12 Series

The word 'expand' means 'write out term by term'. So expanding $(x + 1)^2$ gives $x^2 + 2x + 1$.

In the questions on this page, simplify the terms in your expansions as far as possible.

1 Expand the following binomial expressions:

a) $(1 + x)^5$ **b)** $(1 - x)^5$ **c)** $(1 + 2x)^5$

2 Expand the following binomial expressions:

a) $(2x + y)^3$ **b)** $(2x - y)^3$ **c)** $(2x + 3y)^3$

3 Find the first three terms, in ascending powers of x, in the expansions of:

a) $(3 - x)^5$ **b)** $\left(3 - \frac{x}{2}\right)^5$

4 Find the first three terms, in descending powers of x, in the expansion of the following:

a) $\left(2 - \frac{1}{x}\right)^4$ **b)** $\left(3 - \frac{2}{x}\right)^4$

5 Find:

a) the coefficient of x^2 in the expansion of $(1 + 2x)^6$

b) the coefficient of x^3 in the expansion of $(1 + 2x)^7$.

6 **a)** Expand $(1 – 2x)^4$.

b) Hence expand $(1 + x)(1 – 2x)^4$.

7 Identify which of the following sequences are arithmetic, stating the common difference where appropriate.

	Sequence	Arithmetic? Yes / No	Common difference
a)	1, 5, 9, 13, …		
b)	2, 4, 8, 16 ,…		
c)	5, 3, 1, –1, …		
d)	1, 1, 2, 2, 3, 3, …		

8 The first term of an arithmetic sequence is 5 and the fourth term is 14. Find:

a) the common difference

b) the tenth term

c) the sum of the first ten terms.

9 An arithmetic progression of 15 terms has first term 7 and last term –49.

a) What is the common difference?

b) Find the sum of the arithmetic progression.

10 The 8th term of an arithmetic progression is 9 times the 2nd term.
The sum of the 2nd and 3rd terms is 10.

a) Write down a pair of simultaneous equations for the first term a and the common difference d.

b) Solve the equations to find the values of a and d.

c) Find the sum of the first 20 terms of the progression.

11 A ball rolls down a slope. The distances it travels in successive seconds are 4 cm, 12 cm, 20 cm, 28 cm, etc., and are in an arithmetic progression. How many seconds elapse before it has travelled 9 metres?

12 a) How many terms of the arithmetic progression 15, 13, 11, … make a total of 55?

b) Explain why there are two possible answers to this question.

13 Are the following sequences geometric? If so, state the common ratio and calculate the eighth term.

	Sequence	Geometric? Yes / No	8th term
a)	2, 6, 18, 54, …		
b)	2, 6, 10, 14, …		
c)	1, –1, 1, –1, …		
d)	4, –12, 36, –108, …		
e)	8, 4, 2, 0, …		
f)	1, 0, 0, 0, …		

14 A geometric sequence has first term –2 and common ratio 2. The sequence has 10 terms.

a) Find the last term.

b) Find the sum of the terms in the sequence.

15 a) How many terms are there in the sequence $27, 9, 3, \ldots \frac{1}{27}$?

b) Find the sum of the terms in this sequence.

16 The 1st term of a geometric progression is positive, the 5th term is 128 and the 11th term is 524 288.

a) Find two possible values for the common ratio.

b) Find the first term.

c) Find two possible values for the sum of the first seven terms.

17 The first three terms of an infinite geometric sequence are 100, –60 and 36.

a) Write down the common ratio of the progression.

b) Find the sum of the first 10 terms.

c) Find the sum to infinity of its terms.

18 In each month, the growth of a bush is three-quarters of the growth the previous month. The bush is initially 1.2 m tall and grows 12 cm in the first month.

a) What is the tallest the bush will grow?

b) After how many months is it within 5% of its maximum height?