15 Integration

1 Find y for each of the following gradient functions:

a) $\frac{dy}{dx} = 4x - 5$ **b)** $\frac{dy}{dx} = 2x^2 - 5x - 4$ **c)** $\frac{dy}{dx} = (x + 2)(2x - 3)$

2 Find $f(x)$ for each of the following gradient functions.

a) $f'(x) = x^3 - 3$ **b)** $f'(x) = 4 + 3x - x^2$ **c)** $f'(x) = (2x + 3)^2$

3 Find the following indefinite integrals.

a) $\int(4x + 3)dx$ **b)** $\int(2x^4 - 1)dx$ **c)** $\int(x^3 - 2x)dx$

4 Find the following indefinite integrals.

a) $\int(2x - 3)^2dx$ **b)** $\int(x + 3)(x - 2)dx$ **c)** $\int(1 - 2x)^2dx$

5 Find the equation of the curve $y = f(x)$ that passes through the specified point for each of the following gradient functions.

a) $\frac{dy}{dx} = 4x + 1; (1, 3)$

b) $\frac{dy}{dx} = 1 - 2x^3; (4, 0)$

c) $f'(x) = (3x - 2)^2; (0, -4)$

d) $f'(x) = (x - 2)(x + 3); (-1, -2)$

6 Curve C passes through the point $(4, 10)$; its gradient at any point is given by $\frac{dy}{dx} = 3x^2 - 6x + 1$.

a) Find the equation of the curve C.

b) Show that the point $(2, -12)$ lies on the curve.

7 Evaluate the following definite integrals. Do not use a calculator.

a) $\int_1^3 4x\,dx$

b) $\int_{-1}^5 6x^2\,dx$

c) $\int_{-2}^1 (x-3)\,dx$

d) $\int_{-1}^2 (x^2-3x)\,dx$

e) $\int_{-4}^{-2} (x^3+x)\,dx$

f) $\int_{-1}^0 (5-4x)\,dx$

g) $\int_0^3 (2x+1)^2\,dx$

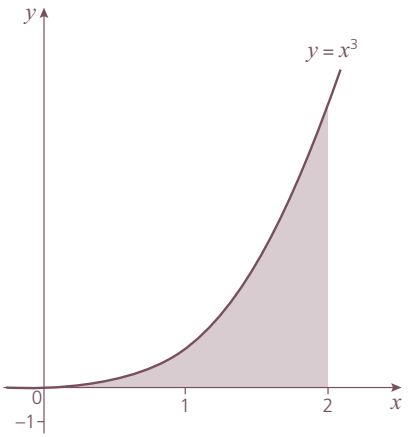
h) $\int_{-2}^2 (2x-3)^2\,dx$

i) $\int_{-1}^1 (x+1)(2x-1)\,dx$

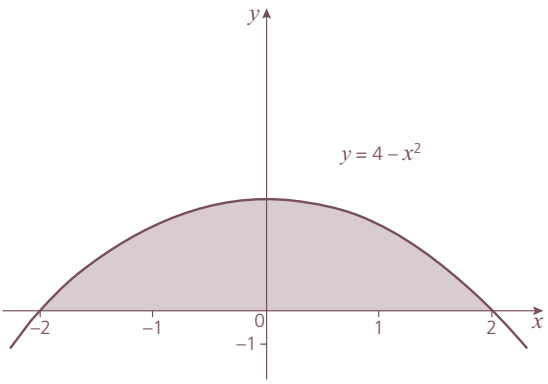
j) $\int_1^3 x(x+1)(x+2)\,dx$

8 Find the area of each of the shaded regions. Do not use a calculator.

a)



c)



