7 Logarithmic and exponential functions

> Remember that, for example, the relationship $2^3 = 8$ can be written as the equation $\log_2 8 = 3$.

1 In each part of this question, find the values of x and y.

a) $4^x = 16, y = \log_4 16$

b) $x^4 = 81, y = \log_3 81$

c) $x = 3^{-2}, y = \log_3 x$

2 Using your knowledge of indices, and without using your calculator, find the values of:

a) $\log_2 32$

b) $\log_3 9$

c) $\log_5 \frac{1}{25}$

3 Use the rules for manipulating logarithms to write each of the following as a single logarithm.

> For example, $\log 4 + \log 3$ can be written as $\log(4 \times 3) = \log 12$

a) $\log 6 + \log 2$

b) $\log 36 - \log 9$

c) $5 \log 2$

d) $\frac{1}{2}\log 9 - \log 3$

e) $\frac{1}{2}\log 25 + \frac{1}{3}\log 27$

f) $\log 5 + \log 4 - \log 2$

4 Express each of the following in terms of $\log x$.

a) $\log x^3 - \log x^2$

b) $2\log x^3 + 3\log x^2$

c) $2\log\sqrt{x} + \log x$

d) $6\log\sqrt[3]{x} - 4\log\sqrt{x}$

5 **a)** Express $\log_5 \frac{1}{x^2} + \log_5 x^4$ as a multiple of $\log_5 x$.

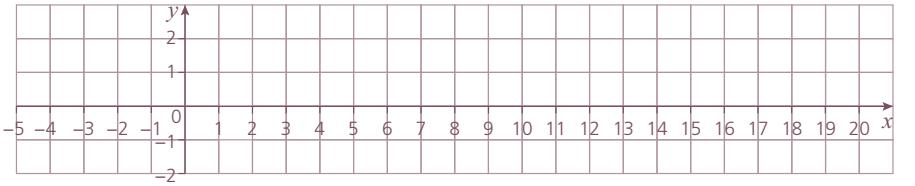
b) Hence solve the equation $\log_5 \frac{1}{x^2} + \log_5 x^4 = 2$ without using your calculator.

6 Draw each of the following graphs on the axes below.
In each case show the vertical asymptote and the coordinates of the points where the graph crosses the x axes.

a) $y = \log_{10} x$

b) $y = \log_{10}(x + 5)$

c) $y = \log_{10}(x - 10)$



7 For each of the following graphs:

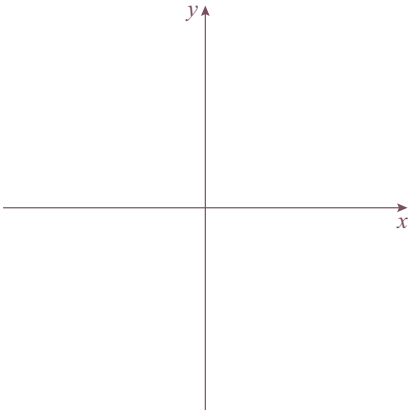
a) Starting with the curve $y = \lg x$ state the transformation (in order when more than one is needed) required to sketch the curve:

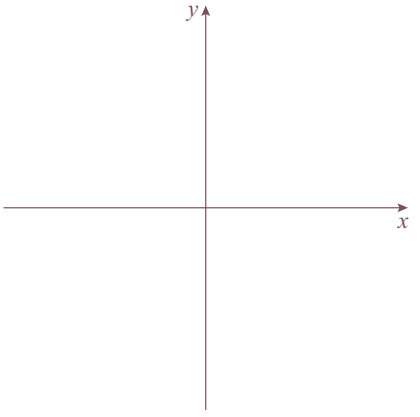
Notice that, as in this question, $\log_{10} x$ is often written as $\lg x$.

(i) $y = 2\lg x$

(ii) $y = \lg 2x$.

b) Sketch the curves on the axes below, together with the curve $y = \lg x$ in each case.





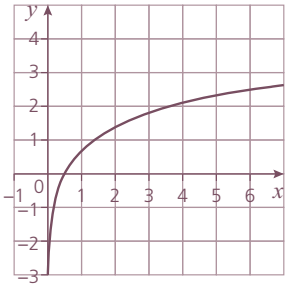
c) Would the results be the same, or different, if logarithms to a different base were used?

8 Match the correct equation to each graph.

Equation	Graph	Equation	Graph
$y = \ln(x + 2)$		$y = \ln x + 2$	
$y = \ln 2x$		$y = \ln(2 - x)$	
$y = \ln(x - 2)$		$y = \ln x - 2$	

Notice that, as in this question, $\log_e x$ is usually written as $\ln x$.

a)



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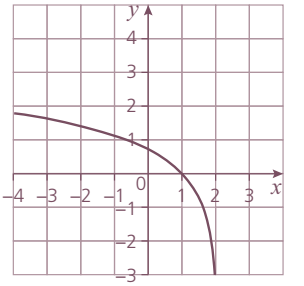

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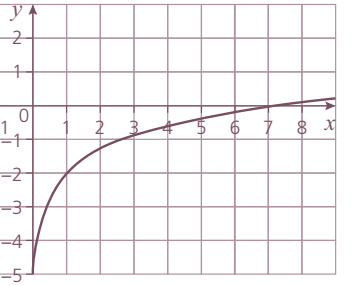

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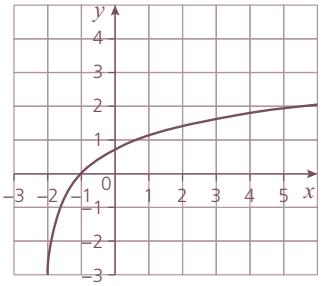

