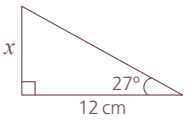
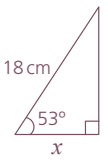
10 Trigonometry

1 Find the length of x in each triangle. Give your answer to 2 d.p.

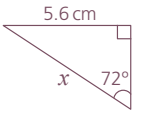
a)



b)



c)



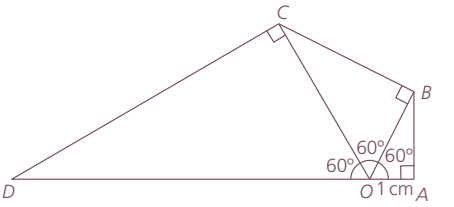
2 Write the following in terms of a single trigonometric function.

a) $\dfrac{\sin\theta}{\cos\theta}$ **b)** $\dfrac{\sin\theta}{\tan\theta}$ **c)** $\cos\theta \times \tan\theta$

3 Simplify:

a) $\cos^2\theta(1 + \tan^2\theta)$ **b)** $\tan^2\theta(1 - \sin^2\theta)$

4 In the diagram, OA = 1 cm, angle AOB = angle BOC = angle COD = 60° and angle OAB = angle OBC = angle OCD = 90°.



a) Find the length of OD.

b) Show that the perimeter of OABCD is $\left(9 + 7\sqrt{3}\right)$ cm.

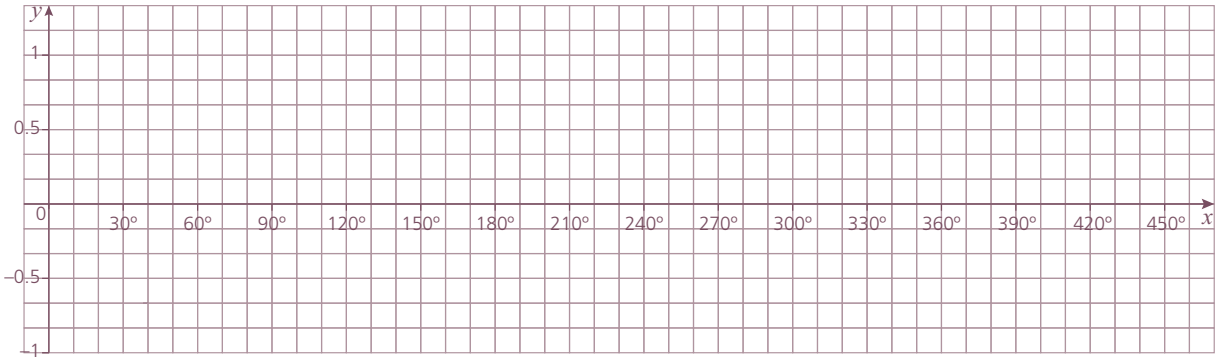
5 Work out the values of the following quantities without using a calculator. Show your working carefully.

a) $\sin^2 30° - \cos^2 30° \tan^2 30°$

b) $\sin^2 \frac{\pi}{4} - \cos^2 \frac{\pi}{4} \tan^2 \frac{\pi}{4}$

c) $\sin^2 60° - \cos^2 60° \tan^2 60°$

6 **a)** By plotting suitable points, draw the curve of $y = \cos x$ for $0° \leqslant x \leqslant 360°$ on the grid below.



b) Solve the equation $\cos x = 0.4$ for $0° \leqslant x \leqslant 360°$ and illustrate the roots on your sketch.

c) Write down, without using your calculator, the solution to the equation $\cos x = -0.4$ for $0° \leqslant x \leqslant 360°$.

7 Without using your calculator, write the following as fractions or using surds.

a) $\sin 60°$ **b)** $\cos 120°$ **c)** $\tan 150°$

8 Solve the following equations for $0 \leqslant x \leqslant 2\pi$ without using your calculator.

a) $\sin\theta = \frac{\sqrt{3}}{2}$ **b)** $\cos\theta = \frac{\sqrt{3}}{2}$ **c)** $\tan\theta = -1$

9 Without using a calculator show that:

a) $\sin^2 30° + \sin^2 45° = \sin^2 60°$

b) $3\cos^2 \frac{\pi}{3} = \sin^2 \frac{\pi}{3}$

10 Solve the following equations for $-360° \leqslant x \leqslant 360°$.

a) $\sin(x - 30°) = 0.6$

b) $\cos(x + 60°) = 0.4$

c) $\tan(x - 45°) = 1$

11 Starting with the graph of $y = \sin x$, state the transformations that can be used to sketch each of the following curves.

