}{5}t}$	**A1**	
	PI: $x=p+qt \Rightarrow \dot{x}=q \Rightarrow \ddot{x}=0$	**M1**	Forms PI and differentiates.
	$3q-p-qt=t+2 \Rightarrow q=-1, p=-5.$	**M1 A1**	Substitutes.
	GS: $x=Ae^{-\frac{1}{2}t}+Be^{\frac{1}{5}t}-t-5$	**A1**	States general solution.
	$\dot{x}=-\frac{1}{2}Ae^{-\frac{1}{2}t}+\frac{1}{5}Be^{\frac{1}{5}t}-1$	**M1**	Differentiates.
	$A+B=5$ $-\frac{1}{2}A+\frac{1}{5}B=1$	**M1**	Forms simultaneous equations.
	$A=0,\ B=5$	**A1**	
	$x=5e^{\frac{1}{5}t}-t-5$	**A1**	States PS.
		10	

Question	Answer	Marks	Guidance
8(i)	$1=\dfrac{z^1-1}{z-1}$ So true when $n=1$.	**B1**	Shows base case.
	Assume that $1+z+\ldots+z^{k-1}=\dfrac{z^k-1}{z-1}$	**B1**	States inductive hypothesis.
	Then $1+z+\ldots+z^{k-1}+z^k=\dfrac{z^k-1}{z-1}+z^k=\dfrac{z^k-1+z^k(z-1)}{z-1}=\dfrac{z^{k+1}-1}{z-1}$, so true when $n=k+1$	**M1 A1**	Combines fractions.
	$H_k \to H_{k+1}$ Hence, by induction, true for all positive integers.	**A1**	States conclusion.
		5	

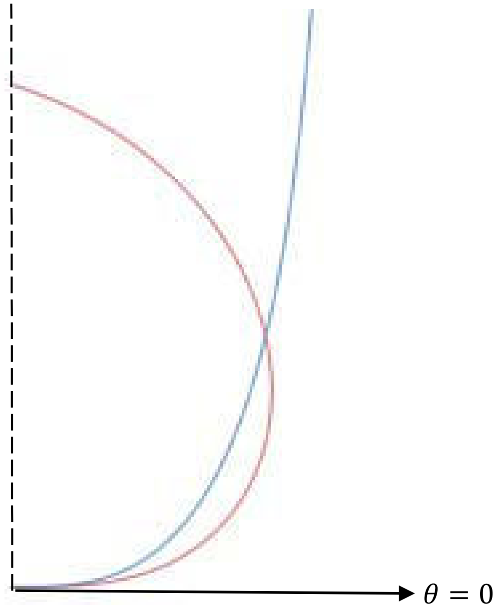
Question	Answer	Marks	Guidance
8(ii)	Since $\|z\|<1$, $\sum_{m=0}^{\infty} z^m = \frac{-1}{z-1}$	B1	States $\|z\|<1$ and uses formula for sum to infinity of geometric progression.
	$\sum_{m=1}^{\infty} 2^{-m} \sin m\theta = \mathrm{Im}\left(\sum_{m=0}^{\infty} z^m\right) = \mathrm{Im}\left(\frac{-1}{\frac{1}{2}\cos\theta + \mathrm{i}\frac{1}{2}\sin\theta - 1}\right)$	M1 A1	Uses de Moivre's theorem.
	$\mathrm{Im}\left(\frac{-\left(\frac{1}{2}\cos\theta - 1 - \mathrm{i}\frac{1}{2}\sin\theta\right)}{\frac{1}{4}\cos^2\theta - \cos\theta + 1 + \frac{1}{4}\sin^2\theta}\right)$	M1	Multiply numerator and denominator by conjugate.
	$\frac{\frac{1}{2}\sin\theta}{\frac{5}{4} - \cos\theta} = \frac{2\sin\theta}{5 - 4\cos\theta}$	A1	States imaginary part, AG.
		5	