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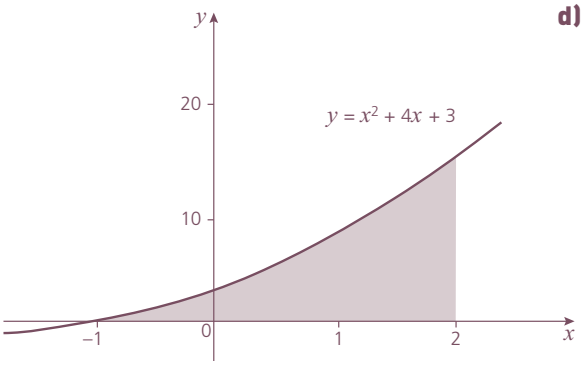
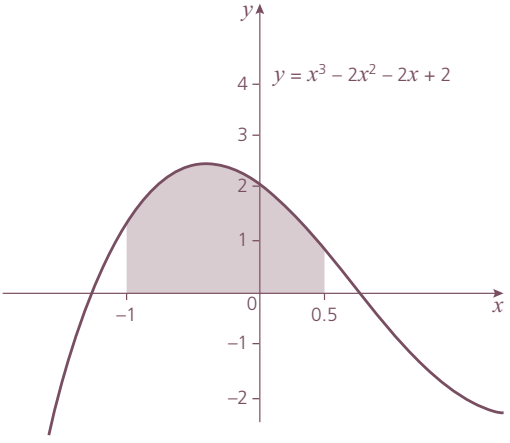
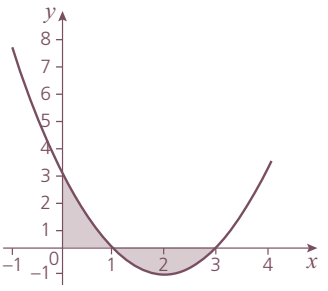


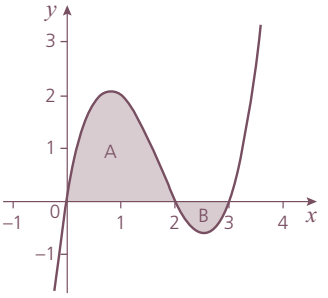
d)



9 The graph shows the curve $y = x^2 - 4x + 3$. Calculate the area of the shaded region.



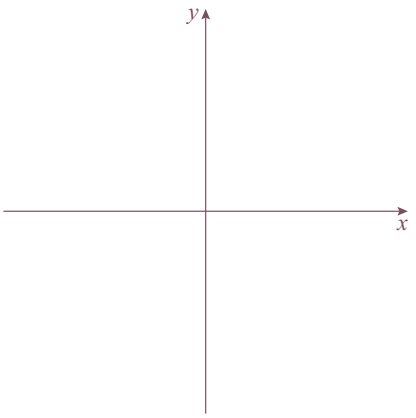
10 The graph shows the curve $y = x^3 - 5x^2 + 6x$.



a) Find the area of each shaded region, A and B. Do not use a calculator.

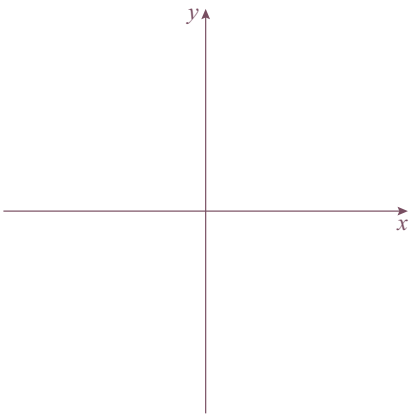
b) State the total area enclosed between the curve and the x-axis for $0 \leqslant x \leqslant 3$. Do not use a calculator.

11 a) Sketch the curve $y = (x + 1)(x - 1)(x - 3)$ and shade the areas enclosed between the curve and the x-axis.



b) Find the total area enclosed between the curve and the x-axis. Do not use a calculator.

12 a) Sketch the curve $y = (x + 1)^2(x - 2)$ and shade the areas enclosed between the curve and the x-axis.



b) Find the area you have shaded. Do not use a calculator.

13 Find the following indefinite integrals.

a) $\int \frac{1}{2x-3}\,dx$

b) $\int e^{2x-3}dx$

c) $\int (2x-3)^3 dx$

d) $\int \sin(2x-3)dx$

e) $\int \cos(2x-3)dx$

f) $\int \sec^2(2x-3)dx$

14 Evaluate the following definite integrals. Do not use a calculator.

a) $\int_1^4 \frac{2}{2x+1}\,dx$

b) $\int_1^4 e^{2x+1}dx$

c) $\int_1^4 (2x+1)^3 dx$

d) $\int_0^{\frac{\pi}{2}} \sin\left(2x+\frac{\pi}{4}\right)dx$

e) $\int_0^{\frac{\pi}{2}} \cos\left(2x+\frac{\pi}{4}\right)dx$

f) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\left(\frac{x}{2}-\frac{\pi}{4}\right)dx$