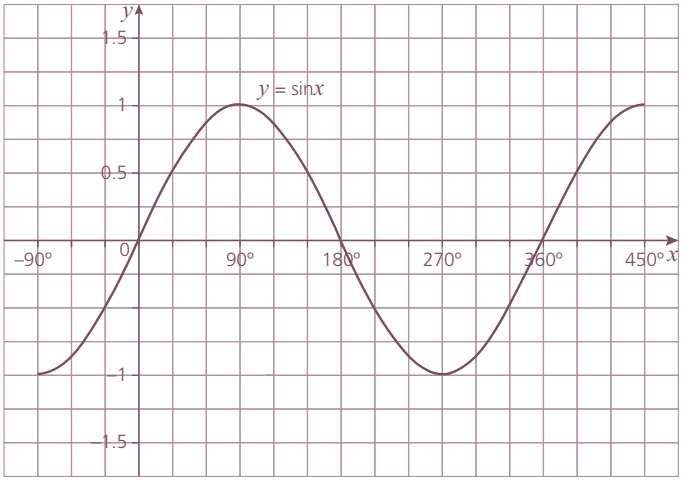
a) $y = \sin 3x$

b) $y = 2\sin 3x$

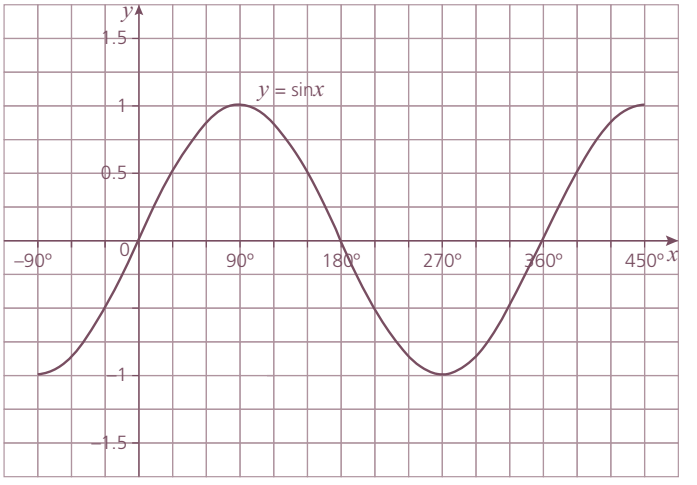
c) $y = 2\sin 3x - 1$

12 Apply these transformations to the graph of $y = \sin x$. State the equation, amplitude and period of each transformed graph.

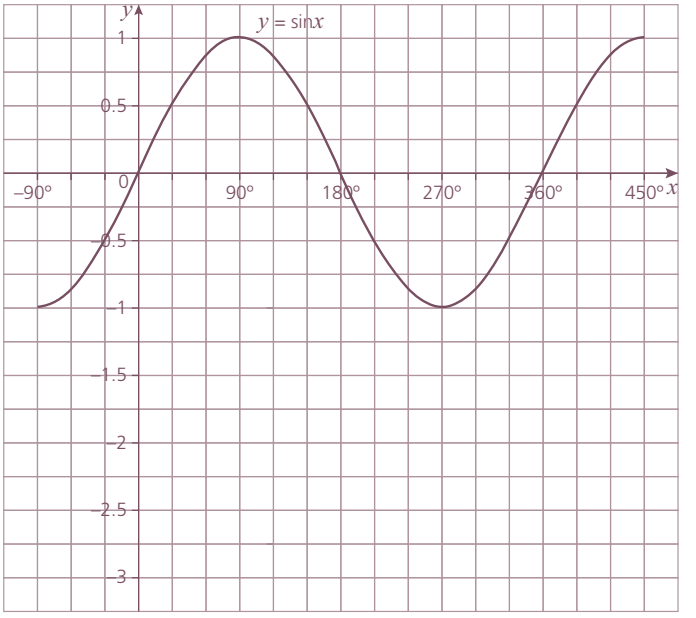
a) A stretch of scale factor $\frac{1}{2}$ parallel to the x-axis.



