

14

Differentiation

In questions 1 to 5 differentiate the given functions with respect to x .

1 a) $y = x^6$

b) $y = 3x^2$

c) $y = -2x$

d) $y = -4$

2 a) $y = x^{\frac{1}{3}}$

b) $y = -3x^{\frac{2}{3}}$

c) $y = 2\sqrt[4]{x}$

d) $y = \frac{2\sqrt{x}}{3}$

3 a) $y = 4x^3 - 3x^4$

b) $y = 7x^2 + x - 5$

c) $y = 2x^3 - 3x^2 + 4$

4 a) $f(x) = \frac{2}{x^3}$

b) $f(x) = 2\sqrt{x} - x\sqrt{x}$

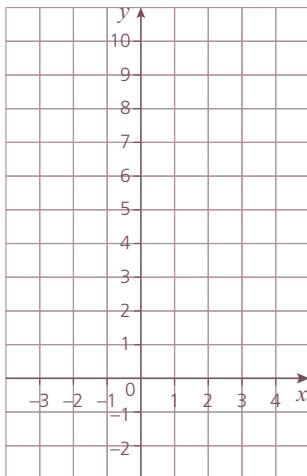
c) $f(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$

5 a) $y = (x - 1)(2x + 1)$

b) $y = 3x^2 - \frac{5}{x}$

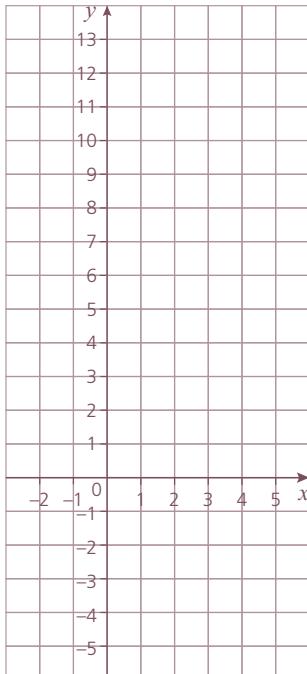
c) $y = x^2(2x - 3)$

6 a) Draw the curve $y = (x + 1)(x - 2)$ b) Find the gradient of the curve at the points of intersection with the x and y axes.



7 a) Draw the curve $y = x^2 - 4$ and the line $y = 3x$ on the same axes.

b) Use algebra to find the coordinates of the points where the two graphs intersect.



c) Find the gradient of the curve at the points of intersection.

8 Complete the following for equations a and b.

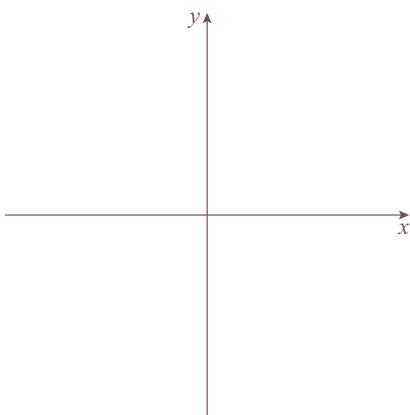
a) $y = x^3 - 3x^2 - 9x + 15$

(i) Find $\frac{dy}{dx}$ and the values of x for which $\frac{dy}{dx} = 0$

(ii) Classify the points on the curve with these x -values

(iii) Find the corresponding y -values

(iv) Sketch the curve.



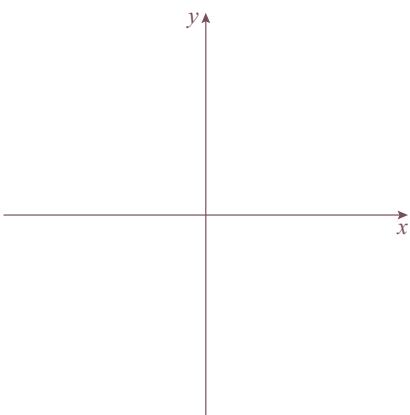
b) $y = x^4 - 8x^2 + 16$

(i) Find $\frac{dy}{dx}$ and the values of x for which $\frac{dy}{dx} = 0$

(ii) Classify the points on the curve with these x -values

(iii) Find the corresponding y -values

(iv) Sketch the curve.



9 The graph of $y = x^2 + ax + b$ passes through the point $(-1, 10)$ and its gradient at that point is -7 .

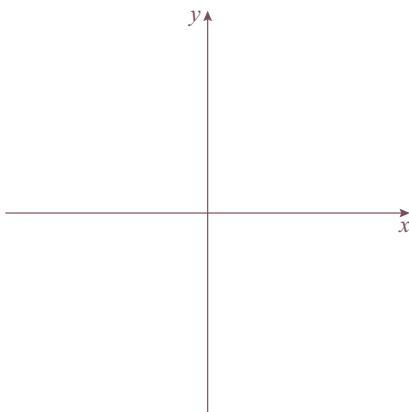
a) Find the values of a and b .

b) Find the coordinates of the stationary point of the curve.

10 a) Find the stationary points of the function $y = (x + 1)^2(x - 1)$ and classify them.

Classifying a stationary point means determining whether it is a maximum, a minimum or a point of inflection.

b) Sketch the curve.