Question	Answer	Marks	Guidance
9(i)	$\mathbf{A}^2\mathbf{e} = \mathbf{A}(\mathbf{Ae}) = \lambda\mathbf{Ae} = \lambda^2\mathbf{e}$	**M1 A1**	AG
		2	
9(ii)	Eigenvalues of $\mathbf{A}$ are $n, 2n$ and $3n$.	**B1**	
	$\lambda = n$: $\mathbf{e}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 3 \\ 0 & n & 0 \end{vmatrix} = \begin{pmatrix} -3n \\ 0 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	**M1 A1**	Uses vector product (or equations) to find corresponding eigenvectors.
	$\lambda = 2n$: $\mathbf{e}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -n & 1 & 3 \\ 0 & 0 & n \end{vmatrix} = \begin{pmatrix} n \\ n^2 \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ n \\ 0 \end{pmatrix}$	**A1**	
	$\lambda = 3n$: $\mathbf{e}_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2n & 1 & 3 \\ 0 & -n & 0 \end{vmatrix} = \begin{pmatrix} 3n \\ 0 \\ 2n^2 \end{pmatrix} = t\begin{pmatrix} 3 \\ 0 \\ 2n \end{pmatrix}$	**A1**	
	Eigenvalues of $\mathbf{A} + n\mathbf{I}$ are $2n, 3n$ and $4n$	**B1**	
	Thus $\mathbf{P} = \begin{pmatrix} 1 & 1 & 3 \\ 0 & n & 0 \\ 0 & 0 & 2n \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} (2n)^2 & 0 & 0 \\ 0 & (3n)^2 & 0 \\ 0 & 0 & (4n)^2 \end{pmatrix}$	**M1 A1**	Or correctly matched permutations of columns.
		8	

Question	Answer	Marks	Guidance
10(i)	$x=-5$ and $y=a$	**B1 B1**	
		2	
10(ii)	$x^2+(a+10)x+5a+26=(x+5)(x+a+5)+1$	**M1**	By inspection or long division.
	oblique asymptote is $y=x+a+5$	**A1**	
		2	
10(iii)	$x^2+10x+5a+26=0$	**M1**	Puts y–values equal and forms quadratic equation.
	$10^2-4(5a+26)=-4-20a<0$ so no intersection point	**A1**	Correct discriminant and conclusion.
		2	
10(iv)	$(x+5)(2x+a+10)-x^2-ax-10x-5a-26=0$	**M1**	Differentiates and forms quadratic equation.
	$x^2+10x+24=0$	**A1**	
	Stationary points are $(-4,a+2)$ and $(-6,a-2)$	**A1**	Must have both points.
		3	

Question	Answer	Marks	Guidance
10(v)		B1	Asymptotes drawn, intersection correct.
		B1	C_1 correct.
		B1	C_2 correct.
		3	

Question	Answer	Marks	Guidance
11E(i)	$x=\sqrt{2}\theta^{\frac{1}{2}}\cos\theta$	M1	Uses $x=r\cos\theta$.
	$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sqrt{2}\theta^{\frac{1}{2}}\cos\theta\right)=\sqrt{2}\left(-\theta^{\frac{1}{2}}\sin\theta+\frac{1}{2}\theta^{-\frac{1}{2}}\cos\theta\right)=0$	M1 A1	Sets derivative of $r\cos\theta$ equal to zero.
	$-\theta^{\frac{1}{2}}\sin\theta+\frac{1}{2}\theta^{-\frac{1}{2}}\cos\theta=0\Rightarrow\cos\theta=2\theta\sin\theta\Rightarrow2\theta\tan\theta=1$	A1	AG
	$2(0.6)\tan(0.6)-1=-0.179$ and $2(0.7)\tan(0.7)-1=0.179$	B1	Shows sign change.
		5	

Question	Answer	Marks	Guidance
11E(ii)	$2\theta=\theta\sec^2\theta\Rightarrow\theta^{\frac{1}{2}}\left(\sec\theta-\sqrt{2}\right)=0\Rightarrow\theta=\frac{\pi}{4}$	**M1 A1**	Finds value of θ.
		2	
11E(iii)		**B1**	Correct shape.
		B1	Intersection correct.
		2	