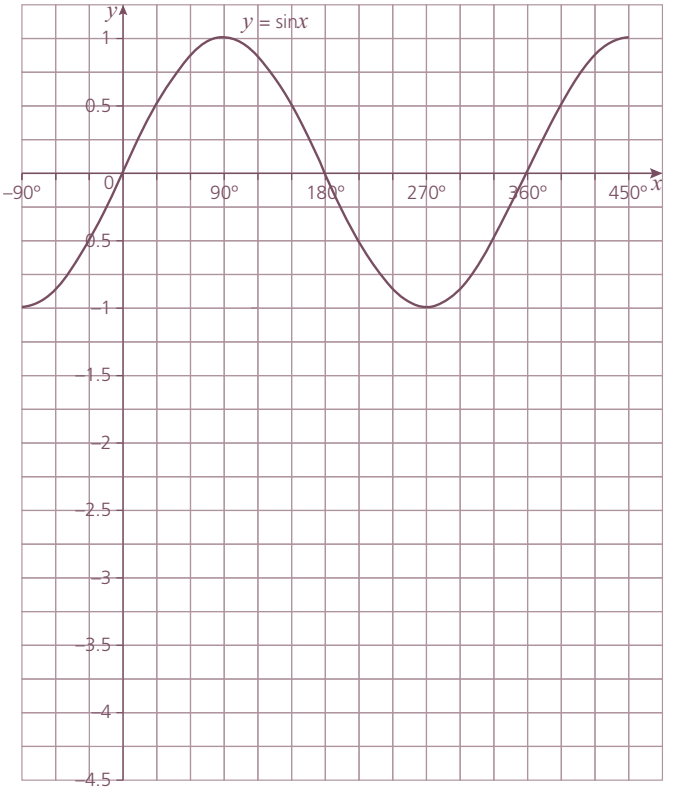
b) A translation of 90° in the negative x direction.



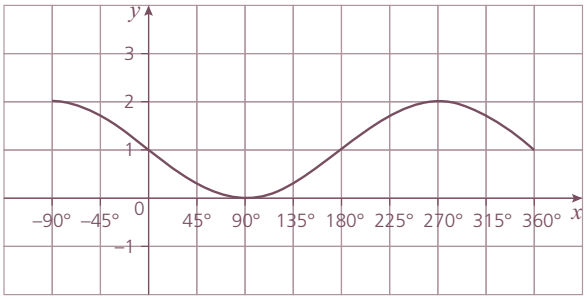
c) A stretch of scale factor 2 parallel to the y-axis followed by a translation of 1 unit vertically downwards.



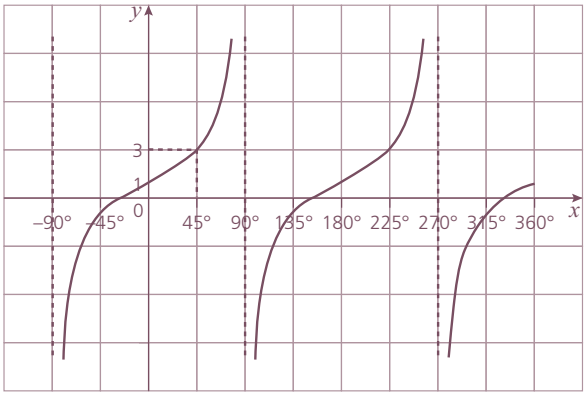
d) A translation of 1 unit vertically downwards followed by a stretch of scale factor 2 parallel to the y-axis.



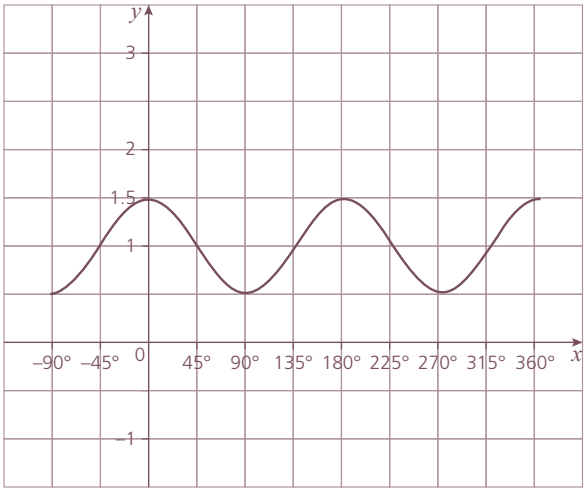
13 State the transformations required, in the correct order, to obtain the graph below from the graph of $y = \sin x$.



14 State the transformations required, in the correct order, to obtain the graph below from the graph of $y = \tan x$.



15 State the transformations required, in the correct order, to obtain the graph below from the graph of $y = \cos x$.



16 Simplify:

a) $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

b) $\frac{\sqrt{(1+\tan^2\theta)}}{\sqrt{(1-\sin^2\theta)}}$

c) $\frac{1}{\cos\theta\sqrt{(1+\tan^2\theta)}}$

d) $\frac{1-\sec^2\theta}{1-\operatorname{cosec}^2\theta}$

17 Solve $\cot x = \sin x$ for $0° \leqslant x \leqslant 360°$.

18 Solve $\tan x + \cot x = 2\sec x$ for $0 \leqslant x \leqslant 2\pi$.