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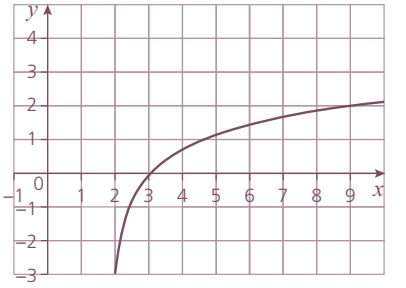

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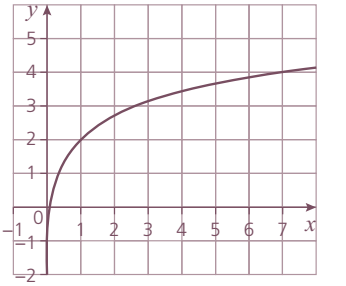


9 Solve the following equations for x, given that $\lg p = 5$.

a) $p = 10^x$

b) $p^{2x} - 6p^x + 8 = 0$

10 Use logarithms to solve the equation $3^{2x-1} = 2^{3x+1}$ giving your answer to 3 s.f.

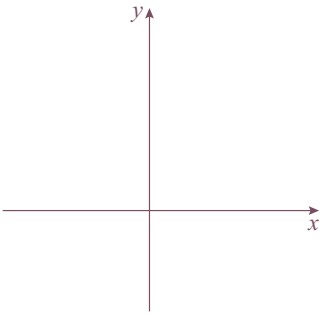
11 The formula for compound interest is $A = P\left(1 + \frac{R}{100}\right)^n$ where A represents the final amount, P the principal (the amount invested), R the rate of interest and n the number of years.

a) For how long, to the nearest month, would \$10 000 need to be invested to produce a final amount of \$15 000 if the rate of interest is 2.8%?

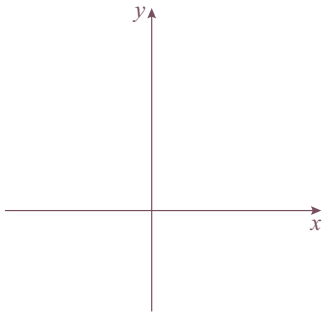
b) \$10 000 is invested for 5 years. What was the rate of interest if the final amount is \$12 000? Answer to 1 decimal place.

12 Sketch the curves of the given functions on the grids below.
Show any asymptotes and give the coordinates of any points of intersection with the axes.

a) $y = e^x$, $y = e^x - 1$ and $y = e^{x-1}$

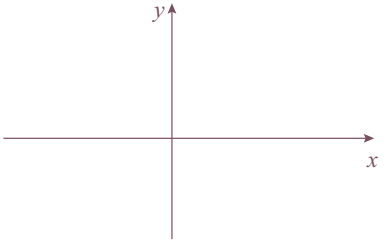


b) $y = e^x$, $y = 3e^x$ and $y = e^{3x}$

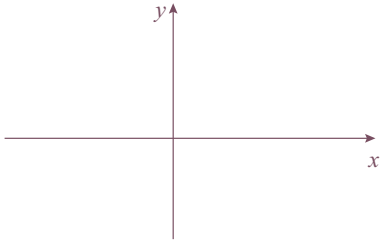


13 Sketch each of the following curves. In each case write the equation of the asymptote and the coordinates of the point where they cross the y-axis.

a) $y = e^x + 1$



b) $y = e^{-x} + 1$



14 Solve the following equations giving your answers to 3 s.f.

a) $4e^{5t} = 30$

b) $5e^{4t} = 30$

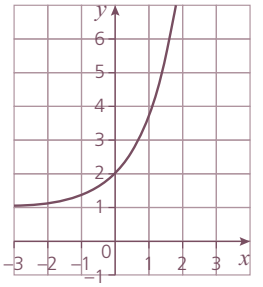
c) $e^{t+3} = 30$

d) $e^{t-3} = 30$

15 Match the correct equation to each of the graphs.

Equation	Graph
$y = e^{x-1}$	
$y = e^{x+1}$	
$y = e^{x} + 1$	
$y = e^{x} - 1$	

a)



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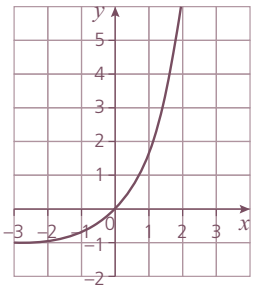

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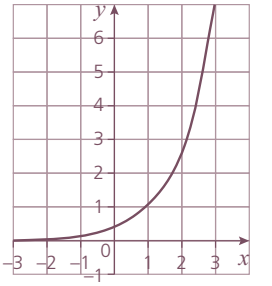

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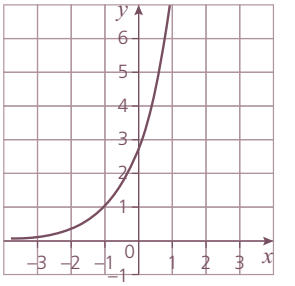

16 In general, continuous compound interest on an investment is given by the formula $A = Pe^{\frac{r}{100} \times t}$, where $\$P$ is the amount invested for t years at a rate of $r\%$, giving a final amount of $\$A$.

a) To the nearest $, how much will an amount of $20 000 invested at a rate of 4% be worth in 5 years' time?

b) How long would $20 000 need to be invested at 4% to be worth $30 000? Give your answer to the nearest month.

17 A radioactive substance of mass 250 g is decaying so that the mass M left after t days is given by the formula $M = 250e^{-0.0035t}$.

a) On the axes below, sketch the graph of M against t.

b) How much, to the nearest gram, is left after one year (i.e. 365 days)?

c) What is the rate of decay after 365 days?

d) After how long, to the nearest day, will there be less than 10 g remaining?