11 For each of the following curves

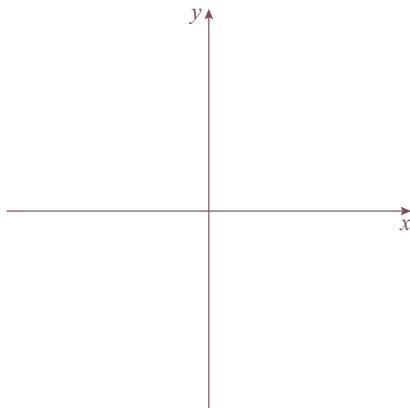
ay = 3x^3 - 4x - 4

(i) find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(ii) find any stationary points

(iii) use the second derivative test to determine their nature

(iv) sketch the curve.



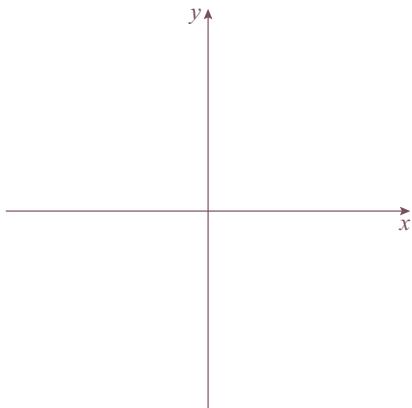
by = x^4 - 6x^2 + 8x - 5

(i) find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(ii) find any stationary points

(iii) use the second derivative test to determine their nature

(iv) sketch the curve.



12 An open tank with a capacity of 32 m^3 is to be constructed with a square base and vertical sides.

- a)** Find an expression for the height $h\text{ m}$ of the tank in terms of the length $x\text{ m}$ of a side of the base.

- b)** To reduce costs, it will be constructed using the smallest possible area of sheet metal. Find its dimensions and use the second derivative test to show that your answer is a minimum.

13 A curve has equation $y = x^2 - 7x + 10$.

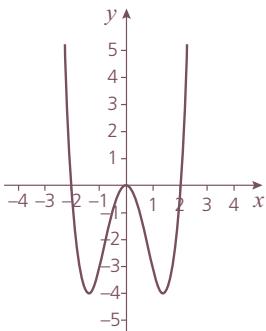
- a)** Find the gradient function $\frac{dy}{dx}$.

- b)** Find the gradient of the curve at the point $P(4, -2)$.

- c)** Find the equation of the tangent at P .

- d)** Find the equation of the normal at P .

14 The diagram shows a sketch of the curve $y = x^4 - 4x^2$.



a) Differentiate $y = x^4 - 4x^2$.

b) Find the equations of the tangent and normal to the curve at the point $(2, 0)$.

c) Find the equations of the tangent and normal to the curve at the point $(-2, 0)$.

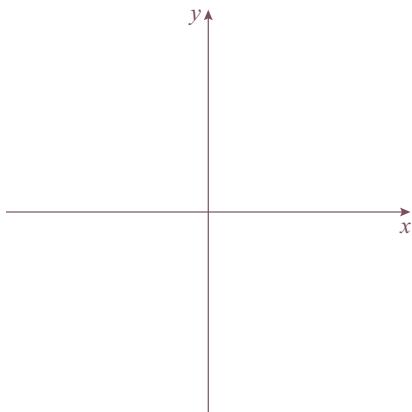
d) State the equations of the tangent and normal to the curve at the point $(0, 0)$.

15 a) The equation of a curve is $y = x^4 + x^3$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b) Find the coordinates of the stationary points on the curve.

c) Classify the stationary points, using the second derivative test where possible.

d) Sketch the curve $y = x^4 + x^3$ labelling the stationary points.



16 Differentiate each of the following functions with respect to x .

a) $y = 2 \sin x + 3 \cos x$

d) $y = \ln 3x$

b) $y = 2 \tan x - 3 \cos x$

e) $y = 4e^x$

c) $y = 3 \ln x$

f) $y = 3e^{-x}$

17 Use the product rule to differentiate each of the following functions with respect to x .

a) $y = xe^x$

d) $y = x \tan x$

b) $y = xe^{-x}$

e) $y = x^2 \sin x$

c) $y = x \ln x$

f) $y = x^3 \cos x$

18 Use the quotient rule to differentiate each of the following functions with respect to x .

a) $y = \frac{e^x}{x}$

d) $y = \frac{e^x}{\sin x}$

b) $y = \frac{x}{e^x}$

e) $y = \frac{\ln x}{x}$

c) $y = \frac{\sin x}{e^x}$

f) $y = \frac{x}{\ln x}$

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19 Use the chain rule to differentiate each of the following functions with respect to x .

a) $y = \sin^4 x$

c) $y = \sqrt{1+2x}$

b) $y = \tan^2 x$

d) $y = \sqrt[3]{1+3x}$

20 Use an appropriate method to differentiate each of the following functions with respect to x .

a) $y = e^x \cos x$

c) $y = \frac{\tan \theta}{1 - \cos \theta}$

b) $y = \frac{\ln x}{e^x}$

d) $y = (1 - \cos \theta)^2$

21 You are given that $y = \sqrt{u}$ and that $u = 2x^2 + 1$.

a) Show that the point $(2, 3)$ lies on the graph of y against x .

b) Find the values of $\frac{du}{dx}$, $\frac{dy}{du}$ and $\frac{dy}{dx}$ at the point $(2, 3)$.