Question	Answer	Marks	Guidance
11E(iv)	$\frac{1}{2}\int_0^{\frac{\pi}{4}} 2\theta\,\mathrm{d}\theta - \frac{1}{2}\int_0^{\frac{\pi}{4}} \theta\sec^2\theta\,\mathrm{d}\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}} \theta\left(2-\sec^2\theta\right)\mathrm{d}\theta$	**M1**	Forms correct integral.
	$\frac{1}{2}\left[\theta(2\theta-\tan\theta)\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\int_0^{\frac{\pi}{4}}(2\theta-\tan\theta)\,\mathrm{d}\theta$	**M1 A1**	Integrates by parts.
	$\frac{1}{2}\left[\theta(2\theta-\tan\theta)\right]_0^{\frac{\pi}{4}} - \frac{1}{2}\left[\theta^2+\ln\cos\theta\right]_0^{\frac{\pi}{4}}$	**A1**	
	$\frac{\pi}{8}\left(\frac{\pi}{2}-1\right)-\frac{1}{2}\left(\left(\frac{\pi}{4}\right)^2+\frac{1}{2}\ln 2\right)=\frac{1}{4}\ln 2+\frac{\pi}{8}\left(\frac{\pi}{4}-1\right)$.	**A1**	AEF, must be exact.
		5	
11O(i)(a)	$\begin{array}{cccc} \multicolumn{4}{c}{\begin{pmatrix}-1 & 2 & 3 & 4\\ 1 & 0 & 1 & -1\\ 1 & -2 & -3 & a\\ 1 & 2 & 5 & 2\end{pmatrix}} \\ v_1 & v_2 & v_3 & v_4\end{array} \to\cdots\to \begin{array}{cccc} \multicolumn{4}{c}{\begin{pmatrix}-1 & 2 & 3 & 4\\ 0 & 2 & 4 & 3\\ 0 & 0 & 0 & a+4\\ 0 & 0 & 0 & 0\end{pmatrix}} \\ v_1' & v_2' & v_3' & v_4'\end{array}$	**M1A1**	Reduces **M** or $\mathbf{M}^{\mathrm{T}}$ to echelon form.
	$\dim V =$ rank $= 3$.	**A1**	
	$c_1v_1'+c_2v_2'+c_3v_4'=0 \Rightarrow c_1=c_2=c_3=0$	**M1**	Shows v_1',v_2',v_4' are linearly independent.
	Thus v_1,v_2,v_4 are linearly independent (and so form a basis for V).	**A1**	
		5	

Question	Answer	Marks	Guidance
11O(i)(b)	$\begin{pmatrix} -1 & 2 & 4 & x \\ 1 & 0 & -1 & y \\ 1 & -2 & a & z \\ 1 & 2 & 2 & t \end{pmatrix} \to \cdots \to \begin{pmatrix} -1 & 2 & 4 & x \\ 0 & 2 & 3 & y+x \\ 0 & 0 & a+4 & z+x \\ 0 & 4 & 6 & t+x \end{pmatrix}$	M1 A1	Uses row operations or $\begin{aligned} x &= -\alpha + 2\beta + 4\gamma \\ y &= \alpha - \gamma \\ z &= \alpha - 2\beta + a\gamma \\ t &= \alpha + 2\beta + 2\gamma \end{aligned}$
	System is consistent when $t + x = 2(y + x) \Rightarrow x + 2y = t$ Or $(-\alpha + 2\beta + 4\gamma) + 2(\alpha - \gamma) = \alpha + 2\beta + 2\gamma = t$	M1 A1	AG
		4	
11O(ii)	$-x + 2y + 3z + 4t = 0$ $2y + 4z + 3t = 0$	M1	Finds basis for null space.
	$t = \mu, \quad z = \lambda, \quad y = -2\lambda - \frac{3}{2}\mu, \quad x = -\lambda + \mu$	A1	
	A basis is $\left\{ \begin{pmatrix} -1 \\ -2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ 0 \\ 2 \end{pmatrix} \right\} = \{\mathbf{e}_1, \mathbf{e}_2\}$	A1	AEF
	$\mathbf{M}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ so particular solution is $\mathbf{e} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$	B1	Finds particular solution.
	General solution is $\mathbf{x} = \mathbf{e} + \lambda\mathbf{e}_1 + \mu\mathbf{e}_2$	A1	**FT** Accept *their* basis. Must have correct particular solution.
		5	

